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ABSTRACT

Piezoelectric bimorph cantilever is a typical structure for vibration energy harvesting. This paper studies the method to improve the accuracy of FEA (finite element analysis) simulation for piezoelectric bimorph cantilever. The COSMOL simulation methods based on 1 D (dimension) model of Euler Bernoulli beam, as well as 2 D and 3 D models of solid mechanics are proposed respectively. Compared with the theory and experiment results given by Erturk A, the influences of 1 D and 3 D piezoelectric material parameters on simulation accuracy in FEA modeling is discussed. The simulation error can be reduced to less than 1.2% by using 1D piezoelectric material parameters in COMSOL simulation, while the maximum simulation error can be up to 28% by using 3D piezoelectric material parameters. Comparing serial and parallel configurations of piezoelectric bimorph cantilever, the output electrical powers, varying with the excitation frequency and load resistance, are also discussed in COMSOL simulation. Performance comparation of three kinds of piezoelectric bimorph cantilevers with tip mass shows that the modelling and simulation in finite element methods by COMSOL are simple and convenient, and are suitable for the electromechanical coupling analysis and optimization of complex topological structures.

I. INTRODUCTION

Piezoelectric vibration energy harvesting technology is promising in power self-supply for IoT (Internet of Things) devices. It uses piezoelectric inertia devices to vibrate resonantly with the ambient vibration frequency and convert the vibration energy into electrical energy based on the piezoelectric effect. Piezoelectric bimorph cantilever is a typical vibration energy harvesting structure. It consists of an upper and lower piezoelectric layer and an intermediate substrate layer. Under the excitation of ambient vibration, the transverse resonance of the composite cantilever produces large longitudinal stress in the upper and lower piezoelectric layers, and the piezoelectric material generates electricity in d_{31} mode.

In order to design piezoelectric bimorph cantilever and analyze its electromechanical coupling characteristics, many modeling methods have been proposed, such as distributed parameter method (DPM),[1] finite element method,[2] and lumped parameter method.[3]

DPM is a more accurate modeling method, which usually uses Newton's or Hamilton's principles[4] to obtain coupled vibration equation. The analytical solution[5] can be obtained by combining the mode shapes of Euler-Bernoulli beams with piezoelectric constitutive equations, and the numerical solution[6] can be obtained by Rayleigh Ritz method. Modeling process of DPM is complex for irregular beam and cantilever with additional mass.[7] It is difficult to obtain analytical solutions of modal modes. Kim[8] investigated the distributed parameter model of piezoelectric cantilever with additional mass. Shengxi Zhou[9] investigated the distributed parameter model for the zigzag piezoelectric cantilever structure.

Finite element analysis (FEA) is a numerical simulation method, which is more suitable for solving complex beam model. Based on Kirchhoff plate theory, Carlos De Marqui Junior modeled and simulated the piezoelectric energy harvester plate by FEA.[10] It is time-consuming and labor-consuming to compile the FEA program. At present, the main commercial FEA softwares, such as ANSYS, Abaqus and COMSOL, all support piezoelectric coupling analysis of structures.

FEA has the advantage in the electromechanical coupling modeling and simulation of complex topological structures. Vinayaga K.[11] discussed the ANSYS modeling and simulation method of smart piezoelectric beam. Yang[12] used FEA to optimize the structural parameters of a clamped piezoelectric circular diaphragm. Qian[13] used ANSYS to carry out modal analysis of a torsional piezoelectric vibration energy harvesting system. Fey[14] used ANSYS to model and simulate a piezoelectric auxetic PZT lattice structure. Yang[15] used ANSYS to model and simulate a two-dimensional piezoelectric energy harvester. Zhou[16] used ANSYS to simulate a flexible longitudinal zigzag structure. Gohari[17] used quadratic piezoelectric multi-layer shell element for analysis of smart laminated composite plates in ABAQUS. Bath[18] used COMSOL to model and simulate 3D folded zigzag piezoelectric energy harvester. Dauksevicius[19] used COMSOL to model and simulate a nonlinear piezoelectric vibration energy harvester with frequency-tuned impacting resonators.

In addition to electromechanical coupling simulation, FEA can also simulate the coupling of magnetic field and piezoelectric structure vibration, which is conducive to the design of nonlinear vibration system. Upadrashta[20] used ANSYS to model and simulate nonlinear piezoelectric energy harvesters with magnetic interaction. Dauksevicius[21] used COMSOL to analyse magnetic plucking dynamics in a frequency up-converting piezoelectric energy harvester.

Another advantage of FEA is the whole system modeling and simulation of electromechanical coupling structure and circuit. Wu[22] used COMSOL to analyze electrically rectified piezoelectric energy harvesters. Elvin N. G[6] established a coupled finite element-circuit simulation model for analyzing piezoelectric energy generators in ABAQUS.

Although FEA has been widely used in the modeling and simulation of piezoelectric vibration energy harvester (PVEH), and some studies claim that the COMSOL simulation results fit well with the experimental results, but the detailed parameter values and modeling methods are not given, and the simulation is difficult to reproduce.

It is still necessary to analyze some basic and detailed problems, such as getting the appropriate modeling parameters for FEA, improving the accuracy of calculation and reducing the calculation time. In addition, the results of FEA simulation should be compared with the theoretical model with distributed parameters and the experimental results to verify the reliability of FEA.

This study will discuss a FEA method for the piezoelectric bimorph cantilever by COMSOL Multiphysics, aiming at structural eigenfrequency analysis and electromechanical coupling analysis. Compared with the DPM simulation results of Erturk A,[23] the reliability of FEA in COMSOL is verified.

II. THEORY

The typical piezoelectric bimorph cantilever for energy harvesting is shown in figure 1. The piezoelectric ceramic layers of the piezoelectric bimorph cantilever are tensioned and compressed axially (in the direction of piezoelectric material d_{31}) by inertia force under transverse vibration or rotational excitation of the fixed-end base. Mechanical energy is converted into electric energy based on the piezoelectric effect. Adding tip mass at the free end can effectively

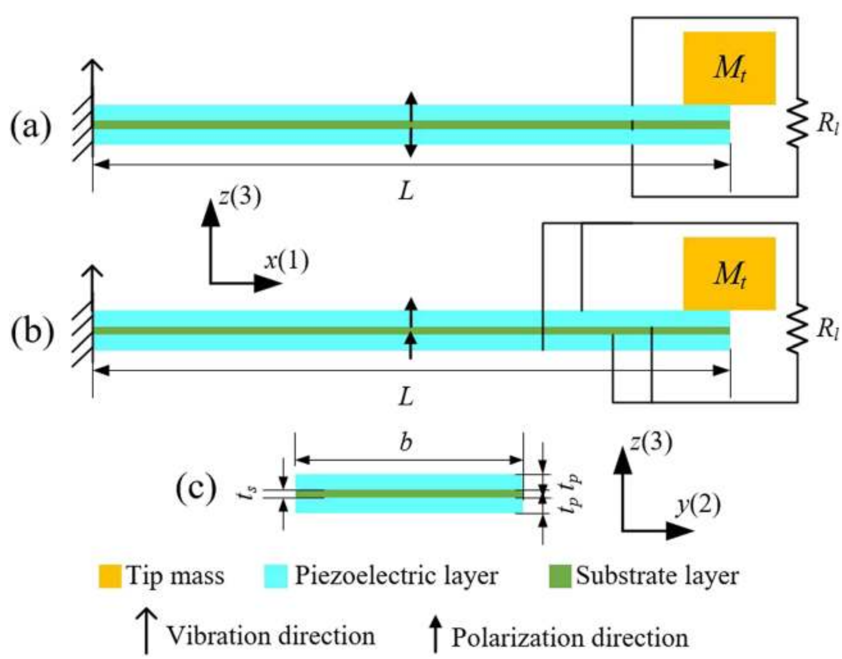

FIG. 1. Geometrical configurations of uniform bimorph piezoelectric energy harvester.[23] (a) Series type. (b) Parallel type. (c) Cross-sectional view of bimorph.

reduce the resonant frequency of the device, and increase inertia and piezoelectric output. The upper and lower piezoceramic layers of bimorph can be connected in series or in parallel. Series type can produce larger voltage output and parallel type can produce larger current output.

A distributed parameter electromechanical coupling model given by Erturk A[23] well described the vibration energy harvesting characteristics of piezoelectric bimorph cantilever. The electromechanical frequency response functions were given both in series type and in parallel type respectively. DPM model reflecting the internal relationship between the steady-state electrical and mechanical responses and the excitations of base translational and rotational accelerations. For the common first-order resonance excitation problem, the multi-modal solution is simplified to a single-modal form. When the base is only excited by translational acceleration, the single mode voltage frequency response function is:

$$V_{DP}(\omega) = \frac{-j\omega R_l \gamma_1 \chi_1}{(1 + j\omega R_l C_p)\left(\omega_1^2 - \omega^2 + j2\zeta_1\omega_1\omega\right) + j\omega R_l \chi_1^2} \quad (1)$$

where in this example the first modal mass term γ_1 is calculated as -0.0224. The first modal electromechanical coupling term χ_1 is calculated as 0.0842 in series type and 0.1684 in parallel type, respectively. The internal capacitance term C_p is calculated as 7.5661 nF in series type and 30.264 nF in parallel type, respectively. The calculation method can be seen in Ref. 23.

For the first-order eigenfrequency analysis of piezoelectric bimorph cantilever, the composite cantilever beam can be equivalent to a homogeneous cantilever beam. The theoretical derivation of the bending section coefficient YI of composite cantilever beam can be expressed as follows:

$$YI = b\left[\frac{Y_s t_s^3}{12} + Y_p\left(\frac{2t_p^3}{3} + t_s t_p^2 + \frac{t_s^2 t_p}{2}\right)\right] \quad (2)$$

For a homogeneous cantilever beam, its inertial moment is:

$$I = b\frac{(2t_p + t_s)^3}{12} \quad (3)$$

Therefore, the equivalent elasticity modulus of homogeneous cantilever beam is:

$$Y = YI/I = \left[\frac{Y_s t_s^3}{12} + Y_p\left(\frac{2t_p^3}{3} + t_s t_p^2 + \frac{t_s^2 t_p}{2}\right)\right] \Big/ \left[\frac{(2t_p + t_s)^3}{12}\right] \quad (4)$$

The equivalent density of homogeneous cantilever beam is:

$$\rho = \frac{(2\rho_p t_p + \rho_s t_s)}{(2t_p + t_s)} \quad (5)$$

The first order eigenfrequency of the theoretical cantilever beam is:

$$\omega_1 = \left(\frac{\lambda_1}{l}\right)^2 \sqrt{\frac{YI}{m}} \quad (6)$$

Here, $\lambda_1 = 1.875$, m is the mass of the cantilever beam of unit length. The explanation of parameters in all above formulas can be seen in Tables I–III.

TABLE I. Geometry parameters of bimorph.

Symbol	Parameters	Value
L	Bimorph length	24.53 mm
b	Bimorph width	6.4 mm
t_p	Piezoelectric layer thickness	0.265 mm
t_s	Substrate layer thickness	0.140 mm

TABLE II. Material parameters of brass.

Symbol	Parameters	Value
ρ_s	Density	9000 kg/m^3
Y_s	Young's modulus	105 GPa
μ	Poisson ratio	0.35
η_s	Loss factor	0.01748

TABLE III. Material parameters of PZT-5H in 1D.

Symbol	Parameters	Value
ρ_p	Density	7500 kg/m^3
Y_p	Young's modulus	60.6 GPa
$\bar{e}_{31}$	Piezo stress coefficient	-16.6 C/m^2
$\bar{\varepsilon}_{33}^S/\varepsilon_0$	Relative permittivity	2886
η_s	Loss factor	0.01748

III. FINITE ELEMENT MODELLING IN COMSOL

An example of piezoelectric bimorph cantilever for theoretical analysis and experimental verification has been given by Erturk A.[23] In this paper, the piezoelectric bimorph cantilever with the same geometry and material parameters as the example is used, then its 1D (dimension), 2D and 3D finite element models are built in COMSOL, respectively. The eigenfrequency and voltage frequency response functions of the model are simulated, and the model parameters are discussed.

A. Geometric parameters

PZT-5H is chosen as piezoelectric material, brass as substrate material, and nickel electrode as top electrode and bottom electrode of piezoceramics. The thickness of nickel electrode is negligible in the simulation. As shown in Table I, structural geometry modeling parameters can be parameterized in the global definition of COMSOL.

B. Geometric modelling

According to the above geometric design parameters, 1D, 2D and 3D finite element parametric modeling are carried out in COMSOL as shown in figure 2. A 1D homogeneous Euler-Bernoulli cantilever beam model is established using beam module. The equivalent parameters of composite materials are calculated by formula for material properties. The piezoelectric double crystal cantilever models of 2D and 3D are built by solid mechanics module. The

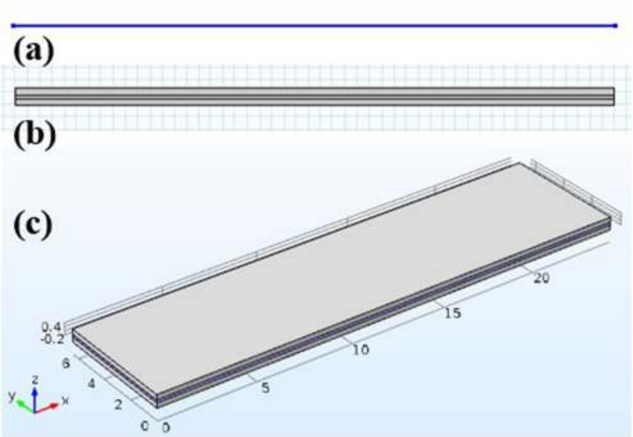

FIG. 2. COMSOL finite element models of piezoelectric bimorph cantilever (a) 1D beam model, (b) 2D solid mechanics model, (c) 3D solid mechanics model.

piezoelectric material parameters of PZT-5H are from COMSOL and material parameters of brass are user-defined. For geometric model, fixed constraint boundary conditions are added to the fixed end of double crystal cantilever beam, and the rest are free. For mesh generation, line segment mesh is used in 1D model, triangle mesh is used in 2D model and tetrahedron mesh is used in 3D model. Routine mesh size is chosen for 1D and 2D models. Ultra-coarsening mesh size is chosen for 3D model to save the calculation time.

C. Definition of material

The material of piezoelectric bimorph cantilever is composed of the upper and lower piezoelectric layers of PZT-5H and the intermediate substrate layer of brass.

Brass substrate is a linear elastic material defined in 1D. The material parameters in Table II are suitable for distributed parameter modeling and 1D, 2D and 3D finite element modeling.

For the definition of PZT-5H piezoelectric material parameters, the parameters of distributed parameter model are simplified in the form of 1D by Euler-Bernoulli beam theory as shown in Table III.

However, in the piezoelectric material library of COMSOL, PZT-5H is defined with 3D material parameters. For the stress-charge constitutive relation, the elastic matrix of PZT-5H is: $\begin{bmatrix} 127.20 & 80.21 & 84.67 & 0 & 0 & 0 \\ 80.21 & 127.20 & 84.67 & 0 & 0 & 0 \\ 84.67 & 84.67 & 117.44 & 0 & 0 & 0 \\ 0 & 0 & 0 & 22.99 & 0 & 0 \\ 0 & 0 & 0 & 0 & 22.99 & 0 \\ 0 & 0 & 0 & 0 & 0 & 23.47 \end{bmatrix}$ GPa. The electromechanical coupling matrix of PZT-5H is: $\begin{bmatrix} 0 & 0 & 0 & 0 & 17.035 & 0 \\ 0 & 0 & 0 & 17.035 & 0 & 0 \\ -6.623 & -6.623 & 23.240 & 0 & 0 & 0 \end{bmatrix}$ C/m^2. The relative permittivity matrix of PZT-5H is: $\begin{bmatrix} 1704.4 & 0 & 0 \\ 0 & 1704.4 & 0 \\ 0 & 0 & 1433.6 \end{bmatrix}$.

IV. EIGENFREQUENCY SIMULATION IN COMSOL

Definition of linear elastic materials in 1D is enough for eigenfrequency simulation in COMSOL. For the selection of physical field, 1D FEA model chooses beam module to build homogeneous Euler-Bernoulli cantilever beam. Its equivalent material properties are calculated by formulas (3-5). Solid mechanics modules are selected to build solid models for 2D and 3D FEA models.

When the piezoelectric parameters of PZT-5H material are defined in 3D as material library of COMSOL, the eigenfrequency simulation results of COMSOL, DPM and experiment are compared in Table IV.

The first order eigenfrequency simulation results in DPM and COMSOL 1D model have almost no error compared with the experimental results. The simulation results obtained by COMSOL 2D and COMSOL 3D models with PZT-5H parameters in 3D are higher than the experimental result.

In order to explore the reason why the simulation results in COMSOL 2D and COMSOL 3D models with PZT-5H parameters in 3D are too high, the piezoelectric material parameters of PZT-5H are modified and replaced with the PZT-5H parameters in 1D as shown in Table III, which are consistent in the DPM model. The exchange formula for the 1D piezoelectric material parameters from 3D piezoelectric material parameters are:[23]

$$Y_p = 1/s_{11}^E, \bar{e}_{31} = d_{31}/s_{11}^E, \bar{\varepsilon}_{33}^S = \varepsilon_{33}^T - d_{31}^2/s_{11}^E \qquad (7)$$

In 1D COMSOL piezoelectric material parameters setting, the elastic matrix and the relative permittivity matrix are all set to be isotropic. All the parameters in electromechanical coupling matrix is set to be 0 except for e_{31}: $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -16.6 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ C/m^2.

The simulation results of the first order eigenfrequency are shown in Table V.

When the piezoelectric material parameters of PZT-5H is simplified in 1D, the simulation results in COMSOL 2D and 3D models are both consistent with those in the DPM model and experiment. So COMSOL models with PZT-5H defined in 1D are more credible. In addition, the error is also related to the meshing accuracy. The denser the meshing, the higher the simulation accuracy and the longer the computational time.

TABLE IV. Comparation of first order eigenfrequency (PZT-5H in 3D).

	Test[23]	DPM[23]	COMSOL 1D	COMSOL 2D	COMSOL 3D
Eigenfrequency/Hz	502.5	502.6	502.44	515.8	513.55
Relative error	0%	+0.02%	−0.01%	+2.65%	+2.2%

TABLE V. Comparation of first order eigenfrequency (PZT-5H in 1D).

	Test	DPM	COMSOL 1D	COMSOL 2D	COMSOL 3D
Eigenfrequency/Hz	502.5	502.6	502.44	503.0	503.26
Relative error	0%	+0.02%	−0.01%	+0.10%	+0.15%

V. ELECTROMECHANICAL COUPLING SIMULATION IN COMSOL

The electromechanical coupling simulation of piezoelectric bimorph cantilever using COMSOL requires three physical fields: solid mechanics, electrostatic and circuit fields. Therefore, 1D beam model cannot handle electromechanical coupling analysis, and only 2D and 3D FEA models are promised to do electromechanical coupling simulation.

In the physical field of solid mechanics, an inertia load is applied to the whole structure, with the acceleration of 1 g (9.8 m/s^2). Isotropic loss factors are chosen for the damping of linear elastic materials on the intermediate substrate and mechanical damping of piezoelectric materials on the upper and lower piezoelectric layers. The polarization direction of piezoelectric materials in the upper and lower layers of bimorph in series type is opposite. The method is to rotate the relative coordinate system of piezoelectric materials in Z direction by 180^0. In the electrostatic physical field, attention should be paid to define the top electrode of the upper piezoelectric layer as terminal 1, the connection between the bottom electrode of the upper piezoelectric layer and the top electrode of the lower piezoelectric layer as terminal 2. The type of the terminal should be chosen as circuit type, and the bottom electrode of the lower piezoelectric layer as grounding. In the circuit physical field, attention should be paid to select the potential of load resistance as the voltage of terminal 1. In the multi-physical field interface, a piezoelectric coupling interface is applied to the piezoelectric bimorph.

For bimorph in parallel type, the polarization directions of piezoelectric materials in the upper and lower layers are the same. In the electrostatic physical field, the top electrode of the upper piezoelectric layer and the bottom electrode of the lower piezoelectric layer are defined as terminal 1, and the bottom electrode of the upper piezoelectric layer and the top electrode of the lower piezoelectric layer are defined as grounding.

A. Voltage frequency response simulation

Voltage frequency response is simulated in frequency domain. Load resistance of 470 Ω is selected for short circuit condition and 1 MΩ is selected for open circuit condition. The excitation frequency is near the first order eigenfrequency and changed with the step length of 0.1 Hz. The voltage-frequency response curves of piezoelectric bimorph in series type built by 2D and 3D COMSOL models and DPM model are simulated and shown in figure 3.

The peak point of the piezoelectric voltage frequency response curve represents the peak voltage at resonant frequency. The simulation curves of 2D and 3D COMSOL models almost coincide. The short-circuit and open-circuit resonant frequencies and corresponding voltages are analyzed and compared with DPM and experimental values, as shown in Table VI.

The 2D &3D COMSOL simulation results of resonance frequency and peak voltage are little higher than those of the test, but the maximum relative error is less than 1.21%. It can be considered that COMSOL FEA electromechanical coupling simulation can well describe the voltage-frequency response characteristics of the system.

If the 2D &3D COMSOL models use the piezoelectric material parameters of PZT-5H in 3D from material library of COMSOL, the simulation results and relative error comparison with experimental results are shown in Table VII.

The largest relative error of peak voltage is calculated in 2D COMSOL model with the value of +28.33%, and the largest relative error of resonant frequency is obtained in 3D model with the value

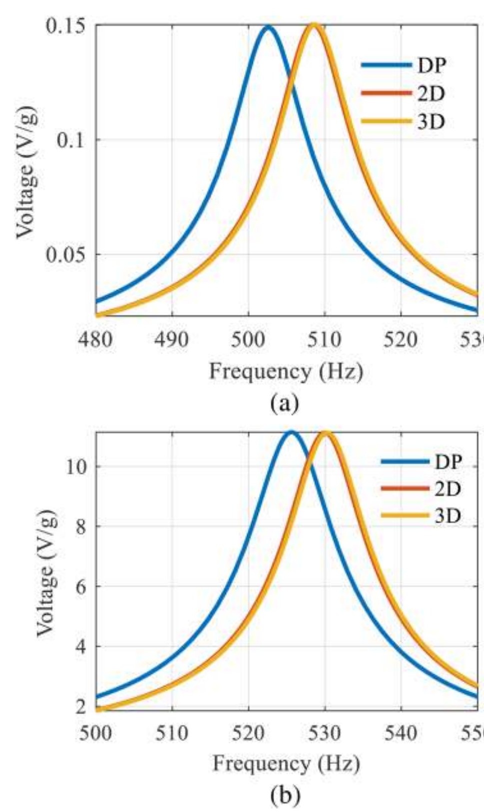

FIG. 3. The voltage-frequency response curves of piezoelectric bimorph in series type. (a) Load resistance of 470 Ω in short circuit condition. (b) Load resistance of 1 MΩ in open circuit condition.

TABLE VI. Peak voltage and resonant frequency of piezoelectric bimorph in series type (PZT-5H in 1D).

	Resistance	Test[23]	DPM[23]	COMSOL 2D &3D	Relative error
Resonant frequency/Hz	470 Ω	502.5	502.6	508.6	+1.21%
	1 MΩ	524.7	525.7	530.2	+1.05%
Peak voltage/V	470 Ω	0.15	0.149	0.1502	+0.13%
	1 MΩ	11.0	11.14	11.12	+1.09%

TABLE VII. Peak voltage and resonant frequency and relative error of piezoelectric bimorph in series type (PZT-5H in 3D).

	Resistance	COMSOL 2D	Relative error	COMSOL 3D	Relative error
Resonant frequency/Hz	470 Ω	541.3	+7.72%	522.6	+4.0%
	1 MΩ	594.1	+13.23%	547.5	+4.35%
Peak voltage/V	470 Ω	0.1925	+28.33%	0.1518	+1.2%
	1 MΩ	14.062	+27.84%	11.231	+2.1%

of +4.35% by using piezoelectric material parameters of PZT-5H in 3D. The eigenfrequency simulation and electromechanical coupling simulation both indicate that the 1D piezoelectric material parameters consistent in the DPM are suitable in COMOSL modelling to achieve the desired simulation accuracy.

B. Contrastive analysis of series and parallel connections

1D piezoelectric material parameters of PZT-5H are applied to 2D and 3D COMSOL models of piezoelectric bimorph in parallel type. The voltage-frequency response curves of piezoelectric bimorph in parallel type built by 2D and 3D COMSOL models as well as DPM model are simulated and shown in figure 4.

The simulation curves of 2D and 3D COMSOL models in parallel type are almost coincident. The simulation time of 2D COMSOL model is shorter than that of 3D COMSOL model. Therefore, priority should be given to 2D modelling and simulation in COMSOL.

The short-circuit and open-circuit resonant frequencies and corresponding voltages are analyzed. Since Alper Erturk did not give the experimental values of bimorph in parallel type, the COMSOL simulation results and relative error are compared with the DPM values, as shown in Table VIII.

The maximum relative errors of resonant frequency and peak voltage based on COMSOL simulation results are both less than 1.24% compared with the DPM values. It can be concluded that COMSOL FEA electromechanical coupling simulation can well describe the voltage-frequency response characteristics of the piezoelectric bimorph both in series type and parallel type.

In addition, the resonant frequency of bimorph is the same whether in series type or parallel type. The open circuit peak voltage in series type is almost twice as much as that in parallel type, which is consistent with the theoretical results.

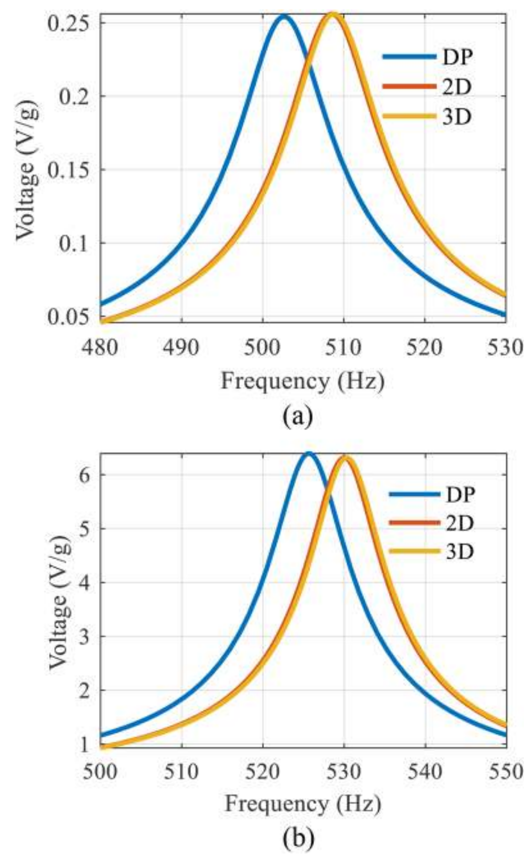

FIG. 4. The voltage-frequency response curves of piezoelectric bimorph in parallel type. (a) Load resistance of 470 Ω in short circuit condition. (b) Load resistance of 1 MΩ in open circuit condition.

TABLE VIII. Peak voltage and resonant frequency of piezoelectric bimorph in series type and parallel type (PZT-5H in 1D).

	Resistance	DPM series type	PM parallel D type	COMSOL 2D &3D	Relative error
Resonant frequency/Hz	470 Ω	502.6	502.6	508.6	+1.21%
	1 MΩ	525.7	525.7	530.2	+1.05%
Peak voltage/V	470 Ω	0.149	0.2542	0.256	+0.71%
	1 MΩ	11.14	6.393	6.314	−1.24%

C. Analysis of output power and optimal load resistance

The output power and optimal load resistance of 2D COMSOL model in series type and parallel type are analysed separately. In frequency domain analysis, the excitation frequency range is varying from 480 Hz to 550 Hz with the step length of 0.5 Hz. The excitation acceleration is still 1 g. The load resistance parameters are scanned by exponential step. The curves of output electric power, excitation frequency and load resistance are observed, as shown in figure 5 and figure 6. The average output power is $P = V^2/2R_l$, where R_l is the load resistance and V is the voltage of load resistance.

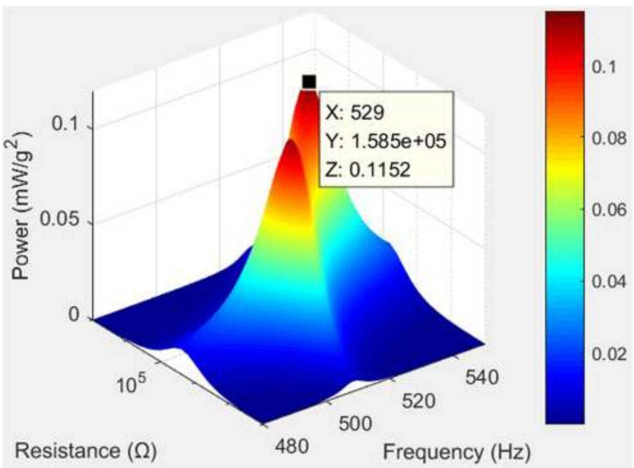

FIG. 5. Electrical power simulation curve of bimorph in series type.

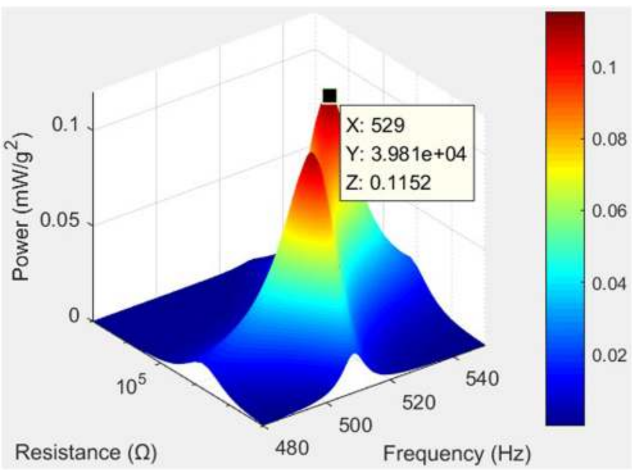

FIG. 6. Electrical power simulation curve of bimorph in parallel type.

There are two peak power for bimorph in both series and parallel type from the simulation curves of 2D COMSOL model. The comparison results are shown in Table IX.

The peak power of bimorph in series type and parallel type are the same, and the corresponding resonant frequency are the same, but the optimal load resistance in series type is four times than that in parallel type, which is consistent with the theory. The above situations are also suitable for sub peak. Interestingly, the resonant frequencies of main peaks are close to the open-circuit resonant frequencies, and the resonant frequencies of sub peaks are close to the short-circuit resonant frequencies. Both can achieve peak power output due to impedance matching.

TABLE IX. Resonant frequency, optimal load resistance and peak power of piezoelectric bimorph in series type and parallel type.

Bimorph type	Peak point	Resonant frequency/Hz	optimal load resistance/kΩ	peak power/mW
Series type	Main peak	529	158.5	0.1152
	Sub peak	510	10.0	0.110
Parallel type	Main peak	529	39.81	0.1152
	Sub peak	510	2.512	0.110

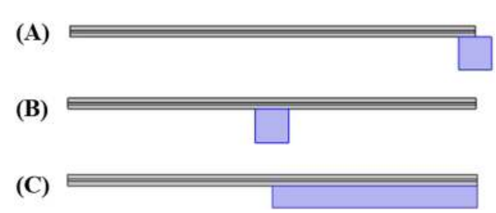

FIG. 7. Three kinds of piezoelectric bimorph cantilever with tip mass. (A) Structure A, (B) Structure B, (C) Structure C.

TABLE X. Performance comparation of three kinds of piezoelectric bimorph cantilever with tip mass.

	Structure A			Structure B	Structure C
	Test[23]	DPM[23]	COMSOL 2D	COMSOL 2D	COMSOL 2D
Eigenfrequency/Hz	338.4	338.5	339.74	475.55	413.06
Resonant frequency/Hz	356.3	355.4	360.0	498.8	431.9
Peak voltage/V	19.2	19.2	19.22	13.61	15.38

VI. PIEZOELECTRIC BIMORPH CANTILEVER WITH TIP MASS

Although Erturk A and Kim have given the DPM model of piezoelectric bimorph cantilever with tip mass, it is very difficult for the theoretical model to consider the vibration mode and mode electromechanical coupling coefficient under the influence of mass location, shape and moment of inertia. However, the advantage of FEA lies in the modeling and simulation of complex topological structures.

As shown in figure 7, this study establishes 2D COMSOL models of three kinds of piezoelectric bimorph cantilever with tip mass. The structural damping is changed to 0.00845 and the loss factor is 0.0169. Structure A is identical to DPM model of Erturk A. The material of mass block is brass and is located at the tip of the cantilever. Structure B moves the mass block to the center of the cantilever. Structural C replaces the mass block material with silicon, and the bonding length of the mass block occupies half of the cantilever length. All the weights of tip mass in the three structures are 0.239 g. The first-order eigenfrequency, resonant frequency and peak voltage of the three structures are simulated in open-circuit state, as shown in Table X.

The simulation results of structure A in COSMOL are consistent with those of DPM model by Erturk A, and the relative error is small, which shows the reliability of FEA modeling and simulation of piezoelectric bimorph cantilever with tip mass. Comparing the simulation results of the first-order eigenfrequency, resonant frequency and peak voltage of the three structures in open-circuit state, the difference between the structure B and structure A shows that the position of mass block has the great influence on the vibration and power generation performance of the system. The difference between the structure C and structure A shows that the bonding length of the mass block has the great influence on the vibration and power generation performance of the system. In addition, the clamping style of cantilever will also affect the performance of PVEH, and the specific analysis can be seen in the literature.[24] The above analyses prove the advantages in modeling and simulation of complex topological structures by COMSOL FEA.

VII. CONCLUSION

COMSOL modelling and simulation method of bimorph cantilever for piezoelectric vibration energy harvesting is put forward in this study. 1D, 2D and 3D geometry models of piezoelectric bimorph cantilever in series and parallel types are built in COMSOL Multiphysics software. The structural eigenfrequency is simulated by linear elastic model. The resonant frequency and corresponding voltage in open-circuit and short-circuit states are simulated by electromechanical coupling simulation both in 2D and 3D model. The COMSOL results are compared with the experiment data and DPM model simulation results. A conclusion can be drawn that the 1D piezoelectric material parameters of PZT-5H, as same in DPM model, instead of the 3D piezoelectric material parameters of COMSOL, is more suitable for 1D, 2D and 3D COMSOL modelling, due to more accuracy of simulation results.

The simulation results of 2D model and 3D model are almost the same, Therefore, priority should be given to 2D modelling and simulation in COMSOL for simulation time saving.

The comparative analyses of bimorph in series and parallel connection show that the resonant frequencies of the two types are the same, and the open-circuit voltage in series type is almost twice than that in parallel type.

The simulation results of electric power, excitation frequency and load resistance show that there are two peak powers of bimorph in both series and parallel type. The peak output powers in series and in parallel are the same, and the corresponding excitation frequency is the same. The optimal load resistance in series type is four times than that in parallel type.

Performance comparation of three kinds of piezoelectric bimorph cantilever with tip mass indicates the position of the mass block and the bonding length of the mass block have a great influence on the vibration and power generation performance of the system. COMSOL FEA proves the advantages in modeling and simulation of complex topological structures.

FEA modelling and simulation methods of piezoelectric bimorph cantilever in COMSOL proposed in this study are accurate and simple. It has practical reference value for electromechanical coupling analysis and piezoelectric vibration energy harvesting design.

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