High frequency volatility co-movements in cryptocurrency markets

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Abstract

Through the application of Diagonal BEKK and Asymmetric Diagonal BEKK methodologies to intra-day data for eight cryptocurrencies, this paper investigates not only conditional volatility dynamics of major cryptocurrencies, but also their volatility co-movements. We first provide evidence that all conditional variances are significantly affected by both previous squared errors and past conditional volatility. It is also shown that both methodologies indicate that cryptocurrency investors pay the most attention to news relating to Neo and the least attention to news relating to Dash, while shocks in OmiseGo persist the least and shocks in Bitcoin persist the most, although all of the considered cryptocurrencies possess high levels of persistence of volatility over time. We also demonstrate that the conditional covariances are significantly affected by both cross-products of past error terms and past conditional covariances, suggesting strong interdependencies between cryptocurrencies. It is also demonstrated that the Asymmetric Diagonal BEKK model is a superior choice of methodology, with our results suggesting significant asymmetric effects of positive and negative shocks in the conditional volatility of the price returns of all of our investigated cryptocurrencies, while the conditional covariances capture asymmetric effects of good and bad news accordingly. Finally, it is shown that time-varying conditional correlations exist, with our selected cryptocurrencies being strongly positively correlated, further highlighting interdependencies within cryptocurrency markets.

Keywords: Cryptocurrencies; High-frequency data; Asymmetric Diagonal BEKK; MGARCH; Volatility.

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1. Introduction

As cryptocurrency markets and exchanges continue to evolve, it is vital to further develop our understanding as to the way in which these markets operate. One avenue of particular interest is based on the conditional volatility dynamics along with the interlinkages and conditional correlations between the largest international cryptocurrencies. Through an investigation of these market interlinkages, we can help to answer key questions which have been asked of not only the integrity of cryptocurrency markets, but indeed, questions based on time-varying effects and the underlying fundamentals of these new exchanges and financial market products. Regulatory bodies and policy-makers alike have observed the growth of cryptocurrencies with a certain amount of scepticism based on the growing potential for illegality and malpractice through the use of cryptocurrencies[1].

While cryptocurrency price dynamics have been extensively studied in the literature, the potential for market manipulation appears to have been broadly identified in cryptocurrency cross-correlations and market interdependencies (Katsiampa et al. [2019]). For instance, Griffins and Shams [2018] examined whether Tether influenced Bitcoin and other cryptocurrency prices to find that purchases with Tether were timed following market downturns and resulted in significant increases in the price of Bitcoin. Further, less than 1% of the hours in which Tether experienced significant transactions were found to be associated with 50% of the increase of Bitcoin prices and 64% of other top cryptocurrencies, drawing the damning conclusion that Tether was used to provide price support and manipulate cryptocurrency prices. Furthermore, Gandal et al. [2018] identified the impact of suspicious trading activity on the Mt.Gox Bitcoin exchange theft when approximately 600,000 Bitcoins were attained. The authors demonstrated that the suspicious trading likely caused the spike in price in late 2013 from $150 to $1,000, most likely driven by one single actor. These two significant pieces of research have fine-tuned the focus of regulators, policy-makers and academics alike, as the future growth of cryptocurrencies cannot be sustained at pace with such significant questions of abnormality remaining unanswered. To develop on this we must focus on interdependencies within cryptocurrency markets, which continue to remain relatively under-explored. Despite the fact that the interconnectedness of cryptocurrencies has been studied by, e.g., Fry and Cheah [2016], Ciaian et al. [2018], Corbet et al. [2018], Katsiampa [2017], Katsiampa et al. [2019], and Koutmos [2018], all of whom em-

[1] Some many regulatory authorities such as the International Monetary Fund (IMF) have expressed their satisfaction with the product's development and the benefits that are contained within its continued growth (An Even-handed Approach to Cryptocurrencies, IMF blogpost written by Christine Lagarde, Head of the International Monetary Fund, available at: https://blogs.imf.org/2018/04/16/an-even-handed-approach-to-crypto-assets/), the Securities and Exchange Commission (SEC) in 2018 have backtracked on earlier positivity to warn of the inherent potential for spoofing and other market manipulation techniques (US Securities and Exchange Commission, Public Statement, Statement on Potentially Unlawful Online Platforms for Trading Digital Assets, Available at: https://www.sec.gov/news/public-statement/enforcement-tm-statement-potentially-unlawful-online-platforms-trading)

ployed daily data, there has been limited research conducted on volatility interdependencies within cryptocurrency markets - especially while allowing for asymmetric effects of positive and negative shocks in cryptocurrencies' volatility dynamics - although volatility modelling is important for many option pricing, portfolio selection, and risk management applications (Fleming et al. [2003]), while understanding covariances and correlations is important for determining the risk of an investor's portfolio (Coudert et al. [2015]). What is more, to the best of the authors' knowledge, no previous study has examined interdependencies within cryptocurrency markets using high-frequency data, which can provide cryptocurrency users and investors with better insights into market behaviours and dynamics.

Consequently, using intraday data for eight cryptocurrencies, namely Bitcoin, Ethereum, Litecoin, Dash, Ethereum Classic, Monero, Neo, and OmiseGO, in this study we utilise the Diagonal BEKK-MGARCH and Asymmetric Diagonal BEKK-MGARCH models which can be used to examine volatility co-movements. The Diagonal BEKK methodology itself is similar to the BEKK model of Engle and Kroner [1995]. However, in the Diagonal BEKK model the number of parameters to be estimated is considerably reduced, while guaranteeing the positive definiteness of the conditional covariance matrix (Terrell and Fomby [2006]). On the other hand, the Asymmetric Diagonal BEKK methodology allows for asymmetric responses of positive and negative shocks to the cryptocurrencies' conditional volatility and covariances, while still guaranteeing the positive definiteness of the conditional covariance matrix (Katsiampa [2018]). It is shown that our selected cryptocurrencies' pairwise price returns are strongly positively correlated and all conditional variances are significantly affected by both previous squared errors and past conditional volatility. Both the Diagonal BEKK and Asymmetric Diagonal BEKK methodologies indicate that cryptocurrency investors pay the most attention to news relating to Neo and the least attention to news relating to Dash, while, although all investigated cryptocurrencies possess high levels of persistence of volatility over time, shocks in OmiseGo persist the least and shocks in Bitcoin persist the most. Furthermore, we find evidence that all estimates of asymmetry are positive and statistically significant, indicating significant asymmetric effects of both positive and negative shocks in the conditional volatility of the price returns of all of our investigated cryptocurrencies. The conditional covariances are found to be significantly affected by cross products of past error terms and past conditional covariances using both methodologies, which is indicative of strong interdependencies between cryptocurrencies, while the significant estimates for the asymmetry terms indicate that the conditional covariances also capture asymmetric effects of both good and bad news accordingly.

The structure of the remainder of the paper is as follows: Section 2 investigates related previous literature. Section 3 describes the data and Multivariate GARCH methodology employed. The empirical findings are discussed in section 4. Finally, some concluding remarks are given in section 5.

2. Previous Literature

Multivariate GARCH models have been used throughout a host of financial market-based research in recent years. For instance, Choudhry and Wu [2008] investigated the forecasting ability of GARCH methodologies inclusive of BEKK-GARCH, in comparison to the Kalman filter methodology on UK time-varying beta's, while Trujillo-Barrera et al. [2012] considered the same methodology when investigating volatility spillovers in the United States from crude oil futures prices to find that the share of corn and ethanol price variability directly attributed to volatility in the crude oil market was generally between 10% and 20%, but reached almost 45% during the international financial crisis. Moreover, Chng [2009] investigated the cross-market trading dynamics in futures contracts on the Tokyo Commodity Exchange using a BEKK-GARCH methodology to find that natural rubber, palladium and gasoline futures are driven in principal by a common industry which is found to be the automobile industry, while Mensi et al. [2014] used the VAR-BEKK-GARCH methodology on the daily spot prices of eight international commodity markets to provide evidence of significant links between energy and cereal markets, with OPEC new announcements being found to exert influence on oil markets and the oil and cereal relationship. Research of such nature can be used to improve the risk-adjusted performance by having more diversified portfolios to hedge risk more effectively. Haixia and Shiping [2013] identified evidence of uni-directional spillover effects from crude oil markets to corn and fuel ethanol markets when analysing commodity markets in China using a BEKK-MGARCH model with further evidence of double-directional spillovers between corn and fuel ethanol, while in another Chinese-based study, Arouri et al. [2015] studied the hedging and diversification effectiveness of gold and stocks in China to find evidence of significant return and volatility cross-effects, while gold acted as a hedge for stocks and a safe haven during financial crisis. Using BEKK, DCC, DECO and ADCC-GARCH methodologies, Mimouni et al. [2016] explored diversification benefits in developed, emerging, GCC and global portfolio stock markets to find that correlations and diversification benefits are time-varying and that such trends and correlations reversed in 2012. Another interesting use of the BEKK-MGARCH methodology was that in the study of Bekiros [2014] who investigated the influence of the US financial crisis on BRIC markets to find that BRICs are integrated, contagion is substantiated and there is little evidence of decoupling using an MGARCH methodology. On the other hand, the use of the Diagonal BEKK-GARCH methodology has been used in research such as that of Bai and Koong [2018] who investigated the time-varying trilateral relationships between oil prices, exchange rate changes and stock market returns in China and the US between 1991 and 2015. Moreover, Boldanov et al. [2016] investigated the dynamic correlation between oil prices and stock markets for six major oil-importing and oil-exporting countries between 2000 and 2014 to find heterogeneous patterns and strong correlations to major economic and geopolitical events using a Diagonal BEKK model. Using a similar methodology Basher et al. [2012] studied the relationship between oil prices, exchange rates and emerging stock markets and Degiannakis and Floros

[2010] investigated hedge ratios in South African stock index futures.

Other than financial and commodity markets, recently there has been an increased interest in studying the price volatility of cryptocurrencies as well as the interconnectedness between cryptocurrencies and various economic and financial assets. While investigating the general behavioural aspects of cryptocurrencies, Corbet et al. [2018] examined the reaction of a broad set of digital assets to US Federal Fund interest rates and quantitative easing announcements to find a broad range of differing volatility responses dependent on the type of cryptocurrency investigated and as to whether the cryptocurrency was mineable or not. Other studies of the volatility of cryptocurrency price returns include those of Corbet et al. [2019], Chu et al. [2017], Katsiampa [2017], Baur and Dimpfl [2018] and Phillip et al. [2018], among others, which have employed a broad range of volatility models in order to examine the volatility behaviour of cryptocurrencies. On the other hand, examples of studies on the interconnectedness between cryptocurrencies and other assets include, e.g., those of Bouri et al. [2017], Corbet et al. [2018], Giudici and Abu-Hashish [2018], Guesmi et al. [2018], Ji et al. [2018], and Chuen et al. [2018], the findings of which suggest that cryptocurrencies are isolated from other markets. More specifically, using a dynamic conditional correlation model, Chuen et al. [2018] studied co-movements between the Cryptocurrency Index (CRIX) and mainstream assets, such as SP 500, T-Note, Gold, Oil and REITs, and concluded that the return correlations between cryptocurrencies and other assets are low, while Bouri et al. [2017] examined as to whether Bitcoin could act as a hedge and safe haven for four major world stock indices, bond, oil, gold, the general commodity index and the US dollar index finding that it is a poor hedge and is suitable for diversification purposes only. Moreover, Corbet et al. [2019] found evidence of the relative isolation of cryptocurrencies from financial and economic assets and that cryptocurrencies may offer diversification benefits for investors with short investment horizons, and Guesmi et al. [2018] analysed the conditional cross effects and volatility spillovers between Bitcoin and other financial assets providing further evidence that Bitcoin can offer diversification benefits and hedging opportunities for investors, while finding that hedging strategies involving gold, oil, emerging stock markets and Bitcoin reduce considerably a portfolio's variance in comparison to the variance of a portfolio composed of gold, oil and stocks from emerging stock only. The relationships between Bitcoin and other assets have been also analysed by Ji et al. [2018], who employed a data-driven methodology, the so-called direct acyclic graph, applied to daily index values for Bitcoin, stock, bonds, commodities and currencies and found that Bitcoin is isolated from other assets, none of which can significantly influence the Bitcoin market, as well as by Giudici and Abu-Hashish [2018], who developed an extended Vector Autoregressive model based on network models that introduce a contemporaneous contagion component that describes contagion effects between asset prices and also concluded that correlation of Bitcoin prices with traditional assets is low. In contrast, Urquhart and Zhang [2018] assessed the relationship between Bitcoin and currencies at the hourly frequency and found that Bitcoin can be an intraday hedge for the CHF, EUR and GBP, but acts as a diversifier for the AUD, CAD and JPY. The authors also found that Bitcoin is a safe haven during periods of extreme market turmoil for the CAD, CHF and GBP.

Recently there has also been an increased interest in interdependencies of cryptocurrencies. Among few authors who have studied the interconnectedness of cryptocurrency markets are Fry and Cheah [2016], Cheah et al. [2018], Ciaian et al. [2018], Corbet et al. [2018] Katsiampa [2017], Katsiampa [2018] and Katsiampa et al. [2019]. More specifically, Cheah et al. [2018] modelled cross market Bitcoin prices as long-memory processes and studied dynamic interdependence in a fractionally cointegrated VAR framework. The authors not only found long memory in both the individual markets and the system of markets depicting non-homogeneous informational inefficiency, but also that Bitcoin markets are fractionally cointegrated, with uncertainty negatively affecting this type of cointegration relationship. On the other hand, Fry and Cheah [2016] tested for contagion during bubbles and found a spillover from Ripple to Bitcoin, whereas Ciaian et al. [2018] employed an Autoregressive Distributed Lag model to study interlinkages within cryptocurrency markets in the short- and long-run and concluded that the markets are interconnected with significantly stronger interdependencies in the short-run, though. Corbet et al. [2018] used a VAR model and the Diebold and Yilmaz [2012] methodology, in order to measure the direction and intensity of spillovers across selected cryptocurrencies. However, the Diebold and Yilmaz [2012] methodology does not distinguish the potential asymmetry in spillovers that originates due to bad and good uncertainty (Baruník et al. [2016]). Moreover, despite the fact that understanding volatility co-movements is imperative to cryptocurrency users and investors in order to make more informed decisions, none of the aforementioned studies examined cryptocurrencies' conditional volatility interdependencies. To the best of the authors' knowledge, only Katsiampa [2017], Katsiampa [2018], and Katsiampa et al. [2019] examined volatility dynamics and conditional correlations between cryptocurrencies using multivariate GARCH models. Whereas Katsiampa [2017] and Katsiampa [2018], used Diagonal BEKK models for daily data of selected cryptocurrencies and found volatility co-movements between the considered cryptocurrencies, Katsiampa et al. [2019], using daily data for Bitcoin, Ether and Litecoin, applied three pair-wise bivariate BEKK models to examine the conditional volatility dynamics along with interlinkages and conditional correlations between three pairs of cryptocurrencies, namely Bitcoin-Ether, Bitcoin-Litecoin, and Litecoin-Ether. The authors found evidence of bi-directional shock transmission effects between Bitcoin and both Ether and Litecoin, and uni-directional shock spillovers from Ether Litecoin as well as bi-directional volatility spillover effects between all the three pairs.

Nevertheless, none of the aforementioned studies on interdependencies within cryptocurrency markets has considered intra-day data, although there seems to be a relative reduction of intra-day volatility while daily volatility remains high in cryptocurrency markets (Corbet et al. [2019]). Moreover, the importance of intra-day return information has been highlighted in several studies in the finance literature (see, e.g., Fleming et al. [2003]; Hanousek et al. [2009]; Maheu and McCurdy [2011]). Furthermore, return volatility varies systematically over a trading day (Andersen et al. [1997]), while the value of switching from daily to intra-day returns to estimate the conditional covariance matrix can be substantial (Fleming et al. [2003]). For this reason, intra-day data have found several financial applications such as in forecasting (see, e.g., Sévi [2014]) and in the field of option pricing (see, e.g., Heston and Nandi [2000]). Consequently, we develop on such research through the application of the Diagonal BEKK-MGARCH and Asymmetric Diagonal BEKK-MGARCH models to intra-day data for eight cryptocurrencies, namely Bitcoin, Ethereum, Litecoin, Dash, Ethereum Classic, Monero, Neo, and OmiseGO. This study therefore aims to contribute to the literature on the volatility behaviour and interlinkages within cryptocurrency markets by investigating not only volatility dynamics but also interdependencies within cryptocurrency markets and correlations between cryptocurrencies.

3. Data and Methodology

The dataset consists of hourly closing prices for Bitcoin (BTC), Ethereum (ETH), Litecoin (LTC), Dash (DASH), Ethereum Classic (ETC), Monero (XMR), Neo (NEO), and OmiseGO (OMG) from 15 September 2017 at 11:00p.m. to 1 July 2018 at 12:00a.m. The sample thus consists of 6907 observations for each cryptocurrency. The data were sourced at Bittrex and the prices are listed in US dollars. The hourly price returns of Bitcoin (i=1), Ether (i=2), Litecoin (i=3), Dash (i=4), Ethereum Classic (i=5), Monero (i=6), Neo (i=7), and OmiseGO (i=8) are calculated as:

$$R_{i,t} = ln(p_{i,t}) - ln(p_{i,t-1}), \tag{1}$$

where $P_{i,t}$ is the hourly closing price of cryptocurrency i on hour t. We start our analysis by producing descriptive statistics for the cryptocurrency price returns and by performing the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit-root tests to assess the stationarity of the returns series. We also perform Engle's ARCH-LM test for ARCH effects in order to examine whether volatility modelling is required for the price returns. For the returns series exhibiting volatility clustering, multivariate GARCH methodology is employed to model the cryptocurrencies' volatility interdependencies. As will be shown in the next section, the price returns of all the cryptocurrencies considered in this study exhibit ARCH effects, and hence we proceed with multivariate GARCH modelling in order to model the eight cryptocurrencies' conditional variances as well as their volatility co-movements.

Following Corbet et al. [2018] and Katsiampa (2018a, b), we employ a simple specification for the conditional mean equation of the returns series as follows

$$r_t = c + \varepsilon_t, \tag{2}$$

where r_t is the vector of price returns, c is a vector of parameters estimating the mean of the returns series, and ε_t is the vector of residuals with a conditional covariance matrix

H_t given the available information set I_{t-1}. Since we consider eight cryptocurrencies in this study, all these three components of the mean equation are (8 x 1) vectors.

In this study, we employ the Diagonal BEKK specification for the conditional covariance matrix, H_t, which is given as:

$$H_t = W'W + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B \tag{3}$$

where W, A and B are matrices of parameters with appropriate dimensions, with W being an upper triangular matrix, and A and B being restricted to be diagonal. The diagonal elements of H_t, $h_{ii,t}$, $i = 1, , 8$, represent the conditional variances which are given as:

$$h_{ii,t} = \tilde{w_{ii}} + a_{ii}^2\varepsilon_{it-1}^2 + \beta_{ii}^2 h_{it-1} \tag{4}$$

while the off-diagonal elements of H_t, $h_{ij,t}$, $i \neq j$, i,$j = 1, ..., 8$, represent conditional covariances between two cryptocurrencies, i and j, and are given as:

$$h_{ij,t} = \tilde{w_{ij}} + a_{ii}a_{jj}\varepsilon_{it-1}\varepsilon_{jt-1} + \beta_{ii}\beta_{jj}h_{ijt-1} \tag{5}$$

where $\tilde{w_{ij}}$ is the ij^{th} element of $W'W$.

It is worth noting that the Diagonal BEKK model is similar to the BEKK model of Engle and Kroner [1995]. However, in the Diagonal BEKK model the number of parameters to be estimated is considerably reduced, while guaranteeing the positive definiteness of the conditional covariance matrix (Terrell and Fomby [2006]).

In this study, we also employ the asymmetric Diagonal BEKK model of Kroner and Ng [1998] which allows for asymmetric responses of conditional variances and covariances to positive and negative shocks while maintaining the positive definiteness of the conditional covariance matrix. In the asymmetric Diagonal BEKK model, the conditional covariance matrix is expressed as:

$$H_t = W'W + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + D'\eta_{t-1}\eta'_{t-1}D + B'H_{t-1}B \tag{6}$$

where $\eta_t = (\eta_{1,t}, \eta_{2,t}, ...)'$ and $\eta_{i,t} = min(\eta_{i,t}, 0)$. The conditional variance of cryptocurrency i, h_{it}, $(i = 1, ..., 8)$, is therefore:

$$h_{it} = \tilde{w}_{ii} + a_{ii}^2\varepsilon_{it-1}^2 + d_{ii}^2\eta_{it-1}^2 + \beta_{ii}^2 h_{it-1} \tag{7}$$

with the current volatility of cryptocurrency i reacting to negative shocks, η_{it}, as determined by the estimated asymmetry parameter, d_{ii}. On the other hand, the conditional covariance between two cryptocurrencies i and j, h_{ijt} (where $i, j = 1, ..., 8$ and $i \neq j$), is given as:

$$h_{ijt} = \tilde{w}_{ij} + a_{ii}a_{jj}\varepsilon_{it-1}\varepsilon_{jt-1} + d_{ii}d_{jj}\eta_{it-1}\eta_{jt-1} + \beta_{ii}\beta_{jj}h_{ijt-1} \tag{8}$$

and captures asymmetric effects between positive and negative shocks correspondingly. The parameters of the conditional mean, variance and covariance equations are estimated simultaneously under maximum likelihood using the BHHH algorithm. Similar to Urquhart and Zhang (2018), we then use three information criteria, namely Akaike (AIC), Bayesian (BIC) and Hannan-Quinn (HQ) as performance criteria as well as the statistical significance of the estimated asymmetry parameters in order to see which model between the two is the preferred one.

Once the model parameters are estimated, the conditional correlations between two cryptocurrencies i and j, r_{ijt}, $(i, j = 1, ..., 8)$, are calculated as follows:

$$r_{ij,t} = \frac{h_{ij,t}}{\sqrt{h_{ii,t}}\sqrt{h_{jj,t}}} \tag{9}$$

4. Empirical Findings

Figure 1 presents evidence of the significant and rapid growth in cryptocurrencies in late Q4 2017 for all of our investigated cryptocurrencies. There is clear evidence of correlation during this phase of growth, and also through the continued decline that quickly followed. However, there is evidence that not all cryptocurrencies shared the same trend of decline. While Bitcoin for example started to fall sharply in late Q4 2017, having fallen from high values of almost $20,000 to approximately $7,000 by February 2018, its price has stabilised somewhat. Litecoin somewhat mirrors this decline, however, currencies such as ETH, XMR, NEO and OMG continue to fall throughout the period. ETC, XMR, NEO and OMG actually present evidence of a secondary phase of price appreciation during mid-Q1 2018 before quickly falling again. Figure 2 echoes such sentiment while depicting the plots of the hourly closing price returns series, indicating the presence of volatility clustering in all the price returns series. There is visual evidence of significant and sustained increases in volatility during Q4 2017 and early Q1 2018.

Insert Figures 1 and 2 about here

The descriptive statistics for the returns of the eight cryptocurrencies considered in this study are given in Table 1. The hourly average closing price returns are negative for Dash and OMG, but positive for all the remaining cryptocurrencies. The standard deviation ranges between 1.33% (BTC) and 2.11% (NEO). Furthermore, although the price returns of Bitcoin are skewed to the left, the price returns of all the altcoins considered in this study are all positively skewed, indicating a longer right tail. It can also be noticed that Dash displays the highest (23.69) and Bitcoin exhibits the lowest (10.05) excess kurtosis, with all returns series being leptokurtic, though. The results of the Jarque-Bera (JB) test reject the null hypothesis of normal price returns and therefore further confirm the non-normality of the price returns series. In addition, the ARCH(1) and ARCH(5) test results clearly suggest the presence of ARCH effects in the hourly price returns of all the eight cryptocurrencies considered. We can thus proceed with multivariate GARCH modelling of the volatility dynamics of the price returns of the eight cryptocurrencies. It can be noticed that the result of volatility clustering are in accordance with the studies of for example, Katsiampa (2018a, b), Phillip et al. [2018] and Zhang et al. [2018], among others.

Insert Table 1 about here

Table 2 presents the unit-root test results. According to the results, both the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests reject the null hypothesis of a unit root and thus confirm the stationarity of all the price returns series. Consequently, the hourly closing price returns of all the eight cryptocurrencies considered in this study are appropriate for further analysis.

Insert Table 2 about here

Table 3 reports the correlation matrix between the different pairs of cryptocurrency price returns. We notice that all the correlations are positive, a finding which is consistent with the study of Katsiampa (2018b). It can also be noticed that all the correlations are above 0.53 suggesting a rather strong positive linear relationship. It is worth noting, though, that in this study we have found higher correlations compared to those found in Katsiampa (2018b). This could be explained by the fact that in this study we have used hourly data and our sample covers a more recent period.

Insert Table 3 about here

Next we proceeded by estimating the parameters of the Diagonal BEKK and Asymmetric Diagonal BEKK models. Since we found evidence of non-normal returns according to the descriptive statistics as well as Jarque-Bera test results, the model parameters were estimated under the multivariate Student's t error distribution. The estimation results of the Diagonal BEKK model are presented in Table 4, while the conditional variance and covariance equations with substituted coefficients are given in Tables 5 and 6, respectively. On the other hand, the estimation results of the Asymmetric Diagonal BEKK model are reported in Table 7, with the conditional variance and covariance equations with substituted coefficients being presented in Tables 8 and 9, accordingly.

It can be noticed that according to the estimation results of the conditional variance equations of both models (Tables 4 and 7), all the parameter estimates are statistically significant at the 1% level. Consequently, all cryptocurrencies' current conditional variances are significantly affected by both previous squared errors and past conditional volatility. By inspecting the substituted coefficients in the conditional variance equations (Tables 5 and 8), we notice that in both models the lowest estimated values of the ARCH coefficient are obtained for Dash while the highest estimated values of the ARCH coefficient are given for Neo. This result indicates that cryptocurrency users pay the most attention to news related to Neo and the least attention to news related to Dash. On the other hand, the lowest estimated values of the GARCH coefficient are given for OmiseGO while the highest estimated values of the GARCH coefficient are given for Bitcoin, suggesting that shocks in the OmiseGO market persist the least, while shocks in the Bitcoin market persist the most, although the high values of the estimated GARCH coefficient indicate high persistence of volatility over time for all the eight cryptocurrencies. With regards to the asymmetry parameter of the Asymmetric Diagonal BEKK model (Table 7), we have found that all the estimates of the asymmetry term are positive and statistically significant at the 1% level. Consequently, there are statistically significant asymmetric effects of positive and negative shocks in the conditional volatility of the price returns of all the eight cryptocurrencies.

Insert Tables 4 through 9 about here

It can also be noticed that similar results are obtained for the conditional covariances which are significantly affected by cross products of past error terms as well as by past conditional covariance terms in both the Diagonal BEKK and Asymmetric Diagonal BEKK models, suggesting significant volatility co-movements and hence significant interlinkages. This finding is thus in accordance with the studies of Ciaian et al. [2018], Katsiampa (2018a, b) and Katsiampa et al. [2019] on interdependencies between cryptocurrencies. Moreover, in view of statistically significant estimates for the asymmetry terms, it is shown that the conditional covariances also capture asymmetric effects of good and bad news accordingly, a result which is again consistent with the study of Katsiampa (2018b).

What is more, in terms of model selection, all the three performance criteria (i.e., Akaike Information Criteria, Bayesian Information Criterion and Hannan-Quinn Criterion) are lower for the Asymmetric Diagonal BEKK model and therefore suggest that the Asymmetric Diagonal BEKK model is superior to the Diagonal BEKK model. This result is also confirmed by the log-likelihood value (LL) which is maximised under the Asymmetric Diagonal BEKK model.

Insert Figures 3 through 5 about here

Finally, the plots of the conditional variances and covariances of the price returns of the eight cryptocurrencies under the Asymmetric Diagonal BEKK model are presented in Figures 3 and 4, respectively, while the conditional correlations plots are depicted in Figure

5. Regarding the conditional volatility plots (presented in Figure 3), we notice several spikes in all the cryptocurrencies' conditional variances, with significant spikes observed for most cryptocurrencies in Q4 2017 and early Q1 2018. It is also worth noting that BTC, XMR and OMG present the most frequent spikes in conditional variances as presented by the Asymmetric Diagonal BEKK methodology, while ETC and NEO present the least number of individual spikes in conditional variance. We also notice several spikes in the conditional covariances between the different pairs of cryptocurrencies (presented in Figure 4), which are time-varying and mostly positive. The plots of the conditional correlations (presented in Figure 5) further confirm time-varying conditional correlations between the different pairs of cryptocurrencies, with positive correlations mostly prevailing as might have been expected. More specifically, there is evidence of exceptionally volatile correlations in 2017. However, at the start of Q1 2018, the correlation between Bitcoin and all investigated cryptocurrencies sharply increases to levels that remain elevated and far more stable throughout 2018. There is a similar trend when investigating all other pairwise conditional correlations, however, there is evidence of substantially less volatility in conditional correlations in 2017.

5. Conclusions

In this study, we applied the Diagonal BEKK-MGARCH and Asymmetric Diagonal BEKK-MGARCH models to intra-day data for eight cryptocurrencies, namely Bitcoin, Ethereum, Litecoin, Dash, Ethereum Classic, Monero, Neo, and OmiseGO, in order to study volatility dynamics of cryptocurrencies as well as interdependencies within cryptocurrency markets and correlations between cryptocurrencies. According to the results, our selected cryptocurrencies' pairwise price returns are strongly and positively correlated. Moreover, we found that all the conditional variances are significantly affected by both previous squared errors and past conditional volatility. Furthermore, both the Diagonal BEKK and Asymmetric Diagonal BEKK methodologies indicated that cryptocurrency investors pay the most attention to news relating to Neo and the least attention to news relating to Dash, while shocks in OmiseGo persist the least and shocks in Bitcoin persist the most, although it was noted that all of the considered cryptocurrencies possess high levels of persistence of volatility over time. Similar results were obtained for the conditional covariances which were found to be significantly affected by cross products of past error terms and past conditional covariances using both methodologies, suggesting strong interdependencies between cryptocurrencies. It was also noticed that the Asymmetric Diagonal BEKK model is a superior choice of methodology due to a superior log likelihood value and lower information criteria values. Using the Asymmetric Diagonal BEKK methodology, our results suggested significant asymmetric effects of positive and negative shocks in the conditional volatility of the price returns of all of our investigated cryptocurrencies, while the conditional covariances capture asymmetric effects of good and bad news accordingly.

The above results combined provide strong evidence supporting the progress and development of cryptocurrency markets in terms of the new product's integration. Further research on cryptocurrency price volatility behaviour and the interlinkages between price volatility and changes in liquidity is vital to support and develop our understanding of the dynamics in which these relatively youthful products operate. This is ever more important due to the recent increase in research providing substantial evidence of market manipulation and other broad trading abnormalities.

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Figure 1: Closing prices of cryptocurrencies (in US dollars)

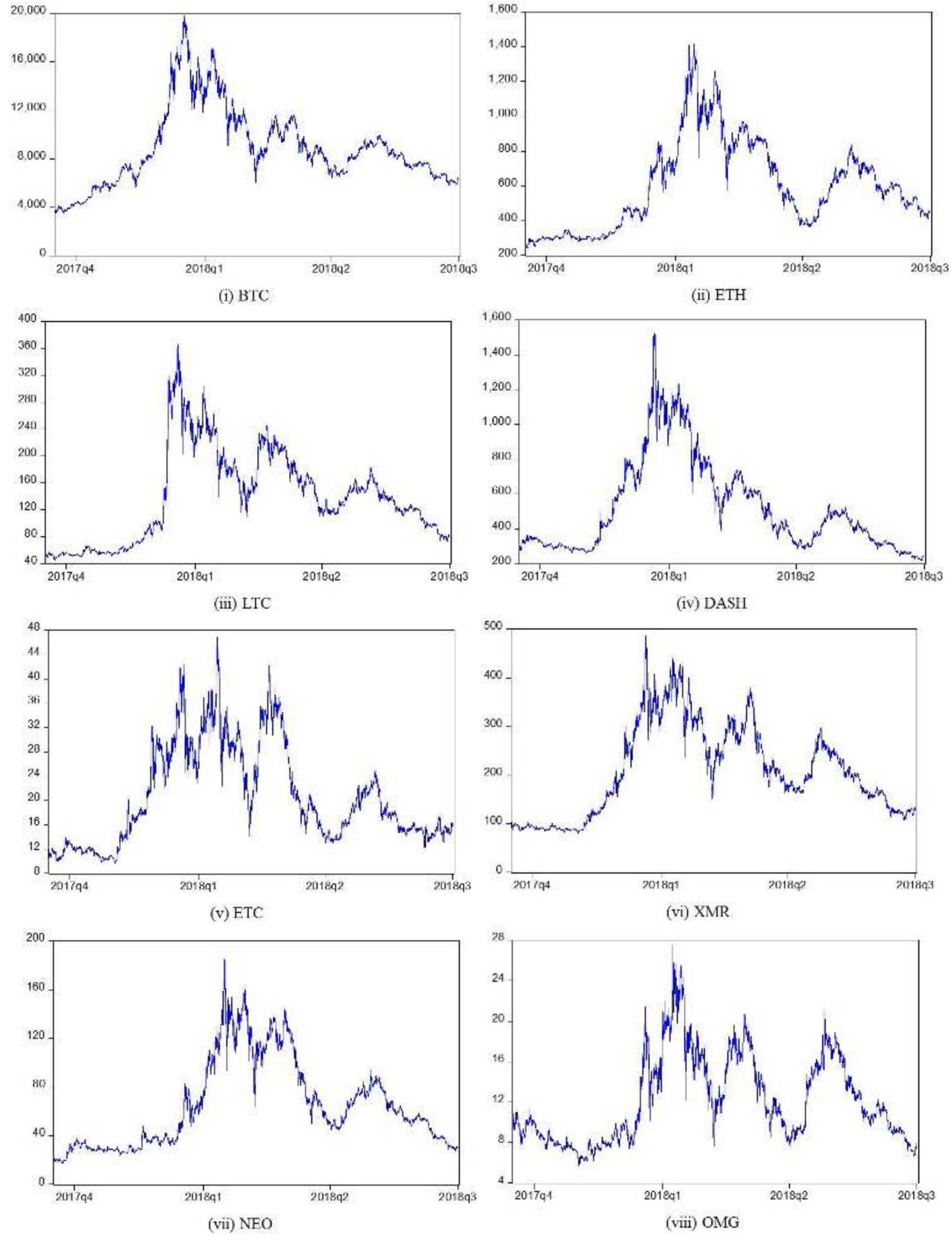
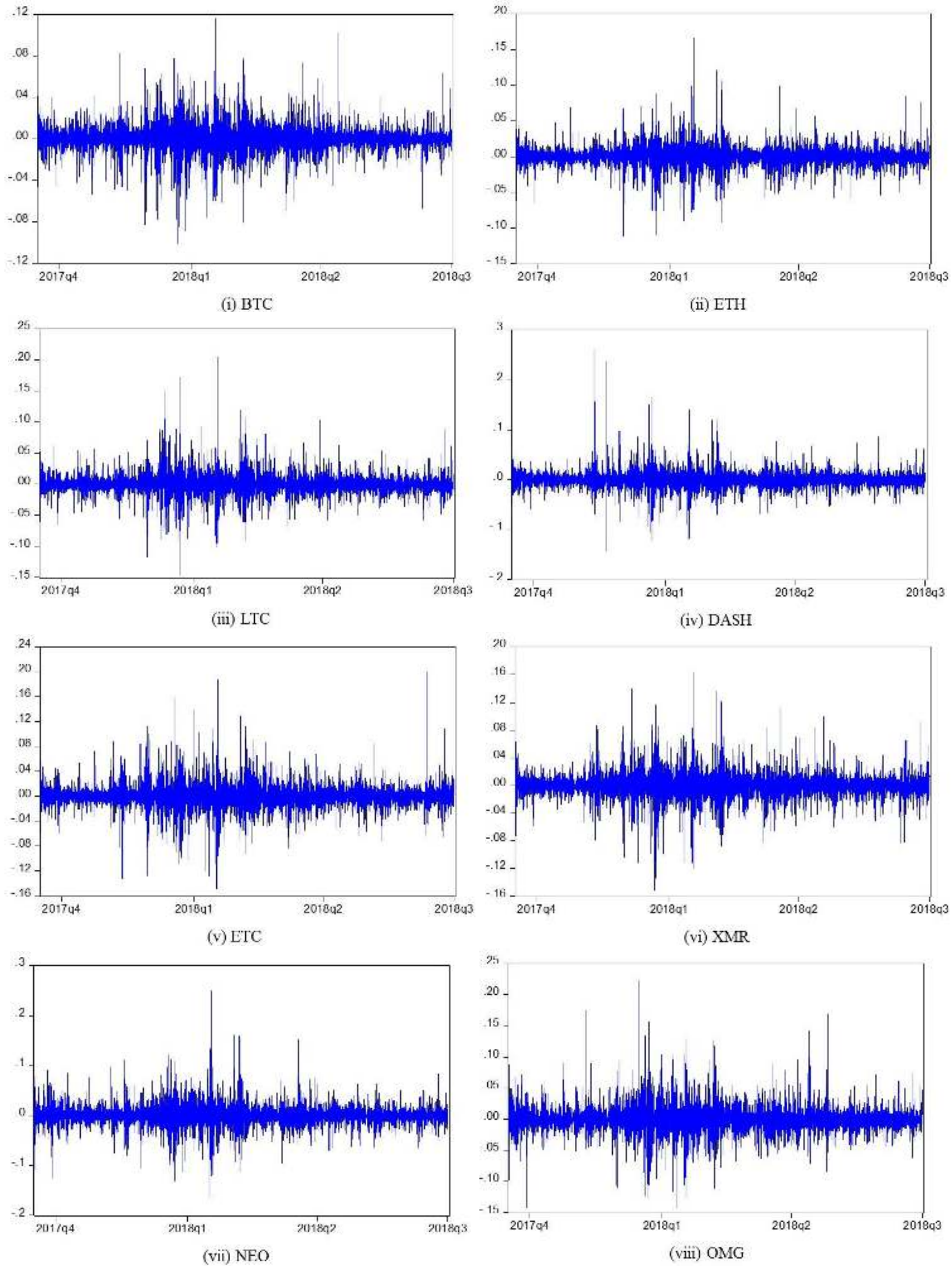


Figure 2: Closing price returns of cryptocurrencies (in US dollars)



(i) BTC

(ii) ETH

(iii) LTC

(iv) DASH

(v) ETC

(vi) XMR

(vii) NEO

(viii) OMG

Figure 3: Asymmetric Diagonal BEKK model conditional variances

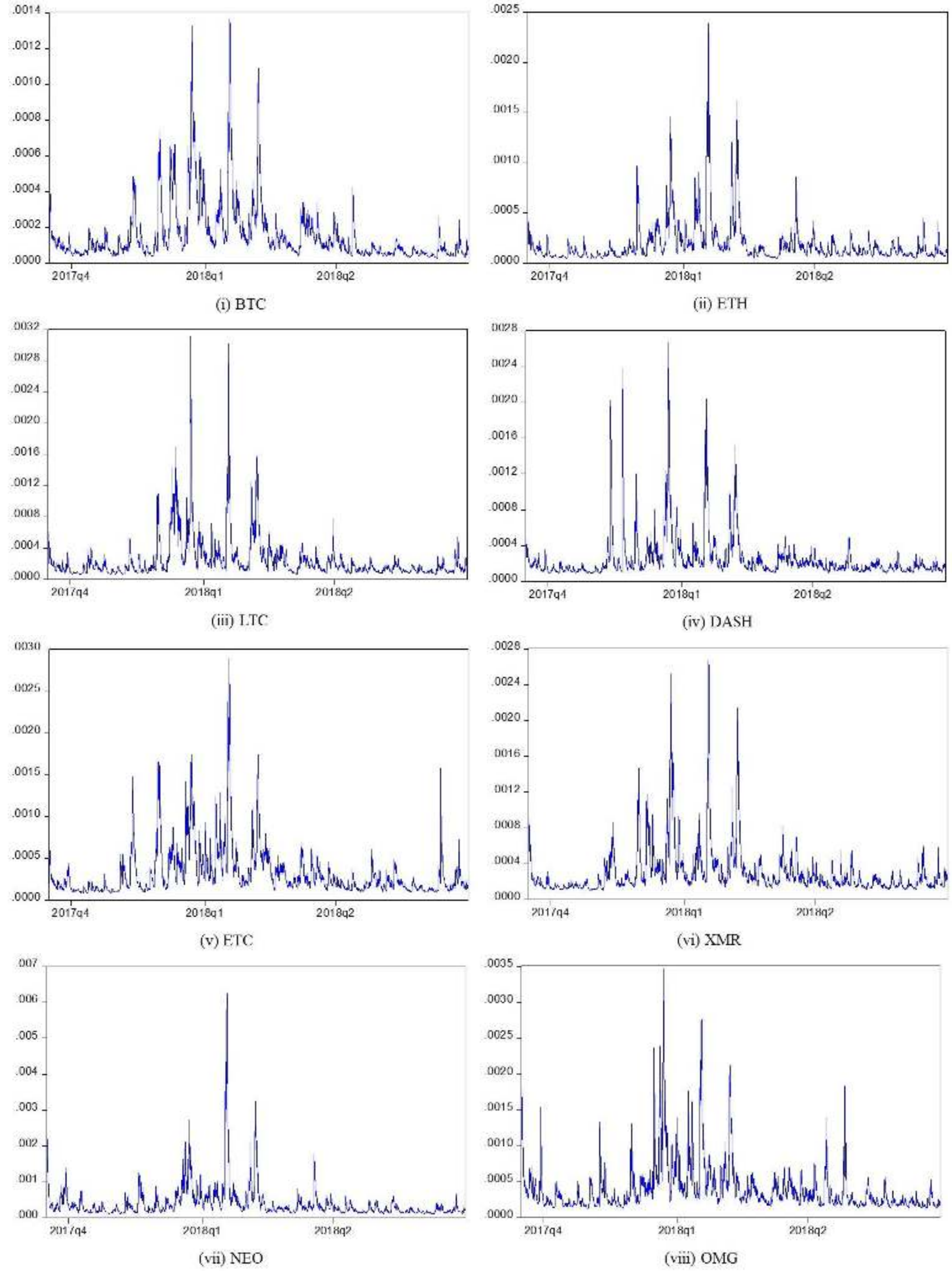


Figure 4: Asymmetric Diagonal BEKK model conditional variances and covariances

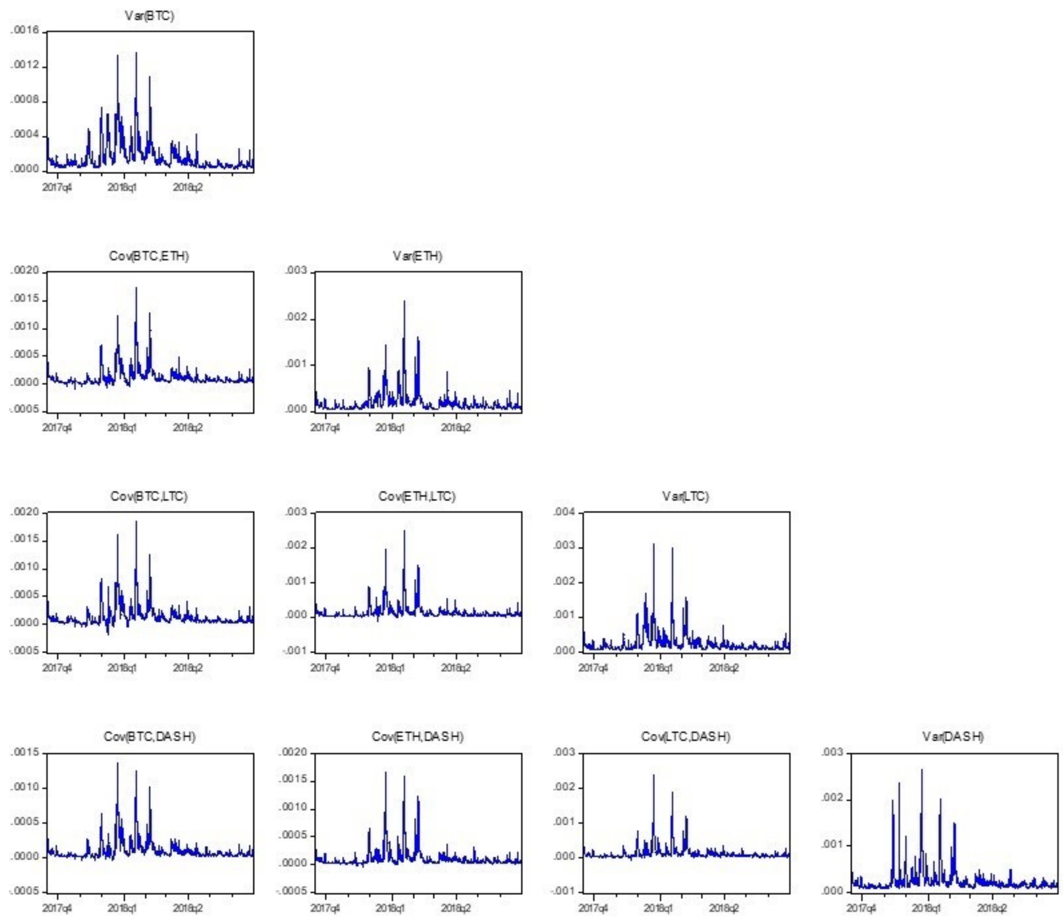


Figure 4: Asymmetric Diagonal BEKK model conditional variances and covariances (continued)

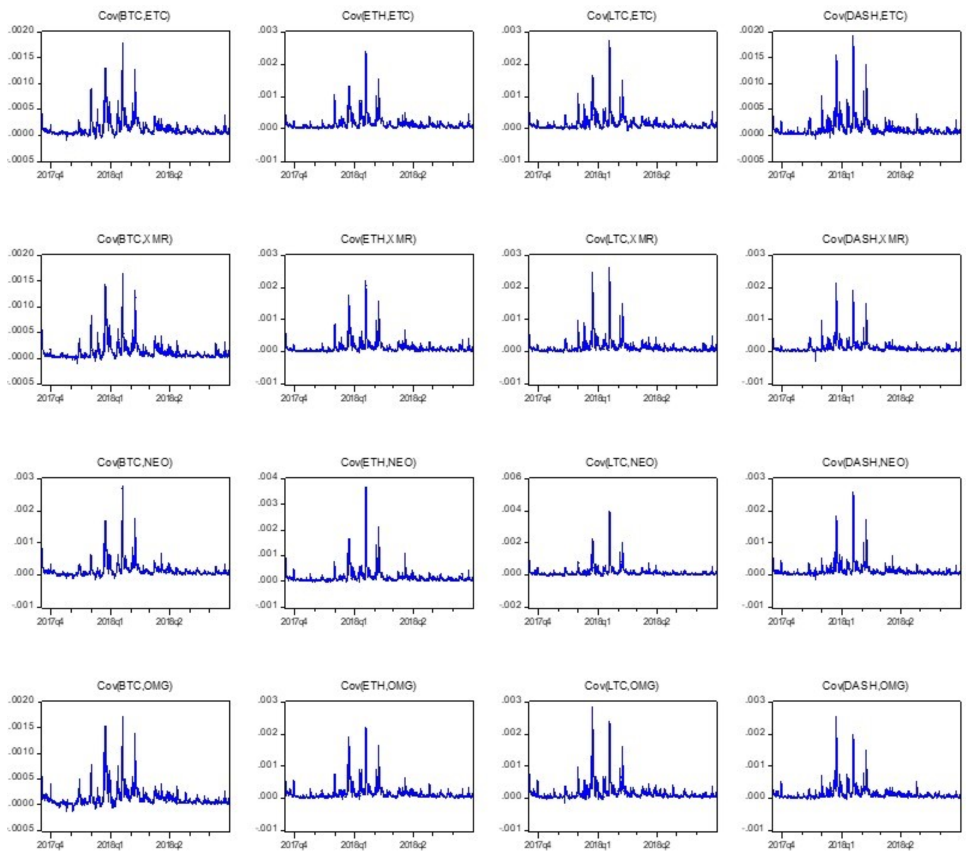


Figure 4: Asymmetric Diagonal BEKK model conditional variances and covariances (continued)

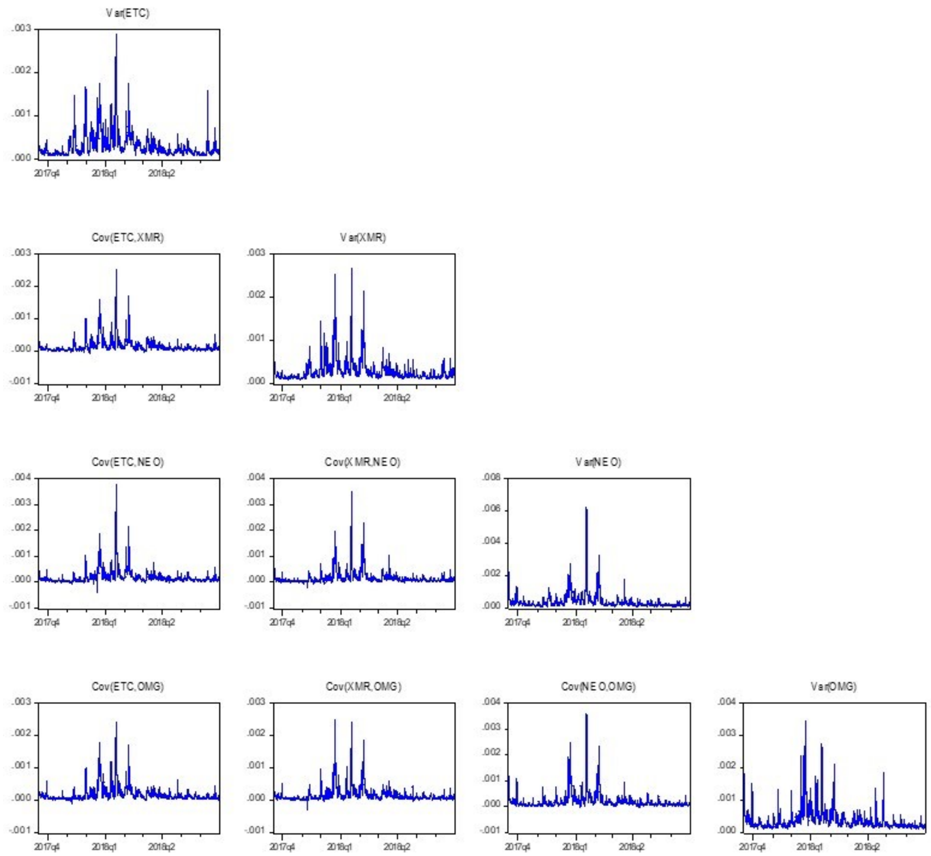


Figure 5: symmetric Diagonal BEKK model conditional correlations

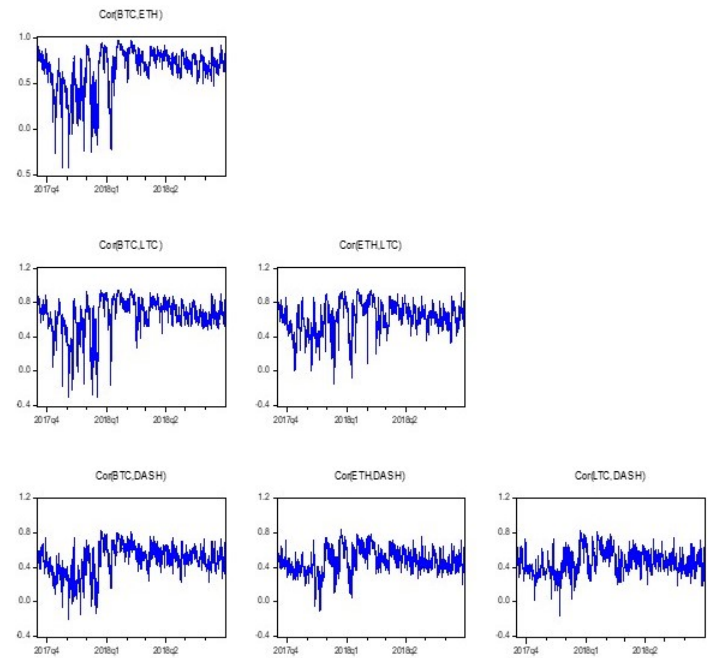


Figure 5: symmetric Diagonal BEKK model conditional correlations (continued)

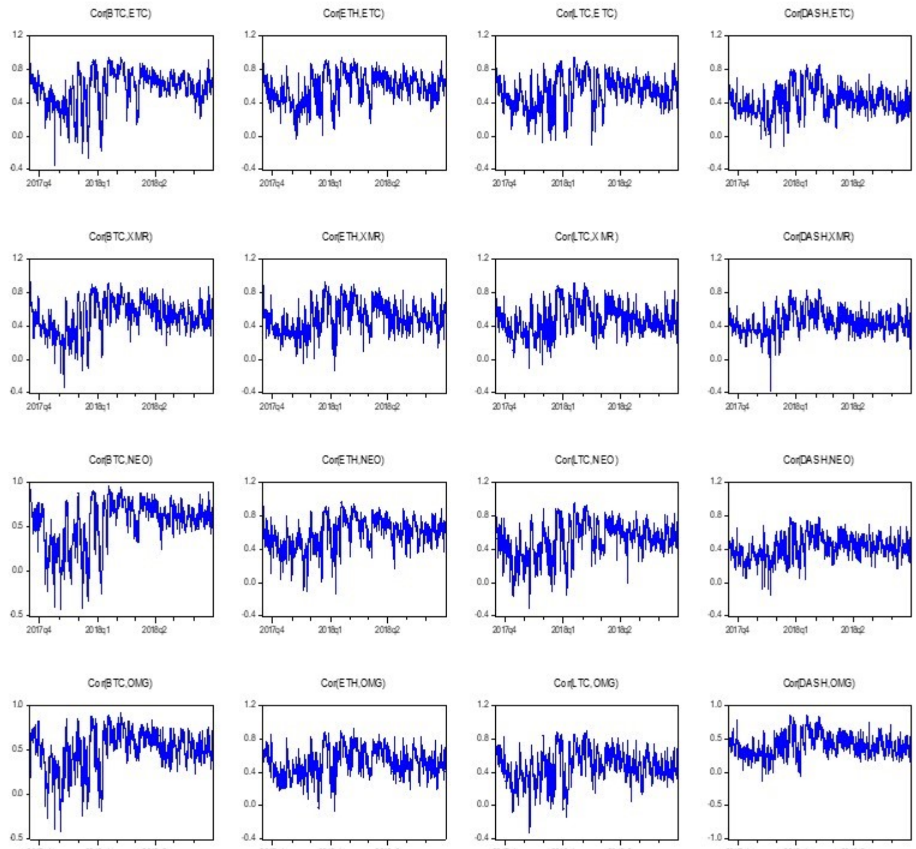


Figure 5: symmetric Diagonal BEKK model conditional correlations (continued)

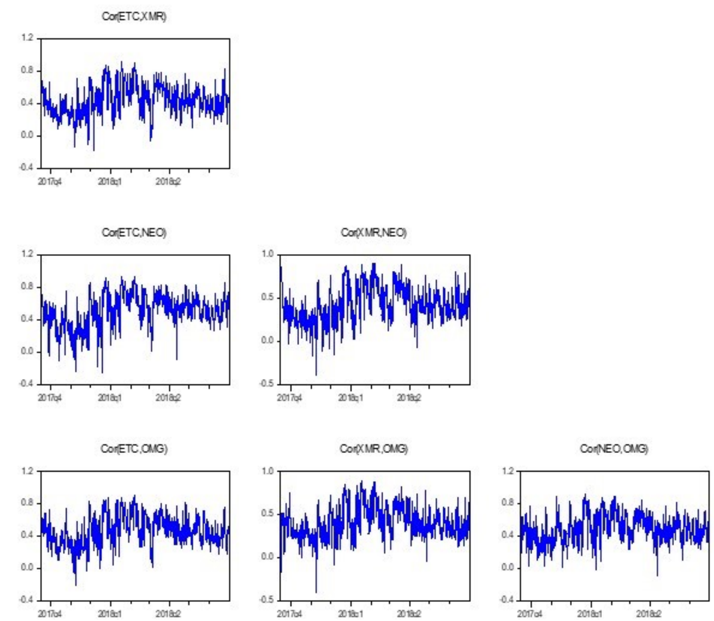


Table 1: Descriptive statistics and unit roots tests

	BTC	ETH	LTC	DASH	ETC	XMR	NEO	OMG
Mean	0.000079	0.000082	0.000064	-0.000025	0.000051	0.000042	0.000071	-0.000046
Median	0.000267	0.000205	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Maximum	0.116163	0.166442	0.204928	0.262319	0.199753	0.163857	0.249618	0.222285
Minimum	-0.101548	-0.112372	-0.145954	-0.143031	-0.149532	-0.152231	-0.161109	-0.141784
Std. Dev.	0.013332	0.014612	0.017001	0.017291	0.019261	0.018312	0.021098	0.02107
Skewness	-0.06904	0.089372	0.545695	1.089521	0.349	0.033457	0.38852	0.424667
Kurtosis	10.05281	12.67596	14.17153	23.68967	12.38947	10.96365	12.45704	11.16148
JB	14320.87***	26953.44***	36260.08***	124559.3***	25512.54***	18252.96***	25912.58***	19377.32***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
ARCH(1)	154.9987***	201.6735***	187.8901***	194.8761***	132.4726***	180.8777***	286.9126***	125.7106***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
ARCH(5)	645.0306***	794.9226***	682.1493***	375.5439***	323.3849***	547.2313***	870.4487***	457.7877***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Note: *** indicates significance at the 1% levels.

Table 2: Unit roots tests

	BTC	ETH	LTC	DASH	ETC	XMR	NEO	OMG
Panel A: Constant								
ADF	-65.08667***	-67.04200***	-65.92221***	-65.86838***	-66.80983***	-66.27712***	-67.43115***	-65.13645***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
PP	-91.14544***	-89.22259***	-91.95355***	-94.75469***	-91.91765***	-94.20914***	-91.56312***	-92.90983***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Panel B: Constant and Linear Trend								
ADF	-65.12712***	-67.06723***	-65.95701***	-65.89514***	-66.81428***	-66.29267***	-67.47250***	-65.13536***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
PP	-91.26687***	-89.25480***	-92.04179***	-94.73471***	-92.00168***	-94.24149***	-91.58902***	-92.91432***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Note: *** indicates significance at the 1% levels.

Table 3: Correlation matrix

	BTC	ETH	LTC	DASH	ETC	XMR	NEO
BTC							
ETH	0.720126***						
LTC	0.717526***	0.721969***					
DASH	0.587829***	0.610081***	0.584058***				
ETC	0.637788***	0.694112***	0.622325***	0.533959***			
XMR	0.650972***	0.654170***	0.627955***	0.594604***	0.584800***		
NEO	0.625587***	0.697842***	0.611454***	0.536131***	0.611769***	0.574041***	
OMG	0.599800***	0.650059***	0.585716***	0.532199***	0.584866***	0.551860***	0.622412***

Note: *** indicates significance at the 1% levels.

Table 4: Diagonal BEKK model parameter estimates

Panel A								
	BTC	ETH	LTC	DASH	ETC	XMR	NEO	OMG
c_{ii}	0.000162*	0.000173*	-0.000077	-0.000084	-0.0000114	-0.000007	-0.000011	-0.000068
	(0.0573)	(0.0762)	(0.4944)	(0.5044)	(0.9309)	(0.9559)	(0.9362)	(0.6575)
$\tilde{w}_{ij}$	0.000001***	0.000001***	0.000001***	0.000001***	0.000001***	0.000001***	0.000001***	0.000002***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
		0.000002***	0.000002***	0.000001***	0.000002***	0.000001***	0.000002***	0.000002***
		(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
			0.000004***	0.000002***	0.000002***	0.000002***	0.000002***	0.000002***
			(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
				0.000006***	0.000002***	0.000002***	0.000002***	0.000002***
				(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
					0.000004***	0.000002***	0.000002***	0.000002***
					(0.0000)	(0.0000)	(0.0000)	(0.0000)
						0.000006***	0.000002***	0.000002***
						(0.0000)	(0.0000)	(0.0000)
							0.000006***	0.000003***
							(0.0000)	(0.0000)
								0.000009***
								(0.0000)
α_{ii}	0.194989***	0.203280***	0.209218***	0.189576***	0.197673***	0.193945***	0.217344***	0.215437***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{ii}	0.973748***	0.968175***	0.965475***	0.965596***	0.969270***	0.966775***	0.963915***	0.961451***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
t-Distribution		5.966613***						
(d.o.f.)		(0.0000)						
Panel B: Squared Standardized Residuals								
Q-Stat(5)	6.7524	15.813***	100.81***	2.6055	6.1626	73.527***	102.13***	10.880*
Q-Stat(10)	8.2715	17.696*	110.58***	2.6715	6.4259	86.066***	119.20***	11.982
Q-Stat(20)	15.837	21.552	120.82***	2.8134	8.8928	124.58***	156.94***	14.208

Note: *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively. The variance specification is presented as $h_{it} = \tilde{w}_{ii} + \alpha_{ii}^2 \varepsilon_{it-1}^2 + \beta_{ii}^2 h_{it-1}$ and the covariance specification is presented as $h_{ijt} = \tilde{w}_{ij} = \alpha_{ii}\alpha_{jj}\varepsilon_{it-1}\varepsilon_{jt-1} + \beta_{ii}\beta_{jj}h_{ijt-1}$

Table 5: Diagonal BEKK model conditional variance equations - substituted coefficients

Ticker	Diagonal BEKK model conditional variance equations
BTC	$h_{11,t} = 1.1752e^{-6} + 0.0380\varepsilon_{1,t-1}^2 + 0.9482h_{11,t-1}$
ETH	$h_{22,t} = 2.4628e^{-6} + 0.0413\varepsilon_{2,t-1}^2 + 0.9374h_{22,t-1}$
LTC	$h_{33,t} = 3.7033e^{-6} + 0.0438\varepsilon_{3,t-1}^2 + 0.9321h_{33,t-1}$
DASH	$h_{44,t} = 6.0579e^{-6} + 0.0359\varepsilon_{4,t-1}^2 + 0.9324h_{44,t-1}$
ETC	$h_{55,t} = 4.3733e^{-6} + 0.0391\varepsilon_{5,t-1}^2 + 0.9395h_{55,t-1}$
XMR	$h_{66,t} = 6.1016e^{-6} + 0.0376\varepsilon_{6,t-1}^2 + 0.9347h_{66,t-1}$
NEO	$h_{77,t} = 6.2394e^{-6} + 0.0472\varepsilon_{7,t-1}^2 + 0.9291h_{77,t-1}$
OMG	$h_{88,t} = 8.9018e^{-6} + 0.0464\varepsilon_{8,t-1}^2 + 0.9244h_{88,t-1}$

Table 6: Diagonal BEKK model conditional covariance equations - substituted coefficients

Ticker	Diagonal BEKK model conditional covariance equations
	BTC
ETH	$h_{12,t} = 1.0429e^{-6} + 0.0396\varepsilon_{1,t-1}\varepsilon_{2,t-1} + 0.9428h_{12,t-1}$
LTC	$h_{13,t} = 1.2300e^{-6} + 0.0408\varepsilon_{1,t-1}\varepsilon_{3,t-1} + 0.9401h_{13,t-1}$
DASH	$h_{14,t} = 1.1642e^{-6} + 0.0370\varepsilon_{1,t-1}\varepsilon_{4,t-1} + 0.9402h_{14,t-1}$
ETC	$h_{15,t} = 1.1237e^{-6} + 0.0385\varepsilon_{1,t-1}\varepsilon_{5,t-1} + 0.9438h_{15,t-1}$
XMR	$h_{16,t} = 1.1051e^{-6} + 0.0378\varepsilon_{1,t-1}\varepsilon_{6,t-1} + 0.9414h_{16,t-1}$
NEO	$h_{17,t} = 1.4243e^{-6} + 0.0424\varepsilon_{1,t-1}\varepsilon_{7,t-1} + 0.9386h_{17,t-1}$
OMG	$h_{18,t} = 1.5248e^{-6} + 0.0420\varepsilon_{1,t-1}\varepsilon_{8,t-1} + 0.9362h_{18,t-1}$
	ETH
LTC	$h_{23,t} = 1.5915e^{-6} + 0.0425\varepsilon_{2,t-1}\varepsilon_{3,t-1} + 0.9347h_{23,t-1}$
DASH	$h_{24,t} = 1.4580e^{-6} + 0.0385\varepsilon_{2,t-1}\varepsilon_{4,t-1} + 0.9349h_{24,t-1}$
ETC	$h_{25,t} = 1.5258e^{-6} + 0.0401\varepsilon_{2,t-1}\varepsilon_{5,t-1} + 0.9384h_{25,t-1}$
XMR	$h_{26,t} = 1.4488e^{-6} + 0.0394\varepsilon_{2,t-1}\varepsilon_{6,t-1} + 0.9360h_{26,t-1}$
NEO	$h_{27,t} = 1.9287e^{-6} + 0.0442\varepsilon_{2,t-1}\varepsilon_{7,t-1} + 0.9332h_{27,t-1}$
OMG	$h_{28,t} = 1.9625e^{-6} + 0.0438\varepsilon_{2,t-1}\varepsilon_{8,t-1} + 0.9309h_{28,t-1}$
	LTC
DASH	$h_{34,t} = 1.7061e^{-6} + 0.0397\varepsilon_{3,t-1}\varepsilon_{4,t-1} + 0.9323h_{34,t-1}$
ETC	$h_{35,t} = 1.7375e^{-6} + 0.0414\varepsilon_{3,t-1}\varepsilon_{5,t-1} + 0.9358h_{35,t-1}$
XMR	$h_{36,t} = 1.6942e^{-6} + 0.0406\varepsilon_{3,t-1}\varepsilon_{6,t-1} + 0.9334h_{36,t-1}$
NEO	$h_{37,t} = 2.0372e^{-6} + 0.0455\varepsilon_{3,t-1}\varepsilon_{7,t-1} + 0.9306h_{37,t-1}$
OMG	$h_{38,t} = 2.1983e^{-6} + 0.0451\varepsilon_{3,t-1}\varepsilon_{8,t-1} + 0.9283h_{38,t-1}$
	DASH
ETC	$h_{45,t} = 1.6475e^{-6} + 0.0375\varepsilon_{4,t-1}\varepsilon_{5,t-1} + 0.9359h_{45,t-1}$
XMR	$h_{46,t} = 2.0348e^{-6} + 0.0368\varepsilon_{4,t-1}\varepsilon_{6,t-1} + 0.9335h_{46,t-1}$
NEO	$h_{47,t} = 2.0300e^{-6} + 0.0412\varepsilon_{4,t-1}\varepsilon_{7,t-1} + 0.9308h_{47,t-1}$
OMG	$h_{48,t} = 2.2628e^{-6} + 0.0408\varepsilon_{4,t-1}\varepsilon_{8,t-1} + 0.9284h_{48,t-1}$
	ETC
XMR	$h_{56,t} = 1.6383e^{-6} + 0.0383\varepsilon_{5,t-1}\varepsilon_{6,t-1} + 0.9371h_{56,t-1}$
NEO	$h_{57,t} = 2.1356e^{-6} + 0.0430\varepsilon_{5,t-1}\varepsilon_{7,t-1} + 0.9343h_{57,t-1}$
OMG	$h_{58,t} = 2.2329e^{-6} + 0.0426\varepsilon_{5,t-1}\varepsilon_{8,t-1} + 0.9319h_{58,t-1}$
	XMR
NEO	$h_{67,t} = 1.9758e^{-6} + 0.0422\varepsilon_{6,t-1}\varepsilon_{7,t-1} + 0.9319h_{67,t-1}$
OMG	$h_{68,t} = 2.2443e^{-6} + 0.0418\varepsilon_{6,t-1}\varepsilon_{8,t-1} + 0.9295h_{68,t-1}$
	NEO
OMG	$h_{78,t} = 2.7775e^{-6} + 0.0468\varepsilon_{7,t-1}\varepsilon_{8,t-1} + 0.9268h_{78,t-1}$

Table 7: Asymmetric Diagonal BEKK model parameter estimates

Panel A								
	BTC	ETH	LTC	DASH	ETC	XMR	NEO	OMG
c_{ii}	0.000117	0.000128	0.000134	0.000263**	0.000095	0.000064	0.000071	0.000208
	(0.1683)	(0.1882)	(0.2328)	(0.0386)	(0.4727)	(0.6337)	(0.6171)	(0.1781)
$\tilde{w}_{ij}$	0.000001***	0.000001***	0.000001***	0.000001***	0.000001***	0.000001***	0.000001***	0.000001***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
		0.000002***	0.000001***	0.000001***	0.000001***	0.000001***	0.000002***	0.000002***
		(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
			0.000004***	0.000002***	0.000002***	0.000002***	0.000002***	0.000002***
			(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
				0.000005***	0.000001***	0.000002***	0.000002***	0.000002***
				(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
					0.000004***	0.000002***	0.000002***	0.000002***
					(0.0000)	(0.0000)	(0.0000)	(0.0000)
						0.000006***	0.000002***	0.000002***
						(0.0000)	(0.0000)	(0.0000)
							0.000006***	0.000003***
							(0.0000)	(0.0000)
								0.000008***
								(0.0000)
α_{ii}	0.188340***	0.200821***	0.205384***	0.143682***	0.187134***	0.193782***	0.218407***	0.196767***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
d_{ii}	0.095413***	0.089058***	0.094597***	0.196164***	0.111626***	0.086226***	0.086724***	0.141849***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{ii}	0.973067***	0.967009***	0.964582***	0.965853***	0.968643***	0.965037***	0.962347***	0.961223***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
t-Distribution		6.077609						
(d.o.f.)		(0.0000)						
Panel B: Squared Standardized Residuals								
Q-Stat(5)	6.7524	15.813***	100.81***	2.6055	6.1626	73.527***	102.13***	10.880*
Q-Stat(10)	8.2715	17.696*	110.58***	2.6715	6.4259	86.066***	119.20***	11.982
Q-Stat(20)	15.837	21.552	120.82***	2.8134	8.8928	124.58***	156.94***	14.208

Note: *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively. Values in parentheses denote p-values. The variance specification is presented as $h_{it} = \tilde{w}_{ii} + \alpha_{ii}^2 \varepsilon_{it-1}^2 + d_{ii}^2 \eta_{it-1}^2 + \beta_{ii}^2 h_{it-1}$ and the covariance specification is presented as $h_{ijt} = \tilde{w}_{ij} = \alpha_{ii}\alpha_{jj}\varepsilon_{it-1}\varepsilon_{jt-1} + d_{ii}d_{jj}\eta_{it-1}\eta_{jt-1} + \beta_{ii}\beta_{jj}h_{ijt-1}$

Table 8: Asymmetric Diagonal BEKK model conditional variance equations - substituted coefficients

Ticker	Asymmetric Diagonal BEKK model conditional variance equations
BTC	$h_{11,t} = 1.1085e^{-6} + 0.0355\varepsilon_{1,t-1}^2 + 0.0091\eta_{1,t-1}^2 + 0.9469h_{11,t-1}$
ETH	$h_{22,t} = 2.3935e^{-6} + 0.0401\varepsilon_{2,t-1}^2 + 0.0079\eta_{2,t-1}^2 + 0.9351h_{22,t-1}$
LTC	$h_{33,t} = 3.5812e^{-6} + 0.0422\varepsilon_{3,t-1}^2 + 0.0089\eta_{3,t-1}^2 + 0.9304h_{33,t-1}$
DASH	$h_{44,t} = 5.0811e^{-6} + 0.0206\varepsilon_{4,t-1}^2 + 0.0385\eta_{4,t-1}^2 + 0.9329h_{44,t-1}$
ETC	$h_{55,t} = 4.2000e^{-6} + 0.0350\varepsilon_{5,t-1}^2 + 0.0125\eta_{5,t-1}^2 + 0.9382h_{55,t-1}$
XMR	$h_{66,t} = 6.1090e^{-6} + 0.0376\varepsilon_{6,t-1}^2 + 0.0074\eta_{6,t-1}^2 + 0.9313h_{66,t-1}$
NEO	$h_{77,t} = 6.0841e^{-6} + 0.0477\varepsilon_{7,t-1}^2 + 0.0075\eta_{7,t-1}^2 + 0.9261h_{77,t-1}$
OMG	$h_{88,t} = 8.3483e^{-6} + 0.0387\varepsilon_{8,t-1}^2 + 0.0201\eta_{8,t-1}^2 + 0.9239h_{88,t-1}$

Table 9: Asymmetric Diagonal BEKK model conditional covariance equations - substituted coefficients

Ticker	Asymmetric Diagonal BEKK model conditional covariance equations
	BTC
ETH	$h_{12,t} = 9.7320e^{-6} + 0.0378\varepsilon_{1,t-1}\varepsilon_{2,t-1} + 0.0085\eta_{1,t-1}\eta_{2,t-1} + 0.9410h_{12,t-1}$
LTC	$h_{13,t} = 1.1437e^{-6} + 0.0387\varepsilon_{1,t-1}\varepsilon_{3,t-1} + 0.0090\eta_{1,t-1}\eta_{3,t-1} + 0.9386h_{13,t-1}$
DASH	$h_{14,t} = 1.0630e^{-6} + 0.0271\varepsilon_{1,t-1}\varepsilon_{4,t-1} + 0.0187\eta_{1,t-1}\eta_{4,t-1} + 0.9398h_{14,t-1}$
ETC	$h_{15,t} = 1.0025e^{-6} + 0.0352\varepsilon_{1,t-1}\varepsilon_{5,t-1} + 0.0107\eta_{1,t-1}\eta_{5,t-1} + 0.9426h_{15,t-1}$
XMR	$h_{16,t} = 1.0293e^{-6} + 0.0365\varepsilon_{1,t-1}\varepsilon_{6,t-1} + 0.0082\eta_{1,t-1}\eta_{6,t-1} + 0.9390h_{16,t-1}$
NEO	$h_{17,t} = 1.3310e^{-6} + 0.0411\varepsilon_{1,t-1}\varepsilon_{7,t-1} + 0.0083\eta_{1,t-1}\eta_{7,t-1} + 0.9264h_{17,t-1}$
OMG	$h_{18,t} = 1.3318e^{-6} + 0.0372\varepsilon_{1,t-1}\varepsilon_{8,t-1} + 0.0135\eta_{1,t-1}\eta_{8,t-1} + 0.9353h_{18,t-1}$
	ETH
LTC	$h_{23,t} = 1.4913e^{-6} + 0.0412\varepsilon_{2,t-1}\varepsilon_{3,t-1} + 0.0084\eta_{2,t-1}\eta_{3,t-1} + 0.9328h_{23,t-1}$
DASH	$h_{24,t} = 1.4330e^{-6} + 0.0289\varepsilon_{2,t-1}\varepsilon_{4,t-1} + 0.0175\eta_{2,t-1}\eta_{4,t-1} + 0.9340h_{24,t-1}$
ETC	$h_{25,t} = 1.4140e^{-6} + 0.0376\varepsilon_{2,t-1}\varepsilon_{5,t-1} + 0.0099\eta_{2,t-1}\eta_{5,t-1} + 0.9367h_{25,t-1}$
XMR	$h_{26,t} = 1.3592e^{-6} + 0.0389\varepsilon_{2,t-1}\varepsilon_{6,t-1} + 0.0077\eta_{2,t-1}\eta_{6,t-1} + 0.9332h_{26,t-1}$
NEO	$h_{27,t} = 1.8109e^{-6} + 0.0439\varepsilon_{2,t-1}\varepsilon_{7,t-1} + 0.0077\eta_{2,t-1}\eta_{7,t-1} + 0.9306h_{27,t-1}$
OMG	$h_{28,t} = 1.8003e^{-6} + 0.0395\varepsilon_{2,t-1}\varepsilon_{8,t-1} + 0.0126\eta_{2,t-1}\eta_{8,t-1} + 0.9295h_{28,t-1}$
	LTC
DASH	$h_{34,t} = 1.6502e^{-6} + 0.0295\varepsilon_{3,t-1}\varepsilon_{4,t-1} + 0.0186\eta_{3,t-1}\eta_{4,t-1} + 0.9316h_{34,t-1}$
ETC	$h_{35,t} = 1.5914e^{-6} + 0.0384\varepsilon_{3,t-1}\varepsilon_{5,t-1} + 0.0106\eta_{3,t-1}\eta_{5,t-1} + 0.9343h_{35,t-1}$
XMR	$h_{36,t} = 1.5728e^{-6} + 0.0398\varepsilon_{3,t-1}\varepsilon_{6,t-1} + 0.0082\eta_{3,t-1}\eta_{6,t-1} + 0.9309h_{36,t-1}$
NEO	$h_{37,t} = 1.8933e^{-6} + 0.0449\varepsilon_{3,t-1}\varepsilon_{7,t-1} + 0.0082\eta_{3,t-1}\eta_{7,t-1} + 0.9283h_{37,t-1}$
OMG	$h_{38,t} = 1.9874e^{-6} + 0.0404\varepsilon_{3,t-1}\varepsilon_{8,t-1} + 0.0134\eta_{3,t-1}\eta_{8,t-1} + 0.9272h_{38,t-1}$
	DASH
ETC	$h_{45,t} = 1.3493e^{-6} + 0.0269\varepsilon_{4,t-1}\varepsilon_{5,t-1} + 0.0219\eta_{4,t-1}\eta_{5,t-1} + 0.9356h_{45,t-1}$
XMR	$h_{46,t} = 2.0095e^{-6} + 0.0278\varepsilon_{4,t-1}\varepsilon_{6,t-1} + 0.0169\eta_{4,t-1}\eta_{6,t-1} + 0.9321h_{46,t-1}$
NEO	$h_{47,t} = 2.0571e^{-6} + 0.0314\varepsilon_{4,t-1}\varepsilon_{7,t-1} + 0.0170\eta_{4,t-1}\eta_{7,t-1} + 0.9295h_{47,t-1}$
OMG	$h_{48,t} = 1.6677e^{-6} + 0.0283\varepsilon_{4,t-1}\varepsilon_{8,t-1} + 0.0278\eta_{4,t-1}\eta_{8,t-1} + 0.9284h_{48,t-1}$
	ETC
XMR	$h_{56,t} = 1.4964e^{-6} + 0.0363\varepsilon_{5,t-1}\varepsilon_{6,t-1} + 0.0096\eta_{5,t-1}\eta_{6,t-1} + 0.9348h_{56,t-1}$
NEO	$h_{57,t} = 1.9930e^{-6} + 0.0409\varepsilon_{5,t-1}\varepsilon_{7,t-1} + 0.0097\eta_{5,t-1}\eta_{7,t-1} + 0.9322h_{57,t-1}$
OMG	$h_{58,t} = 1.8744e^{-6} + 0.0368\varepsilon_{5,t-1}\varepsilon_{8,t-1} + 0.0158\eta_{5,t-1}\eta_{8,t-1} + 0.9311h_{58,t-1}$
	XMR
NEO	$h_{67,t} = 1.8383e^{-6} + 0.0423\varepsilon_{6,t-1}\varepsilon_{7,t-1} + 0.0075\eta_{6,t-1}\eta_{7,t-1} + 0.9287h_{67,t-1}$
OMG	$h_{68,t} = 1.0286e^{-6} + 0.0381\varepsilon_{6,t-1}\varepsilon_{8,t-1} + 0.0122\eta_{6,t-1}\eta_{8,t-1} + 0.9276h_{68,t-1}$
	NEO
OMG	$h_{78,t} = 2.5902e^{-6} + 0.0430\varepsilon_{7,t-1}\varepsilon_{8,t-1} + 0.0123\eta_{7,t-1}\eta_{8,t-1} + 0.9250h_{78,t-1}$