High-Order Methods for Incompressible Fluid Flow

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Fluid Mechanics and Computation: An Introduction

According to the Greek philosopher Heraclitus, who used to say "$\pi\alpha\nu\tau\alpha\ \rho\epsilon\iota\ldots$,"[1] daily life is concerned with the flow of ordinary fluids: water, air, blood, and so forth, in very common situations like breathing, coffee drinking, and hand washing.

Most flows are generated by nature (e.g., oceans, winds, rivers) and by human industrial activity (e.g., planes, cars, materials processing, biomedical engineering). There is a need to model fluid flow problems in order to improve the basic understanding of these complex phenomena and to increase the design quality of technological applications. With the advent of large and powerful computational tools, modeling has become more and more a substitute for direct experimentation. In some circumstances, experimentation may be too expensive – particularly if it leads to the destruction of the facility – or even impossible to perform, so that modeling is the only reasonable way to get answers and to study a range of parameters for optimal design.

1.1 Viscous Fluid Flows

We know from experience that many flows are set into motion by shear forces, and hence viscous effects play a vital role in fluids. In general, the viscosity depends on the shear rate (roughly speaking, the velocity gradient), as is explained by non-Newtonian theory. In this book, however, we will restrict ourselves mainly to the case of viscous Newtonian incompressible fluids in isothermal situations or under the influence of thermal convection as described by the Boussinesq approximation.

[1] Everything flows....

The conceptual framework through which the governing equations will be derived is the axiomatic presentation based on the principles of the mechanics of continuous media (see, e.g., Gurtin [179], Truesdell [385], Truesdell and Toupin [387], and, more recently, Truesdell and Rajagopal [386]), although the equations also can be obtained from the principles of statistical mechanics (Cercignani [77]). This process will deliver the well-known Navier–Stokes equations, a set of nonlinear partial differential equations (PDEs) subject to both initial and boundary conditions.

A major parameter characterizing the flow features is given by the Reynolds number, which is a nondimensional measure of the importance of inertial forces relative to viscous forces. Starting from a zero value (creeping Stokes flow), where the physics is straightforward because the model is basically linear, the sweep to increasing values leads from laminar flow to the transition to turbulence. Later stages are weak turbulence and fully developed turbulence; correspondingly the Reynolds number goes up from a few hundred to several million. In this latter situation, the physics becomes quite complicated, even in simple geometrical configurations, because the flow structure is highly nonlinear and time-dependent, and involves spatial scales spread over several orders of magnitude. These circumstances render the use of computers essential.

The advent of computers led to the emergence of numerical fluid mechanics, also known as computational fluid dynamics (CFD). At the beginning, this new discipline brought some hope that computation would open the door to simulations of transient, three-dimensional (3D) flows in or around industrial (complex) geometries at high Reynolds numbers. These flows correspond to internal or external flows, respectively. Reality has disappointed the expectations of this naive picture. We will analyze the difficulties faced by computational scientists at the present time and will study carefully the advantages (and drawbacks) of applying high-order methods to CFD. Despite spectacular breakthroughs in software and hardware, the present tools are still insufficient to cope with direct numerical simulation (DNS) at moderate ($\approx 10^4$–10^5) and high ($\approx 10^6$–10^7) Reynolds numbers. Therefore, large-eddy simulation (LES), where the gross structures of the flow are resolved spatially and temporally and where small scales are modeled through a subgrid-scale approximation, constitutes a viable compromise between the expensive DNS and the first- or second-order moment closure models applied to the Reynolds-averaged Navier–Stokes equations (RANS). Fortunately, even though it is more complicated to model, LES retains the same computational complexity as the classical Navier–Stokes equations.

Before turning to mathematical models, we mention examples of flows arising in nature and industrial processing. The following list is by no means exhaustive:

- Creeping flows with thermal convection: plate tectonics, glass flow in a furnace.
- Materials processing: crystal growth, mold filling, thermal convection in a molten tin bath of a float glass process.
- Biofluid mechanics: blood flow in stenosed arteries (see, for example, Tu et al. [388]) through artificial valves.
- Hydraulic machines: flow through a Francis runner.
- Turbulent flows with thermal convection: liquid-metal processing, planetary sciences (Jupiter's red spot; see, for example, Marcus [266]).

As examples of DNS, we mention the computation of a flow in a square duct (e.g., Gavrilakis [150]), in a 3D parallelepipedic cavity (e.g., Deville et al. [98]), and in a cubical cavity (e.g., Leriche and Gavrilakis [240]).

In this chapter, we review the general principles of fluid mechanics: mass conservation (Section 1.2), momentum and angular momentum conservation (Section 1.3), and the first and second principles of thermodynamics (Sections 1.4 and 1.5, respectively). Then, we discuss the fluid flow equations (Section 1.6) and the relevant dimensionless numbers characterizing the physical situations (Section 1.7). Section 1.8 presents the vorticity equations. In Section 1.9, we discuss simplified models that can be derived as limit cases of the general equations when some dimensionless numbers go to extreme values. The challenge raised by turbulence then is examined. Several turbulence models are given, and large-eddy simulation is also summarized (Section 1.10). Because these last problems lead to large-scale, computation-intensive applications, the questions coming from scientific computing are evoked in a broad sense, that is, in the context of software and hardware interactions (Section 1.11). In particular, parallel computing and second-generation languages are scrutinized.

1.2 Mass Conservation

The principle of mass conservation states: *In a given deforming material volume $\Omega(t)$, the mass $M(t)$ is constant with respect to time t*. Translated into mathematical terms, this principle yields the relation

$$\frac{d}{dt}M(t) = 0, \tag{1.2.1}$$