High-Precision Muonic X-Ray-Measurement of the Rms Charge Radius of C-12 with a Crystal Spectrometer

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[15]The actual sign of A_y could be calculated, but requires a detailed treatment of the channel distortions, which goes beyond the scope of this report.
[16]Spin and/or isospin coupling coefficients may differ for specific intermediate states reached in the reactions on $^{13,14}C$, but in the summation over all such states these differences are subsumed in the scaling with occupancy.

High-Precision Muonic X-Ray Measurement of the rms Charge Radius of ^{12}C with a Crystal Spectrometer

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A precision measurement with the SIN bent-crystal spectrometer of the wavelength of the $2p_{3/2}$-$1s_{1/2}$ transition in muonic ^{12}C yields $\lambda = 16.473\,766(89)$ pm, the accuracy being an order of magnitude higher than that of earlier investigations. The rms charge radius of ^{12}C is deduced as $\langle r^2 \rangle^{1/2} = 2.4832(18)$ fm, differing by 2.4 standard deviations from the most accurate electron-scattering results. Consequences of attributing this discrepancy to a μ-N interaction beyond QED are discussed.

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The work described in this paper was carried out with the purpose of determining the charge radius of ^{12}C with the highest precision available at present. Moreover, a comparison of the charge radius determined in the muonic atom with the one derived from elastic electron-scattering

data permits us to obtain information on a possible presence of muon-nucleon interactions beyond standard QED.

The rms charge radius of ^{12}C was determined from a high-precision measurement of the $2p$-$1s$ transition wavelength in muonic ^{12}C. The measurement was performed with the bent-crystal spectrometer facility at Schweizerisches Institut für Nuklearforschung (SIN). The spectrometer and the continuous scanning method used for this measurement are described in detail elsewhere.[1,2] Two 3.2-mm-thick quartz crystals reflecting from the 310 planes were used (see Fig. 1). The target consisted of natural graphite. For the wavelength calibration the 84-keV line in the decay of ^{170}Tm was used.[3] In order to eliminate possible aberrational effects the data were analyzed with the center-of-gravity method.[2]

The corrections for nuclear recoil and the ^{13}C contribution amount to $-0.000\,056$ and $0.000\,118$ pm, respectively. In the calculation of the correction for the $2p$ fine structure ($-0.000\,886$ pm) we assumed statistical population of the fine-structure levels corrected by an E^3 dependence. In muonic atoms with Z around 13 this has been tested experimentally to 5% (corresponding to 0.2 eV in ^{12}C). No theoretical error in the fine-structure population is assumed here, however, since in carbon no resonances between muonic and nuclear states are possible. For the transition wavelength we thus find

$$\lambda(2p_{3/2}\text{-}1s_{1/2}) = 16.473\,766(89)\ \text{pm}. \tag{1}$$

In order to deduce from (1) the rms charge radius with high precision it is necessary to expand the major finite-size effects—those on the Coulomb energy and on the vacuum polarization of order $\alpha(Z\alpha)$—into a series of charge moments. This is done by use of first- and second-order perturbation theory. Since the finite-size contributions depend primarily on $\langle r^2\rangle$ (90%), the less important contributions from other charge moments can be calculated with sufficient accuracy by use of information on the charge distribution $\rho(r)$ from a charge-model–independent analysis of electron scattering data.[4] We will show that the remaining part of the finite-size effects (0.3%), representing the effects beyond the expansion in charge moments, has a negligible dependence on $\rho(r)$.

The experimental finite-size shift is obtained by subtracting from (1) the point-nucleus Dirac wavelength[5] and those corrections—given in the upper part of Table I—which are insensitive to the exact shape of the nuclear charge distribution. The result is transformed into an energy:

$$\Delta E_{fs}(\text{expt.}) = -409.02(53)\ \text{eV}. \tag{2a}$$

The theoretical value for this shift can be written[8] as

$$\Delta E_{fs}(\text{theor.}) = \Delta E_{fs}(\text{perturb.}) + \Delta E_{fs}(\text{vac. pol.}) + \Delta E_{fs}(\text{rest}), \tag{2b}$$

where the first two terms represent the finite-size effects on the Coulomb energy (first- and second-order perturbation calculation) and on the vacuum polarization of order $\alpha(Z\alpha)$. Following a procedure similar to that of Friar,[9] we can write the $1s$ part of the first term as

$$\begin{aligned}\Delta E_{fs}(\text{perturb.}) = -\tfrac{2}{3}(\alpha Z)^2 m_\mu c^2 \big[&\langle r^2\rangle/a^2 - \langle r^3\rangle/2a^3 - \langle r\rangle\langle r^2\rangle/a^3 - \langle 1/r\rangle\langle r^4\rangle/10a^3 + \langle r^4\rangle/10a^4 \\ &- \tfrac{2}{3}(\tfrac{7}{3}-\gamma)\langle r^2\rangle^2/a^4 + \langle r\rangle\langle r^3\rangle/a^4 + \langle 1/r\rangle\langle r^5\rangle/9a^4 + \tfrac{2}{3}\langle r^2\rangle\langle r^2\ln(2r/a)\rangle/a^4 \\ &+ (\alpha Z)^2\{(2-\gamma)\langle r^2\rangle/a^2 - \langle r^2\rangle\langle\ln(2r/a)\rangle/a^2 - \langle 1/r\rangle\langle r^3\rangle/3a^2\}\big],\end{aligned} \tag{3}$$

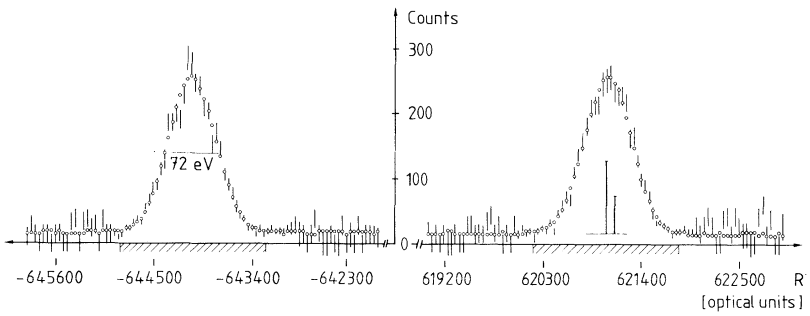

FIG. 1. Right and left reflection of the $2p$-$1s$ transition of one crystal; the dots are the result of a fit—not used in the analysis—showing that the left and right reflections have the same line shape. The two fine-structure components are shown.

TABLE I. Corrections to the point-nucleus transition energy of the $2p_{3/2}$-$1s_{1/2}$ transition in ^{12}C in electronvolts. Corrections A do not depend on the exact shape of the nuclear charge distribution. Corrections B are due to the finite nuclear charge distribution, evaluated for $\langle r^2\rangle^{1/2} = 2.4832$ fm.

Corrections A	
Vacuum polarization $\alpha(Z\alpha)$ (point nucleus)	378.64(2)
Vacuum polarization $\alpha^2(Z\alpha)$	2.72(3)
Vacuum polarization $\alpha(Z\alpha)^3$	−0.03(1)
Vacuum polarization μ pairs	0.21(1)
Vacuum polarization hadrons	0.14(2)
Vertex corrections	−4.80(14)
Virtual Delbrueck scattering (Ref. 6)	0.02(0)
Relativistic recoil	0.67(3)
Two-photon recoil	−0.22(2)
Nuclear polarization (Ref. 7)	2.50(30)
Electron screening	−0.04(4)
Total	379.81(34)
Corrections B	
ΔE_{fs}(perturb.), $\langle r^2\rangle$ part	−446.15
ΔE_{fs}(perturb.), other moments	44.48(4)
ΔE_{fs}(vac. pol.)	−6.12
ΔE_{fs}(rest)	−1.23(14)
ΔE_{fs}	−409.02

where $a = \hbar c/(\alpha Z m_\mu c^2)$, $\gamma = 0.5772$, and m_μ is the reduced μ mass. The $2p_{3/2}$ part amounts to 0.006 eV only; its ρ dependence is negligible. We convert (3) into a polynomial in $\langle r^2\rangle^{1/2}$ by using the ratios of nuclear charge moments given in Table II.

The second term of (2b) can be shown[8] to be proportional to $\langle r^2\rangle$ in very good approximation:

$$\Delta E_{fs}(\text{vac. pol.}) = (0.992\ \text{eV/fm}^2)\langle r^2\rangle . \quad (4)$$

The third (rest) term is now determined (i) by solving the Dirac equation exactly with the code MURKS[8,10] for a charge distribution from Ref. 4 including vacuum polarization of order $\alpha(Z\alpha)$ and (ii) by summing the perturbation results (3) and (4) for the same charge distribution. This yields

$$\Delta E_{fs}(\text{rest}) = \Delta E_{fs}(\text{exact}) - \Delta E_{fs}^{\text{tot}}(\text{perturb.}) = -1.24(13)\ \text{eV}. \quad (5)$$

The smallness of the uncertainty in ΔE_{fs}(rest) —reflecting the influence of variations of the charge distribution—shows that the rms radius can be determined with the full experimental precision of the finite-size shift.

Comparison of (2a) and (2b) now permits one to solve the polynomial in $\langle r^2\rangle^{1/2}$, yielding

$$(\langle r^2\rangle^{1/2})_\mu = 2.4832 \pm 0.0018\ \text{fm}. \quad (6)$$

TABLE II. Charge-moment ratios obtained from a model-independent analysis of elastic electron scattering experiments (Ref. 4).

k	$\langle r^k\rangle^{1/k}/\langle r^2\rangle^{1/2}$
−1	0.7637(6)
1	0.9321(3)
3	1.0628(6)
4	1.1224(14)
5	1.1799(27)
$\langle r^2 \ln(r/a)\rangle^{1/2}/\langle r^2\rangle^{1/2}$	−1.4267(10)
$\langle \ln(r/a)\rangle$	−2.3210(13)

In Table III this value is compared with other recent determinations of the rms charge radius of ^{12}C. Taking the average from Refs. 4 and 12 we obtain

$$(\langle r^2\rangle^{1/2})_e = 2.4703 \pm 0.0050\ \text{fm}. \quad (7)$$

Comparing with (6) we find a difference of 2.3 standard deviations:

$$(\langle r^2\rangle^{1/2})_\mu - (\langle r^2\rangle^{1/2})_e = 0.0129 \pm 0.0053\ \text{fm}. \quad (8)$$

This corresponds to an energy shift of -4.0 ± 1.6 eV, the error originating almost entirely from the electron-scattering results.

It seems tempting to try to attribute the discrepancy of (8) to some additional interaction not taken into account by the present QED calculations. Such an interaction would be caused, e.g., by the exchange of a hadronic scalar or vector boson.[14] The coupling between boson and muon could be either direct or through some more complicated—but more conventional—mechanism as proposed, e.g., by Barshay[15] for scalar bosons. In both cases the resulting Yukawa-type interaction can be characterized by the coupling to the nucleus (g_N), by the (effective) coupling to the muon (g_μ), and by the mass of the boson (M).

A limited series of other experiments also give information on such an interaction. Comparison

TABLE III. Recent results on the rms charge radius of ^{12}C.

Experiment	$\langle r^2\rangle^{1/2}$ (fm)	Reference
Muonic atom	2.4826(18)	This work
Muonic atom	2.472(16)	11
e scattering	2.468(12) [a]	12
e scattering	2.4708(55) [a,b]	4
e scattering	2.472(15)	13

[a] Dispersion corrections considered.
[b] Reanalysis of data from among others Ref. 13.

of our experiment with the results of the muonic $g-2$ experiment[16] and with the measurements of muonic $3d$-$2p$ transitions[2] restricts the boson mass region to $0.05 \lesssim M \lesssim 20$ GeV. Interpretation of our discrepancy (8) along these lines then yields $g_N g_\mu / e^2 M^2 = -0.137(56)$ GeV^{-2}. On the other hand analysis of the muonic Lamb-shift experiment[17] in ^{4}He with a new rms radius[4] yields $g_N g_\mu / e^2 M^2 = +0.020(52)$ GeV^{-2}. It is interesting to note that an interaction of a strength as deduced from our experiment with a vector boson of a few gigaelectronvolts not only explains the discrepancy (8), but also the observed (though not always openly admitted) trend in experimental comparisons of μ-proton and e-proton scattering cross sections.[8,18]

In the present work the rms charge radius of ^{12}C has been determined with an accuracy of better than 2×10^{-16} cm under the assumption that no additional interactions beyond standard QED are present. Comparison with the charge radius obtained from electron scattering indicates a difference. The exciting possibility of attributing this difference to an additional muon-hadron interaction requires further experimental and theoretical work.

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Calculation of the $4d$ Subshell Photoabsorption Spectra of Ba, Ba$^+$, and Ba^{++}

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Calculations of the photoabsorption spectra for Ba, Ba$^+$, and Ba^{++} near the $4d$ ionization threshold have been performed with the time-dependent local density approximation. The results are in quantitative agreement with recent experimental data and indicate that the sharp resonant structure below threshold is due to transitions to hybridized f states which are strongly modified by electron-electron interactions.

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Recently the vacuum-ultraviolet photoabsorption spectra for the $4d$ subshell in Ba, Ba$^+$, and Ba^{++} have been reported.[1] Of particular interest in the sequences is the Ba^{++} spectrum which exhibits several sharp resonances, each containing an appreciable oscillator strength, below the $4d$