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Higher-derivative supergravity and moduli stabilization

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Abstract: We review the ghost-free four-derivative terms for chiral superfields in $\mathcal{N} = 1$ supersymmetry and supergravity. These terms induce cubic polynomial equations of motion for the chiral auxiliary fields and correct the scalar potential. We discuss the different solutions and argue that only one of them is consistent with the principles of effective field theory. Special attention is paid to the corrections along flat directions which can be stabilized or destabilized by the higher-derivative terms. We then compute these higher-derivative terms explicitly for the type IIB string compactified on a Calabi-Yau orientifold with fluxes via Kaluza-Klein reducing the $(\alpha')^3 R^4$ corrections in ten dimensions for the respective $\mathcal{N} = 1$ Kähler moduli sector. We prove that together with flux and the known $(\alpha')^3$-corrections the higher-derivative term stabilizes all Calabi-Yau manifolds with positive Euler number, provided the sign of the new correction is negative.

Keywords: Flux compactifications, Superstring Vacua, Supersymmetric Effective Theories, Supergravity Models

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Contents

1 Introduction

In many applications supersymmetric field theories or supergravities are considered as an effective description of a more fundamental theory, such as string theory. Most properties of this low energy effective theory are captured by the leading two-derivative Lagrangian

$\mathcal{L}_{(0)}$. It can, however, happen that specific couplings vanish in $\mathcal{L}_{(0)}$ and then higher order corrections do become important. A particular class of corrections are higher-derivative terms which in supersymmetric theories can simultaneously induce corrections of the scalar potential. It is the purpose of this paper to analyse supersymmetric higher-derivative operators with this property — both conceptually and as a new tool to stabilize moduli in string theory. Such terms were also studied in [1–5], while [6–10] started looking at their implications for cosmology.

More precisely, we focus on $\mathcal{N} = 1$ supersymmetry and supergravity in four space-time dimensions and within such theories on ghost-free higher-derivative operators. In non-supersymmetric theories it is well-known that the unique ghost-free four-derivative operator for a scalar field ϕ is given by $(\partial_\mu \phi\, \partial^\mu \phi)^2$.[1] Several distinct superspace-operators exist which induce such terms. However, there is a unique ghost-free operator given by [2]

$$\mathcal{L}_{(1)} \sim \int \mathrm{d}^4\theta\, (D^\alpha \Phi)(D_\alpha \Phi)(\bar{D}_{\dot{\alpha}} \Phi^\dagger)(\bar{D}^{\dot{\alpha}} \Phi^\dagger)\,, \tag{1.1}$$

where $D_\alpha, \bar{D}_{\dot{\alpha}}$ denote the superspace derivatives, $\mathrm{d}^4\theta = \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}$ denotes the integration over the Grassmann variables and Φ is a chiral superfield. We will see that the equation of motion for the auxiliary field F is cubic instead of linear after including $\mathcal{L}_{(1)}$. This in turn implies up to three inequivalent solutions for F and, hence, three inequivalent on-shell theories. The presence of this multiplet of theories is somewhat puzzling as one seems to loose predictability. However, studying the explicit solutions we find that only one out of the three theories is consistent with the principles of effective field theory (EFT).

There is a notable example in which higher-derivative operators such as $\mathcal{L}_{(1)}$ have been computed from radiative corrections in a manifest off-shell scheme, namely the effective one-loop superspace Lagrangian of the Wess-Zumino model [13–15]. These references focused purely on those higher-derivative operators that contribute to the scalar potential and in [15] an infinite tower of such higher-derivative operators, denoted as the effective auxiliary field potential (EAFP), was explicitly computed. To lowest order in superspace-derivatives this EAFP coincides with $\mathcal{L}_{(1)}$ given in eq. (1.1). The full non-local EAFP turns out to imply a unique on-shell theory. When truncating this EAFP to a finite number of terms, the truncation naively produces multiple on-shell theories. Applying the truncation at higher order even increases the number of solutions. However, we will show that at any order of the truncated EAFP there is a unique Lagrangian which reproduces the dynamics of the non-local theory at that order and which is consistent with the principles of EFT. The remaining theories can be regarded as artefacts of the truncation of the infinite tower of higher-derivative operators similar to the emergence of ghosts in truncated theories [16].

Apart from addressing this conceptual issue we proceed to compute the on-shell Lagrangians for models with arbitrarily many chiral superfields both in global and local supersymmetry. In particular we focus on the induced correction to the scalar potential and

[1]The ghost-free higher-derivative operators are in general those that do not induce more than two derivatives acting on fields in the equations of motion. Additionally, in supersymmetric theories ghostlike degrees of freedom can occur when a kinetic term for the auxiliary field is induced [11, 12].

analyze the situation where the two-derivative theory has a minimum with a flat direction which can (or cannot) be lifted by the presence of $\mathcal{L}_{(1)}$.

In the second part of this paper we will purely focus on the effective action obtained from type IIB flux compactifications on Calabi-Yau orientifolds. The background fluxes are able to stabilize the complex structure moduli and the dilaton [17, 18]. In contrast, all Kähler moduli are described at leading order by a no-scale supergravity and thus are flat directions of the potential. Perturbative corrections for the Kähler moduli are induced from α'- and g_s-corrections in the ten-dimensional action. An important example is the leading order $(\alpha')^3$-correction to the Kähler potential which is computed by reducing higher-curvature terms in ten dimensions [19]. This correction breaks the no-scale property, but by itself does not lead to a stabilization. When non-perturbative effects are taken into account scenarios with supersymmetric [20] or non-supersymmetric minima can be found [21].[2] There is an intrinsic merit to demonstrate the existence of various classes of meta-stable de Sitter (dS) vacua as explicitly as possible in well-controlled examples of string compactifications. Thus, we find it worthwhile to explore further possibilities of moduli stabilization using only fully perturbative and explicitly computable contributions.

Hence, it is of interest to pursue the question to what extent additional $(\alpha')^3$-corrections of the ten-dimensional theory can lead to corrections to the scalar potential and, thus, potentially to a stabilization of moduli without taking into account non-perturbative effects. Indeed, several such corrections to the scalar potential are expected to appear and have not been discussed in detail, owing to the fact that the explicit structure of the ten-dimensional analogues are still unknown. We will argue that some of these $(\alpha')^3$-corrections to the scalar potential can be matched to higher-derivative operators of the type of $\mathcal{L}_{(1)}$ as off-shell completions.[3] Therefore, even though it is not possible to compute the corrections to the scalar potential directly, one can determine the respective four-derivative terms, which as we will show decend from the explicitly known R^4-terms in ten dimensions [25, 26], and, hence, infer the correction to the scalar potential $V_{(1)}$ indirectly. By computing these four-derivative terms we find

$$V_{(1)} \sim \frac{\Pi_i\, t^i}{\mathcal{V}^4}\,, \tag{1.2}$$

where the t^i denote the two-cycle volumes, $\mathcal{V}$ the overall volume and the Π_i are topological numbers defined as

$$\Pi_i = \int c_2 \wedge \hat{D}_i\ . \tag{1.3}$$

They encode information of the second Chern class c_2 and $\hat{D}_i$ form a basis of $H^{1,1}(M,\mathbb{Z})$.[4]

We then proceed to study the minima of $V_{(1)}$ taken together with the potential obtained from the α'-corrected Kähler potential. We show the existence of a model-independent non-supersymmetric minimum of this potential where all four-cycle volumes are fixed to values

[2] For reviews on moduli stabilization, flux compactifications and de Sitter vacua, see e.g. [22–24].

[3] We will show that it is in fact impossible to supersymmetrically complete these corrections within a two-derivative theory, that is via additional terms in the Kähler and/or superpotential.

[4] We perform this computation without determining numerical factors and for the simple case of $h^{1,1} = 1$ but argue that $V_{(1)}$ given in (1.2) also holds for arbitrary $h^{1,1}$ as long as $W = \text{const}$.

$\tau_i \sim \Pi_i$ for any Calabi-Yau threefold with $\chi(M) > 0$.[5] This result suggests the existence of many new non-supersymmetric vacua within the landscape, where stabilization occurs purely from the leading order α'-corrections, but a more detailed discussion of all possible α'-corrections will be necessary to support this. Furthermore, the minimum only exists if the overall sign of $\mathcal{L}_{(1)}$ is negative. This sign is universal and does not depend on the choice of the Calabi-Yau. Unfortunately, determining this sign requires the knowledge of the particular linear combination of all additional 4D higher-derivative operators contributing to the 4D four-derivative kinetic terms. This is beyond the scope of this paper and we leave it for future work.

This paper is organized as follows. In section 2 we study $\mathcal{L}_{(1)}$ in effective theories with global supersymmetry. The conceptual discussion of the on-shell theories is performed for theories with a single chiral superfield in section 2.2 and in appendix A, where we also display the exact solutions for the chiral auxiliary field and prove the absence of ghosts. In section 2.3 we illustrate the interpretation of the higher-derivative operators and the respective on-shell theories with the one-loop Wess-Zumino model. In section 2.4 we then display the physical on-shell Lagrangian for arbitrarily many chiral superfields and make some statements regarding the structure of the resulting minima, providing an explicit example for the lifting of flat directions in section 2.5. In section 3 we show the respective Lagrangians for the case of supergravity and again discuss the structure of the minima with an explicit example in section 3.2. Finally in section 4 we turn to the discussion of flux compactifications of Type IIB on Calabi-Yau orientifold, where the details of the reduction of the curvature-terms in ten dimensions can be found in appendix B and appendix C. Furthermore, in appendix D we prove that $V_{(1)}$ given in eq. (1.2) cannot be off-shell completed via corrections to the Kähler potential and/or superpotential. At the end we provide some conclusions in section 5.

2 Higher-derivative terms in $\mathcal{N} = 1$ supersymmetry

2.1 Preliminaries

In this section we consider globally supersymmetric theories with n_c chiral superfields Φ^i, $i = 1, \ldots, n_c$ whose couplings are encoded in a Kähler potential K, a superpotential W and the higher-derivative operator $\mathcal{L}_{(1)}$. In the following we adopt the conventions and notation of [27]. Thus, the total superspace Lagrangian is of the form[6]

$$\begin{aligned} &\mathcal{L} = \mathcal{L}_{(0)} + \mathcal{L}_{(1)}\,, \\ \text{where} \qquad &\mathcal{L}_{(0)} = \int d^4\theta\, K(\Phi, \Phi^\dagger) + \int d^2\theta\, W(\Phi) + \text{h.c.}\,, \\ &\mathcal{L}_{(1)} = \frac{1}{16} \int d^4\theta\, T_{ij\bar{k}\bar{l}}(\Phi, \Phi^\dagger)\, D^\alpha \Phi^i D_\alpha \Phi^j \bar{D}_{\dot{\alpha}} \Phi^{\dagger \bar{k}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger \bar{l}}\,. \end{aligned} \tag{2.1}$$

[5] We estimate the typical size of the Π_i for a specific Calabi-Yau threefold to be $\mathcal{O}(10\text{–}100)$.

[6] Here and henceforth we drop brackets, which would indicate explicitly on which fields certain superspace-derivatives act. More precisely this means $D^\alpha\Phi D_\alpha\Phi = (D^\alpha\Phi)(D_\alpha\Phi)$.

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In the spirit of [3] we allow for an arbitrary hermitian four-tensor superfield $T_{ij\bar{k}\bar{l}}(\Phi,\Phi^\dagger)$ which we assume to depend only Φ and $\Phi^\dagger$ but not on any derivative.[7] We will often refer to this mass dimension -4 quantity, respectively its scalar component as coupling tensor. From the structure of $\mathcal{L}_{(1)}$ one infers the symmetry properties

$$T_{ij\bar{k}\bar{l}} = T_{ji\bar{k}\bar{l}} = T_{ji\bar{l}\bar{k}} \; . \tag{2.2}$$

In order to obtain the component expression of $\mathcal{L}$ we use the well known θ-expansion of the chiral superfields

$$\Phi^i = A^i + \sqrt{2}\theta\psi^i + \theta^2 F^i + i\theta\sigma^\mu\bar{\theta}\partial_\mu A^i - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi^i\sigma^\mu\bar{\theta} + \frac{1}{4}\theta^2\bar{\theta}^2\Box A^i \;, \tag{2.3}$$

where A^i are scalars, ψ^i chiral fermions and F^i auxiliary components. From the form of the superspace derivatives

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial x^\mu} \qquad \text{and} \qquad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\frac{\partial}{\partial x^\mu}\,, \tag{2.4}$$

one finds that the bosonic part of $\mathcal{L}_{(1)}$ only has a contribution at order $\theta^2\bar{\theta}^2$ which is given by

$$\begin{aligned} &T_{ij\bar{k}\bar{l}}(\Phi,\Phi^\dagger)D^\alpha\Phi^i D_\alpha\Phi^j\bar{D}_{\dot{\alpha}}\Phi^{\dagger\bar{k}}\bar{D}^{\dot{\alpha}}\Phi^{\dagger\bar{l}}|_{\text{bos}} = \\ &\qquad 16T_{ij\bar{k}\bar{l}}(A,\bar{A})\left[(\partial_\mu A^i\partial^\mu A^j)(\partial_\nu\bar{A}^{\bar{k}}\partial^\nu\bar{A}^{\bar{l}}) - 2F^i\bar{F}^{\bar{k}}(\partial_\mu A^j\partial^\mu\bar{A}^{\bar{l}}) + F^iF^j\bar{F}^{\bar{k}}\bar{F}^{\bar{l}}\right]\theta^2\bar{\theta}^2 \; . \end{aligned} \tag{2.5}$$

Performing the θ integration in eq. (2.1) one obtains the Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{bos}} = &- G_{i\bar{j}}\,\partial_\mu A^i\partial^\mu\bar{A}^{\bar{j}} + G_{i\bar{j}}\,F^i\bar{F}^{\bar{j}} + F^i\,W_{,i} + \bar{F}^{\bar{i}}\,\bar{W}_{,\bar{i}} \\ &+ T_{ij\bar{k}\bar{l}}(A,\bar{A})\left[(\partial_\mu A^i\partial^\mu A^j)(\partial_\nu\bar{A}^{\bar{k}}\partial^\nu\bar{A}^{\bar{l}}) - 2F^i\bar{F}^{\bar{k}}(\partial_\mu A^j\partial^\mu\bar{A}^{\bar{l}}) + F^iF^j\bar{F}^{\bar{k}}\bar{F}^{\bar{l}}\right]\,, \end{aligned} \tag{2.6}$$

where $G_{i\bar{j}} = \partial_i\partial_{\bar{j}}K$ and $W_{,i}$ denotes the holomorphic derivative of the superpotential. We indeed see that no derivative terms for F^i appear and, thus, their equations of motion stay algebraic such that the F^i remain non-propagating auxiliary fields. However, $\mathcal{L}_{\text{bos}}$ contains quartic terms in the F^i which lead to cubic contributions to the bosonic part of the respective equations of motion

$$G_{i\bar{k}}F^i + \bar{W}_{,\bar{k}} + 2F^i(F^j\bar{F}^{\bar{l}} - \partial_\mu A^j\partial^\mu\bar{A}^{\bar{l}})T_{ij\bar{k}\bar{l}} = 0 \; . \tag{2.7}$$

Determining all solutions to this equation in all generality is a delicate task and therefore we first turn to a theory with a single chiral multiplet where we can solve the cubic equation (2.7) exactly.

[7]Note that if T would depend on space-time or superspace-derivatives of the chiral multiplets the resulting theory would either involve more than four derivatives for the component fields and/or not correct the scalar potential.

2.2 Theory with one chiral multiplet

For one chiral multiplet eqs. (2.7) reduce to

$$G_{A\bar{A}}\,\bar{F} + W_{,A} + 2T\bar{F}\left(|F|^2 - \partial_\mu A\partial^\mu \bar{A}\right) = 0\,, \tag{2.8}$$

where we defined $T = T_{AA\bar{A}\bar{A}}$ for brevity. In appendix A.1 we solve eq. (2.8) exactly and show that depending on T and the specific region in the phase space of A one or three solutions for F exist. Expanding the solutions for small T and inserting into eq. (2.6) keeping only the leading terms one obtains, in the case where all three solutions exist, the following three Lagrangians

$$\begin{aligned}\mathcal{L}_{F_1} =& - G_{A\bar{A}}\left(1 + 2\hat{T}V_{(0)}\right)\partial_\mu A\partial^\mu\bar{A} + \hat{T}G^2_{A\bar{A}}(\partial_\mu A\partial^\mu A)(\partial_\nu\bar{A}\partial^\nu\bar{A})\\ &- V_{(0)} + \hat{T}V^2_{(0)} + \mathcal{O}(\hat{T}^2)\,,\\ \mathcal{L}_{F_{2,3}} =& -\frac{1}{4}\hat{T}^{-1} + \frac{1}{2}V_{(0)} + \mathcal{O}(\hat{T}^{1/2})\,,\end{aligned} \tag{2.9}$$

where for convenience we defined $\hat{T} = TG^{-2}_{A\bar{A}}$ and $V_{(0)} = G^{A\bar{A}}|W_{,A}|^2$ is the scalar potential of $\mathcal{L}_0$.[8] In the following we will sometimes refer to the individual Lagrangians in eq. (2.9) as branches. We observe that $\mathcal{L}_{F_1}$ is analytic in $\hat{T}$ and reproduces $\mathcal{L}_0$ at leading order. At linear order in $\hat{T}$ it induces a correction to the kinetic energy, which is proportional to $V_{(0)}$, as well as to the potential, proportional to $V^2_{(0)}$. $\mathcal{L}_{F_{2,3}}$ on the other hand have a pole-like term in $\hat{T}$ and at order $\hat{T}^0$ only have a contribution to the potential, which differs from $V_{(0)}$ by a factor $-1/2$.

In summary the theory defined by (2.1) can lead to three different and independent on-shell Lagrangians. However, a multiplet of theories is dissatisfying, since it predicts several inequivalent evolutions of fields for a given set of initial data. Furthermore, suppose we include additional off-shell higher-derivative operators with more than four superspace-derivatives then the equations of motion for the chiral auxiliaries admit more than three solutions, rendering the problem even more severe. Let us now argue how to resolve this issue in the context of an effective field theory.

When performing the limit $T \to 0$ in the off-shell Lagrangian given in eq. (2.1) we recover the ordinary, two-derivative theory $\mathcal{L}_{(0)}$. For consistency this should also hold in the on-shell theories given in eq. (2.9). For example suppose that the higher-derivative operator arises by integrating out massive states associated with a mass scale M from a UV theory. Then to lowest order in fields one has $T \sim M^{-4}$ and hence the operator should decouple as M becomes large compared to the masses of the light states as dictated by the decoupling principle, see for instance [28]. We see that $\mathcal{L}_{F_1}$ given in (2.9) is analytic in T, while $\mathcal{L}_{F_{2,3}}$ contain a non-analytic part and thus violate the decoupling limit. Based on this observation we propose to regard only $\mathcal{L}_{F_1}$ as the physical on-shell Lagrangian since it is the unique Lagrangian compatible with the principles of effective field theory. We

[8] Expanding eq. (A.10) one observes that $\hat{T}$ is the correct expansion parameter only for $\mathcal{L}_{F_1}$, while for $\mathcal{L}_{F_{2,3}}$ it is $\sqrt{\hat{T}}$. Note that at the displayed order in $\sqrt{\hat{T}}$ the solutions F_2 and F_3 induce the same Lagrangian while at higher order we find $\mathcal{L}_{F_2} \neq \mathcal{L}_{F_3}$.

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will substantiate this proposition with the example of the effective one-loop Wess-Zumino model in the next section. Notably we will show that the non-analytic theories not only fail to obey the decoupling limit, but furthermore are incapable of reproducing the on-shell Lagrangian of the full, non-local theory. To some extent this is already visible in eq. (2.9). More precisely the non-analytic branches fail to reproduce the terms in $\mathcal{L}_{(0)}$. In fact they neither include the kinetic terms nor the scalar potential of $\mathcal{L}_{(0)}$. On the other hand the $\mathcal{O}(T^0)$ contributions in $\mathcal{L}_{F_1}$ exactly coincide with the terms in $\mathcal{L}_{(0)}$. In summary, this observation and the results of the next section suggest that the non-analytic solutions should be regarded as mere artefacts of the truncation of an infinite sum of higher-derivatives. Note that the above observation is reminiscent of the discussion of theories with higher-derivative terms in the equations of motion where ghost-like degrees of freedom emerge. Similarly the ghosts arise from truncating an infinite series of higher-derivative terms to a finite sum and violate EFT-reasoning in as much as the inclusion of higher order operators should merely induce a small correction to the dynamics of some IR-Lagrangian. A ghost-free theory can then be obtained by demanding analyticity of the solutions to the equations of motion in EFT-control parameters [16, 29], identical to our reasoning above.

In the rest of this paper we will therefore only discuss the analytic theory. Furthermore, recall that besides the operator in eq. (2.1) superspace higher-derivative terms with more than four superspace-derivatives exist and they contribute higher polynomial powers of the auxiliary field to the Lagrangian (next section we display the one-loop Wess-Zumino model as an explicit example where infinitely many superspace-derivative operators are present). These operators are further mass-suppressed and hence modify the equations of motion for the auxiliary fields at order $\mathcal{O}(T^2)$.[9] This implies that without including such higher-derivative terms into the superspace Lagrangian, we can trust the resulting on-shell Lagrangian only up to linear order in T.[10] Fortunately this greatly simplifies the structure of the on-shell Lagrangian and makes a proper discussion of the multi-field case feasible.

To conclude this section let us describe why the theory is free of ghosts. The absence of ghosts is not immediately clear, but can be understood with the exact solution for the auxiliary field at hand. The sign of the ordinary kinetic term is affected by the presence of the higher-derivative operator through eq. (2.8). In appendix A.2 the absence of ghosts is explicitly demonstrated for the theory obtained by solving eq. (2.8) exactly and reinserting the result into eq. (2.6). Nevertheless, one might still worry about the sign of the ordinary kinetic term in the truncated theory after inspection of eq. (2.9). More precisely one finds that the theory becomes ghost-like once $\hat{T}V_{(0)} \sim -1$. However, in that regime we cannot trust our truncation at linear order in T any longer as we illustrate in appendix A. In other words, studying the exact solutions of eq. (2.8) shows that if $\hat{T}V_{(0)} \sim -1$, the analytic solution ceases to exist and one enters a regime, in which only non-perturbative solutions can be found. To summarize, the analytic theory breaks down before it would become ghostlike.

[9] They might also induce modifications at order $\mathcal{O}(T^{3/2})$.

[10] In models, where the EAFP is solely given in terms of the four-derivative operator, it is sensible to regard the full solution for F^i and the respective Lagrangians along the lines of appendix A.

2.3 One-loop Wess-Zumino model

After the general discussion of the previous section let us now turn to an explicit example, where the truncation of the infinite sum of higher-derivatives and the structure of the equations of motion for the auxiliary field can be explicitly studied. This example is given by the one-loop Wess-Zumino model in superspace, for which the full, non-local effective auxiliary field potential (EAFP) was recently computed in [15] following up on earlier works [13, 14]. More precisely the model consists of a single chiral superfield Φ with Kähler potential and superpotential of the form

$$K = \Phi\Phi^\dagger \,, \qquad W = \frac{1}{2}m\Phi^2 + \frac{1}{6}\lambda\Phi^3 \,. \tag{2.10}$$

According to [15] the only contributions to the effective superspace potential at one-loop come from corrections to the Kähler potential as well as an EAFP, which we denote as $\mathbb{F}$. More precisely it consists of an infinite tower of higher-derivatives of the form

$$\mathbb{F} = \int \mathrm{d}^4\theta \, \frac{D\Psi D\Psi \bar{D}\Psi^\dagger \bar{D}\Psi^\dagger}{(\Psi\Psi^\dagger)^2} \, G\left(\frac{D^2\Psi \bar{D}^2\Psi^\dagger}{(\Psi\Psi^\dagger)^2}\right) \quad , \tag{2.11}$$

where $\Psi = m + \lambda\Phi = W''$ and G is a known real-valued analytic function with non-vanishing coefficients in the respective series expansion at all orders [15]. The lowest order contribution arises from the constant term in the series expansion of G and comparing with (2.1) we have

$$T \sim |W''|^{-4} \,. \tag{2.12}$$

Expanding T as a geometric series, we identify that to lowest order we have $T \sim m^{-4}$.

Let us now proceed by performing the superspace integration in eq. (2.11). From eq. (2.5) we infer that the bosonic part of the superfield multiplying G has only a $\theta^2\bar{\theta}^2$ contribution and hence the remaining superfields have to be evaluated at their scalar component. This yields

$$\mathbb{F}_{\text{bos}} = \frac{(D\Psi D\Psi \bar{D}\Psi^\dagger \bar{D}\Psi^\dagger)|_{\theta^4}}{|m+\lambda A|^4} \, G\left(\frac{|\lambda F|^2}{|m+\lambda A|^4}\right) \,. \tag{2.13}$$

For simplicity let us set $\lambda = 1$ from now on. $\mathbb{F}_{\text{bos}}$ displays an infinite sum in the auxiliary field F and $\bar{F}$. Additional powers of the auxiliary field are in a one-to-one correspondence with additional powers of superspace-derivatives. We can identify

$$\epsilon \equiv |m + A|^{-4} \tag{2.14}$$

as the parameter controlling the infinite series of higher-derivatives and powers of the auxiliary field, respectively. We immediately observe that eq. (2.13) comprises an analytic function in ϵ. Using the full (and explicitly known) function G we checked numerically that the solution to the equations of motion for F derived from the standard Lagrangian plus $\mathbb{F}_{\text{bos}}$ is unique and analytic in ϵ.[11]

[11] The numerical analysis was performed with the help of Mathematica 10.

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The non-local theory with $\mathbb{F}$ in eq. (2.13) can be regarded as a UV-theory for a local theory after truncating the infinite sum of higher-derivatives to a finite sum. Moreover, the control parameter ϵ is non-polynomial, which in turn makes the Lagrangian non-local even after truncation of the tower of higher-derivative operators. Thus, it would also be necessary to expand ϵ in a small parameter and truncate this expansion at an appriopriate order to obtain a local theory. We omit this here, as it does not provide additional insight into the structure of the series in higher-derivatives.

It is interesting to discuss the equations of motion for the auxiliary field once the theory is truncated at a given order in ϵ. In the following let G_n denote the truncation of the series expansion of G at order n. If we truncate G at $\mathcal{O}(\epsilon)$, the discussion reduces to the familiar cubic in eq. (2.8), which admits only one analytic solution. For arbitrary n the contribution of eq. (2.13) to the scalar potential reads

$$\mathbb{F}_{\text{bos}} \sim \epsilon |F|^4 G_n(\epsilon |F|^2) \ . \tag{2.15}$$

Taking into account the remaining, ordinary terms in the Lagrangian, i.e. $\mathcal{L}_{(0)}$ in eq. (2.1), the equation of motion for F reads

$$F + \bar{W}' + 2\epsilon F|F|^2 G_n(\epsilon|F|^2) + \epsilon^2 F|F|^4 G_n'(\epsilon|F|^2) = 0\,, \tag{2.16}$$

where we only took into account terms that contribute to the scalar potential. G_n induces monomials in $|F|^2$ up to degree n and, hence, eq. (2.16) admits up to $(2n+3)$ independent solutions. In other words the number of solutions is increasing with the order of the truncation. To solve eq. (2.16) we first redefine the auxiliary field via

$$F = \bar{W}' f \ . \tag{2.17}$$

Inserted into eq. (2.16) one observes that f has to be real and, hence, eq. (2.16) reduces to

$$f + 1 + 2\epsilon f^3 |W'|^2 G_n(\epsilon f^2 |W'|^2) + \epsilon^2 f^5 |W'|^4 G_n'(\epsilon f^2 |W'|^2) = 0 \ . \tag{2.18}$$

We make an ansatz of the form

$$f = \sum_{i=-1}^{\infty} \epsilon^{i/2} f_i \,, \tag{2.19}$$

such that eq. (2.18) at lowest order in ϵ reads

$$f_{-1} + f_{-1}^3 |W'|^2 G_n(f_{-1}^2 |W'|^2) + f_{-1}^5 |W'|^4 G_n'(f_{-1}^2 |W'|^2) = 0 \ . \tag{2.20}$$

Since G_n is a polynomial of degree n with non-vanishing coefficients we see that only the branch given by $f_{-1} = 0$ is analytic. All other solutions, which are defined at lowest order by the remaining $2n+2$ solutions of eq. (2.20) and necessarily fulfill $f_{-1} \neq 0$, are non-analytic in ϵ for any n.

In effective field theory one generally expects to be able to compute observables with higher precision by including more and more operators. Indeed since the unique solution of the non-local theory was analytic, the analytic solution of the truncated theory is able to reproduce the Lagrangian of the non-local theory at order ϵ^{n+1} and, thus, mimics the non-local theory with better precision for larger n. However, regardless of the order of the truncation the non-analytic theories fail to reproduce the non-local theory to that specific order. One can explicitly check this for the first components in the expansion in eq. (2.19). At lowest order this was also already visible in eq. (2.9).

It is worth noting that the existence of a unique analytic solution for F in the truncated theory does not depend on the details of the $\mathbb{F}$, but we expect it to hold in general as long as the coefficient of the $|F|^2$ term in the Lagrangian is non-vanishing. Indeed the EAFP is correcting the Lagrangian by at least cubic powers of F and $\bar{F}$ [13] so that one would always expect the analytic solution to be unique.

After the above conceptual discussion we can now proceed to study theories with more than one chiral multiplet.

2.4 Multi-field case and analysis of scalar potential

Given the results of the previous sections we constrain the discussion of the multi-field case to the analytic solution of eq. (2.7). Solving eq. (2.7) using perturbation theory yields at linear order in T

$$\begin{aligned} F^i =& F^i_{(0)} + F^i_{(1)} \,, \qquad \text{where} \qquad F^i_{(0)} = -G^{i\bar{l}}\,\bar{W}_{,\bar{l}} \,, \\ F^i_{(1)} =& 2\,T^{\bar{k}\bar{l}ij}\,\bar{W}_{,\bar{k}}\,\bar{W}_{,\bar{l}}\,W_{,j} - 2T^{\bar{k}}{}_{j}{}^{i}{}_{\bar{l}}\,(\partial_\mu A^j \partial^\mu \bar{A}^{\bar{l}})\,\bar{W}_{,\bar{k}} \,. \end{aligned} \tag{2.21}$$

Insertion of the auxiliary field into the Lagrangian in eq. (2.6) yields

$$\mathcal{L}_{\text{bos}} = -\left(G_{i\bar{k}} + 2T^{\bar{l}}{}_{i}{}^{j}{}_{\bar{k}}\,W_{,j}\,\bar{W}_{,\bar{l}}\right)\partial_\mu A^i \partial^\mu \bar{A}^{\bar{k}} + T_{ij\bar{k}\bar{l}}\,(\partial_\mu A^i \partial^\mu A^j)(\partial_\mu \bar{A}^{\bar{k}} \partial^\mu \bar{A}^{\bar{l}}) - V(A,\bar{A}) \,. \tag{2.22}$$

The resulting scalar potential at linear order in T reads

$$V = V_{(0)} + V_{(1)} \,, \qquad \text{where} \qquad V_{(0)} = G^{i\bar{j}} W_{,i} \bar{W}_{,\bar{j}} \,, \quad V_{(1)} = -T^{ij\bar{k}\bar{l}} W_{,i} W_{,j} \bar{W}_{,\bar{k}} \bar{W}_{,\bar{l}} \,. \tag{2.23}$$

Before we analyse this potential, let us make a comment regarding the ordinary kinetic term in the Lagrangian in eq. (2.22). The metric multiplying the kinetic term is corrected by

$$\delta G_{i\bar{k}} = 2T^{\bar{l}}{}_{i}{}^{j}{}_{\bar{k}}\,W_{,j}\,\bar{W}_{,\bar{l}} \,. \tag{2.24}$$

Since we added a new operator in the superspace Lagrangian in eq. (2.1), there is no reason for the metric multiplying the two-derivative term in eq. (2.22) to be Kähler. The complex structure on the manifold spanned by the chiral scalars is unchanged and continues to be locally defined by the chiral superfields. Indeed, it was shown in [1] that for the following special

$$T_{ij\bar{k}\bar{l}} = \frac{T}{2}\left(G_{i\bar{k}} G_{j\bar{l}} + G_{i\bar{l}} G_{j\bar{k}}\right) \,, \tag{2.25}$$

with constant T the hermitian connection has non-vanishing torsion and, thus, the metric multiplying the two-derivative term in eq. (2.22) is not Kähler.

Since the off-shell supersymmetry transformations of the chiral multiplets do not change, the order parameter for supersymmetry breaking continues to be $\langle F^i \rangle$. Therefore the supersymmetric minima of V are found at

$$\langle F^i \rangle = 0 \,. \tag{2.26}$$

From eq. (2.7) we see that the supersymmetric locus in field space $\langle A^i \rangle$ which solves (2.26) is determined by $\langle F^i_{(0)} \rangle = \langle W_{,i} \rangle = 0$ and, thus, is not corrected by the presence of the higher-derivative terms under the condition that T is non-singular.[12] Indeed it was shown that for arbitrary higher-derivative theories the structure of the supersymmetric vacua is unchanged [1]. In particular this implies that any flat direction of $V_{(0)}$ is not lifted.

If supersymmetry is broken by some $\langle F^i_{(0)} \rangle \neq 0$ the higher-derivative correction can become important. Still $V_{(1)}$ is a perturbation of $V_{(0)}$ and therefore the minimum $\langle A^i_{(0)} \rangle$ of $V_{(0)}$ will at best be shifted to a nearby field value $\langle A^i_{(0)} \rangle \to \langle A^i_{(0)} \rangle + \langle \delta A^i \rangle$. However, if the non-supersymmetric minimum of $V_{(0)}$ has a flat direction the contribution from $V_{(1)}$ becomes the leading term in this direction and may lift its flatness. A possible exception to this occurs when the flatness is due to a symmetry, such as a perturbatively unbroken shift-symmetry. Further exceptions are models in which supersymmetry breaking occurs due to a spontaneously broken R-symmetry [30]. In this case there always exists a flat direction, the R-axion, associated with the Goldstone boson of the broken R-symmetry. Here the existence of higher-derivative corrections does not lift the flatness.

If the flatness is lifted, then depending on the structure and sign of T the flat direction can be stabilized or destabilized. It is difficult to make a general statement, and in the end a case-by-case analysis is necessary. Nevertheless, before we proceed, let us offer some general observations.

A (real) flat direction ϕ is characterized by the fact the all ϕ-derivatives of V vanish in the background, or in other words

$$\langle \partial^n_\phi V \rangle = 0 \,, \quad \forall n \in \mathbb{N} \,. \tag{2.27}$$

Let us assume that $V_{(0)}$ has a flat direction and thus satisfies (2.27). A special (and simple) case of this situation is that $V_{(0)}$ does not depend on ϕ at all, i.e. $\partial^n_\phi V_{(0)} \equiv 0, \forall n$. In this case the flat direction is lifted for generic T but preserved if T is also independent of ϕ. A slight generalization occurs when $W_{,i}$ and only the matrix element of $G^{i\bar{j}}$ in the direction of the supersymmetry breaking F-term, say F^0, are independent of ϕ. In this case the flat direction is preserved if also $T^{00\bar{0}\bar{0}}$ is independent of ϕ. As a final example let us discuss a specific form of the coupling tensor given in eq. (2.25). In this case we have $V_{(1)} = -TV^2_{(0)}$ and thus any flat direction of $V_{(0)}$ remains flat with respect to $V_{(1)}$, given that the scalar function T does not depend upon it.

2.5 Example: O'Raifeartaigh model

For concreteness let us discuss a specific example of a model with flat directions within non-supersymmetric vacua. The simplest case is given by the O'Raifeartaigh model. This is defined via a Kähler and superpotential, which read

$$K = |A_0|^2 + |A_1|^2 + |A_2|^2 \,, \qquad W = \lambda A_0 + m A_1 A_2 + Y A_0 A_1^2 \,. \tag{2.28}$$

[12]This can also be inferred from eq. (2.21). However, care must be taken as eq. (2.21) suggests that up to two additional solutions to eq. (2.26) exist for which $\langle W_{,i} \rangle \neq 0$. Yet these would be due to a non-trivial cancellation between $F^i_{(0)}$ and $F^i_{(1)}$ that will be spoiled once higher order corrections in T to F^i are considered. More precisely these solutions would only exist because we truncate the auxiliary field at a certain order and are, thus, artefacts of this truncation.

Here λ, m, Y are real parameters such that $m^2 > 2\lambda Y$. The resulting potential is minimized at $\langle A_1 \rangle = \langle A_2 \rangle = 0$ leaving A_0 unfixed. Since $\langle V_0 \rangle = \langle |F_0|^2 \rangle = \lambda^2$, supersymmetry is broken in the vacuum. Eq. (2.28) has a $\mathbb{Z}_2$-symmetry in A_1 and A_2 and furthermore an R-symmetry, if we assign R-charges as follows

$$R(A_0) = R(A_2) = 2\,, \qquad R(A_1) = 0\,. \tag{2.29}$$

For the continuum of vacua labeled by $\langle A_0 \rangle$ there exists one vacuum, namely $\langle A_0 \rangle = 0$, in which the R-symmetry is not spontaneously broken. Thus, the O'Raifeartaigh model is an exception to the generic expectation that supersymmetry breaking occurs due to R-symmetry breaking in models, which reduce to Wess-Zumino models in the low energy regime and respect the principles of EFT [30].

Let us proceed by switching on the higher-derivative operator. We consider vacua in which $\langle A_1 \rangle = \langle A_2 \rangle = 0$ as in the ordinary theory. The respective potential at the point $A_1 = A_2 = 0$ is extremized, if the following holds

$$\partial_i V = -T^{00\bar{0}\bar{0}}_{,i}\lambda^4 - 2m\lambda^3(1-\delta_{i,0})(T^{i0\bar{0}\bar{0}} + T^{00\bar{i}\bar{0}}) = 0\,. \tag{2.30}$$

We see that the flatness of A_0 is lifted, if certain components of the tensor require a specific value for extremization.

Inspecting eq. (2.1) we find that the higher-derivative Lagrangian is R-symmetric, if

$$R(T_{ij\bar{k}\bar{l}}) = 0\,. \tag{2.31}$$

The most general coupling tensor at quadratic order in fields respecting the $\mathbb{Z}_2$- and R-symmetry is given by

$$T = T_{(0)} + T_{(1)}|A_0|^2 + T_{(2)}|A_1|^2 + T_{(3)}|A_2|^2 + T_{(4)}(A_1^2 + \bar{A}_1^2)\,. \tag{2.32}$$

For simplicity we suppressed the tensor indices of T and $T_{(0)}, \ldots, T_{(4)}$ here. From eq. (2.30) we see that A_0 is fixed in the minimum to the value $\langle A_0 \rangle = 0$, in which the R-symmetry is preserved, unless the following couplings vanish

$$T^{00\bar{0}\bar{0}}_{(1)} = T^{10\bar{0}\bar{0}}_{(1)} + T^{00\bar{1}\bar{0}}_{(2)} = T^{20\bar{0}\bar{0}}_{(1)} + T^{00\bar{2}\bar{0}}_{(1)} = 0\,. \tag{2.33}$$

In a generic effective field theory there is no reason why these couplings could be zero and so one concludes that indeed A_0 is fixed. Note furthermore that if the R-symmetry would have been broken in the minimum, then a flat direction associated with the respective Goldstone boson would have persisted. Finally, note that the flatness of A_0 can also be lifted by including higher-dimensional operators into the Kähler- or superpotential.

3 Higher-derivative terms in $\mathcal{N} = 1$ supergravity

3.1 Preliminaries

Let us now couple the theory specified in (2.1) to supergravity. We will only reproduce the essential steps here and refer the reader for a detailed derivation to the original paper [3].

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Without any higher-derivative operator the Lagrangian is given by [27]

$$\mathcal{L}_{(0)} = \int \mathrm{d}^2\Theta\, 2\mathcal{E}\left[\frac{3}{8}(\bar{\mathcal{D}}^2 - 8R)\mathrm{e}^{-K(\Phi^i,\Phi^{\dagger j})/3} + W(\Phi_i)\right] + \text{h.c.}\,, \tag{3.1}$$

where $\mathcal{E}$ denotes the chiral density, R the curvature superfield and $\bar{\mathcal{D}}^2 = \bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}$ with $\bar{\mathcal{D}}_{\dot\alpha}$ being the covariant spinorial derivative. To obtain the Einstein-frame Lagrangian for the scalar fields A_i, it is necessary to perform a Weyl transformation of the vielbein and successively integrate out all the auxiliary fields. This results in the familiar scalar potential

$$V_{(0)} = \mathrm{e}^K\left(G^{i\bar{j}}D_iW\bar{D}_{\bar{j}}\bar{W} - 3|W|^2\right)\,, \tag{3.2}$$

where $D_iW = W_{,i} + K_{,i}W$ is the Kähler covariant derivative of the superpotential.

To couple the higher-derivative operator of eq. (2.1) to supergravity one can either add the term [3]

$$\mathcal{L}_{(1)} = -\frac{1}{64}\int \mathrm{d}^2\Theta\, \mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}\Phi^i\mathcal{D}\Phi^j\bar{\mathcal{D}}\Phi^{\dagger\bar{k}}\bar{\mathcal{D}}\Phi^{\dagger\bar{l}}T_{ij\bar{k}\bar{l}} + \text{h.c.} \tag{3.3}$$

to (3.1) or modify the Kähler potential as[13]

$$K(\Phi^i,\Phi^{\dagger\bar{j}}) \to K(\Phi^i,\Phi^{\dagger\bar{j}}) + \frac{1}{16}T_{ij\bar{k}\bar{l}}\mathcal{D}\Phi^i\mathcal{D}\Phi^j\bar{\mathcal{D}}\Phi^{\dagger\bar{k}}\bar{\mathcal{D}}\Phi^{\dagger\bar{l}}\,. \tag{3.4}$$

Due to (2.5) the bosonic Lagrangians obtained by the two methods coincide up to a Kähler factor, which can be absorbed in a redefinition of T. Here we assume that $T_{ij\bar{k}\bar{l}}$ only depends on the chiral and anti-chiral superfields Φ and $\Phi^\dagger$ but not on the gravitational multiplet.

In the Lagrangian $\mathcal{L} = \mathcal{L}_{(0)} + \mathcal{L}_{(1)}$ one performs the same Weyl-transformation as before and integrates out the auxiliary fields in the gravitational multiplet. This procedure is not affected by the presence of $\mathcal{L}_{(1)}$. One is then left with the Lagrangian [3]

$$\begin{aligned}\frac{\mathcal{L}_{\text{bos}}}{\sqrt{-g}} =& -\frac{1}{2}\mathcal{R} - G_{i\bar{k}}\partial_\mu A^i\partial^\mu\bar{A}^{\bar{k}} + G_{i\bar{k}}\mathrm{e}^{K/3}F^i\bar{F}^{\bar{k}} + \mathrm{e}^{2K/3}\left[F^iD_iW + \bar{F}^{\bar{k}}\bar{D}_{\bar{k}}\bar{W}\right] + 3\mathrm{e}^K|W|^2\\ &+T_{ij\bar{k}\bar{l}}(\partial_\mu A^i\partial^\mu A^j)(\partial_\nu\bar{A}^{\bar{k}}\partial^\nu\bar{A}^{\bar{l}}) - 2T_{ij\bar{k}\bar{l}}\mathrm{e}^{K/3}F^i\bar{F}^{\bar{k}}(\partial_\mu A^j\partial^\mu\bar{A}^{\bar{l}}) + T_{ij\bar{k}\bar{l}}\mathrm{e}^{2K/3}F^iF^j\bar{F}^{\bar{k}}\bar{F}^{\bar{l}}.\end{aligned} \tag{3.5}$$

The equations of motion for F^i now read

$$G_{i\bar{k}}F^i + \mathrm{e}^{K/3}\bar{D}_{\bar{k}}\bar{W} + 2F^i(\mathrm{e}^{K/3}F^j\bar{F}^{\bar{l}} - \partial_\mu A^j\partial^\mu\bar{A}^{\bar{l}})T_{ij\bar{k}\bar{l}} = 0\,. \tag{3.6}$$

After the discussion in the previous section we only focus on the analytic solution of (3.6).[14] Here it is sufficient to know the auxiliary fields up to linear order in the coupling tensor. They read

$$\begin{aligned}F^i &= F^i_{(0)} + F^i_{(1)}\,, \qquad F^i_{(0)} = -\mathrm{e}^{K/3}G^{i\bar{k}}\,\bar{D}_{\bar{k}}\bar{W},\\ F^i_{(1)} &= 2\mathrm{e}^{4K/3}\,T^{\bar{k}\bar{l}ij}\,\bar{D}_{\bar{k}}\bar{W}\,\bar{D}_{\bar{l}}\bar{W}\,D_jW - 2\mathrm{e}^{K/3}T^{\bar{k}}{}_j{}^i{}_{\bar{l}}\,(\partial_\mu A^j\partial^\mu\bar{A}^{\bar{l}})\,\bar{D}_{\bar{k}}\bar{W}\,.\end{aligned} \tag{3.7}$$

[13] This type of procedure of coupling a higher-derivative operator to supergravity was also used in [31].

[14] For the special case in eq. (2.25) we determine the exact analytic solution in appendix A.3.

Inserting the above auxiliary field into the Lagrangian in eq. (3.5) yields

$$\frac{\mathcal{L}_{\text{bos}}}{\sqrt{-g}} = -\frac{1}{2}\mathcal{R} - \left(G_{i\bar{k}} + 2\mathrm{e}^K T^{\bar{l}}{}_i{}^j{}_{\bar{k}}\, D_j W\, \bar{D}_{\bar{l}}\bar{W}\right) \partial_\mu A^i \partial^\mu \bar{A}^{\bar{k}} \\ + T_{ij\bar{k}\bar{l}}(\partial_\mu A^i \partial^\mu A^j)(\partial_\nu \bar{A}^{\bar{k}} \partial^\nu \bar{A}^{\bar{l}}) - V(A, \bar{A}) \ . \tag{3.8}$$

The scalar potential is corrected as follows

$$V = V_{(0)} + V_{(1)} \, , \tag{3.9}$$

where $V_{(0)}$ is given in (3.2) while

$$V_{(1)} = -\mathrm{e}^{2K} T^{\bar{i}\bar{j}kl} \bar{D}_{\bar{i}}\bar{W} \bar{D}_{\bar{j}}\bar{W} D_k W D_l W \ . \tag{3.10}$$

Analogous to eq. (2.22) the metric multiplying the ordinary kinetic term receives a correction. From eq. (3.8) we read off its form

$$\delta G_{i\bar{k}} = 2\mathrm{e}^K T^{\bar{l}}{}_i{}^j{}_{\bar{k}}\, D_j W\, \bar{D}_{\bar{l}}\bar{W} \ . \tag{3.11}$$

As in the global case this correction in general renders the metric non-Kähler.

3.2 Fate of flat directions and simple no-scale examples

Let us begin the analysis with the supersymmetric minima of the potential given in (3.2), (3.9) and (3.10). $\langle F^i \rangle$ denotes the order parameter for supersymmetry breaking. Analogous to the discussion with global supersymmetry eq. (3.6) implies that unbroken supersymmetry imposes the exact same condition as in a standard two-derivative supergravity, that is

$$\langle F^i \rangle = \langle D_i W \rangle = 0 \, , \qquad \langle V \rangle = -3\langle \mathrm{e}^K |W|^2 \rangle \ . \tag{3.12}$$

Thus, the location of the supersymmetric minima in field space are determined by $F^i_{(0)} = 0$ and they are unaffected by the presence of $F^i_{(1)}$. In particular, any flat direction of $V_{(0)}$ is preserved by $V_{(1)}$. In addition, $\langle W \rangle = 0$ corresponds to a Minkowski vacuum while $\langle W \rangle \neq 0$ corresponds to an AdS vacuum.

Let us now turn to minima with spontaneously broken supersymmetry. As in the global case $V_{(1)}$ is considered to be a perturbation of $V_{(0)}$ and the minimum $\langle A^i_{(0)} \rangle$ of $V_{(0)}$ is shifted to a nearby field value $\langle A^i_{(0)} \rangle \to \langle A^i_{(0)} \rangle + \langle \delta A^i \rangle$. Therefore qualitatively nothing changes except for the flat directions. Contrary to the case of global supersymmetry in the local case non-trivial models with vanishing potential exist. These are the no-scale models. The no-scale property is generally expected to be lost when higher-derivative corrections are taken into account, thus making it possible to lift flat directions. In the rest of this section we present a simple example to illustrate the fate of flat directions and make a first step towards the potential relevance to moduli stabilization.

More precisely we consider a model specified by a constant superpotential $W(A) = W_0$ and the Kähler potential

$$K(A, \bar{A}) = -p \ln(A + \bar{A}) \, , \tag{3.13}$$

where $p > 0$. This K is of the no-scale type in that it satisfies

$$G^{A\bar{A}} K_{,A} K_{,\bar{A}} = p \ . \tag{3.14}$$

In this case $V_{(0)}$ given in (3.2) is positive (negative) for $p > 3$ ($p < 3$) and vanishes identically for $p = 3$. Adding $V_{(1)}$ given in (3.10) and redefining $\hat{T} = T(A, \bar{A}) G^{-2}_{A\bar{A}}$ one obtains

$$V = V_{(0)} + V_{(1)} = (A + \bar{A})^{-p}(p-3)|W_0|^2 - \hat{T}(A + \bar{A})^{-2p} p^2 |W_0|^4 \ . \tag{3.15}$$

For $p = 3$ both real and imaginary parts of A are flat directions of $V_{(0)}$. We see that generically both flat directions are lifted unless the combination $\hat{T}(A + \bar{A})^{-6}$ is constant in $\text{Re}(A)$ and/or $\text{Im}(A)$. For example a continuous shift symmetry $A \to A+$ i const. which often holds perturbatively in string theory would protect the flat direction along $\text{Im}(A)$ in that $\hat{T}$ could not depend on $\text{Im}(A)$. In order to say something about the stability, however, one has to make some assumptions about the functional dependence of $\hat{T}$.

Let us now consider a very simple situation, in which the inclusion of $V_{(1)}$ stabilizes a certain direction. For instance if $p < 3$ and $\hat{T} = \text{const.}$,[15] the two terms in eq. (3.15) can balance for $\hat{T} < 0$ with a non-supersymmetric AdS minimum at

$$\langle A + \bar{A} \rangle = \left(\frac{2p^2}{p-3} \hat{T} \, |W_0|^2 \right)^{1/p} , \qquad \text{and} \qquad \langle V \rangle = \frac{(p-3)^2}{4p^2 \hat{T}} < 0 \ . \tag{3.16}$$

Furthermore we have to check whether the field-value in eq. (3.16) is within the regime, where the perturbative solution for the auxiliary field converges. An estimate for the boundary between the perturbative and non-perturbative regime can be obtained from the results of appendix A. Indeed, from eq. (A.11) one infers that the boundary lies at

$$\langle A + \bar{A} \rangle = \left(-\frac{27}{2} p \hat{T} |W_0|^2 \right)^{1/3} \ . \tag{3.17}$$

We see that $|\hat{T}| \, |W_0|^2$ has to be sufficiently large for some given p to ensure that the minimum in eqs. (3.16) still lies within the perturbative regime. For example, for $p = 1$ one needs $|\hat{T}||W_0|^2 \gtrsim 10^{-3}$.

The existence of the minima in eq. (3.16) are of particular interest in string theory, where the Kähler potential in (3.13) for $p = 1$ typically describes the geometry of the dilaton. For example in Calabi-Yau compactifications of the heterotic string the perturbative superpotential does not depend on the dilaton and background fluxes can generate a superpotential W_0, which is sufficiently big to ensure perturbativity. Of course a proper discussion of the dilaton in such scenarios lies outside the scope of this paper. We leave this to future research.

[15] $\hat{T} = \text{const.}$ can be motivated by the explicit computation of four-derivative terms in [32]. There the one-loop corrections to the typical no-scale supergravity inspired by the heterotic string were computed.

4 Consequences for moduli stabilization in type IIB

In this section we consider type IIB Calabi-Yau orientifold compactifications with background fluxes and the dynamics of the respective four-dimensional $\mathcal{N} = 1$ Kähler moduli sector. At lowest order in the effective action appropriate fluxes can stabilize the dilaton and complex structure moduli supersymmetrically, but the Kähler moduli are flat directions described by a no-scale model. The leading order $(\alpha')^3$-corrections in the bosonic ten-dimensional action include specific contractions of four Riemann-tensors [25, 26]. It was shown that these terms induce a correction to the Kähler potential of the Kähler moduli in the four-dimensional theory, that lifts the no-scale property [19]. Furthermore, the Kähler potential can receive certain string-loop corrections. These have been explicitly computed for toroidal orientifolds, such as $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ in [33] and for arbitrary Calabi-Yau threefolds their functional form has been inferred in [34].

Besides the R^4-term the action of the type IIB superstring in ten-dimensions receives several additional eight-derivative corrections at order $(\alpha')^3$. A subset of these terms accounts for the $(\alpha')^3$-piece of the 4D scalar potential, which was indirectly inferred in [19]. However, additional $(\alpha')^3$-corrections in 10D exist, which can contribute to the scalar potential in four dimensions. Even though the existence of these 10D terms is required by supersymmetry, their explicit structure is still unknown and so we cannot compute the respective corrections to the scalar potential directly. Still, one can argue that the proper off-shell completion of some of these corrections to the scalar potential is provided by the higher-derivative operator in eq. (3.3). In appendix D we explicitly prove that the respective corrections to the scalar potential cannot be described via a two-derivative theory, i.e. via a correction to the Kähler potential. Now similarly to [19] where the scalar potential was inferred indirectly by computation of α'-corrections to the two-derivative term, our strategy will be to compute the bosonic four-derivative term originating from the 10D $(\alpha')^3$-correction which generically contains a contribution from one of the additional derivative-type terms in eq. (3.8). This determines the respective correction to the scalar potential via supersymmetry using eq. (3.10). It turns out that the respective four-derivative term for the chiral scalars is descending from the explicitly known R^4-correction in 10D and, hence, can be computed exactly.[16] This identification is unique and will be discussed in a forthcoming publication. The detailed computation of the four-derivative terms of the four-dimensional theory can be found in appendix B. In this section we present the action in ten-dimensions and illustrate the influence of the individual terms on the theory in four dimensions. Afterwards we will display the resulting potential, which emerges from the results of appendix B and study the possible implications for moduli stabilization these novel corrections might bring.

[16]Strictly speaking this is only true at lowest order in superspace-derivatives. More precisely, one expects additional four-derivative terms involving factors of the flux superpotential and hence the overall volume. These have to be merged into off-shell operators with more than four superspace-derivatives. Thus, the respective correction to the scalar potential from such terms is subleading.

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4.1 Type IIB action and perturbative corrections

The low energy effective action of type IIB receives perturbative corrections in α' as well as in g_s. The leading order corrections to the action of the bulk fields arise at order $(\alpha')^3$ and consist of several eight-derivative terms. More specifically, the bosonic action takes the form

$$S_{IIB} = S_{b,0} + (\alpha')^3 S_{b,3} + \dots\,, \tag{4.1}$$

where $S_{b,0}$ denotes the tree level bosonic action of the bulk fields in the string-frame

$$S_{b,0} = -\frac{1}{\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-g}\mathrm{e}^{-2\phi}\left(R + 4(\partial\phi)^2 - \frac{1}{2\cdot 3!}H_3^2\right) + S_R + S_{cs}\,. \tag{4.2}$$

Eq. (4.2) contains the ordinary kinetic terms for the bosonic fields of the type IIB superstring as well as the Chern-Simons term. Here we displayed explicitly the NS-NS sector which includes the metric g, the ten-dimensional dilaton ϕ and a two-form with field strength H_3. In eq. (4.1) we neglected terms associated with localised sources, such as D3/D7 branes or O3/O7 orientifold planes. The contribution of the D3-branes to the scalar potential is cancelled by the tension of the O-planes. Wrapped D7-branes on the other hand contribute and the leading order $(\alpha')^2$-corrections to their action are relevant and were discussed in [17]. These corrections induce effective D3-brane charge and tension. Higher order α'-corrections to the action of the localised sources can be ignored here [35]. Recently additional (apparent) $(\alpha')^2$-corrections to the Kähler potential for the Kähler moduli were inferred from F-theory [36, 37]. These corrections are related to a redundancy in the underlying M-theory description [38] and can be absorbed via field-redefinitions [36, 37].

The term $S_{b,3}$ in eq. (4.1) contains the leading order, eight-derivative α'-corrections to the action of the bulk fields. The full explicit structure of $S_{b,3}$ is unknown. Nevertheless one can infer their general form to be schematically [35]

$$\begin{aligned} S_{b,3} \sim \frac{1}{\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-g}\Big[&R^4 + R^3\left(G_3G_3 + G_3\bar{G}_3 + \bar{G}_3\bar{G}_3 + F_5^2 + (\nabla\tau)^2\right)\\ &+ R^2\left(G_3^4 + G_3^2\bar{G}_3^2 + \dots + (\nabla G_3)^2 + (\nabla F_5)^2 + \dots\right)\\ &+ R\left(G_3^6 + \dots + G_3^2(\nabla G_3)^2 + \dots\right) + G_3^8 + \dots\Big]\,. \end{aligned} \tag{4.3}$$

Here G_3 is given by

$$G_3 = F_3 - \tau H_3\,, \tag{4.4}$$

where F_3 denotes the field strength of the RR two-form and τ is the axiodilaton, cf. eq. (B.18). Moreover, R schematically denotes the Riemann tensor and ∇G_3 the covariant derivative (defined with respect to the metric g). Besides G_3 and g the bosonic sector includes the axiodilaton τ as well as the self-dual five form field strength F_5. All indices within the terms in eq. (4.3) are suppressed. Note that expressions with a single factor of G_3 or F_5 are forbidden. The precise structure of some of the contributions in eq. (4.3) is explicitly known. Notably this is the case for the R^4 term to which we turn in a moment, but also all remaining quartic terms have been determined [39, 40]. Furthermore couplings of the type $R^3H_3^2$ and $R^2H_3^4$ are required to ensure supersymmetry [41]. These terms imply the existence of the $R^3G_3^2$, $R^3G_3\bar{G}_3$, $R^3\bar{G}_3\bar{G}_3$, $R^2G_3^2\bar{G}_3^2$ and further contributions in eq. (4.3).

Note that the R^4 contribution in eq. (4.3) is known exactly and has been determined in [25].[17] This particular sum of contractions of four Riemann tensors is usually denoted as[18]

$$J_0 = t_8t_8R^4 + \frac{1}{8}\epsilon_{10}\epsilon_{10}R^4 \ . \tag{4.5}$$

For the specific contractions in eq. (4.5) we refer to [25].

Eq. (4.3) implies that contributions to the scalar potential in four dimensions exist, which involve four powers of the three-form flux G_3.[19] Since the explicit form of the quartic terms in G_3 in ten dimensions are unknown, one cannot compute the respective correction to the potential directly. However, these terms can be supersymmetrically completed off-shell by the higher-derivative operator in eq. (3.3) which induces also four-derivative term. From eq. (3.8) and eq. (3.10) we observe that $V_{(1)}$ already has the correct superpotential-dependence, such that the corresponding coupling tensor must be independent on the flux-superpotential. Hence, the respective four-derivative terms in eq. (3.8) can only be generated via J_0. Since J_0 is known exactly, one can determine the four-derivative terms and infer the structure of the correction to the potential from eq. (3.8).[20] The explicit computation of the four-derivative term is performed in appendix B. Here we will only summarize the main steps. For simplicity only a single Kähler class deformation is turned on. However, we expect the inferred form of the correction to the scalar potential $V_{(1)}$ in eq. (4.21) to hold also in the case of arbitrarily many Kähler moduli (we explain this at the end of appendix B), as long as the superpotential does not receive non-perturbative corrections. Moreover, due to the presence of background fluxes the background metric has to involve a warp factor. Here we are interested in the behaviour of the potential at large volume and, therefore, we work in a weak-warping approximation in which we neglect all warping effects.

We set the four-dimensional piece of the metric to a Minkowski-form. Neglecting the warping the ten-dimensional metric in the string-frame then reads

$$\mathrm{d}s^2_{(10)} = \eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \hat{\mathcal{V}}^{1/3}(x)\mathrm{d}s^2_{(6)} \,, \tag{4.6}$$

[17] Note that also loop corrections contribute R^4-type terms which have been computed for instance in [42]. The tensor structure of these corrections is precisely the same as the tree-level term as required by supersymmetry [25].

[18] At tree level this contraction is present for both IIA and IIB and all factors coincide.

[19] Note that in the situation with localised sources and background fluxes turned on we expect these contributions to be present. On the other hand, in the context of $\mathcal{N}=2$ compactifications these corrections will be absent as no scalar potential for the moduli is generated. Indeed the corrections to the potential that will be computed in this section vanish when turning off fluxes.

[20] Naturally there can also be contributions to the scalar potential which arise from 10D terms with more than four powers of G_3. These can also be off-shell completed via eq. (3.3), but are of higher-order in superspace derivatives. The respective four-derivative terms now carry a dependence on the flux-superpotential and are obtained from G_3-dependent terms in the ten-dimensional action.

where $\hat{\mathcal{V}}$ describes the Kähler type deformation of the (string-frame) background metric of the Calabi-Yau threefold denoted by $\mathrm{d}s^2_{(6)}$. The next step then involves the computation of the components of the Riemann tensor and finally we determine the four-derivative terms for $\hat{\mathcal{V}}$, which emerge from J_0. Afterwards it is necessary to express the result in terms of the appropriate $\mathcal{N}=1$ variables and match to the four-derivative term inside the Lagrangian in eq. (3.8) to determine the form of $T_{ij\bar{k}\bar{l}}$. We will present the result in the next section. First it is necessary to establish the notation of the $\mathcal{N}=1$ theory and discuss the known contributions to the scalar potential.

4.2 Structure of scalar potential

Let us now proceed to discuss the general structure of the scalar potential including the known leading order α'- and string-loop corrections to the Kähler potential as well as the new, 'higher-derivative' α'-corrections. Concretely we consider the Kähler and superpotential that arises after integrating out the complex structure moduli and the dilaton

$$\begin{aligned} K &= -2\ln(\mathcal{V}+\frac{1}{2}\hat{\xi}) + \delta K^{KK}_{(g_s)} + \delta K^{W}_{(g_s)}\,, \\ W &= \frac{1}{\sqrt{2}}\sqrt{g_s}\mathrm{e}^{\langle K_{cs}\rangle/2}\,W_0 = \frac{1}{\sqrt{2}}\sqrt{g_s}\mathrm{e}^{\langle K_{cs}\rangle/2}\Big\langle \int_{M_3} G_3\wedge\Omega\Big\rangle\,. \end{aligned} \tag{4.7}$$

Here W_0 denotes the flux superpotential, which is the Gukow-Vafa-Witten superpotential evaluated at the supersymmetric minimum of the complex structure moduli and the dilaton. Furthermore, G_3 is given in eq. (4.4) and Ω is the $(3,0)$ form of the Calabi-Yau. $\langle K_{cs}\rangle$ denotes the Kähler potential for the complex structure moduli evaluated at their minimum, which reads $\langle K_{cs}\rangle = -\ln\langle -i\int_{M_3}\Omega\wedge\bar{\Omega}\rangle$. The additional factor of $\sqrt{g_s/2}\,\mathrm{e}^{\langle K_{cs}\rangle/2}$ in the superpotential stems from the Kähler potentials of the complex structure moduli and the dilaton after performing a Kähler transformation. Moreover, the total (Einstein-frame) volume modulus $\mathcal{V}$ can be expressed in terms of the (completely symmetric) triple intersection numbers k_{ijk} of the Calabi-Yau M_3 as well as the 2-cycle volumes t^i as follows

$$\mathcal{V} = \frac{1}{6}k_{ijk}t^it^jt^k\,. \tag{4.8}$$

The 4-cycle volumes τ_i, that constitute the imaginary components of the Kähler moduli T^i, are derived via

$$\tau_i = \frac{\partial\mathcal{V}}{\partial t^i} = \frac{1}{2}k_{ijk}t^jt^k\,. \tag{4.9}$$

From these definitions one infers

$$\mathcal{V} = \frac{1}{3}\tau_i t^i\,. \tag{4.10}$$

Furthermore $\hat{\xi}$ parametrizes the leading α'-corrections to the Kähler potential and is given by

$$\hat{\xi} = \xi g_s^{-3/2} = -\frac{(\alpha')^3\zeta(3)\chi(M_3)}{2(2\pi)^3 g_s^{3/2}}\,, \tag{4.11}$$

where $\chi(M_3) = 2(h^{1,1}-h^{2,1})$ is the Euler characteristic of M_3, g_s denotes the string-coupling and the Hodge numbers $h^{1,1}, h^{2,1}$ count the number of Kähler and complex structure moduli. The corrections $\delta K^{KK}_{(g_s)}$ and $\delta K^{W}_{(g_s)}$ in eq. (4.7) denote the leading order string-loop corrections. Their general form for arbitrary Calabi-Yau threefolds has been argued to be [34]

$$\delta K^{KK}_{(g_s)} \sim g_s \sum_{i=1}^{h^{1,1}} \frac{C_i(a_{ij}t^j)}{\mathcal{V}}\,, \qquad \delta K^{W}_{(g_s)} \sim \sum_{i=1}^{h^{1,1}} \frac{D_i(a_{ij}t^j)^{-1}}{\mathcal{V}}\,. \tag{4.12}$$

The first term is interpreted as coming from exchange of closed strings carrying Kaluza-Klein momentum, while the latter is coming from the exchange of winding strings. The coefficients C_i and D_i are expected to be functions of the complex structure moduli and the dilaton. However, since we assume the latter have already been stabilized, we treat C_i, D_i as constants. The matrix a_{ij} consists of combinatorial constants.

The scalar potential derived from eq. (4.7) including the higher-derivative term $\mathcal{L}_{(1)}$ in (3.3) can be split up as follows

$$V = V_{(0)} + V_{(1)} = V_{(\alpha')} + V_{(g_s)} + V_{(1)}\,. \tag{4.13}$$

The first term describes the scalar potential obtained from the Kähler potential in eq. (4.7) without string-loop corrections. It reads [43]

$$V_{(\alpha')} = \mathrm{e}^K 3\,\hat{\xi}|W|^2 \,\frac{\hat{\xi}^2 + 7\hat{\xi}\mathcal{V} + \mathcal{V}^2}{(\mathcal{V}-\hat{\xi})(2\mathcal{V}+\hat{\xi})^2}\,, \tag{4.14}$$

and has a runaway behaviour at large $\mathcal{V}$. Expanding around large volume yields

$$V_{(\alpha')} = \frac{3\hat{\xi}|W|^2}{4\mathcal{V}^3} + \mathcal{O}((\alpha')^6)\,. \tag{4.15}$$

When expanding the string-loop contribution to the potential, one obtains the following terms at leading order [44]

$$V_{(g_s)} = \sum_i \frac{|W|^2}{\mathcal{V}^2}\left[g_s^2 C_i^2 K_{(0),ii} - 2\delta K^{W}_{(g_s),\tau_i}\right]\,, \tag{4.16}$$

where $K_{(0)} = -2\ln(\mathcal{V})$.

Let us now turn to the higher-derivative operator. Inserting eq. (4.7) into eq. (3.10) the higher-derivative contribution generally has the form

$$V_{(1)} = -\mathrm{e}^{2K} T^{\bar{i}\bar{j}kl} K_{,\bar{i}} K_{,\bar{j}} K_{,k} K_{,l} |W|^4\,. \tag{4.17}$$

The result of appendix B are four-derivative terms for the four-cycle volumes, which when matched to eq. (3.8) yield the following coupling tensor

$$T_{ij\bar{k}\bar{l}} = \hat{\lambda}_0 (\Pi_m\, t^m) K_{(0),i} K_{(0),j} K_{(0),\bar{k}} K_{(0),\bar{l}}\,, \tag{4.18}$$

where we introduced $\hat{\lambda}_0 = (\alpha')^3 g_s^{-3/2}\lambda$ with λ being a universal combinatorial number that is not computed at this stage. A direct 4D reduction of the partially unknown 10D terms with four powers in the fluxes and their derivatives which contribute to the scalar potential of $\mathcal{L}_{(1)}$ would determine the sign of λ. Thus, at this point we treat it as an unknown

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real number. Note furthermore that $\hat{\lambda}_0 \sim (\alpha')^3 g_s^{-3/2}$ includes precisely the same expansion parameter that shows up in the correction to the Kähler potential via $\hat{\xi}$ given in (4.11). This is expected since both corrections originate from the same term in ten dimensions. However, the fact that $V_{(1)}$ is multiplied by an additional power of $|W|^2$ compared to $V_{(0)}$ implies that $V_{(1)}$ is subleading in g_s compared to $V_{(0)}$. Let us stress again that the result in eq. (4.18) holds only for $h^{1,1} = 1$. However, we expect the respective correction to the potential to be correct in general as we discuss at the end of appendix B and in appendix C. The numbers Π_i encode the topological information of the second Chern class c_2 of M_3. Specifically let us choose a basis $\hat{D}_i$ of harmonic $(1,1)$-forms, such that the Kähler form is expressed as

$$J = \sum_{i=1}^{h^{1,1}} \hat{D}_i t^i \ . \tag{4.19}$$

Then we have that

$$\Pi_i = \int_{M_3} c_2 \wedge \hat{D}_i \, , \qquad \Pi_i t^i = \int_{M_3} c_2 \wedge J \ . \tag{4.20}$$

In appendix B we discuss this term further. In particular we have $\int_{M_3} c_2 \wedge J > 0$ unless M_3 is a torus T^6. The variables in which J takes the form of eq. (4.19) span the Kähler cone and, thus, we have $t_i \geq 0$ independently for all two-cycle volumes, see e.g. [23]. Accordingly, in order to ensure $\int_{M_3} c_2 \wedge J > 0$, we must have that $\Pi_i \geq 0$ in this basis. In section 4.4 we compute these topological numbers for an explicit example.

Inserting (4.18) into eq. (4.17) we can read off the correction to the potential

$$V_{(1)} = -\hat{\lambda} \frac{|W|^4}{\mathcal{V}^4} (\Pi_i \, t^i) \, , \tag{4.21}$$

where we abbreviate $\hat{\lambda} = 3^4 \hat{\lambda}_0$. To understand the volume-behaviour of the individual terms in the potential (4.13) in the large volume limit we set $h^{1,1} = 1$. In this case we obtain

$$\frac{V}{|W|^2} \sim \hat{\xi} \mathcal{V}^{-3} + (g_s^2 C_1^2 + D_1) \mathcal{V}^{-10/3} - \hat{\lambda} |W|^2 \Pi_1 \mathcal{V}^{-11/3} \, , \tag{4.22}$$

where we ignored numerical factors.[21] The higher-derivative contribution scales slightly steeper with the volume than the two string-loop contributions. Moreover, it differs by a factor of $|W_0|^2$ and by powers of the string coupling $g_s^{-1/2}$ and $g_s^{-5/2}$ with respect to the string-loop corrections.

Before studying the implication of $V_{(1)}$ for Kähler moduli stabilization, we should pause for a moment to present a better understanding of the individual pieces of the ten-dimensional action, which was displayed in eq. (4.1) and eq. (4.3). As already mentioned the remaining terms in eq. (3.8), such as the corrections to the scalar potential and to the ordinary kinetic term are related to different, partially unknown terms in the ten dimensional action, which are connected by supersymmetry. Furthermore we wish to analyze the relevance of further higher-order corrections in the large-volume expansion. Let us

[21] We thank Michele Cicoli and Francisco Pedro for comments regarding the g_s-dependence and the correct form of the string-loop corrections.

also stress again that the correction in eq. (4.21) cannot be off-shell completed within a two-derivative theory. More precisely it is not possible to describe it via a correction to K and W. We provide an explicit proof of this statement in appendix D.

Let us see how the individual terms in eq. (4.3) contribute to the four-dimensional action after compactification. J_0 generates derivative terms for the Kähler moduli, but does not contribute to the potential if warping is neglected. This is due to the fact, that after turning off fluxes this term is still present. However, in the respective $\mathcal{N} = 2$ theory, no potential can be generated for the moduli as all α'-corrections merely renormalise the definition of the tree-level moduli. Thus, J_0 induces derivative-corrections, such as the $\hat{\xi}$-contribution to the Kähler potential in eq. (4.7), the four-derivative terms that we computed in appendix B and further six- and eight-derivative terms. The $\hat{\xi}$-corrections imply the existence of the potential in eq. (4.15) in the $\mathcal{N} = 1$ theory. It was noted in [19] that after transforming into the Einstein-frame in the four-dimensional action the H_3^2 term in eq. (4.2) indeed produces the correct functional form of eq. (4.15). However, to obtain the correct prefactors it was concluded that necessarily also $R^3G_3^2$ terms have to be present. Indeed, we have

$$V_{(\alpha')} \sim \underbrace{\chi(M_3)}_{R^3}\underbrace{|W_0|^2}_{G_3\bar{G}_3}\,, \tag{4.23}$$

where we used that $\chi(M_3) \sim \int \mathrm{d}^6y\sqrt{g}Q$ with the six-dimensional Euler integrand Q being a contraction of three Riemann tensors. The corrections of the type $R^2(\nabla G_3)^2$ also contribute to $V_{(\alpha')}$.

Next let discuss the additional terms in the Lagrangian which accompany the four-derivative terms in eq. (3.8). The non-Kähler correction to the two-derivative term in eq. (3.11) is induced by the terms of the type $R^3G_3\bar{G}_3$. These corrections have the form

$$\delta G_{i\bar{j}}\,\partial_\mu T^i\partial^\mu\bar{T}^{\bar{j}} \sim \frac{\hat{\lambda}}{\mathcal{V}^2}\underbrace{|W_0|^2}_{\sim G_3\bar{G}_3}\underbrace{\Pi_m t^m}_{\sim R^2}\underbrace{K_{(0),i}K_{(0),\bar{j}}\partial_\mu T^i\partial^\mu\bar{T}^{\bar{j}}}_{\sim R}\,, \tag{4.24}$$

where we used eq. (B.11). Furthermore the terms with two Riemann tensors in eq. (4.3) generate $V_{(1)}$ in eq. (4.21), since

$$V_{(1)} \sim \frac{\hat{\lambda}}{\mathcal{V}^4}\underbrace{|W_0|^4}_{\sim G_3^2\bar{G}_3^2}\underbrace{(\Pi_i\, t^i)}_{\sim R^2}\ . \tag{4.25}$$

Furthermore $V_{(1)}$ can be induced by terms of the type $RG_3^2(\nabla G_3)^2$.[22]

Let us now make a few remarks regarding the terms we did not discuss so far. To begin with there exist corrections with additional derivatives of the dilaton. These terms do not contribute to the scalar potential, but are important for the consistency of the equations of motion. More precisely the presence of the R^4 terms demands the addition of terms of the type $R^3(\nabla\tau)^2$ [19]. Furthermore we have terms involving the self-dual five-form F_5.

[22] Note that in [35] a naive estimate for the volume dependence of the potential induced by the $R^2G_3^4$ terms was found to be $\mathcal{V}^{-11/3}$. This is in agreement with eq. (4.21).

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In compactifications with imaginary self-dual fluxes warping effects generate a flux for the five-form [17]. Since we ignore the warp factor here, we will not discuss this term further. However, in principle warping-induced corrections to the scalar potential are relevant, since naive dimensional arguments suggest that these contribute at $\mathcal{O}(\mathcal{V}^{-11/3})$ [35]. A proper accounting of such effects is outside the scope of this paper and will be left to future investigations. Moreover we have terms of the type RG_3^6 and G_3^8 in eq. (4.3). Dimensional analysis yields that the contributions to the scalar potential coming from both terms are suppressed by additional powers of $\mathcal{V}^{-2/3}$ and $\mathcal{V}^{-4/3}$ with respect to $V_{(1)}$ [35]. Furthermore naively one finds that a reduction of RG_3^6 yields a factor of $c_1(M_3)$, which vanishes for a Calabi-Yau orientifold compactification at order $(\alpha')^3$, see also appendix B.

The potential in eq. (4.14) also induces subleading terms at the level of $(\alpha')^6$, which scale as $\mathcal{V}^{-4}$. Besides the fact that their volume-dependence is slightly suppressed compared to $V_{(1)}$, they involve a factor $\hat{\xi}^2$, which is rather small for CY threefolds with small Euler number and moderate g_s-values.

To conclude this section let us make a remark regarding the expansion in higher-derivatives of the action in eq. (4.3). The expansion in α' in ten dimensions is indeed an expansion in higher-derivatives. However, when compactifying the R^4 term in eq. (4.3), one obtains two, four, six and eight-derivative terms for the volume modulus, cf. appendix B. Thus, in the four dimensional theory the α'-expansion is still roughly controlling the expansion in higher-derivatives, but several higher-derivative terms might appear at the same order in α'. This implies that the coupling tensor in eq. (4.18) cannot control all higher-derivatives, but possibly only a subclass. Moreover, let us briefly revisit the general discussion in section 2.2, as we now have an example with an explicit expansion parameter given by α'. Recall that we identified the analytic branch as the unique physical theory. Since $T_{ij\bar{k}\bar{l}} \sim (\alpha')^3$ we find evidence for this once more. In particular the non-analytic branches would require the presence of terms in ten-dimensions, which are $\mathcal{O}(\alpha'^{-3})$. Furthermore note that it would not be meaningful to discuss the corrections at order $\mathcal{O}(T^2)$, as we would have to include ten-dimensional terms of order $(\alpha')^6$ into the analysis.

4.3 Stabilization of the volume for $h^{1,1} = 1$

Using the leading order α'-correction to the Kähler potential accompanied by non-perturbative corrections to the superpotential there exist scenarios, where all Kähler moduli can be frozen [20, 21, 43, 45]. Later works incorporated also string-loop-corrections in the Kähler potential into the analysis [33, 34, 44, 46, 47]. In all of these scenarios the non-perturbative superpotential is necessary for the stability of the overall volume. Attempts to stabilize the volume modulus without the non-perturbative superpotential including string-loop corrections were made in [46], but a significant amount of fine-tuning of the complex-structure moduli was required. In addition the structure of the string-loop corrections is very model-dependent and a case-by-case study is necessary.

In the following we will entertain the possibility that the overall volume and all four-cycle volumes are stabilized purely by α'-corrections instead of the non-perturbative corrections to the superpotential. The leading order $(\alpha')^3$-corrections are partially captured by the higher-derivative corrections together with the known corrections to the Kähler potential. It is instructive to discuss a stabilization first in the simple case of $h^{1,1} = 1$. We will generalize the analysis to an arbitrary number of four-cycles in the next section. In the following we neglect string-loop corrections to the scalar potential. Since these are suppressed by powers $g_s^{1/2}$ and $g_s^{5/2}$, respectively, relative to both $(\alpha')^3$-contributions, a moderate tuning of $g_s < 1$ should suffice to parametrically suppress them. Moreover, note that from the discussion in refs. [33, 46] it is also expected that the coefficients C_i and D_i in eq. (4.12) are small. Indeed, in the explicit computations they are suppressed by loop factors of $1/(128\pi^4)$ and thus small, unless the complex structure moduli are frozen at large values. Note that fluxes typically stabilize the complex structure moduli at smaller values $\langle Z^a \rangle < 1$. In this case one also finds that $\mathrm{e}^{\langle K_{cs} \rangle} \gtrsim 1$, which leads to an additional enhancement of $V_{(1)}$ over the string-loop corrections.

The potential in eq. (4.22) is then minimized at

$$\langle \mathcal{V} \rangle \sim \left(g_s\, \mathrm{e}^{\langle K_{cs} \rangle} \frac{\hat{\lambda} |W_0|^2 \Pi_1}{\hat{\xi}} \right)^{3/2} \tag{4.26}$$

under the assumption that $\hat{\lambda} < 0$ and $\hat{\xi} < 0$. The latter requirement is fulfilled for any Calabi-Yau with $\chi(M_3) > 0$ or in other words $h^{1,1}(M_3) > h^{2,1}(M_3)$. Note that background fluxes require $h^{2,1}(M_3) \geq 1$. Hence, we need at least two Kähler moduli to satisfy $\chi(M_3) > 0$. Thus, the above analysis is not realistic. However, based on this simple example one would naively expect that in the case of $h^{1,1} > 1$ one finds a stabilized volume only if $\chi(M_3) > 0$ and $\lambda < 0$. This is indeed confirmed in the next section.

Supersymmetry is broken in the vacuum given in eq. (4.26) which can be seen as follows. From section 3.2 we know that supersymmetry is broken, if it is broken at two-derivative level. Suppose supersymmetry was unbroken, then one could derive the minimum from eq. (3.12). However, necessarily all such points would be $\hat{\lambda}$-independent, which is not satisfied for our minimum. Thus, supersymmetry is indeed broken in the vacuum in eq. (4.26). Furthermore computation shows that it is an AdS vacuum with a value of the cosmological constant given by

$$\langle V \rangle \sim \frac{\hat{\xi} |W|^2}{\langle \mathcal{V} \rangle^3} < 0 \tag{4.27}$$

In the next section we generalize to the case $h^{1,1} > 1$ and prove the existence of a general minimum. Finally, we note that this minimum does not arise by balancing two terms in the same expansion at different order. Instead, both terms originate from 10d terms which are of the same order in α' and g_s. Moreover, in the four-dimensional theory we formally have an expansion in the coupling tensor, which controls the higher-derivative corrections, as well as in $\hat{\xi}$, which controls $V_{(\alpha')}$ in eq. (4.14). From this point of view, in the minimization we are comparing leading order terms, which are associated with different expansions.

4.4 Existence of model-independent minimum

Neglecting string-loop corrections and taking the large-volume limit the potential given in (4.13) reads

$$V = \frac{3\hat{\xi} |W|^2}{4\mathcal{V}^3} - \hat{\lambda} |W|^4 \frac{\Pi_i t^i}{\mathcal{V}^4} \,. \tag{4.28}$$

For $\hat{\lambda} < 0$ we will now show that V has a non-supersymmetric AdS minimum for any orientifolded Calabi-Yau threefold with $\chi(M_3) > 0$ where *all* four-cycles are fixed as

$$\langle\tau_i\rangle = \mathcal{C}\,\Pi_i\,, \qquad \text{with} \qquad \mathcal{C} = \frac{44\hat{\lambda}|W|^2}{9\hat{\xi}}\,. \tag{4.29}$$

The volume in this minimum is given by

$$\langle\mathcal{V}\rangle = \frac{1}{3}\mathcal{C}\,\Pi_k\langle t^k\rangle = \frac{44}{27}\left\langle\int c_2\wedge J\right\rangle\frac{\hat{\lambda}|W|^2}{\hat{\xi}} \sim \Pi_k\langle t_0^k\rangle\left(\frac{\hat{\lambda}|W|^2}{\hat{\xi}}\right)^{3/2}, \tag{4.30}$$

where $\langle t_0^i\rangle$ do not depend on $\mathcal{C}$, but are implicit functions of the Π_i. Moreover, positivity of the four-cycles requires that $\Pi_i > 0$ for all $i = 1,\ldots,h^{1,1}$. As we already mentioned when choosing the correct Kähler cone variables one has $\Pi_i \geq 0$, so we have to require that $\Pi_i \neq 0$.

In order to prove the existence of this minimum it is sufficient to show that the potential in eq. (4.28) is minimal as a function of the two-cycle volumes t^i as it is then also minimal in terms of the four-cycle volumes τ_i. The first derivatives of eq. (4.28) read

$$\frac{\partial V}{\partial t^i} = \frac{|W|^2}{\mathcal{V}^5}\left[-\frac{3}{4}\hat{\xi}\tau_i\left(t^i\tau_i\right) - \frac{1}{3}\hat{\lambda}|W|^2\Pi_i\left(t^j\tau_j\right) + 4\hat{\lambda}|W|^2\tau_i\left(\Pi_j t^j\right)\right], \tag{4.31}$$

where we used eq. (4.10). Inserting the values of the four-cycle volumes given in eq. (4.29) one finds that indeed $\langle\partial V/\partial t^i\rangle = 0$. From eq. (4.10) we also obtain the first equality in eq. (4.30). To determine the overall dependence of $\langle\mathcal{V}\rangle$ on $\mathcal{C}$, note that the two-cycles are implicitly defined via eq. (4.9), which at the extremal point is given by

$$k_{ijk}\langle t^j\rangle\langle t^k\rangle = 2\mathcal{C}\Pi_i\,. \tag{4.32}$$

This implies $\langle t^i\rangle = \sqrt{\mathcal{C}}\langle t_0^i\rangle$, where t_0^i do not depend on $\mathcal{C}$. With this we obtain the scaling of the volume with respect to $|W|, \hat{\xi}$ and $\hat{\lambda}$ in eq. (4.30).

It remains to analyse the matrix of second derivatives. In general it reads

$$\begin{aligned}\frac{\partial^2 V}{\partial t^i\partial t^j} = \frac{|W|^2}{\mathcal{V}^6}\bigg[&9\hat{\xi}\mathcal{V}\tau_i\tau_j + 4\hat{\lambda}|W|^2\mathcal{V}\left(\tau_i\Pi_j + \Pi_i\tau_j\right) - 20\hat{\lambda}|W|^2(\Pi_k t^k)\tau_i\tau_j \\ &+ \frac{\partial\tau_j}{\partial t^i}\left(4\hat{\lambda}|W|^2\mathcal{V}(\Pi_k t^k) - \frac{9}{4}\hat{\xi}\mathcal{V}^2\right)\bigg]\,.\end{aligned} \tag{4.33}$$

Making use of eq. (4.10) we find that at the extremal point this simplifies to

$$\left\langle\frac{\partial^2 V}{\partial t^i\partial t^j}\right\rangle = a\Pi_i\Pi_j + bk_{ijk}\langle t^k\rangle\,, \quad \text{where} \qquad a = -\frac{8\hat{\lambda}|W|^4\mathcal{C}}{\langle\mathcal{V}\rangle^5}\,, \qquad b = \frac{9}{44}\frac{\hat{\xi}|W|^2}{\langle\mathcal{V}\rangle^4}\,. \tag{4.34}$$

For $\lambda < 0$ and $\chi(M_3) > 0$ we see that $a > 0$ and $b < 0$. For any vector with components x_i we have $(x_i\Pi_i)(x_j\Pi_j) \geq 0$ and so $a\Pi_i\Pi_j$ is a positive-semidefinite matrix. The matrix $k_{ijk}t^k$ was studied in [48] and shown to have signature $(1, h^{1,1}-1)$. In other words there exists an orthogonal decomposition of the $h^{1,1}$-dimensional vector space into a one-dimensional subspace, on which $k_{ijk}t^k$ is positive definite and an $(h^{1,1}-1)$-dimensional complement on which it is negative definite. Here orthogonality is defined with respect to the inner product determined by $k_{ijk}t^k$. The one-dimensional subspace is spanned by the vector with components t^i, as the volume has to be positive. Since we have $b<0$ the signature of $bk_{ijk}\langle t^k\rangle$ reads $(h^{1,1}-1,1)$. On the $(h^{1,1}-1)$-dimensional subspace the sum $a\Pi_i\Pi_j+bk_{ijk}\langle t^k\rangle$ must hence be positive-definite. On the one-dimensional subspace we find

$$\langle t^i\rangle \left\langle \frac{\partial^2 V}{\partial t^i \partial t^j}\right\rangle \langle t^j\rangle = -\frac{22\hat{\lambda}|W|^4\mathcal{C}}{3\langle\mathcal{V}\rangle^5}\Big(\Pi_k\langle t^k\rangle\Big)^2 > 0\,, \tag{4.35}$$

which shows that the matrix of second derivatives is also positive definite there.

It remains to be shown, that the matrix (4.34) is positive definite on the whole space. A generic non-zero vector with components x^i can be decomposed as $x^i=\mu\langle t^i\rangle+x^i_\perp$, where $\mu\in\mathbb{R}$ and $x^i_\perp$ is the component of x^i in the subspace orthogonal to the one-dimensional space spanned by $\langle t^i\rangle$. Since

$$\Pi_i\Pi_j\langle t^j\rangle \sim \Pi_i \sim k_{ijk}\langle t^j\rangle\langle t^k\rangle \tag{4.36}$$

we have the following orthogonality relations

$$x^i_\perp k_{ijk}\langle t^j\rangle\langle t^k\rangle = x^i_\perp \Pi_i\Pi_j\langle t^j\rangle = 0\ . \tag{4.37}$$

With this we find

$$x^i\left\langle \frac{\partial^2 V}{\partial t^i \partial t^j}\right\rangle x^j = x^i_\perp\Big(a\Pi_i\Pi_j+bk_{ijk}\langle t^k\rangle\Big)x^j_\perp+\mu^2\langle t^i\rangle\Big(a\Pi_i\Pi_j+bk_{ijk}\langle t^k\rangle\Big)\langle t^j\rangle>0\,, \tag{4.38}$$

since the matrix is positive on the respective subspaces. We conclude that the matrix in eq. (4.34) is positive definite.

In addition we have to establish that the locus specified in eq. (4.29) is a minimum of the potential, which includes also the dilaton as well as the complex structure moduli. The answer can be easily obtained in the spirit of [21]. Indeed the potential including the dilaton and complex-structure moduli reads [19]

$$V=\mathrm{e}^K(G^{a\bar{b}}D_aWD_{\bar{b}}\bar{W}+G^{\tau\bar{\tau}}D_\tau WD_{\bar{\tau}}\bar{W})+\mathrm{e}^K\frac{\xi}{2\mathcal{V}}(WD_{\bar{\tau}}\bar{W}+\bar{W}D_\tau W)+V_{(\alpha')}+V_{(1)}\,, \tag{4.39}$$

where a,b label complex structure moduli directions. W denotes the the Gukov-Vafa-Witten superpotential. The first term in the above potential is positive definite and has a $\mathcal{V}^{-2}$ behaviour at large volume. At the extremal condition $D_aW=D_\tau W=0$, it vanishes identically and is positive around this value. Since it dominates over the subleading $\mathcal{O}(\mathcal{V}^{-3})$ and $\mathcal{O}(\mathcal{V}^{-11/3})$ terms coming from $V_{(\alpha')}+V_{(1)}$, eq. (4.29) represents a minimum of the full potential. Of course also the dilaton and complex structure moduli will receive higher-derivative corrections, which contribute in $V_{(1)}$. However, these terms have a subleading volume-dependence compared to the first terms in eq. (4.39) and thus do not spoil the argument.

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As in the preceding section, supersymmetry is broken in the minimum. Up to numerical factors the value of the potential in the minimum reads

$$\langle V\rangle \sim \frac{\hat{\xi}}{|W|^7}\left(\frac{\hat{\xi}}{\hat{\lambda}}\right)^{9/2} . \tag{4.40}$$

We can estimate the gravitino mass from the ordinary two-derivative theory. It reads

$$m_{3/2} \sim \mathrm{e}^{K/2}|W| \sim \frac{|W|}{\mathcal{V}} \sim \frac{\hat{\xi}^{3/2}}{\hat{\lambda}^{3/2}|W|^2\Pi_i\langle t_0^i\rangle} . \tag{4.41}$$

Note that the corrections $F_{(1)}$ contribute only subleading here. Let us compare the gravitino mass with the string scale and Kaluza-Klein scale [35]

$$m_s \sim \frac{1}{\sqrt{\mathcal{V}}} , \qquad m_{KK} \sim \frac{1}{\mathcal{V}^{2/3}} . \tag{4.42}$$

Direct computation reveals that

$$\frac{m_{3/2}}{m_s} \sim \frac{\hat{\xi}^{3/4}}{\hat{\lambda}^{3/4}\sqrt{|W|\Pi_i\langle t_0^i\rangle}} . \tag{4.43}$$

Furthermore, from eq. (4.32) we find that roughly $\langle t_0^i\rangle \sim \sqrt{\Pi_i}$. Let Π denote a typical value for the topological numbers Π_i, then we can estimate

$$\Pi_i\langle t_0^i\rangle \sim h^{1,1}\Pi^{3/2} . \tag{4.44}$$

In the next section we show that $\Pi \sim \mathcal{O}(10\ldots 100)$. Furthermore, we can estimate the size of $\hat{\lambda}$ by the combinatorial part of $\hat{\xi}$. In other words we roughly expect that $|\hat{\lambda}|\sim |\hat{\xi}/\chi(M_3)|$. Altogether, the scale-quotients read[23]

$$\frac{m_{3/2}}{m_s} \sim \mathrm{e}^{-\langle K_{cs}\rangle/4} g_s^{-1/4} \frac{\chi(M_3)^{3/4}}{\sqrt{|W_0|\, h^{1,1}\,\Pi}} \lesssim \mathcal{O}(10^{-1}) , \qquad \frac{m_{3/2}}{m_{KK}} \sim \frac{\chi(M_3)^{1/2}}{(h^{1,1})^{1/3}\sqrt{\Pi}} < 1 . \tag{4.45}$$

To obtain more accurate expressions for $m_{3/2}/m_s$ and $m_{3/2}/m_{KK}$, it will be necessary to compute $\hat{\lambda}$ and study the minimum for explicit examples. Note furthermore, that $m_{3/2}/m_{KK} \ll 1$ in order to ensure that higher superspace-derivative corrections and hence higher corrections to the scalar potential of the type $(F_{(0)})^n$ with $n > 4$ are under control [49]. This can be achieved best by choosing a geometry with $\chi(M_3) \sim \mathcal{O}(1)$ and $h^{1,1} \gg 1$.[24]

Let us finish this section with some remarks. Firstly let us stress again that the stabilization of the four-cycle volumes proposed here does not require any non-perturbative effects, but occurs purely from considering the leading order $(\alpha')^3$-corrections in the potential. Note furthermore that even though a Calabi-Yau might have some $\Pi_i = 0$, the overall volume is stabilized at a positive value. In such cases it could still happen that string-loop

[23]We thank Shanta de Alwis for helpful comments and discussions regarding this point.

[24]We thank Michele Cicoli for helpful comments regarding this point.

or other α'-corrections shift the minimum to a point at which all four-cycles are positive and the overall volume is roughly the same.

Consequently, we might now worry about the size of the flux density

$$\rho_{flux} = \frac{1}{\alpha'} \left(\int d^6y \, G_3 \cdot \bar{G}_3 \right)^{1/2} \sim \frac{W_0}{\mathcal{V}} \quad . \tag{4.46}$$

While supersymmetric flux stabilization of the type IIB axio-dilaton and the complex structure moduli has vanishing F-terms $D_\tau W = D_a W = 0$ which removes their contribution to the flux density (see e.g. section 2.3 in [49]), volume stabilization requires the $(0,3)$-piece of G_3 to be non-zero in order to generate the VEV for W in the first place. Hence, the F-terms of the Kähler moduli still produce a flux density

$$\rho_{flux} \sim \left(e^K K^{i\bar{j}} D_i W_0 \bar{D}_{\bar{j}} \bar{W}_0 \right)^{1/2} \sim \frac{W_0}{\mathcal{V}} \tag{4.47}$$

scaling the same way as the naive ten-dimensional estimate above. However, inserting the scaling of the volume in our vacuum, we note that the fraction $|W_0|/\mathcal{V} \sim |W_0|^{-2}$ and so one expects the flux density to actually decrease with increasing W_0 — quite contrary to the situation known for KKLT or LVS class vacua.

In the minimum eq. (4.29) the value of the cosmological constant is negative. To lift this vacuum to a metastable dS one may introduce an uplifting sector in the same way as it is done for LVS. We do not see any obstacles to an uplifting, since supersymmetry is already broken for eq. (4.29).

4.5 Estimating the size of the Π_i — a simple explicit example

At this point we have established the functional form of the contribution from the higher-derivative correction to the scalar potential. Moreover, we know that the positivity of the 2nd Chern class guarantees the positive semi-definiteness of its expansion coefficients Π_i when using proper Kähler cone variables. In closing our discussion, we should like to have a ballpark estimate of the size of the Π_i in order to assess the generic size of the new correction.

To this end, we will provide results for the coefficients Π_i in the expression $\int c_2 \wedge J = \Pi_i t^i$ for the well-known complete-intersection CY manifold $X_3 = \mathbb{P}^4_{11169} \begin{bmatrix} 18 \\ 4 \end{bmatrix}$ which has $h^{11} = 2$, $h^{21} = 272$, and consequently $\chi = -540 < 0$. This example was presented in [21, 50, 51] and is of the "Swiss-Cheese" type. While this example cannot show volume stabilization due to its negative Euler number, its mirror does stabilize all the volumes, and we use the $\chi < 0$ manifold just as an illustrative example to provide an estimate for the numerical size of the Π_i.

We can describe X_3 as the vanishing locus of the polynomial

$$\xi^2 = P_{18,4}(u_i) \tag{4.48}$$

in the ambient toric variety

$$X_4^{\text{amb}}: \quad \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & \xi \\ \hline 1 & 1 & 1 & 6 & 0 & 9 \\ 0 & 0 & 0 & 1 & 1 & 2 \end{array} \quad . \tag{4.49}$$

Eq. (4.48) arises in Sen's limit as the double cover of the base $B_3 = \mathbb{P}^1 \to \mathbb{P}^2$ with twist $n = -6$ of an elliptically fibred CY 4-fold $Y_4 : T_2 \to B_3$. From the above weight system data we can compute the linear relations and triple intersections of the toric divisors D_{u_i}, D_ξ (given by the vanishing loci $u_i = 0, \xi = 0$), and their restriction to the hypersurface equation (see e.g. [45]). This allows us to compute the total Chern class of X_3 by adjunction in terms of the Chern class of the embedding toric variety and the normal bundle of the hypersurface. Expanding to second order, we get the second Chern class of X_3 in terms of the elements of a basis of toric divisors. Carefully expanding the Kähler form $J = t^i D_i$ into a basis of divisors spanning the Kähler cone, i.e. where all $t_i \geq 0$ simultaneously and independently from each other, we can then compute $\int c_2 \wedge J$ using the known divisor triple intersection numbers on our CY 3-fold. Following the conventions of [45], we write $J = t^1 D_1 + t^5 D^5$ and the Chern class computation produces

$$\int_{X_3} c_2 \wedge J = 36t^1 + 102t^5 \ . \tag{4.50}$$

Hence, this example served us to verify that the $\Pi_i \geq 0$, and provides us with a first estimate of their typical size to be $\mathcal{O}(10 \dots 100)$.

5 Conclusion

In the first part of this paper we revisited the ghost-free four-derivative sector for chiral superfields in $\mathcal{N} = 1$ global supersymmetry as well as supergravity in superspace. This sector is captured by the operator in eq. (2.1). This term does not lead to a propagating auxiliary field, but induces cubic polynomial equations for the chiral auxiliaries and, thus, up to three inequivalent on-shell theories. We showed that within the context of effective field theory there is a unique physical on-shell theory, namely the theory with analyticity in the coupling T. The additional theories can be regarded as mere artefacts of a truncation of an infinite-series of higher-derivative operators in superspace, as was illustrated explicitly by the one-loop Wess-Zumino model in section 2.3. This example furthermore revealed that the non-analytic theories are incapable of reproducing the non-local, untruncated 'UV'-theory. In addition we have demonstrated that in a regime of small kinetic terms all on-shell Lagrangians obtained from eq. (2.1) are ghost-free. After clarification of these conceptual issues we displayed the general on-shell theory in eq. (3.8).

In the second part of this paper we analysed the correction to the scalar potential, which is generated by the operator in eq. (2.1), and the properties of the vacua of the theory. Firstly, in situations in which the ordinary, two-derivative theory possesses a supersymmetric minimum, this minimum persists unchanged in the higher-derivative theory in agreement with the general discussion in [1]. If, one the other hand, supersymmetry was already broken in the two-derivative theory, then the higher-derivative operator might be of interest, specifically in situations, in which flat directions exist within the minimum. Unless a symmetry is protecting this flat direction or the flat direction is a Goldstone boson, as for example if supersymmetry breaking occurs via R-symmetry breaking, we expect that in general the higher-derivative operator lifts the flatness. For the case of global supersymmetry this was exemplified using the O'Raifeartaigh model. Within supergravity we provided a simple one-dimensional no-scale type model as a first example in section 3.2.

Of special interest are theories, which do not have a minimum at two-derivative level. This is for instance the case for the Kähler moduli sector of type IIB flux compactifications on Calabi-Yau orientifolds after inclusion of the leading order α'-corrections to the Kähler potential, but ignoring non-perturbative effects. We extended the analysis of how $(\alpha')^3$-corrections in ten dimensions modify the four-dimensional theory obtained after compactification to the higher-derivative sector. Specifically we found that corrections to the scalar potential, which are induced by terms with four powers of the flux three-form G_3 fit into the off-shell operator in eq. (3.3). The respective four-derivative terms for the Kähler moduli can be found by reducing the ten-dimensional R^4 corrections. Contrary to the terms quartic in G_3, the R^4 term is fully known [25, 26] and, thus, we computed the four-derivative terms and inferred the correction to the scalar potential by matching to eq. (3.8). The result is displayed in eq. (4.21). In this computation we omitted numerical factors. A proper treatment of these factors lies outside the scope of this paper as this requires a systematic understanding of the off-shell higher-derivatives in four dimensions. Notably (ghost-like) operators exist, which do not modify the scalar potential, but induce four-derivative terms of the same type as those obtained from the R^4 correction. However, it is important to note that eq. (3.3) is the only off-shell operator which receives four-derivative terms from R^4 and contributes to the scalar potential at order $(\alpha')^3$, as we will demonstrate elsewhere. Furthermore we proved in appendix D that the correction $V_{(1)}$ in eq. (4.21) cannot be off-shell completed via a two-derivative theory, that is by including a correction to the Kähler potential, but indeed is only consistent with supersymmetry after including higher-derivatives.

Moreover, in our KK-reduction we neglected warping effects. In principle, warping-induced contributions are expected to enter the scalar potential at the same order in powers of inverse volume as the correction in eq. (4.21), for instance via terms $R^3 F_5^2$ in ten-dimensions. On the other hand it was recently shown that large cancellations associated with warping-induced terms occur in the context of α-corrections to the effective action of M-theory [52]. Leading order warping-effects were also studied in [53]. Thus, it will be interesting to test our approximation in the future.

In a second step we assessed whether the correction in eq. (4.21) can lead to a theory with a minimum without taking into account non-perturbative effects. In section 4.4 we indeed found that a model-independent minimum exists, where all four-cycle volumes are frozen to values which are determined by topological numbers encoded in the second Chern class, cf. eq. (4.29). This holds for all Calabi-Yau threefolds with $\chi(M) > 0$ and provided that the undetermined overall numerical factor of the higher-derivative operator has a negative sign. This moduli stabilization scenario is intriguing as the structure and properties of the vacuum are determined purely from topological data of the Calabi-Yau and no additional ingredients are required. Moreover let us compare the vacuum in eq. (4.29) and eq. (4.30) to the results of LVS. We obtain a minimum given that $\chi(M_3) > 0$, contrary to LVS, where it is necessary that $\chi(M_3) < 0$. To ensure a large volume in eq. (4.30) we see that a largish value $|W_0| \gtrsim 1$ is preferred. The ensuing scaling of the stabilized value of the volume with the W_0 also renders both the 3-form flux density and the gravitino mass in eq. (4.41) small at large W_0.

In the future it will be necessary to determine the sign of λ to confirm the existence of the minimum in eq. (4.29). However, a prior systematic understanding of all higher-derivative operators in curved superspace is required. Furthermore, a better understanding of additional $(\alpha')^3$-corrections to the scalar potential, such as for instance warping-induced terms but also the subleading terms in inverse volume, is important in order to fully trust the minimum in eq. (4.29).

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A Exact solutions of the cubic equation for F

A.1 One-dimensional models with arbitrary W

In this appendix we discuss the general solution of the equation of motion for the auxiliary field F of a single chiral multiplet in the context of supergravity. All the results below can be extrapolated to the case of global supersymmetry after reintroducing the factors of the Planck scale M_p and performing the limit $M_p \to \infty$.

Recall that the equation of motion for F is cubic and given by (cf. (3.6))

$$F\left[|F|^2+\mathrm{e}^{-K/3}\left((2T)^{-1}G_{A\bar{A}}-|\partial A|^2\right)\right]+(2T)^{-1}\bar{D}_A\bar{W}=0\ . \tag{A.1}$$

It is possible to rewrite (A.1) as a cubic equation with real coefficients after performing the field redefinition

$$F=f(A,\bar{A})\,\bar{D}_A\bar{W}\,, \tag{A.2}$$

where f is the new auxiliary field variable and we assume $W\neq 0$. Inserted into (A.1) we obtain

$$f\left[|f|^2\,|D_AW|^2+\mathrm{e}^{-K/3}\left((2T)^{-1}G_{A\bar{A}}-|\partial A|^2\right)\right]+(2T)^{-1}=0\ \ . \tag{A.3}$$

Since T and the expression in the square bracket are real we see that also f has to be real. Therefore (A.3) is of the form

$$f^3 + pf + q = 0 \tag{A.4}$$

with

$$p = \frac{e^{-K/3}}{|D_A W|^2}\left(\frac{G_{A\bar{A}}}{2T} - |\partial A|^2\right), \qquad q = \frac{1}{2T|D_A W|^2} \ . \tag{A.5}$$

In the case of global supersymmetry it necessary to note that the Kähler potential has mass dimension two, so that in the limit $M_p \to \infty$ we get

$$p \to \frac{1}{|W_{,A}|^2}\left(\frac{G_{A\bar{A}}}{2T} - |\partial A|^2\right), \qquad q \to \frac{1}{2T|W_{,A}|^2} \ . \tag{A.6}$$

Eq. (A.4) is a cubic equation with real coefficients p, q and its solutions are known. However, in general only one out of the three possible solutions is real. There are different regimes of interest [10]:

(1) $p > 0$: in this case only one real solution exists given by

$$f_{(1)} = -2\sqrt{\frac{p}{3}}\sinh\left[\frac{1}{3}\text{arsinh}\left(\sqrt{x}\right)\right] \ , \tag{A.7}$$

where we defined

$$x \equiv \frac{27q^2}{4p^3} \ . \tag{A.8}$$

(2) $p < 0$ and $4p^3 + 27q^2 > 0$: here also only one real solution exists, which reads

$$f_{(2)} = -2\,\text{sign}(q)\sqrt{-\frac{p}{3}}\cosh\left[\frac{1}{3}\text{arcosh}\left(\sqrt{-x}\right)\right] \ , \tag{A.9}$$

(3) $p < 0$ and $4p^3 + 27q^2 < 0$: in this regime all three solutions are real and can be expressed as follows

$$f_{(3),k} = 2\sqrt{-\frac{p}{3}}\cos\left[\frac{1}{3}\arccos\left(\sqrt{-x}\right) - \frac{2\pi\,k}{3}\right] \ , \quad k = 0, 1, 2 \ . \tag{A.10}$$

In terms of the variable x defined in (A.8) the different regimes can be expressed by

$$(1):\ x > 0\,, \qquad (2):\ x < -1\,, \qquad (3):\ -1 < x < 0\ . \tag{A.11}$$

Let us make a few remarks regarding the different regimes. Suppose that T is a constant. Then, for $T < 0$ we have $p < 0$ and so one is always in regime (2) or (3). For simplicity let us assume that the contribution of the kinetic terms in p is negligible, then the difference between the regions is characterized by

$$27|D_A W|^2 \gtrless -2T^{-1}e^{-K}G^3_{A\bar{A}} \ . \tag{A.12}$$

If we take the kinetic contribution into account, one can directly see that for large kinetic terms p becomes large and negative so that one always reaches regime (3). For $T > 0$ one can be in all three regions. Note, that if we would restrict ourselves to the discussion of the non-derivative component of F, the condition $T > 0$ could only be supported in region (1). Moreover, we see that the different regions are dynamically connected. For instance a theory with $T > 0$ could describe a dynamical field with initially small kinetic terms, thus, being described by the appropriate theory in regime (1). However, it could be that the kinetic terms are growing with the evolution of the field and hence one reaches regimes (2) and finally (3).

A.2 Analysis of kinetic terms

In this appendix we will demonstrate the absence of ghosts in the on-shell theories in all three regimes. To this end we compute the sign of the ordinary kinetic term in the on-shell Lagrangian. We will conduct the analysis in the context of supergravity and the results extrapolate directly to the case of global supersymmetry. Eliminating the auxiliary field F from (3.5) and keeping only terms which contribute to the standard kinetic term we arrive at

$$\frac{\mathcal{L}}{\sqrt{-g}} \supset -G_{A\bar{A}}|\partial A|^2 + f\,\mathrm{e}^{2K/3}\,|D_A W|^2 - T\,\mathrm{e}^{2K/3}\,f^4\,|D_A W|^4 \ , \tag{A.13}$$

where f was determined in the previous section and via (A.5) depends on ∂A. Let us expand the above terms in $|\partial A|^2$ assuming that they are sufficiently small. The coefficient Σ of the first term in the expansion determines the sign of the ordinary kinetic term. Making use of (A.4) Σ is given by

$$\mathcal{L} = \Sigma G_{A\bar{A}}|\partial A|^2\sqrt{-g} + \dots\,, \qquad \Sigma = -\left[1 + \frac{\partial f}{\partial p}\Big|_0\left(2f_0 + 3\frac{p_0}{q}\right)\right] \ . \tag{A.14}$$

Here the subscript zero denotes that a quantity is evaluated at $|\partial A|^2 = 0$. Using the solutions for f in the three regimes given in eqs. (A.7) to (A.10) it is always possible to express Σ as a function of x_0 (defined in (A.8)) only. More precisely, in each regime one finds the following:

(1) $x > 0$: one obtains

$$\Sigma_{(1)} = -\left\{1 + \left[-\sinh\left(\frac{1}{3}\mathrm{arsinh}(\sqrt{x_0})\right) + \sqrt{\frac{x_0}{1+x_0}}\cosh\left(\frac{1}{3}\mathrm{arsinh}(\sqrt{x_0})\right)\right]\right.$$
$$\left.\times\left[-\frac{4}{3}\sinh\left(\frac{1}{3}\mathrm{arsinh}(\sqrt{x_0})\right) + \frac{2}{3}\sqrt{x_0}\right]\right\} \tag{A.15}$$

Inspecting (A.5) and (A.8) one finds that in this regime $T > 0$ has to hold in order to ensure $x > 0$. Thus, we necessarily also have $x_0 > 0$ and then numerical evaluation shows that Σ is always negative implying that this region is ghost-free.

(2) $x < -1$: the computation in this case has to be done more carefully, since $x < -1$ can occur for $T < 0$ and $T > 0$. From (A.5) we see that in the latter case the kinetic term

has to be large and an expansion around zero is not meaningful. For $T<0$ we have that $x_0<0$ and computing Σ yields

$$\Sigma_{(2)} = -\Bigg\{ 1 + \left[\cosh\left(\frac{1}{3}\mathrm{arcosh}(\sqrt{-x_0})\right) + \sqrt{\frac{x_0}{1+x_0}} \sinh\left(\frac{1}{3}\mathrm{arcosh}(\sqrt{-x_0})\right)\right] \\ \times \left[\frac{4}{3}\cosh\left(\frac{1}{3}\mathrm{arcosh}(\sqrt{-x_0})\right) + \frac{2}{3}\sqrt{-x_0}\right]\Bigg\} \; . \tag{A.16}$$

$\Sigma_{(2)}$ is discontinuous at $x_0=-1$, but numerical evaluation shows that it is negative for all $x_0<0$, which again implies the absence of ghosts.

(3) $-1<x<0$: here one finds that

$$\Sigma_{(3),k} = -\Bigg\{ 1 + \left[\cos\left(\frac{1}{3}\arccos(\sqrt{-x_0}) - \frac{2\pi\, k}{3}\right) + \sqrt{\frac{-x_0}{1+x_0}} \sin\left(\frac{1}{3}\arccos(\sqrt{-x_0}) - \frac{2\pi\, k}{3}\right)\right] \\ \times \left[\frac{4}{3}\cos\left(\frac{1}{3}\arccos(\sqrt{-x_0}) - \frac{2\pi\, k}{3}\right) - \mathrm{sign}(T)\,\frac{2}{3}\sqrt{-x_0}\right]\Bigg\} \; . \tag{A.17}$$

Again there are two cases to discuss: for $T<0$, we always have $-1<x<x_0<0$ and all branches are ghost-free. On the other hand for $T>0$ we have that $x_0>0$. It is not expected that Σ is defined here, since this corresponds to large $|\partial A|^2$.

Let us make some additional comments about the appearance of ghosts in those theories, where we truncate the theory to linear order in T. In region (1), the auxiliary field is analytic in T and the lowest order contributions to the Lagrangian generated by the auxiliary can be obtained as in (2.9) and are given by[25]

$$\mathcal{L}_{(1)} \supset -\mathrm{e}^K G^{A\bar{A}}|D_AW|^2 + T\left[\left(\mathrm{e}^K G^{A\bar{A}}|D_AW|^2\right)^2 - 2G_{A\bar{A}}|\partial A|^2 \mathrm{e}^K G^{A\bar{A}}|D_AW|^2\right] + \mathcal{O}(T^2)\; . \tag{A.18}$$

In region (2) on the other hand the auxiliary field has a pole at $T\to 0$ and hence is not analytic. The respective contributions to the Lagrangian are of the form

$$\mathcal{L}_{(2)} \supset -4T^{-1} + \left(\frac{1}{2}\mathrm{e}^K G^{A\bar{A}}|D_AW|^2 + G_{A\bar{A}}|\partial A|^2\right) + \mathcal{O}(\sqrt{T})\; . \tag{A.19}$$

The fact that there exists a region in which the Lagrangian is not analytic in T is not surprising, since the limit $T\to 0$ with fixed K, W automatically implies that one must exit region (2) and enter region (3) as can be seen from (A.5). In the third region the Lagrangian coincides with $\mathcal{L}_{(2)}$ for $k=0,2$ and with $\mathcal{L}_{(1)}$ for $k=1$.

Inspecting $\mathcal{L}_{(1)}$ we observe that the theory becomes ghost-like, once

$$2\left(\mathrm{e}^K G^{A\bar{A}}|D_AW|^2\right)^2 T \lesssim -1 \; . \tag{A.20}$$

[25]In contrast to the previous appendices here we introduced additional factors of the Kähler metric according to eq. (2.25).

Equivalently this reads $x_0 \lesssim -27/4$. However, such values of x_0 correspond to regime (2), where no analytic Lagrangian exists. This indicates that the expansion of the analytic solution fails to converge around values where the theory becomes ghostlike. In the above we have treated the solutions to all orders in T. This is sensible only, if we know that all higher-order contributions to the EAFP vanish. However, even in the situation where we know the EAFP only up to four-derivative level, we expect that the theory becomes unreliable near $x_0 \sim -1$, which coincides with the threshold, where the kinetic term starts to behave ghostlike.

A.3 Analytic solution in arbitrary dimensions

To complete the discussion of the four-derivative operator let us analyse eq. (3.6) for arbitrary dimension. If we assume that the coupling tensor is given by eq. (2.25) and we only look for analytic solutions, then the task is feasible and the answer unique. Inspecting the equations of motion for the F^i shows that all analytic solutions have to be of the form

$$F^i = \mathrm{e}^{K/3} G^{i\bar{j}} \bar{D}_{\bar{j}} \bar{W} f \quad , \tag{A.21}$$

where f is analytic in T.[26] Inserted into eq. (3.6) yields

$$2|f|^2 f T(\mathrm{e}^K G^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W}) + f + 1 = 0\,, \tag{A.22}$$

where for simplicity we ignore the dependence on the derivatives of the scalar fields. As in section A.1 we obtain the additional condition that f has to be real-valued and, hence, the cubic equation reduces to

$$f^3 + pf + p = 0\,, \qquad \text{where} \qquad p = (2T\mathrm{e}^K G^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W})^{-1}\ . \tag{A.23}$$

Thus, the exact solution is be given by

$$F^i = \mathrm{e}^{K/3} G^{i\bar{j}} \bar{D}_{\bar{j}} \bar{W} \sqrt{-\frac{4p}{3}} \cos\left[\frac{1}{3}\arccos\left(\sqrt{-\frac{27}{4p}}\right) - \frac{2\pi}{3}\right]\ . \tag{A.24}$$

B Higher-derivatives for Kähler moduli from string-theoretic α'-corrections

In this appendix we discuss how to compute four-derivative terms for Kähler class deformations from $(\alpha')^3 R^4$ corrections to the action of IIB in the context of flux compactifications on Calabi-Yau orientifolds. These corrections were already presented in eq. (4.5). Notably J_0 generates the $\hat{\xi}$-correction to the Kähler potential in eq. (4.7) as shown in [19]. The following derivation is many ways analogous to the computation in this reference.

Before turning to the explicit analysis let us stress that we will focus on obtaining the overall functional form of the coupling tensor and omit the details of numerical factors. A proper treatment of these factors lies outside the scope of this paper as a complete discussion

[26] One can directly show via induction that the solution must reduce to this by assuming a general analytic expansion in T.

of the four-derivative bosonic action is required. Specifically a full understanding of all off-shell operators in four dimensions is necessary, that contribute four-derivative terms for the scalar fields. For instance off-shell higher-derivative operators exist, which mix with $(\partial\mathcal{V})^4$, but do not correct the scalar potential. Note that the operator in eq. (3.3) is the only higher-derivative operator, that receives four-derivative terms from J_0 and can contribute to the scalar potential at order $\mathcal{O}(\alpha'^3)$, as we will demonstrate elsewhere.

The necessary terms of the 10-dimensional string-frame action for this appendix are given by[27] [25]

$$S_{(10)} \supset -\frac{1}{\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-g^{(10)}}\mathrm{e}^{-2\phi}\left(R + 4(\partial\phi)^2 + \frac{(\alpha')^3\zeta(3)}{3\cdot 2^{11}}J_0\right) . \tag{B.1}$$

Here R denotes Ricci scalar, ϕ the ten-dimensional dilaton, $g^{(10)}$ the metric and J_0 was given in eq. (4.5).[28] There exists a basis of 26 independent contractions of four Riemann tensors, in which J_0 necessarily has to expand [54]. Here we do not compute the exact coefficients of this expansion, but simply argue within this basis of the 26 terms to obtain the functional form of the possible four-derivative terms.

We will not compute the coupling of gravity to the higher-derivatives of the Kähler moduli and, therefore, set the four-dimensional piece of the metric to a Minkowski-form. Furthermore we will neglect the warping-factor, which is non-trivial in the presence of background-fluxes. For simplicity we will conduct the analysis with a single Kähler-type deformation turned on. Altogether the ten-dimensional metric then reads

$$\mathrm{d}s^2_{(10)} = g_{MN}\mathrm{d}x^M\mathrm{d}x^N = \eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + g_{mn}\mathrm{d}y^m\mathrm{d}y^n\,, \tag{B.2}$$

where $M,N = 0,\ldots,9$, the y^m, $m = 1,\ldots,6$ are real coordinates on the compact manifold M_3 and $g_{mn} = \mathrm{e}^{2u(x)}\widetilde{g}_{mn}(y)$. The volume measured by the background metric $\widetilde{g}_{mn}$ is normalized to unity, i.e. we choose $(2\pi\alpha') = 1$. This way the Planck constants in ten and four dimensions can be directly related to each other as $\kappa_{10}^{-2} = \kappa_4^{-2}$. The single volume modulus in the string-frame is normalized as $\mathrm{e}^{6u} = \hat{\mathcal{V}}$.

Note that the higher-curvature terms in eq. (B.1) modify the Einstein equations. More precisely the Einstein equations along the internal directions read [55]

$$R_{\alpha\bar{\beta}} \sim (\alpha')^3\partial_\alpha\partial_{\bar{\beta}}Q\,, \tag{B.3}$$

where we introduced local complex coordinates $(z^\alpha,\bar{z}^{\bar{\beta}})$ with $\alpha,\bar{\beta} = 1,2,3$ on the internal manifold. Furthermore Q denotes the six-dimensional Euler integrand, i.e. $\int d^6y\sqrt{g}Q = \chi(M_3)$. As a consequence of eq. (B.3) the background metric in eq. (B.2) is in general not Ricci-flat. Eq. (B.3) is formally solved by

$$\widetilde{g}_{mn} = \widetilde{g}^{(0)}_{mn} + (\alpha')^3\widetilde{g}^{(1)}_{mn}\,, \tag{B.4}$$

[27] For brevity we do not display the terms for the RR and NSNS field strength forms here.

[28] Note that the dilaton receives higher-derivative corrections [39]. In the following, we shall consider the dilaton only at the two-derivative level and hence stick to the results of [19].

where $\widetilde{g}^{(0)}_{mn}$ is a Ricci-flat metric solving the zeroth-order Einstein equations and $\widetilde{g}^{(1)}_{mn}$ solves eq. (B.3) at order $(\alpha')^3$. When reducing J_0 it is not necessary to take into account the correction $\widetilde{g}^{(1)}_{mn}$ as it enters at order $(\alpha')^6$. The leading $(\alpha')^0$ terms in eq. (B.1) on the other hand induce $(\alpha')^3$-corrections via $\widetilde{g}^{(1)}_{mn}$. However, the respective correction coming from the standard Einstein-Hilbert term is a total derivative. The remaining terms do not correct the kinetic terms. Hence, we will in the following ignore the correction $\widetilde{g}^{(1)}_{mn}$ and treat $\widetilde{g}_{mn}$ as Ricci-flat.

To determine the curvature terms inside J_0 we need to compute the components of the Riemann tensor. In the following we use the conventions

$$\begin{aligned} R^M{}_{NPQ} &= \partial_P \Gamma^M_{QN} - \partial_Q \Gamma^M_{PN} + \Gamma^R_{QN}\Gamma^M_{PR} - \Gamma^R_{PN}\Gamma^M_{QR}\,, \\ \Gamma^M_{PN} &= \frac{1}{2} g^{MQ}\left(\partial_P g_{NQ} + \partial_N g_{PQ} - \partial_Q g_{PN}\right)\ . \end{aligned} \tag{B.5}$$

Up to symmetries there are only two non-vanishing independent pieces of the Riemann tensor computed with respect to the metric in eq. (B.2). They are given by

$$\begin{aligned} R_{m\mu n\nu} &= -g_{mn}(\partial_\mu u \partial_\nu u + \partial_\mu\partial_\nu u)\,, \\ R_{kmnp} &= \mathrm{e}^{2u}\widetilde{R}_{kmnp} + (\partial u)^2 (g_{kp}g_{mn} - g_{kn}g_{pm})\ . \end{aligned} \tag{B.6}$$

Here $\widetilde{R}_{kmnp}$ denotes the Riemann tensor components of the background metric $\widetilde{g}_{mn}$. From the Riemann tensor we can furthermore compute the Ricci-tensor as well as the scalar curvature

$$R_{\mu\nu} = -6(\partial_\mu u\partial_\nu u + \partial_\mu\partial_\nu u)\,, \quad R_{mn} = -g_{mn}(6(\partial u)^2 + \Box u)\,, \quad R = -42(\partial u)^2 - 12\Box u\ . \tag{B.7}$$

It is evident that in the reduction of eq. (4.5) one obtains terms with up to eight derivatives of u. Here we are solely interested in the terms with four-derivatives. Computation of all 26 basis elements in [54] shows that one obtains the following four-derivative terms

$$\begin{aligned} J_0 \supset \mathrm{e}^{-4u}\Big[&\alpha_1(\partial u)^4 + \alpha_2 \Box u(\partial u)^2 + \alpha_3(\Box u)^2 + \alpha_4(\partial_\mu\partial_\nu u)(\partial^\mu\partial^\nu u) \\ &+ \alpha_5(\partial_\mu\partial_\nu u)(\partial^\mu u)(\partial^\nu u)\Big]\widetilde{R}_{kmnp}\widetilde{R}^{kmnp}\,, \end{aligned} \tag{B.8}$$

for some constants α_i. Since for a Calabi-Yau $\widetilde{R}_{mn} = 0$, the only non-vanishing contraction of two Riemann tensors is given by $\widetilde{R}_{kmnp}\widetilde{R}^{kmnp}$. We see that five different four-derivative terms appear here. However, in the four-dimensional action these terms are not independent and related by partial integration.[29] For the purpose of keeping track of the proper coefficients it would be necessary to discuss all five operators in eq. (B.8) jointly. Here we are interested only in the functional behaviour and, thus, confine our attention to the first term in eq. (B.8), since this is the only four-derivative term, which can be matched to the Lagrangian in eq. (3.8). It is convenient to express the Riemann-tensor square with respect to g_{mn} again. Up to derivatives we have

$$R_{kmnp}R^{kmnp} = \mathrm{e}^{-4u}\widetilde{R}_{kmnp}\widetilde{R}^{kmnp} + \dots \tag{B.9}$$

[29] For example partial integration reveals that $(\partial_\mu\partial_\nu u)(\partial^\mu u)(\partial^\nu u)$ can be recast into a combination of $(\partial u)^4$ and $\Box u(\partial u)^2$.

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In the action we obtain at order $(\alpha')^3$

$$S_{(\partial u)^4} = -\frac{1}{2\kappa_4^2}\int \mathrm{d}^4x\sqrt{-g}\,\mathrm{e}^{-2\phi_0}\,\alpha_1\,(\partial u)^4\int_{M_3}\mathrm{d}^6y\sqrt{g}\,R_{kmnp}R^{kmnp}\,. \tag{B.10}$$

It is convenient to rewrite the integral over the compact dimensions as follows

$$\int_{M_3}\mathrm{d}^6y\sqrt{g}\,R_{kmnp}R^{kmnp}\sim\int_{M_3}c_2\wedge J\,, \tag{B.11}$$

where c_2 is the second Chern class of the Calabi-Yau threefold and J its Kähler form. This can be checked directly using local complex coordinates. With respect to these coordinates we have

$$c_2 = \frac{1}{2}\left(\mathrm{Tr}\mathcal{R}^2 - (\mathrm{Tr}\mathcal{R})^2\right)\,, \qquad J \;= ig_{\alpha\bar{\beta}}\mathrm{d}z^\alpha\wedge\mathrm{d}\bar{z}^{\bar{\beta}}\,, \tag{B.12}$$

where $\mathcal{R}$ is the curvature two-form. The traces of the curvature two-form are given by

$$\mathrm{Tr}\mathcal{R} = R^\alpha{}_{\alpha\beta\bar{\gamma}}\,\mathrm{d}z^\beta\wedge\mathrm{d}\bar{z}^{\bar{\gamma}}\,, \qquad \mathrm{Tr}\mathcal{R}^2 \;= R^\alpha{}_{\beta\gamma\bar{\delta}}R^\beta{}_{\alpha\epsilon\bar{\zeta}}\,\mathrm{d}z^\gamma\wedge\mathrm{d}\bar{z}^{\bar{\delta}}\wedge\mathrm{d}z^\epsilon\wedge\mathrm{d}\bar{z}^{\bar{\zeta}}\,. \tag{B.13}$$

On a Calabi-Yau the first Chern class vanishes and, hence, we have $\mathrm{Tr}\mathcal{R} = 0$.[30]

From eq. (B.11) it is evident that

$$\int_{M_3}c_2\wedge J\geq 0\,. \tag{B.14}$$

Here equality holds, if and only if M_3 has constant holomorphic sectional curvature [56]. For Kähler manifolds with constant holomorphic sectional curvature c the Riemann tensor must necessarily take the form [57]

$$R_{\alpha\bar{\beta}\gamma\bar{\delta}} = -\frac{c}{2}\left(g_{\alpha\bar{\beta}}g_{\gamma\bar{\delta}} + g_{\alpha\bar{\delta}}g_{\gamma\bar{\beta}}\right) \tag{B.15}$$

and, thus, for Calabi-Yau manifolds $c = 0 = R_{\alpha\bar{\beta}\gamma\bar{\delta}}$. This is only possible if M_3 is a torus T^6.

The term in eq. (B.10) is expressed in the string frame. In order to transform to the Einstein frame note that the two-derivative part of the bosonic action is given by [19][31]

$$S = -\frac{1}{2\kappa_4^2}\int\mathrm{d}^4x\sqrt{-g}\,\mathrm{e}^{-2\phi_0}\left(\mathrm{e}^{6u}+\frac{\xi}{2}\right)R^{(4)}+\ldots \tag{B.16}$$

where ξ parametrizes the leading α'-corrections and is given in eq. (4.11) and $R^{(4)}$ denotes the scalar curvature in four dimensions.

The next step is to transform into the four-dimensional Einstein frame via a Weyl rescaling. Simultaneously one has to rediagonalize the kinetic terms for the scalar fields. This is achieved by the redefinitions

$$g^{(E)}_{\mu\nu} = \mathrm{e}^{-\phi_0/2}\left(\mathcal{V}+\frac{\hat{\xi}}{2}\right)g_{\mu\nu}\,, \qquad \mathcal{V} = \hat{\mathcal{V}}\mathrm{e}^{-3\phi_0/2} = \mathrm{e}^{-3\phi_0/2}\mathrm{e}^{6u}\,. \tag{B.17}$$

[30] To prove eq. (B.11) it is also helpful to note the relation $\sqrt{\det(\tilde{g}_{mn})} = \det(g_{\alpha\bar{\beta}})$, which links the volume forms of the two different coordinate systems to each other.

[31] We promote $\eta_{\mu\nu}$ to an arbitrary Lorentzian metric $g_{\mu\nu}$ here.

where $\hat{\xi}$ is defined in (4.11). Here one observes that also couplings of the four-dimensional Riemann tensor to the Kähler deformation contribute to the four-derivative term for u after the Weyl rescaling.[32]

Even though we considered only a single volume modulus so far, in the following the results can be generalized to the situation of arbitrarily many Kähler moduli. The proper $\mathcal{N}=1$ field variables are [19]

$$T^i = \frac{1}{3}\left(g^i + i\mathcal{V}^i\right) , \qquad \tau \ = l + i\mathrm{e}^{-\phi_0} , \tag{B.18}$$

where l is the R-R scalar and g^i originate from the R-R four-form. The imaginary components of T^i are given by rescaled four-cycle volumes as follows

$$\mathcal{V}^i = \tau_i = \frac{\partial \mathcal{V}}{\partial t^i} , \qquad \text{where} \qquad t^i = \hat{t}^i \mathrm{e}^{-\phi_0/2} , \tag{B.19}$$

denote the Einstein-frame two-cycle volumes. Furthermore $\hat{t}^i$ are the two-cycle volumes measured in the string-frame. These are related to the overall volume via eq. (4.8). For a generic Calabi-Yau threefold we can expand $J = \sum_{i=1}^{h^{1,1}} \hat{t}^i \hat{D}_i$, where $\hat{D}_i$ form a basis of the Dolbeault cohomology $H^{1,1}(M_3, \mathbb{Z})$. Hence, the integral on the r.h.s. of eq. (B.11) can be understood as

$$\int_{M_3} c_2 \wedge J = \hat{t}^i \int_{M_3} c_2 \wedge \hat{D}_i \equiv \Pi_i \, \hat{t}^i , \tag{B.20}$$

where Π_i is a number encoding the topological information of the second chern class.

Up to terms involving derivatives of the dilaton we can use the above coordinates to rewrite eq. (B.10)

$$\begin{aligned} S_{(\partial u)^4} \sim -\frac{1}{2\kappa_4^2} \int \mathrm{d}^4 x \sqrt{-g^{(E)}} \, (\Pi_m \, t^m) \left[\frac{1}{2i}(\tau - \bar{\tau})\right]^{3/2} \\ \times K_{(0),i} K_{(0),j} K_{(0),k} K_{(0),l} \, (\partial_\mu \tau_i \partial^\mu \tau_j)(\partial_\nu \tau_k \partial^\nu \tau_l) , \end{aligned} \tag{B.21}$$

where $K_{(0)} = -2 \ln(\hat{\mathcal{V}})$ denotes the classical Kähler potential of the underlying $\mathcal{N}=1$ geometry. Finally we can match this result to the Lagrangian in eq. (3.8) and read off the coupling tensor

$$T_{ij\bar{k}\bar{l}} = \lambda(\alpha')^3 (\Pi_m \, t^m) \left[\frac{1}{2i}(\tau - \bar{\tau})\right]^{3/2} K_{(0),i} K_{(0),j} K_{(0),\bar{k}} K_{(0),\bar{l}} , \tag{B.22}$$

where λ denotes the overall unknown numerical factor. Its computation is beyond the scope of this paper as we discussed at the beginning of this appendix.

In the last steps we generalized to the case of arbitrarily many Kähler moduli even though we took into account only a single modulus during the compactification. When arbitrarily many Kähler-class deformations are switched on the coupling tensor might differ

[32] A coupling of the $\mathcal{N}=2$ vector multiplets to four-dimensional curvature invariants is forbidden by supersymmetry [25] and, hence, one might expect these couplings also to be absent in the $\mathcal{N}=1$ sector. However, a coupling of the four dimensional Riemann-tensor to derivatives of the Kähler moduli might be present. Similarly one expects corrections also to the two-derivative term in eq. (4.24).

from eq. (B.22). For instance, just as for the ordinary kinetic term, the Kähler metric $K_{(0),i\bar{j}}$ could appear. Even though the coupling tensor computed for arbitrary $h^{1,1}$ could be different from eq. (B.22), there is evidence that the induced correction to the scalar potential can be inferred from the computation with $h^{1,1} = 1$ without loss of generality. To see this we will make use of the results of appendix C, which we will briefly summarize now. In the large volume limit the correction to the scalar potential in eq. (4.17) behaves as

$$V_{(1)} = -\frac{|W|^4}{\mathcal{V}^4} T_{(0)}{}^{\bar{i}\bar{j}kl} K_{(0),\bar{i}} K_{(0),\bar{j}} K_{(0),k} K_{(0),l} + \dots\,, \tag{B.23}$$

where $T_{(0)}$ is the coupling tensor truncated to the leading order term in the large volume limit. From the above index structure it is clear that $T_{(0)}$ is a tensor in the geometry defined by the Kähler potential $K_{(0)}$. We assume that its tensor structure is derived from $K_{(0)}$, which means that any indexed quantity appearing within $T_{(0)}$ is related to derivatives of $K_{(0)}$ and possibly contractions with the inverse Kähler metric, see appendix C for more details. In appendix C we study eq. (B.23) in detail and provide evidence for the following statement: if $T_{(0)}$ does not involve any scalar function and, hence, only consists of objects with at least one index, then $V_{(1)} \sim \mathcal{V}^{-4}$ up to some constant. Thus, an additional dependence of $V_{(1)}$ upon $\mathcal{V}$ or τ_i can only be generated by scalar functions appearing within $T_{(0)}$.

When reducing J_0 with an arbitrary number of Kähler-type deformations turned on, the four-derivative terms are again obtained from those contractions where two out of the four Riemann tensors have indices along the internal directions and, thus, contribute a factor $\int c_2 \wedge J$. The remaining indices yield contracted metrics or derivatives. We infer that the general coupling tensor should be of the form

$$T_{ij\bar{k}\bar{l}} \sim (\Pi_m\, t^m) \mathcal{T}_{ij\bar{k}\bar{l}} \tag{B.24}$$

where $\mathcal{T}$ is a tensor, that consists purely of indexed quantities. As we consider only terms at order $(\alpha')^3$ this tensor is a tensor in the geometry defined by $K_{(0)}$. Thus, we can apply the results of the appendix C and conclude that the functional behaviour of eq. (B.23) is captured by $\int c_2 \wedge J$, which was already present in the computation with $h^{1,1} = 1$.

C Kähler moduli space and coupling tensor

In this appendix we study the correction to the scalar potential induced by the higher-derivative operator in eq. (B.23) for the geometry of the Kähler moduli at leading order in the large volume limit, that is for $K_{(0)} = -2\ln(\mathcal{V})$, $W = const.$ and $\mathcal{V}$ given by eq. (4.8).[33] Within this appendix we set $K_{(0)} = K$ and $T_{(0)} = T$ for brevity. Up to factors the relevant object of study is given by

$$\mathcal{Z} \equiv T_{ij\bar{k}\bar{l}} K^i K^j K^{\bar{k}} K^{\bar{l}}\,, \tag{C.1}$$

where $K^i = K^{i\bar{j}} K_{\bar{j}}$ and $K^{i\bar{j}}$ denotes the inverse Kähler metric. Due to the shift-symmetry of K in the following we replace anti-holomorphic by holomorphic indices.

[33]Some of the below results can also be found in [58].

We will now provide evidence, but not a rigorous proof, for the following claim: if T_{ijkl} purely consists of quantities carrying at least one index, that is no scalar functions appear, then $\mathcal{Z}$ is a constant. If, one the other hand, explicit scalar quantities, such as K or the curvature R appear, in general this no longer holds.

Let us begin by investigating the possible structure of T_{ijkl}. Since the superpotential is a constant, we can assume that the coupling tensor is built entirely out of derivatives of K. The following list contains the simplest conceivable objects that can be constructed this way:

$$T_{ijkl} = K_{ik}K_{jl} + K_{il}K_{jk} \tag{C.2}$$

$$T_{ijkl} = K_i K_j K_k K_l \tag{C.3}$$

$$T_{ijkl} = K_i K_k K_{jl} + \text{symmetrized} \tag{C.4}$$

$$T_{ijkl} = R_{ijkl} = K_{ijkl} - K_{ijm}K^{mn}K_{nkl} \tag{C.5}$$

$$T_{ijkl} = R_{ik}R_{jl} + R_{il}R_{jk} \tag{C.6}$$

$$T_{ijkl} = R_{ik}K_{jl} + \text{symmetrized} \tag{C.7}$$

$$T_{ijkl} = R_{ik}K_j K_l + \text{symmetrized} \tag{C.8}$$

$$T_{ijkl} = K_j \nabla_l R_{ik} + \text{symmetrized} \tag{C.9}$$

$$T_{ijkl} = \nabla_j \nabla_l R_{ik} + \text{symmetrized} \tag{C.10}$$

Here R_{ijkl} denotes the Riemann tensor, R_{ij} the Ricci tensor and ∇_k the covariant derivative. We will show that for any four-tensor in the upper list of choices $\mathcal{Z}$ is a constant. For the tensors in eq. (C.2) to eq. (C.4) this simply follows from the no-scale condition $K^iK_i = 3$. Note that, if we choose T_{ijkl} according to eq. (C.5), then $\mathcal{Z}$ describes the holomorphic sectional curvature along K^i.

The following identity is essential in order to prove our claim

$$K_{i_1 \dots i_n j_1 \dots j_m} K^{i_1} \dots K^{i_n} \propto K_{j_1 \dots j_m} \ . \tag{C.11}$$

This relation can be shown stepwise. To begin with note that $\mathcal{V}$ is a homogeneous function of degree (3/2) in the four-cycle volumes τ_i. According to Euler's theorem for homogeneous functions it, thus, has to satisfy

$$\frac{3}{2}\mathcal{V} = \sum_i \tau_i \mathcal{V}_i \ . \tag{C.12}$$

Taking iterative derivatives of this equation we obtain

$$\sum_i \tau_i \mathcal{V}_{ij_1 \dots j_n} = \frac{3-2n}{2}\mathcal{V}_{j_1 \dots j_n} \ . \tag{C.13}$$

With this we can prove the following auxiliary result[34]

$$K_{i_1 \dots i_n} K^{i_1} \dots K^{i_n} = \text{const} \ . \tag{C.14}$$

[34] For $n = 2$ this simply corresponds to the no-scale condition.

First note that we have

$$K^i = K^{ij} K_j = -\tau_i \ . \tag{C.15}$$

In general the derivative is of the form

$$K_{i_1 \dots i_n} = -\frac{2}{\mathcal{V}} \mathcal{V}_{i_1 \dots i_n} + \frac{2}{\mathcal{V}^2} (\mathcal{V}_{i_1 \dots i_{n-1}} \mathcal{V}_{i_n} + \text{symm.}) + \dots + 2 \frac{(-1)^n (n-1)!}{\mathcal{V}^n} \mathcal{V}_{i_1} \dots \mathcal{V}_{i_n} \ . \tag{C.16}$$

For each term a successive insertion of eq. (C.13) yields precisely the correct power of $\mathcal{V}$, since there are always as many products of derivatives of $\mathcal{V}$ in the numerator as there are powers of $\mathcal{V}$ in the denominator. Thus, one is left with a combinatorial constant for each term. We conclude that eq. (C.14) is satisfied.

Now we are in a position to show the following

$$K_{i_1 \dots i_n j} K^{i_1} \dots K^{i_n} \propto K_j \ . \tag{C.17}$$

This can be seen via induction in n. For $n = 1$ the above can simply be checked using eq. (C.13). Suppose the statement is true for $(n-1)$. Then, taking the derivative of eq. (C.14) with respect to τ_j, we obtain

$$K_{i_1 \dots i_n j} K^{i_1} \dots K^{i_n} = -K_{j i_2 \dots i_n} K^{i_2} \dots K^{i_n} - K_{i_1 j i_3 \dots i_n} K^{i_1} K^{i_3} \dots K^{i_n} - \dots \tag{C.18}$$

Thus, since the statement is true for $(n-1)$, one infers that eq. (C.17) holds. Now we are in a position to generalize this statement for eq. (C.11). Again the proof uses induction: for $n = 1$ this can be directly deduced by taking derivatives of eq. (C.17). For arbitrary n successive differentiation of eq. (C.17) yields eq. (C.11), if eq. (C.11) holds for $(n-1)$.

Now let us consider for example $\mathcal{Z}$ with T_{ijkl} given by eq. (C.5), then iterative use of eq. (C.11) yields

$$\mathcal{Z} \propto K_i K^{ij} K_j + \text{const.} \,, \tag{C.19}$$

which again gives a constant due to the no-scale property. Similarly one can show that $\mathcal{Z}$ is a constant for the choices in eq. (C.6), (C.7), (C.8). The cases of eq. (C.9) and eq. (C.10) require a little more effort, but can be derived making use of properties, such as $(\partial_k K^{ij}) K_{ij} = -K^{ij} K_{ijk}$.

D Proof of non-Kähler structure of new α'-correction

In this appendix we provide a proof that $V_{(1)}$ as given by eq. (4.21) cannot be captured or induced by a correction to the two-derivative theory, that is to K and/or W. Suppose $V_{(1)}$ could be described as a correction to the two-derivative theory, then possibly only via a new contribution to the Kähler potential since W has to be holomorphic. In addition we have to guarantee Kähler-invariance. Hence, the correction to the Kähler potential has to be a function of $G \equiv K + \ln W + \ln \bar{W}$. The corrected Kähler potential K_c is then of the form

$$K_c = K_0 + (\alpha')^3 \mathcal{K}(Q, T + \bar{T}) \,, \tag{D.1}$$

where $K_0 = -2 \ln(\mathcal{V})$ is the tree-level Kähler potential. Here we chose for convenience $Q \equiv e^G$ and T collectively denotes the Kähler moduli. Note that in eq. (D.1) we do not need to include the $\hat{\xi}$-correction since it already is of order $(\alpha')^3$. From now on we set $\alpha' = 1$ for simplicity. We will assume that $\mathcal{K}$ is an analytic function. In order to reproduce the scalar potential for the theory at $\mathcal{O}(|W|^2)$ the lowest order coefficient of the series expansion in Q has to vanish, such that

$$\mathcal{K}(Q, T+\bar{T}) = Q\,\mathcal{K}_{(1)}(T+\bar{T}) + \mathcal{O}(Q^2)\ . \tag{D.2}$$

Including solely the Kähler moduli and ignoring again the $\hat{\xi}$-correction the scalar potential has the form[35]

$$V = e^{K_c}|W|^2(K_c^{ij}K_{c,i}K_{c,j} - 3)\ . \tag{D.3}$$

We now want to compute the terms in V which are quartic in $|W|$. In other words these are all terms of order $\mathcal{O}(Q^2)$. To this end we compute the following expansion

$$e^{K_c}|W|^2 = Q + Q^2\mathcal{K}_{(1)} + \mathcal{O}(Q^3)\ . \tag{D.4}$$

Furthermore, the Kähler metric reads

$$K_{c,ij} = K_{0,ij} + Q(K_{0,ij}\mathcal{K}_{(1)} + K_{0,i}K_{0,j}\mathcal{K}_{(1)} + K_{0,i}\mathcal{K}_{(1),j} + K_{0,j}\mathcal{K}_{(1),i} + \mathcal{K}_{(1),ij}) + \mathcal{O}(Q^2)\ . \tag{D.5}$$

We find that the inverse Kähler metric is given by

$$K_c^{ij} = K_0^{ij} - Q(K_0^{ij}\mathcal{K}_{(1)} + K_0^iK_0^j\mathcal{K}_{(1)} + K_0^iK_0^{jk}\mathcal{K}_{(1),k} + K_0^jK_0^{ik}\mathcal{K}_{(1),k} + K_0^{ik}K_0^{jl}\mathcal{K}_{(1),ij}) + \mathcal{O}(Q^2)\ . \tag{D.6}$$

Now we are in a position to determine the scalar potential at order Q^2. We find that

$$V_{Q^2} = -Q^2(6\mathcal{K}_{(1)} + 4K_0^i\mathcal{K}_{(1),i} + K_0^iK_0^j\mathcal{K}_{(1),ij})\,, \tag{D.7}$$

where we made extensive use of the no-scale property of K_0. Now, V_{Q^2} has to match $V_{(1)}$ as given by eq. (4.21). This yields

$$6\mathcal{K}_{(1)} + 4K_0^i\mathcal{K}_{(1),i} + K_0^iK_0^j\mathcal{K}_{(1),ij} = \hat{\lambda}\int c_2 \wedge J\ . \tag{D.8}$$

We read this equation as an inhomogeneous linear partial differential equation for $\mathcal{K}_{(1)}$. The solution can always be decomposed into an arbitrary solution to the respective homogeneous differential equation as well as a particular solution to the inhomogeneous one. A particular solution to the inhomogeneous differential equation is given by

$$\mathcal{K}_{(1)} = \frac{4}{31}\hat{\lambda}\Pi_k t^k\ . \tag{D.9}$$

To check that eq. (D.9) indeed solves eq. (D.8) we have to make use of the identity

$$\tau^i\frac{\partial t^j}{\partial\tau^i} = \frac{1}{2}t^j\,, \tag{D.10}$$

[35] As in the previous appendix we do not need to distinguish between holomorphic and antiholomorphic indices.

which can be checked by using

$$K_{0,ij} = \frac{1}{2}\frac{t^i t^j}{\mathcal{V}^2} - \frac{1}{\mathcal{V}}\frac{\partial t^j}{\partial \tau^i} \tag{D.11}$$

as well as

$$K_0^i = -\tau^i \tag{D.12}$$

and finally it is also necessary to note that

$$K_{0,i} = -\frac{t^i}{\mathcal{V}}\,. \tag{D.13}$$

So far we have found that the correction in eq. (D.9) indeed reproduces $V_{(1)}$ in eq. (4.21). However, it demands also a new correction to the two-derivative kinetic term via the formula of the Kähler metric in eq. (D.5). In particular this includes a term in the Lagrangian

$$e^{-1}\mathcal{L} \supset \frac{4}{31}\hat{\lambda}\left(\partial_\mu T^i \partial^\mu \bar{T}^{\bar{j}}\right)\partial_{T^i}\partial_{\bar{T}^{\bar{j}}}\left[\int c_2 \wedge J\right]\,. \tag{D.14}$$

However, this term cannot be obtained by KK-reducing the action in eq. (4.3). The reason for this is, that the action in 10d does not include any terms with derivatives acting on the Riemann-tensor and $\int c_2 \wedge J$ can only descend from contractions of Riemann tensors in 10D. Note, that it is indeed possible to obtain first-derivatives of $\int c_2 \wedge J$ via partial integration of terms such as $\int c_2 \wedge J\,\Box u$, but it is not possible to obtain double-derivatives acting on $\int c_2 \wedge J$ this way. Thus, we have found a contradiction.

One might wonder, whether it is possible to circumvent this argument by allowing for a more general solution to eq. (D.8). This is not possible, since the general solution to the homogeneous differential equation does not entail $\int c_2 \wedge J$ and so could not cancel the term $\partial_{T^i}\partial_{\bar{T}^{\bar{j}}} \int c_2 \wedge J$ in the Kähler metric.

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