Higher order sliding mode control based on optimal approach of an electropneumatic actuator — Source link

Salah Laghrouche, Mohamed Smaoui, Franck Plestan, Xavier Brun

Institutions: Supélec, Institut national des sciences Appliquées de Lyon, École centrale de Nantes

Published on: 01 Feb 2006 - International Journal of Control (Taylor & Francis Group)

Topics: Sliding mode control, Pneumatic actuator, Pressure control and Actuator

Higher order sliding mode control based on optimal approach of an electropneumatic actuator

Salah Laghrouche, Mohamed Smaoui, Franck Plestan, Xavier Brun

Higher order sliding mode control based on optimal approach of an electropneumatic actuator

S. Laghrouche[1], M. Smaoui[3], F. Plestan[2], and X. Brun[3]

[1]L2S, UMR
SUPELEC/CNRS
Gif-sur-Yvette, France
Salah.Laghrouche@lss.supelec.fr

[2]IRCCyN, UMR 6597
Ecole Centrale de Nantes/CNRS
Nantes, France
Franck.Plestan@irccyn.ec-nantes.fr

[3]LAI
INSA de Lyon
Villeurbanne, France
{Mohamed.Smaoui,Xavier.Brun}@lai.insa-lyon.fr

Submitted to *International Journal of Control* **- June 2005**

Abstract

This paper presents the synthesis and the experimental implementation of robust higher order sliding mode controllers for an electropneumatic actuator. These controllers are based on a recent approach and are designed in monovariable (position control) and multivariable (position and pressure control) contexts. The controllers' robustness is analyzed with respect to parameters uncertainties and load disturbances.

Keywords. Higher order sliding mode, LQ approach, robust SISO/MIMO control, electropneumatic actuator, experimental results.

Nomenclature

u input voltage (V)
v Load velocity (m/s)
V Chamber volume (m^3)
F_f Dry friction forces (N)
M Load mass (kg)
T Chamber temperature (K)
S Area of the cylinder piston on a chamber side (m^2)
$_N$ (resp. $_P$) Relative to N (resp. P) chamber
y Load position (m)
p Pressure (Pa)
F_{ext} External force (N)
b Viscous friction coefficient ($N/m/s$)
k Polytropic constant
r Perfect gas constant related to unit mass ($J/kg/K$)
q_m Mass flow rate provided by the servo-distributor to cylinder chamber (kg/s)

1 Introduction

Pneumatic actuators are widely used in industrial fields because of low maintenance cost, lightweight, compliance and good force/weight ratio. But, the position control of pneumatic actuators keeps a problem since due the nonlinearity of its model and behaviour. A number of characteristics, such as friction, variation of the actuators dynamics due to large change of load and piston position along the cylinder stroke evolves the development of high-performance closed-loop controllers [9, 11, 14, 15, 23].
Several works have proposed linear and nonlinear controllers in order to get high performances behavior, in single input-single output (SISO) and multi input-multi output (MIMO) contexts. The main advantages of MIMO control (position and pressure are controlled) versus SISO one (only the position is controlled) are the energy economy, viewed that it is possible to make the same displacement (with same velocity and acceleration) with less energy consumption [7, 8], and the lack of zero dynamics while, in SISO context, zeros dynamics is one-dimensional and its stability is difficult to formally prove [5]. In [6], a comparison between two positioning linear control laws (a fixed gains control law and a control law with scheduling gains) of an electropneumatic

dissymmetrical cylinder is made in point to point displacement aim in SISO context. This work has been extended to nonlinear control in [5, 26] and [8], in which a linearizing controller has been implemented on an experimental set-up in single and multi variable context. Due to uncertainties on the model, robust controllers are necessary to ensure position tracking with high precision. Then, sliding mode SISO controllers have been used for electropneumatic actuators [4, 24, 35]. Their advantages are that they are simply implemented and robust versus parameters variations and exhibit good dynamic response. However, since the sampling frequency of the controller is limited, chattering will be produced (dangerous high-frequency vibrations of the controlled system). In order to reduce the chattering, the control can be modified to a so-called boundary layer control [30]. This type of control implies a small deterioration in accuracy and robustness[1]. Higher order sliding mode control [1,12,16,17,19,20] is a recent approach which allows to remove all the standard sliding mode restrictions, while preserving the main sliding-mode features and improving its accuracy. In [18], a second order sliding mode SISO controller has been successfully implemented on an experimental pneumatic actuator set-up. The controller guarantees a second order sliding mode etablishment with respect to the constraint $\lambda\ \delta y + \delta v = 0$ where λ is positive constant and δy and δv are the position and velocity error respectively. However, only an asymptotically 2-sliding mode with respect to the constraint $\delta y = 0$ is guaranteed. To ensure a finite time convergence to zero of the position error, the sliding variable has to be δy. Then, a 3^{rd} order sliding mode controller is *at least* necessary.

In the first part of the current paper, a 3^{rd} order sliding mode SISO controller is designed and checked on the same experimental set-up. This controller uses standard sliding mode control with linear quadratic (LQ) one over a finite time interval with a fixed final state [17] and ensures the etablishment of a *real* third order sliding mode. The *real* third order sliding mode means that, σ being the sliding variable, $\sigma, \dot{\sigma}$ and $\ddot{\sigma}$ converge in finite time in arbitrarily small vicinity of zero. Good performance, robustness against parameters uncertainties and perturbation and simple implementation (desired dynamic can be easily tuned through two parameters and convergence time can be fixed a priori) are the main features of the proposed method. This latter approach can be used for control of MIMO systems and is applied to the under interest electropneumatic actuator, in order to control *both* its position and the pressure in one of its two chambers. The originality of the current paper lays in the algorithm chosen for the controller design, the comparison between the performances of SISO and MIMO controllers, and the experimental validation.

The paper is organized as follows: Section 2 describes the model of the electropneumatic actuator and states the problem under interest in SISO and MIMO contexts. Section 3 theorically displays the control approach. Sections 4 and 5 discuss respectively the application and implementation of the proposed control schemes on an experimental set-up.

2 Model of the pneumatic system and control objectives

2.1 Description of the experimental set-up

The electropneumatic system under interest is a double acting actuator (Figure 1) composed by two chambers, denoted P (as positive) and N (as negative). The air mass flow rates entering the two chambers are modulated by two three-way servodistributors controlled by a micro-controller with two electrical inputs of opposite signs. The pneumatic jack horizontally moves a load carriage of mass M, has a stroke of 500 mm and is very unsymmetrical since it has an internal diameter of 32 mm with a simple rod of 20 mm diameter. The position sensor of the load cariage is a potentiometer. Velocity is obtained by analog derivation from the position signal and a numerical derivation of the velocity signal gives the acceleration information used by the control law. Two pressure sensors are also implemented in each chamber and used for control synthesis.

2.2 Model

Assumptions [22, 29] used to obtain a model of the pneumatic part of the electropneumatic system are:

- The supply and exhaust pressures are constant,
- The air is a perfect gas and its kinetic energy is negligible in both chambers,
- The pressure and the temperature are homogeneous in each chamber,

[1] Note that this solution is not enough in pneumatic field [3]: indeed, a good compromise between static position error and chattering cannot be found. So, the spool of the valve is exited which induced noise due to the air going from source to exhaust and an undesirable deterioration of the servodistributor.

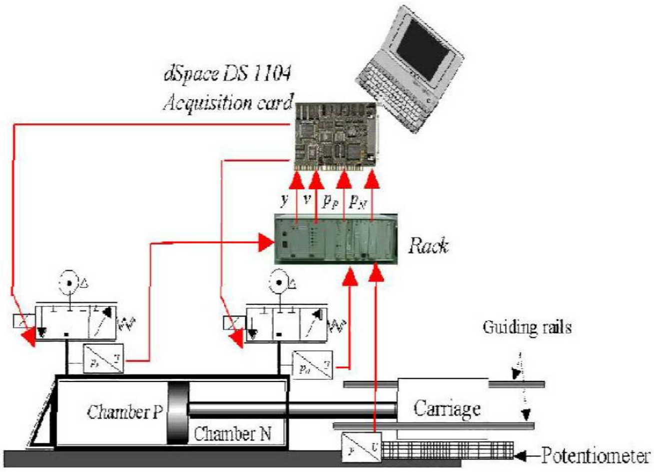

Figure 1: Electropneumatic system

- The thermodynamic evolution of the air in the cylinder chambers is polytropic and characterized by a coefficient k,
- The temperature variations in each chamber are negligible with regards to the mean temperature T,
- There is no mass flow leakage between the two cylinder chambers and outside the actuator,
- The dynamics of the servo-distributor are neglected.

Then, a nonlinear dynamic model of the electropneumatic system reads as:

$$\begin{aligned} \dot{p}_P &= \frac{krT}{V_P(y)}[q_m(u_p, p_P) - \frac{S_P}{rT}p_P v] \\ \dot{p}_N &= \frac{krT}{V_N(y)}[q_m(u_N, p_N) + \frac{S_N}{rT}p_N v] \\ \dot{v} &= \frac{1}{M}[S_P p_P - S_N p_N - bv - F_f - F_{ext}] \\ \dot{y} &= v \end{aligned} \tag{1}$$

with y the load carriage position, v its velocity and p_P and p_N the pressures of P and N chambers. The model of mass flow rate delivered by each servodistributor can be reduced to a static function described by two relationships $q_m(u_P, p_P)$ and $q_m(u_N, p_N)$. The two first equations of (1) concern the pneumatic part of the system and are obtained from the equation of perfect gases, the mass conservation law and the polytropic law under the assumptions given above. The two last equations describe the mechanical dynamics and are derived from the fundamental mechanical equation applied to the moving part. In the current study, the term F_f represents all the dry friction forces which act on the moving part in presence of viscous friction ($b.v$) and an external force only due to atmospheric pressure (F_{ext}). In order to get an affine nonlinear state model, the mass flow rate static characteristic issued from measurements [28] is written as a function of control input u_P and u_N and polynomial functions (fifth order) of p_P and p_N [2] (with $* = P$ or N)

$$q_m(u_*, p_*) = \varphi(p_*) + \psi(p_*, \text{sign}(u_*)) \cdot u_* \tag{2}$$

2.3 Uncertainties

Two kinds of uncertainties are taken into account: uncertainties due to the identification of physical parameters, and variations of environment (see Table 1). The knowledge of the viscous friction coefficient has been identified

Table 1: Uncertainties or variations of system parameters

Viscous friction coefficient	± 20%
Dry friction coefficient	± 90%
Function $\varphi(\cdot)$	± 20%
Function $\psi(\cdot,\cdot)$	± 15%
Load mass variation	± 50%

and the variation of this coefficient around the nominal value has been experimentally evaluated at ±20%. The dry friction coefficient is more difficult to identify: the track surface quality (thus the piston position), the seal wear, the working conditions (temperature, pressure, quality of air) act on its value. By some experimental tests, dry friction variation around the nominal value is evaluated to ±90%. Futhermore, the dry friction variations are supposed to be not instantaneous: the dry friction dynamics are then bounded. The mass flow rate delivered by each servodistributor has been approximated by polynomial functions (2). The uncertainties on $\varphi(\cdot)$ and $\psi(\cdot)$ are evaluated to ±20% and ±15% respectively. Finally, the total mass in displacement can evolve from 17 kg to 47 kg. The nominal mass being 32 kg, the variation is ±50%.

2.4 State model with uncertainties

The formalization of the variations is stated as

$$\begin{array}{lllll} krT\varphi(p_P) & = k_1 = k_{01} + \delta k_1, & krT\psi(p_P, \mathrm{sign}(u_P)) & = k_2 = k_{02} + \delta k_2, \\ -kS_p & = k_3 = k_{03} + \delta k_3, & krT\varphi(p_N) & = k_4 = k_{04} + \delta k_4, \\ krT\psi(p_N, \mathrm{sign}(u_N)) & = k_5 = k_{05} + \delta k_5, & kS_N & = k_6 = k_{06} + \delta k_6, \\ S_P/M & = k_7 = k_{07} + \delta k_7, & -S_N/M & = k_8 = k_{08} + \delta k_8, \\ -b/M & = k_9 = k_{09} + \delta k_9. & & \end{array} \tag{3}$$

where k_{0i} $(1 \leq i \leq 9)$ is the nominal value of the concerned parameter, δk_i the uncertainty on the concerned parameter such that $|\delta k_i| \leq \delta k_{0i}$, with δk_{0i} a known positive bound. Note that, viewed the previous hypotheses, $\delta k_3 = \delta k_6 = 0$. The term $-\frac{F_f + F_{ext}}{M} =: k_{10}$ is viewed as a perturbation which is bounded, as its first time derivative. Let x denote the state $x = [x_1\ x_2\ x_3\ x_4]^T := [p_P\ p_N\ v\ y]^T$ and $u := [u_P\ u_N]^T$ the input. Then, a state space model of the pneumatic actuator is

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} \frac{1}{V_P(x_4)}[k_1 + k_3 \cdot x_1 \cdot x_3] \\ \frac{1}{V_N(x_4)}[k_4 + k_6 \cdot x_2 \cdot x_3] \\ k_7 \cdot x_1 + k_8 \cdot x_2 + k_9 \cdot x_3 + k_{10} \\ x_3 \end{bmatrix}}_{f(x) =: f_N(x) + \Delta f} + \underbrace{\begin{bmatrix} \frac{1}{V_P(x_4)}k_2 & 0 \\ 0 & \frac{1}{V_N(x_4)}k_5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{g(x) =: g_N(x) + \Delta g} \cdot \underbrace{\begin{bmatrix} u_P \\ u_N \end{bmatrix}}_{u} \tag{4}$$

with $x \in \mathcal{X}$ and $u \in \mathcal{U}$ such that $\mathcal{X} = \{x \in I\!R^4 \mid, 0 < x_{min} \leq x_i \leq x_{MAX},\ 1 \leq i \leq 2,\ x_{imin} \leq |x_i| \leq x_{iMAX},\ 3 \leq i \leq 4\}$ and $\mathcal{U} = \{u \in I\!R^2 \mid |u_P| \leq u_{MAX}, |u_N| \leq u_{MAX}\}$, x_{min} and x_{MAX} the minimum/maximum values of the P and N chambers pressures, x_{3min} and x_{3MAX} (resp. x_{4min} and x_{4MAX}) the minimum/maximum values of the load velocity (resp. position) and u_{MAX} the maximum value of the voltage input. $f_N(x)$ (resp. $g_N(x)$) is the nominal (known) value of $f(x)$ (resp. $g(x)$), and Δf (resp. Δg) contains all the uncertain components.

2.5 Control objectives

2.5.1 MIMO problem

The aim of the MIMO control is to ensure a displacement of the load mass with controlled level of pressure by respecting a good accuracy in term of position/pressure tracking for desired trajectories (Figure 2). The amplitude of displacement is equal to 50% of the total stroke around the central position, the maximum desired velocity equals 0.60 m/s and the maximum desired acceleration is 2.2 m/s^2. The outputs of (4) read as

$$\begin{aligned} \sigma_1 &= x_4 - x_4^{ref}(t) \\ \sigma_2 &= x_1 - x_1^{ref}(t) \end{aligned} \quad (5)$$

The relative degree of (4) versus $\sigma := [\sigma_1 \ \sigma_2]^T$ are respectively 3 and 1. Then, it is necessary to design *at least* a 3-1 order sliding mode controller.

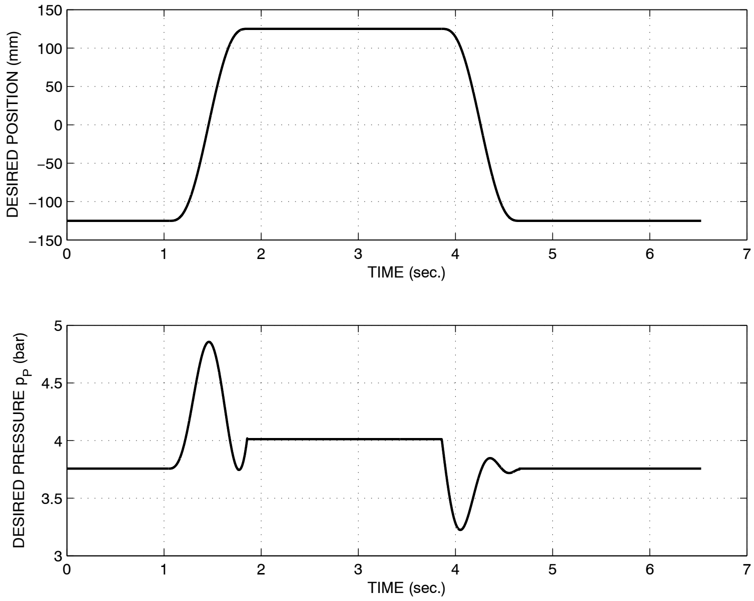

Figure 2: Desired position $x_4^{ref}(t)$ (mm) and desired pressure $x_1^{ref}(t)$ (bar) trajectories versus time (s).

2.5.2 SISO problem

The aim of the control law is to respect a good accuracy in term of only position tracking of the desired trajectory (Top of Figure 2). As the output is the actuator position

$$\sigma = x_4 - x_4^{ref}(t), \quad (6)$$

the two three-way servodistributors are supposed to be the same and their electrical variable inputs are of inverse signs, *i.e.* $u_P = -u_N$. The relative degree of (4) versus σ equals 3. Then, it is necessary to design *at least* a 3^{rd} order sliding mode controller.

3 Higher order sliding mode controller

3.1 Statement

In a sake of clarity, the approach is introduced only in monovariable context. Consider a single-input nonlinear system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ &= f_N(x) + \Delta f(x) + [g_N(x) + \Delta g(x)]u \\ h &= \sigma(x,t) \end{aligned} \quad (7)$$

with $x \in \mathcal{X} \subset I\!R^n$ the state variable and $u \in \mathcal{U} \subset I\!R$ the input, such that $\mathcal{X} = \{x \in I\!R^n \mid |x_i| \leq x_{iMAX}, \ 1 \leq i \leq n\}$ and $\mathcal{U} = \{u \in I\!R \mid |u| \leq u_{MAX}\}$. $\sigma(x,t)$ is the output function, called *sliding variable*. f, g and σ are smooth uncertain functions, f and g being composed by a nominal "well-known" part (f_N, g_N) and a term containing all the uncertain components (Δf and Δg). Suppose that the control objective is to force $\sigma(x,t)$ to zero (or to a arbitrary small vicinity of zero for higher order real sliding mode) in finite time. Assume that

H1. The relative degree ρ of (7) with respect to σ is known and the associated zero dynamics are asymptotically stable.

Definition 1 *[12] Given the sliding variable $\sigma(x,t)$, the "r^{th} order sliding manifold" associated to (7) is defined as*

$$\mathcal{S} \;=\; \{x \in \mathcal{X} \mid \sigma = \dot{\sigma} = \cdots = \sigma^{(r-1)} = 0\} \tag{8}$$

■

Definition 2 *[12] Consider the not-empty r^{th} order sliding set (8), and assume that it is locally an integral set in the Filippov sense, i.e. it consists of Filippov's trajectories of the discontinuous dynamics system. The corresponding behavior of system (7) satisfying (8) is called "r^{th} order sliding mode" with respect to the sliding variable $\sigma(x,t)$.* ■

Definition 2 means that system (7) satisfies a r^{th} order sliding mode with respect to $\sigma(x,t)$ if its state trajectories lie on the intersection of the r manifolds $\sigma = 0$, $\dot{\sigma} = 0$, $\cdots$, and $\sigma^{(r-1)} = 0$ in the state space. The r^{th} order sliding mode control approach allows the finite time stabilization to zero of the sliding variable σ and its $r-1$ first time derivatives by defining a suitable discontinuous control function which is either the actual control if $\rho = r$, or its $(r-\rho)^{th}$ time derivative if $r > \rho$. In a sake of clarity, consider only the case[2] The output σ satisfies $\sigma^{(r)} = \beta(x,t) + \gamma(x)u$, where $\gamma = L_f^r\sigma + \mathrm{d}\sigma/\mathrm{d}t$ and $\beta = L_g L_f^r \sigma$. Assume that

H2. $u \in \mathcal{U} = \{u \in I\!R \mid |u| < u_M\}$ where u_M is a real constant; $u(t)$ is a bounded discontinuous function of time and the solution of the differential equation with discontinuous right-hand side (7) admits a solution in Filippov sense on $\mathcal{S}$ for all t.

H3. Functions $\beta(x,t)$ and $\gamma(x)$ read as

$$\beta(x,t) = \beta_0(x,t) + \beta_\Delta(x), \qquad \gamma(x) = \gamma_0(x) + \gamma_\Delta(x) \tag{9}$$

with $\beta_0(x,t) = L_{f_N}^r\sigma + \mathrm{d}\sigma/\mathrm{d}t$ and $\gamma_0(x) = L_{g_N}L_{f_N}^r\sigma$.

Denoting $\Delta = \beta_\Delta + \gamma_\Delta u$ a term containing all the uncertainties/perturbations, it can be shown [17] that the r^{th} order sliding mode control problem of (7) is equivalent to the finite time stabilization of system

$$\begin{array}{rcl} \dot{Z}_1 & = & A_{11}Z_1 + A_{12}Z_2 \\ \dot{Z}_2 & = & \beta_0 + \gamma_0 u + \Delta \end{array} \tag{10}$$

where $Z_1 = [\sigma \;\cdots\; \sigma^{(r-2)}]'$, $Z_2 = \sigma^{(r-1)}$. A_{11}, A_{12} are defined by

$$A_{11} = \begin{bmatrix} 0 & 1 & \ldots & 0 & \ldots \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & \ddots & \ddots & \ldots & 1 \\ 0 & \ddots & \ddots & \ddots & 0 \end{bmatrix}_{(r-1)\times(r-1)} \qquad A_{12} = [0 \;\;\cdots\;\; 0 \;\;\; 1]'_{1\times(r-1)}\,. \tag{11}$$

H4. Functions γ_0 and Δ are such that, with $C \in I\!R^{+*}$,

$$0 < \gamma_0, \qquad |\Delta| \le C. \tag{12}$$

The first part of the next section is proposed in the context of the *ideal* higher order sliding mode, as previously presented in order to present with a sake of clarity the philosophy of the higher order sliding mode controller design. But, the establishment of an ideal sliding mode needs an infinite frequency of the control switching, which is not possible for practical applications. Then, the *real* higher order sliding mode needs to be introduced. Note that, as this chapter concerns *real* applications of higher order sliding mode control, the main results (Theorem 1 and Algorithm) are given in the context of *real* higher order sliding mode [34].

Definition 3 *Given the sliding variable $\sigma(x,t)$ and a parameter ϵ , the "real r^{th} order sliding manifold" associated to (7) is defined as*

$$\mathcal{S}_\epsilon \;=\; \{x \in \mathcal{X} \mid ||\sigma|| < C_0(\epsilon),\; ||\dot{\sigma}|| < C_1(\epsilon), \cdots,\; ||\sigma^{(r-1)}|| < C_{r-1}(\epsilon)\} \tag{13}$$

with $C_i \to 0$ when $\epsilon \to 0$ $(1 \le i \le r-1)$. ■

[2]The general case $r > \rho$ is studied in [17] and appears as a trivial extension of the case $r = \rho$.

Definition 4 *Consider the not-empty real r^{th} order sliding set (13), and assume that it is locally an integral set in the Filippov sense, i.e. it consists of Filippov's trajectories of the discontinuous dynamics system. The corresponding behavior of system (7) satisfying (13) is called "real r^{th} order sliding mode" with respect to the sliding variable $\sigma(x,t)$.* ■

3.2 A LQ control-based approach

Under Assumption H4, the system (10) can be viewed as a chain of integrators with uncertain bounded terms. The problem is stated as the finite time stabilization of (10) in a linear uncertain context, while considering the nonlinear functions β and φ as bounded non structured parametric uncertainties. Let τ define as $\tau := t - t_0$ with $t \in [t_0, t_0 + t_F]$. A solution consists in stabilizing (10) towards the origin in a finite time $t_F < +\infty$ while minimizing the linear quadratic cost (with $\tau \in [0,\ t_F]$)

$$J \;=\; \frac{1}{2}\int_0^{t_F} Z'Q\,Z\,\mathrm{d}\tau \tag{14}$$

with $Z = [Z_1'\ Z_2]'$ and under the fixed final state constraint $Z(t_F) = 0$. The positive definite symmetric matrix Q is defined as

$$Q = \left[\begin{array}{cc} Q_{11} & Q_{12} \\ Q_{12}' & Q_{22} \end{array}\right] \tag{15}$$

where Q_{11}, Q_{12} and Q_{22} are $((r-1)\times(r-1))$, $((r-1)\times(1))$ and (1×1) dimensional matrices respectively. Criterion (14) becomes

$$J \;=\; \frac{1}{2}\int_0^{t_F} (Z_1'Q_{11}Z_1 + 2Z_1'Q_{12}Z_2 + Z_2'Q_{22}Z_2)\,\mathrm{d}\tau. \tag{16}$$

The idea is to determine a switching manifold resulting in the minimum of the criterion (14), on which a higher order sliding mode occurs. $\tau = 0$ $(t = t_0)$ is the instant for which the sliding mode begins and is viewed as the initial point in (14). In the first equation of (10), consider Z_1 as the state variable, and Z_2 as a fictive control input. Then, the problem leads back to the resolution of the LQ problem (14) for the dynamics of Z_1, under the constraint $Z(t_F) = 0$. The fictive control Z_2, stabilizing Z_1 to $Z(t_F) = 0$ in finite time and minimizing the quadratic cost function (14), is given by [21]

$$Z_2 \;=\; -\left[Q_{22}^{-1}A_{12}'P(\tau) + Q_{22}^{-1}Q_{12}' - Q_{22}^{-1}A_{12}'V(\tau)H(\tau)^{-1}V(\tau)'\right] Z_1 \tag{17}$$

where $P(\tau) \in I\!R^{(r-1)\times(r-1)}$ is the unique solution to the differential Riccati equation

$$-\dot{P} = P(A_{11} - A_{12}Q_{22}^{-1}Q_{12}') - PA_{12}Q_{22}^{-1}A_{12}'P + (A_{11} - A_{12}Q_{22}^{-1}Q_{12}')'P + (Q_{11} - Q_{12}Q_{22}^{-1}Q_{12}') \tag{18}$$

with $P(t_F) =: P_F$ stated by the user. $V \in I\!R^{(r-1)\times(r-1)}$ and $H \in I\!R^{(r-1)\times(r-1)}$ are the solutions of two linear differential equations such $V(t_F) = I_{(r-1)\times(r-1)}$, $H(t_F) = 0_{(r-1)\times(r-1)}$ and

$$\begin{array}{rcl} -\dot{V} & = & (A_{11} - A_{12}Q_{22}^{-1}Q_{12}' - A_{12}Q_{22}^{-1}A_{12}'P)'V \\ \dot{H} & = & V'A_{12}Q_{22}^{-1}A_{12}'V. \end{array} \tag{19}$$

From (17), let $S(Z,\tau)$ defined by

$$S(Z,\tau) \;=\; Z_2 + \left[Q_{22}^{-1}A_{12}'P + Q_{22}^{-1}Q_{12}' - Q_{22}^{-1}A_{12}'VH^{-1}V'\right] Z_1. \tag{20}$$

Equation $S(Z,\tau) = 0$ describes dynamics which satisfy the finite time stabilization of vector Z to zero and minimize the quadratic cost function (14). Then, the *optimal switching manifold* is defined as

$$\mathcal{S}^{opt} = \{Z \mid S(Z,\tau) = 0,\ \tau \in [0, t_F]\}$$

on which system (7) is forced to slide on via the discontinuous control u. Then, an *ideal* higher order sliding mode occurs.

H5. At $\tau = 0$, $S(Z,0) = 0$.

The function $S(Z, \tau)$ is a switching variable with time-varying coefficients depending on $P(\tau)$, $V(\tau)$ and $H(\tau)$. These coefficients do not depend on state variables and then, can be computed *off-line* and stored from resolution of (18)-(19) for each time between $\tau = 0$ and $\tau = t_F$. Then, when the controller is implemented, these coefficients are *fully known*. However, at $\tau = t_F$, the function S can not be evaluated because it is undetermined. As a matter of fact, it depends on the inverse of $H(\tau)$ (with $H(t_F) = 0$) which is multiplied by Z_1 (with $Z_1(t_F) = 0$). From [27], it is known that H^{-1} exists for $\tau \in [0, t_F - \epsilon]$, with ϵ an arbitrarily small constant. Then, via a discontinuous control u, the final control objective consists in forcing, the trajectories of (7) to slide on

$$\mathcal{S}_\epsilon^{opt} = \{Z \mid S(Z, \tau) = 0, \ \tau \in [0, t_F - \epsilon], \ 0 < \epsilon << t_F\}$$

in finite time. For $\tau \in [0, t_F - \epsilon]$, equation $S(Z, \tau) = 0$ describes the desired dynamics which satisfy the finite time convergence of vector Z to an arbitrary small vinicity of the origin (fixed by parameter ϵ), *i.e.* $||\sigma|| < C_0(\epsilon)$, $||\dot{\sigma}|| < C_1(\epsilon), \cdots$, $||\sigma^{(r-1)}|| < C_{r-1}(\epsilon)$. Then, *a real higher order sliding mode occurs.*

The design of a switching control function u, which allows the sliding on $\mathcal{S}_\epsilon^{opt}$, follows the conventional path [34]; the variable structure control u can be selected to satisfy the sliding mode condition

$$S \cdot \dot{S} < -\eta |S|, \tag{21}$$

where $\eta > 0$ is a positive real number.

Theorem 1 *Consider the nonlinear system (7) with a relative degree ρ with respect to $\sigma(x, \tau)$. Suppose that it is minimum phase and that hypotheses H_1, H_2, H_3, H_4 and H_5 are fulfilled. Consider $0 < \epsilon << t_F$. Let $S \in I\!R$ a function defined as (with $0 \le \tau \le t_F - \epsilon$)*

$$S(Z, \tau) \quad = \quad Z_2 + \left[Q_{22}^{-1} A_{12}' P + Q_{22}^{-1} Q_{12}' - Q_{22}^{-1} A_{12}' V H^{-1} V'\right] Z_1 \tag{22}$$

with $r = \rho$ the sliding order, $Z_1 = [\sigma \ \ \dot{\sigma} \ \ \cdots \ \ \sigma^{(r-2)}]'$, $Z_2 = \sigma^{(r-1)}$, the matrix A_{12} defined by (11), P the unique non-negative definite solution of the differential matrix Riccati equation (18) (with a given P_F), V and H the solutions of equations (19), and Q defined by (15). Then, the control input u defined by

$$u \quad = \quad \gamma_0^{-1} \left[-\alpha \ \mathrm{sign}(S(Z, \tau)) - \beta_0\right] \tag{23}$$

with

$$\begin{aligned} &\alpha \ge \eta + \Theta + C, \\ &\Theta > \mathrm{Max} \left[\left| \left| \Psi \cdot \begin{bmatrix} \dot{\sigma} \\ \ddot{\sigma} \\ \vdots \\ \sigma^{(r-1)} \end{bmatrix} + \Sigma \cdot \begin{bmatrix} \sigma \\ \dot{\sigma} \\ \vdots \\ \sigma^{(r-2)} \end{bmatrix} \right| \right| \right], \end{aligned} \tag{24}$$

where

$$\begin{aligned} \Psi &= Q_{22}^{-1} A_{12}' P - Q_{22}^{-1} A_{12}' V H^{-1} V' + Q_{22}^{-1} Q_{12}' \\ \Sigma &= Q_{22}^{-1} A_{12}' \cdot \left[\dot{P} - \dot{V} H^{-1} V' - V (\dot{H^{-1}}) V' - V H^{-1} (\dot{V'})\right] \end{aligned} \tag{25}$$

with $\eta > 0$ a positive real number and C, $\dot{P}$, $\dot{V}$ and $\dot{H}$ defined respectively by H4-(18)-(19), leads in finite time to the establishment of r^{th} order real sliding mode with respect to σ by attracting each trajectory in finite time. ■

Proof. The finite time convergence of vector Z to an arbitrary small vinicity of the origin (fixed by parameter ϵ), *i.e.* $||\sigma|| < C_0(\epsilon)$, $||\dot{\sigma}|| < C_1(\epsilon)$, $\cdots$, $||\sigma^{(r-1)}|| < C_{r-1}(\epsilon)$, via the minimization of (14) is realized by sliding on the optimal switching manifold (for $0 \le \tau \le t_F - \epsilon$)

$$\mathcal{S}_\epsilon^{opt} \quad = \quad \left\{ x \in \mathcal{X} \mid \sigma^{(r-1)} + (Q_{22}^{-1} A_{12}' P(\tau) - Q_{22}^{-1} A_{12}' V(\tau) H(\tau)^{-1} V(\tau)' + Q_{22}^{-1} Q_{12}') \cdot \begin{bmatrix} \sigma \\ \dot{\sigma} \\ \vdots \\ \sigma^{(r-2)} \end{bmatrix} = 0 \right\} \tag{26}$$

The design of a switching control function follows the conventional path [34]: the variable structure control u takes the form

$$u \quad = \quad \gamma_0^{-1} \left[-\alpha \ \ \mathrm{sign}(S) - \beta_0\right] \tag{27}$$

with the gain α tuned such condition (21) holds. One gets

$$\begin{aligned} \dot{S} &= \beta_0 + \gamma_0 u + \Delta + \chi \\ &= -\alpha \text{sign}(S(Z,\tau)) + \Delta + \chi \end{aligned} \tag{28}$$

with χ given by (with Ψ, Σ defined by (25))

$$\chi = \Psi \begin{bmatrix} \dot{\sigma} \\ \ddot{\sigma} \\ \vdots \\ \sigma^{(r-1)} \end{bmatrix} + \Sigma \begin{bmatrix} \sigma \\ \dot{\sigma} \\ \vdots \\ \sigma^{(r-2)} \end{bmatrix}. \tag{29}$$

The inequality

$$\alpha > \eta + \Delta + \chi$$

implies $\dot{S}S < -\eta|S|$ in finite time. A sufficient condition reads as

$$\alpha > \eta + \text{Max}(|\Delta|) + \text{Max}(|\chi|). \tag{30}$$

Since the vector $[\sigma\ \dot{\sigma}\ \cdots\ \sigma^{(r-2)}\ \sigma^{(r-1)}]'$, $P(\tau)$, $V(\tau)$ and $H(\tau)$ are bounded functions, and viewed that $H(\tau)^{-1}$ exists for $\tau \in [0, t_F - \epsilon]$ [27], then function χ can be bounded by a positive real number Θ. From (30), one deduces that gain α has to be tuned so that $\alpha > \eta + \Theta + C$ to ensure (21). ∎

Implementation in practice. The following Steps and algorithm describe the practical implementation of the control law (23).

Step 1. $t \in [0,\ t_0[$. The goal of this Step is to set system (7) to reach the optimal switching surface $S_0 = Z_2(t) + \lambda_0 Z_1(t) = 0$, with λ_0 issued from the *off-line* computations/resolutions of (18)-(19) at $\tau = 0$ ($t = t_0$), *i.e.*

$$\lambda_0 = Q_{22}^{-1} A'_{12} P(\tau = 0) - Q_{22}^{-1} A'_{12} V(\tau = 0) H(\tau = 0)^{-1} V(\tau = 0)' + Q_{22}^{-1} Q'_{12},$$

by applying the control law $u = -\alpha$ sign $(Z_2(t) + \lambda_0 Z_1(t))$. Of course, during all this step, the coefficients vector λ_0 is constant. Obviously, the time $t = t_0$ is finite and defined such that $S_0 = Z_2(t_0) + \lambda_0 Z_1(t_0) = 0$.

Step 2. $t \in [t_0,\ t_0 + t_F - \epsilon]$. The control law $u = -\alpha$ sign $(S) = -\alpha$ sign $(Z_2(t) + \lambda Z_1(t))$ with λ issued from the *off-line* computations/resolutions of (18)-(19) for $t \in [t_0,\ t_0 + t_F - \epsilon]$ ($\tau \in [0,\ t_F - \epsilon]$), *i.e.*

$$\lambda = Q_{22}^{-1} A'_{12} P(\tau) - Q_{22}^{-1} A'_{12} V(\tau) H(\tau)^{-1} V(\tau)' + Q_{22}^{-1} Q'_{12},$$

maintains $S = 0$. Consequently, the equality (17) minimizing (14) under the constraint $Z(t_F) = 0$, holds. Then, at $t = t_0 + t_F - \epsilon$, the trajectories reach an arbitrary small vicinity of the origin: real higher order sliding mode occurs.

Step 3. $t \in]t_0 + t_F - \epsilon,\ \infty[$. The control task consists in maintaining the system trajectories in the vicinity of the origin. This objective is fulfilled by the control law $u = -\alpha$ sign $(S_{F_\epsilon}) = -\alpha$ sign $(Z_2(t) + \lambda_{F_\epsilon} Z_1(t))$ with λ_{F_ϵ} issued from the *off-line* computations/resolutions of (18)-(19) at $\tau = t_F - \epsilon$ ($t = t_0 + t_F - \epsilon$), *i.e.*

$$\begin{aligned} \lambda_{F_\epsilon} &= Q_{22}^{-1} A'_{12} P(\tau = t_F - \epsilon) - Q_{22}^{-1} A'_{12} V(\tau = t_F - \epsilon) H(\tau = t_F - \epsilon)^{-1} \\ &\quad V(\tau = t_F - \epsilon)' + Q_{22}^{-1} Q'_{12}. \end{aligned}$$

An upper bound for the convergence time to the origin vicinity is defined as $t_0 + t_F - \epsilon < \frac{S_0(0)}{\eta} + t_F - \epsilon$. Then, the proposed algorithm can be expressed through the following sequence of steps.

Algorithm. From optimal switching manifold defined in Theorem 1, the control is given by

(i) At $t = 0$, if $S_0 = Z_2(0) + \lambda_0 Z_1(0) \neq 0$, then for $t \geq 0$, apply $u = -\alpha$ sign$(Z_2(t) + \lambda_0 Z_1(t))$

(ii) If $S_0 = 0$, then $t = t_0$. For $t \in [t_0,\ t_0 + t_F - \epsilon]$, $u = -\alpha$ sign $(Z_2(t) + \lambda(t) Z_1(t))$

(iii) If $t \in [t_0 + t_F - \epsilon,\ \infty[$, apply $u = -\alpha$ sign $(Z_2(t) + \lambda_{F_\epsilon} Z_1(t))$ ∎

Remark 1 *As mentioned previously, equations (18) and (19) are three differential equations which do not depend on the state trajectories. Since only their final conditions are available, these equations have to be integrated backward from a a priori final time t_F over a time interval $\tau \in [t_F, 0]$ in order to find the initial conditions of P, V and H at $\tau = 0$. Practically, a possible method is to discretize these equations and to run the resulting difference equations backward. From the knowledge of the initial conditions $P(0)$, $V(0)$ and $H(0)$, two ways are possible. First, the values of P, V and H are computed off-line and stored in computer memory before the use of the controller: of course, a sufficient space of memory is necessary. The second way consists in computing on-line the equations (18)-(19) from the initial conditions: in this case, the computer has to be sufficiently fast.* ■

Remark 2 *The control law (23) is composed by a linearizing controller ("equivalent control" in the sliding mode context) coupled to a high order sliding mode one: as a matter of fact, it has been shown in [10] that, for the class of uncertainties considered here, the use of a linearizing controller leads a lower effort in terms of discontinuous part of the controller (lower gain), which is interesting in terms of chattering and energy.* ■

4 Position control

4.1 Controller design

In order to track the desired load position $x_{4_{ref}}(t)$ (Top of Figure 2), the sliding variable is defined from the position error $\sigma = x_4 - x_{4_{ref}}(t)$. The relative degree of (4) versus σ equals 3. Consider $\sigma^{(3)} = \beta(x,t) + \gamma(x)\bar{u}$ (recall that, in SISO context, $u_P = -u_N := \bar{u}$) where

$$\begin{aligned}
\beta(x,t) &= \frac{k_7}{V_P(x_4)}[k_1 + k_3 x_1 x_3] + \frac{k_8}{V_N(x_4)}[k_4 + k_6 x_2 x_3] + k_9[k_7 x_1 + k_8 x_2 + k_9 x_3 + k_{10}] + \dot{k}_{10} - x_{4ref}^{(3)} \\
&=: \beta_0 + \delta\beta \\
\gamma(x) &= \frac{k_7 k_2}{V_P(x_4)} + \frac{k_8 k_5}{V_N(x_4)} =: \gamma_0 + \delta\gamma
\end{aligned} \tag{31}$$

As previously mentioned, β_0 and γ_0 are the known nominal expressions whereas the expressions $\delta\beta$ and $\delta\gamma$ contain all the uncertainties due to parameters variations and term k_{10}. From (23), the third order sliding mode controller reads as

$$\bar{u} = \gamma_0^{-1}[-\beta_0 - \alpha \cdot \text{sign}(S)], \tag{32}$$

with

$$S = \ddot{\sigma} + (Q_{22}^{-1}A_{12}'P((t) - Q_{22}^{-1}A_{12}'V(t)H(t)^{-1}V(t)' + Q_{22}^{-1}Q_{12}') \begin{bmatrix} \sigma \\ \dot{\sigma} \end{bmatrix}$$

with

$$A_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}' & Q_{22} \end{bmatrix}$$

The gain α is tuned such that sliding mode condition (21) is satisfied. Matrices Q, P_F, t_F and ϵ are tuned to get the desired dynamics, and by taking into account the physical limitations of the system.

4.2 Practical implementation results

The proposed algorithm has been implemented on a dSpace DS1104 controller board with a dedicated digital signal processor with a 1 ms sampling time. Two pressure sensors are fixed in each chamber. The pressures p_N and p_P are such that $x_{1,2_{min}} = 1\ bar$ and $x_{1,2_{MAX}} = 7\ bar$ absolute. The maximum/minimum value of the load position equals $x_{4min/4MAX} = \pm 250\ mm$. The control input is such that $u_{MAX} = 10V$. The objective consists in tracking with high accuracy the desired position $x_{4ref}(t)$ (see Top of Figure 2) in spite of model uncertainties and load variations. t_F is fixed at $0.1s$, $\epsilon = 5\ ms$, $P_F = 0_{2\times 2}$ and

$$Q = \left[\begin{array}{c|c} Q_{11} & Q_{12} \\ \hline Q_{12}' & Q_{22} \end{array}\right] := \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & 7e-3 \end{array}\right].$$

The gain α has been tuned such that condition (21) is satisfied : $\alpha = 520$. These values ensure good static and dynamic performances. Some experimental results are provided here to demonstrate the good performance and robustness of the 3^{rd} order sliding mode controller. Firstly, the total load mass equals 32 kg (nominal case), and the viscous friction coefficient b equals 140 $N/m/s$. Figure 3 displays the tracking position error with desired position described. The maximum position tracking error is about 1.38 mm which is better[3] than with classical nonlinear control [8] and second order sliding mode controller (the maximum value of tracking error in nominal case equals 2.12 mm in [18]). This error equals about 0.5% of the total displacement magnitude. In steady state, the average position error is about 60 μm. Figure 4 displays the control input. Even if the signal exited

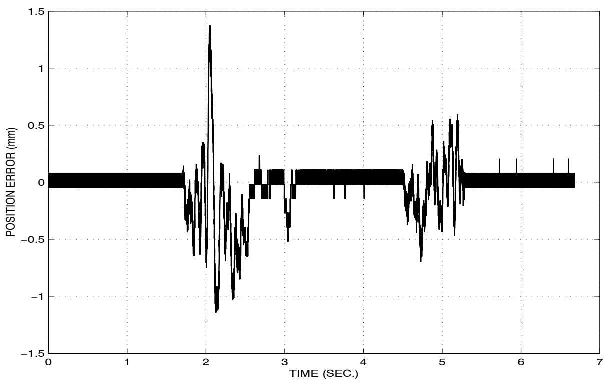

Figure 3: **Nominal mass = 32** kg**.** Tracking position error (mm) versus time (s).

the spool valve during the dynamic stage, no audible noise can be heard[4], which was not the case with first order sliding mode [4]. From these experiment results, good tracking responses are obtained for the position owing to the robust control characteristics of the controller. In order to illustrate the controller robustness, the

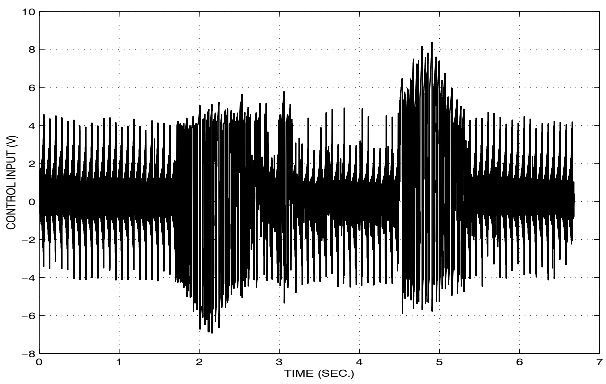

Figure 4: **Nominal mass = 32** kg**.** Control input (V) versus time (s).

total load mass is decreased to 17 kg keeping the same controller computed for nominal case. The robustness of the controller versus the load mass variation can be observed in Figure 5. The maximum position tracking error is about 2.7 mm. In steady state, the maximum position error is about 106 μm.

5 Position and pressure control

5.1 Controller design

In order to track the desired load position $x_{4_{ref}}(t)$ and pressure $x_{1_{ref}}(t)$ given by Figure 2, the sliding variable is defined from the position and pressure errors

$$\sigma \quad := \quad \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} x_4 - x_4^{ref}(t) \\ x_1 - x_1^{ref}(t) \end{bmatrix} \tag{33}$$

[3] The control laws in [8] and [18] have been implemented on the same experimental set-up, in the same conditions.
[4] The noise is due to a very high frequency displacement of the servodistributor mobile part, which can induce a faster wear.

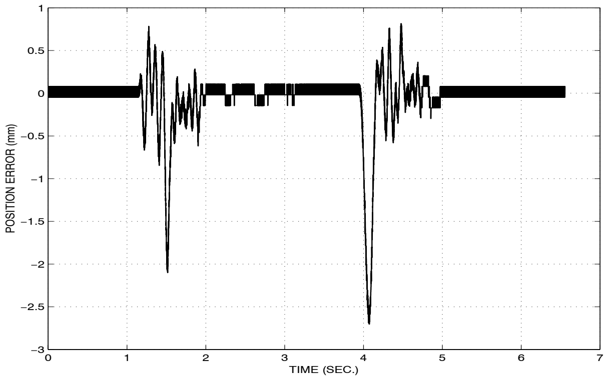

Figure 5: **Modified mass = 17** kg**.** Tracking position error (mm) versus time (s).

In this case, the relative degrees of (4) versus σ are $\rho_1 = 3$ and $\rho_2 = 1$. Given these values, it is necessary to ensure *at least* sliding mode with orders versus each sliding variable as $r_1 = 3$ and $r_2 = 1$. One gets

$$\begin{bmatrix} \sigma_1^{(3)} \\ \sigma_2^{(1)} \end{bmatrix} = \underbrace{\begin{bmatrix} \beta_1(x,t) \\ \beta_2(x,t) \end{bmatrix}}_{\beta} + \underbrace{\begin{bmatrix} \gamma_{11}(x) & \gamma_{12}(x) \\ \gamma_{21}(x) & 0 \end{bmatrix}}_{\gamma} \cdot \underbrace{\begin{bmatrix} u_P \\ u_N \end{bmatrix}}_{\bar{u}} \tag{34}$$

with

$$\begin{aligned}
\beta_1 &= \tfrac{k_7}{V_P(x_4)}\left[k_1 + k_3 x_1 x_3\right] + \tfrac{k_8}{V_N(x_4)}\left[k_4 + k_6 x_2 x_3\right] + k_9\left[k_7 x_1 + k_8 x_2 + k_9 x_3 + k_{10}\right] + \dot{k}_{10} - x_4^{ref^{(3)}}(t) \\
&=: \beta_{10} + \delta\beta_1 \\
\beta_2 &= \tfrac{1}{V_P(x_4)}\left[k_1 + k_3 \cdot x_1 \cdot x_3\right] - x_1^{ref}(t) =: \beta_{20} + \delta\beta_2 \\
\gamma_{11} &= \frac{k_7 k_2}{V_P(x_4)} =: \gamma_{110} + \delta\gamma_{11} \\
\gamma_{12} &= \frac{k_8 k_5}{V_N(x_4)} =: \gamma_{120} + \delta\gamma_{12} \\
\gamma_{21} &= \frac{1}{V_P(x_4)} k_2 =: \gamma_{210} + \delta\gamma_{21}
\end{aligned} \tag{35}$$

where β_{*0} (resp. γ_{**0}) is the known nominal value of β_* (resp. γ_{**}), and $\delta\beta_*$ (resp. $\delta\gamma_{**}$) is the term which contains all the uncertainties. Viewed the form of matrix γ, it is clear that the input u_P must be used in order to control the pressure p_P, and that the outputs are coupled. Consider the feedback, which linearizes system (34) without uncertainties,

$$u = \begin{bmatrix} \gamma_{110} & \gamma_{120} \\ \gamma_{210} & 0 \end{bmatrix}^{-1} \left(\begin{bmatrix} -\beta_{10} \\ -\beta_{20} \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \right) \tag{36}$$

where $v := [v_1 \; v_2]^T$ is the new control. One gets

$$\begin{bmatrix} \sigma_1^{(3)} \\ \dot{\sigma}_2 \end{bmatrix} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} + \begin{bmatrix} \hat{\gamma}_{11} & \hat{\gamma}_{12} \\ 0 & \hat{\gamma}_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{37}$$

with

$$\begin{aligned}\hat{\beta}_1 &= \delta\beta_1 - \frac{\delta\gamma_{12}}{\gamma_{120}}\beta_{10} - \frac{\delta\gamma_{11}}{\gamma_{210}}\beta_{20} + \frac{\gamma_{110}}{\gamma_{120}\gamma_{210}}\delta\gamma_{12}\beta_{20} \\ \hat{\beta}_2 &= \delta\beta_2 - \frac{\delta\gamma_{21}}{\gamma_{210}}\beta_{20} \\ \hat{\gamma}_{11} &= 1 + \frac{\delta\gamma_{12}}{\gamma_{120}} \\ \hat{\gamma}_{12} &= \frac{\delta\gamma_{11}}{\gamma_{210}} - \frac{\delta\gamma_{12}}{\gamma_{120}\gamma_{210}}\gamma_{110} \\ \hat{\gamma}_{22} &= 1 + \frac{\delta\gamma_{21}}{\gamma_{120}}\end{aligned} \tag{38}$$

Then, one gets

$$\begin{bmatrix} \sigma_1^{(3)} \\ \dot{\sigma}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} \tag{39}$$

with Δ_1 and Δ_2 containing all the uncertainties components and given by

$$\begin{aligned}\Delta_1 &= \delta\beta_1 - \frac{\delta\gamma_{12}}{\gamma_{120}}\beta_{10} - \frac{\delta\gamma_{11}}{\gamma_{210}}\beta_{20} + \frac{\gamma_{110}}{\gamma_{120}\gamma_{210}}\delta\gamma_{12}\beta_{20} + \frac{\delta\gamma_{12}}{\gamma_{120}}v_1 + \left(\frac{\delta\gamma_{11}}{\gamma_{210}} - \frac{\delta\gamma_{12}}{\gamma_{120}\gamma_{210}}\gamma_{110}\right)v_2 \\ \Delta_2 &= \frac{\delta\gamma_{21}}{\gamma_{120}}v_2\end{aligned} \tag{40}$$

Given the assumptions of boundness, there exist $C_1 \in I\!R^{+*}$ and $C_2 \in I\!R^{+*}$ such that $|\Delta_1| < C_1$ and $|\Delta_2| < C_2$. Viewed the structure of (39), it follows that the input v_2 is designed to control σ_2, and the input v_1 to control σ_1. Then, v_1 and v_2 are defined such that

$$\begin{aligned}v_1 &= -\alpha_1 \cdot \text{sign}(S_1(\sigma_1, \dot{\sigma}_1, \ddot{\sigma}_1, t)) \\ v_2 &= -\alpha_2 \cdot \text{sign}(\sigma_2)\end{aligned} \tag{41}$$

with

$$\begin{aligned}S_1 &= \ddot{\sigma}_1 + \underbrace{(Q_{22}^{-1}A_{12}'P(t) - Q_{22}^{-1}A_{12}'V(t)H(t)^{-1}V(t)' + Q_{22}^{-1}Q_{12}')}_{\Psi_1}\begin{bmatrix} \sigma_1 \\ \dot{\sigma}_1 \end{bmatrix}, \\ A_{12} &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}' & Q_{22} \end{bmatrix}.\end{aligned} \tag{42}$$

Design of v_2. The control law $v_2 = -\alpha_2 \cdot \text{sign}(\sigma_2)$ has to ensure the establishment of a sliding mode on $\mathcal{S}_2 = \{x \in \mathcal{X} \mid \sigma_2 = 0\}$ leading to desired tracking property for pressure p_P. The gain $\alpha_2 > 0$ is tuned such that the "standard" sliding condition is fulfilled, *i.e.* $\dot{\sigma}_2\sigma_2 < -\eta_2|\sigma_2|$ ($\eta_2 > 0$). From the second equation of (39), one gets

$$\sigma_2\dot{\sigma}_2 = \sigma_2\left(-\alpha_2 \cdot \text{sign}(\sigma_2) + \Delta_2\right) < -\eta_2|\sigma_2|$$

The previous inequality is fulfilled if $\alpha_2 > \eta_2 + C_2$. ■

Design of v_1. The gain α_1 is tuned such that condition $\dot{S}_1S_1 < -\eta_1|S_1|$ is satisfied, which ensures the establishment of a real third order sliding mode w.r.t. σ_1, for stated parameters t_F and ϵ. Then, one gets

$$\begin{aligned}\dot{S}_1S_1 &= \left(\sigma_1^{(3)} + \dot{\Psi}_1\begin{bmatrix} \sigma_1 \\ \dot{\sigma}_1 \end{bmatrix} + \Psi_1\begin{bmatrix} \dot{\sigma}_1 \\ \ddot{\sigma}_1 \end{bmatrix}\right) \cdot S_1 \\ &= \left(-\alpha_1 \cdot \text{sign}(S_1) + \Delta_1 + \dot{\Psi}_1\begin{bmatrix} \sigma_1 \\ \dot{\sigma}_1 \end{bmatrix} + \Psi_1\begin{bmatrix} \dot{\sigma}_1 \\ \ddot{\sigma}_1 \end{bmatrix}\right) \cdot S_1 < -\eta_1|S_1|\end{aligned} \tag{43}$$

The latter inequality is fulfilled if $\alpha_1 \geq \eta_1 + C_1 + \Theta_1$ with

$$\Theta_1 = \text{Max}\left[\left|\dot{\Psi}_1\begin{bmatrix} \sigma_1 \\ \dot{\sigma}_1 \end{bmatrix} + \Psi_1\begin{bmatrix} \dot{\sigma}_1 \\ \ddot{\sigma}_1 \end{bmatrix}\right|\right].$$

■

5.2 Practical implementation results

The experimentation conditions are the same than previously. The objective consists in tracking with high accuracy the desired position and pressure (see Figure 2) in spite of model uncertainties and load variations. For the pressure control, the gain α_2 has been tuned as $\alpha_2 = 4e5$. For the position control, t_F is fixed at $t_F = 0.1s$, $\epsilon = 5\ ms$, $Q_{11} = I_{2\times2}$, $Q_{12} = [0\ 0]'$, $Q_{22} = [0.007]$, and $\alpha_1 = 150$. Figure 6 displays the tracking errors of position and pressure with respect to desired trajectories. The maximum position tracking error is about 1.84 mm. This error equals about 0.7% of the total displacement magnitude. In steady state, the average position error is about 60 μm. The pressure tracking error is very low: in steady state, its average value equals $0.03bar$, and its maximum value about $0.06bar$. Figure 7 displays the control input. The dynamic and static

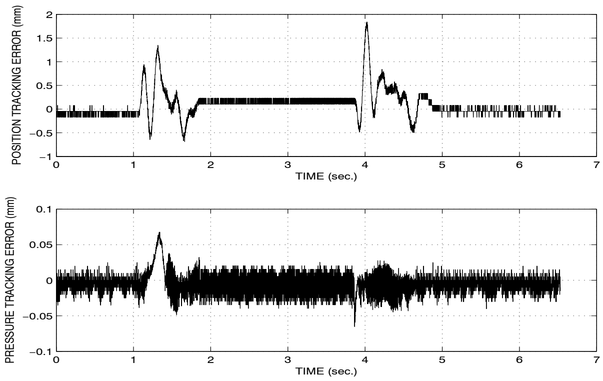

Figure 6: **Nominal mass = 32 kg.** Tracking position (mm) (Top) and pressure (bar) (Bottom) errors versus time (s).

performances are satisfying in terms of robustness. Even if the MIMO controller is interesting in terms of energy [7] and for the absence of zero dynamics, a drawback appears: as a matter of fact, the input signal intensively excites the spool valve during static stage, which is certainly due to the 1-order sliding mode control of the pressure. However, viewed the level of the inputs, this phenomena is not really “dangerous” for the system and could be attenuated by a boundary layer in the discontinuous control part [30]. But, this latter solution would decrease the performances of the controller.

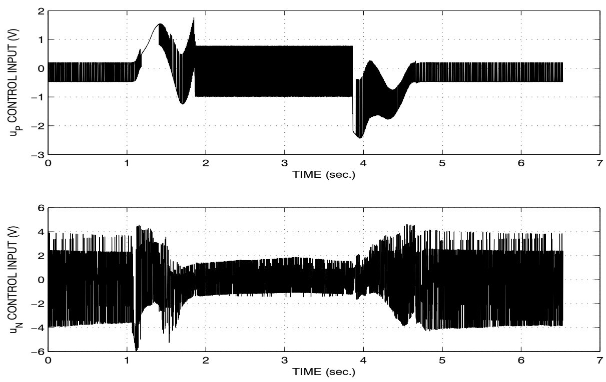

Figure 7: **Nominal mass = 32 kg.** Inputs u_P (Top) and u_N (Bottom) (V) versus time (s).

6 Conclusion

This paper shows the interest and efficiency of higher order sliding mode control in terms of robustness and precision for electropneumatic positioning system. In this kind of systems, parameters as frictions, mass flow rates, ... can evolve in time; other parameters as load mass can be modified by the user. One of the aims of control laws is to ensure repeatability in terms of position accuracy during tracking and static stage. Two kinds of control laws have been developed, one in SISO context (control of loas mass position), the second in MIMO one (control of load mass position and chamber P pressure) . The chattering effect has been decreased in SISO case with a real third order sliding mode controller of position with respect to classical sliding mode control [4]. This improvement is obtained without performances loss, which was not the case with first order sliding mode control when boundary layer is used [30]. However, the energy used by this controller can be reduced by controlling both load mass position and chamber P pressure. The results in MIMO context allow to successfully control the position by using less energy, but need to be improved in order to decrease the chattering due to the 1-order sliding mode control of pressure. Then, two research axes are currently in perspective: the first one consists in designing other control strategies in order to check their robustness, as backstepping [31]; a second one concerns observers synthesis which allows to decrease the signal measurement noise (which is injected in controller) [33].

References

[1] G. Bartolini, A. Ferrara, and E. Usai, "Chattering avoidance by second-order sliding mode control", *IEEE Trans. Automat. Control*, vol.43, no.2, pp.241-246, 1998.

[2] M. Belgharbi, D. Thomasset, S. Scavarda, and S. Sesmat, "Analytical model of the flow stage of a pneumatic servo-distributor for simulation and nonlinear control", in *Scandinavian International Conference on Fluid Power SICFP'99*, pp.847-860, Tampere, Finland, 1999.

[3] M. Bouri, D. Thomasset, and S. Scavarda, "Integral sliding mode controller of a rotational servodrive", in *JHPS International Symposium on Fluid Power*, pp.145-150, Tokyo, Japan, 1996.

[4] M. Bouri, and D. Thomasset, "Sliding control of an electropneumatic actuator using an integral switching surface", *IEEE Trans. Control Syst. Technology*, vol.2, no.2, pp.368-375, 2001.

[5] X. Brun, M. Belgharbi, S. Sesmat, D. Thomasset, and S. Scavarda, "Control of an electropneumatic actuator, comparison between some linear and nonlinear control laws", *Journal of Systems and Control Engineering*, vol.213, no.I5, pp.387-406, 1999.

[6] X. Brun, S. Sesmat, D. Thomasset, and S. Scavarda, "A comparative study between two control laws of an electopneumatic actuator", in *European Control Conference ECC'99*, CD-ROM Ref.F1000-5, Karlsruhe, Germany, 1999.

[7] X. Brun, D. Thomasset, S. Sesmat, and S. Scavarda, "Limited energy consumption in positioning control of electropneumatic actuator", in *Bath Workshop on Power Transmission and Motion Control*, pp.199-211, Bath, England, 1999.

[8] X. Brun, and D. Thomasset, "Choice of control law in electropneumatics. Expertise using an industrial benchmark and some new trends", in *Conference on Decision and Control CDC'00*, CD-ROM Ref.CD009702, Sydney, Australia, 2000.

[9] X. Brun, D. Thomasset, and E. Bideaux, "Influence of the process design on the control strategy: application in electropneumatic field", *Control Engineering Practice*, vol.10, no.7, pp.727-735, 2002.

[10] R. Castro-Linarès, A. Glumineau, S. Laghrouche, and F. Plestan, "Higher order sliding mode observer-based control", in *IFAC Symposium on System, Structure and Control SSSC 2004*, Oaxaca, Mexico, 2004.

[11] K.A. Edge, "The control of fluid power systems - responding to the challenge", *Journal of Systems and Control Engineering*, vol.211, no.I2, pp.91-110, 1997.

[12] S.V. Emelyanove, S.K. Korovin, and A. Levant,"Higher-order sliding modes in control systems", *Differential Equations*, vol.29, no.11, pp.1627-1647, 1993.

[13] A.F. Filippov, *Differential Equations with Discontinuous Right-Hand Side*, Kluwer, Dordrecht, The Netherlands, 1988.

[14] K. Hamiti, A. Voda-Besançon, and H. Roux-Buisson, "Position control of a pneumatic actuator under the influence of stiction", *Control Engineering Practice*, vol.4, no.8, pp.1079-1088, 1996.

[15] T. Kimura, S. Hara, T. Fujita, and T. Kagawa, "Feedback linearization for pneumatic actuator systems with static friction", *Control Engineering Practice*, vol.5, no.10, pp.1385-1394, 1997.

[16] S. Laghrouche, F. Plestan, A. Glumineau, and R. Boisliveau, "Robust second order sliding mode control for a permanent magnet synchronous motor", in *American Control Conference ACC'03*, pp.4071-4076, Denver, Colorado, 2003.

[17] S. Laghrouche, F. Plestan, and A. Glumineau, "Higher order sliding mode control based on optimal linear quadratic control", in *European Control Conference ECC'03*, Cambridge, England, 2003.

[18] S. Laghrouche, M. Smaoui, X. Brun and F. Plestan, "Second order sliding mode controllers for pneumatic actuators", in *American Control Conference ACC'04*, Boston, Massachusetts, 2004.

[19] A. Levant, "Sliding order and sliding accuracy in sliding mode control", *International Journal of Control*, vol.58, no.6, pp.1247-1263, 1993.

[20] A. Levant, "Universal SISO sliding-mode controllers with finite-time convergence", *IEEE Trans. Automat. Control*, vol. 49, no.9, pp.1447-1451, 2001.

[21] F.L. Lewis, and V.L. Syrmos, *Optimal Control - 2nd Edition*, Wiley, New York, 1995.

[22] D. Mac Cloy, and D. Martin, *Control of fluid power: analysis and design*, Ellis Horwood, Chichester, England, 1980.

[23] S. Ming-Chang, and T. Shy-I, "Identification and position control of a servo pneumatic cylinder", *Control Engineering Practice*, vol.3, no.9, pp.1285-1290, 1995.

[24] A.K. Paul, J.K. Mishra, and M.G. Radke, "Reduced order sliding mode control for pneumatic actuator", *IEEE Trans. Control Syst. Technology*, vol.2, no.3, pp.271-276, 1994.

[25] F. Plestan, S. Laghrouche, and A. Glumineau, "Higher order sliding mode control for MIMO nonlinear systems", in *International Workshop on Variable Structure Systems VSS'04*, Vilanova i la Geltrù, Spain, 2004.

[26] E. Richard, and S. Scavarda, "Comparison between linear and lonlinear control of an electropneumatic servodrive", *J. of Dynamic Systems, Measurement, and Control*, vol.118, pp.245-252, 1996.

[27] W.E. Schmitendorf, and S.J. Citron, "On the applicability of the sweep method to optimal control problems", *IEEE Trans. Automat. Control*, vol. 14, no.1, pp.69-72, 1969.

[28] S. Sesmat, and S. Scavarda, "Static characteristics of a three way servovalve", in *Conference on Fluid Power Technology*, pp.643-652, Aachen, Germany, 1996.

[29] J.L. Shearer, "Study of pneumatic processes in the continuous control of motion with compressed air", *Trans. Am. Soc. Mech. Eng.*, vol.78, pp.233-249, 1956.

[30] J.J. Slotine, "Sliding mode controller designe for non-linear systems", *International Journal of Control*, vol.40, no.2, pp.421-434, 1984.

[31] M. Smaoui, X. Brun, and D. Thomasset, "A robust multivariable control for an electropneumatic system using backstepping design", in *IFAC Symposium on Nonlinear Control Systems NOLCOS*, pp.1193-1198, Stuttgart, Germany, 2004.

[32] M. Smaoui, X. Brun, D. Thomasset, and R. de Giorgi, "Expérimentation d'une commande multivariable par modes glissants d'ordre supérieur sur un système électropneumatique" [in French], in *Conférence Internationale Francophone d'Automatique CIFA*, CD-ROM Ref.84, Douz, Tunisia, 2004.

[33] M. Smaoui, "Commandes non linéaires robustes et multidimensionnelles de dispositifs électropneumatiques. Synthèses et application", *Ph.D. Thesis* [in French], INSA de Lyon, 195p, 2004.

[34] V.I. Utkin, *Sliding Mode in Control and optimization*, Springer-Verlag, Berlin, 1992.

[35] L. Yang, and J.H. Lilly, "Sliding mode tracking for pneumatic muscle actuators in bicep/tricep pair configuration", in *American Control Conference ACC'03*, pp.4669-4674, Denver, Colorado, 2003.