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Higher spin extension of cosmological spacetimes in 3D: asymptotically flat behaviour with chemical potentials and thermodynamics

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Abstract: A generalized set of asymptotic conditions for higher spin gravity without cosmological constant in three spacetime dimensions is constructed. They include the most general temporal components of the gauge fields that manifestly preserve the original asymptotic higher spin extension of the BMS_3 algebra, with the same central charge. By virtue of a suitable permissible gauge choice, it is shown that this set can be directly recovered as a limit of the boundary conditions that have been recently constructed in the case of negative cosmological constant, whose asymptotic symmetries are spanned by two copies of the centrally-extended W_3 algebra. Since the generalized asymptotic conditions allow to incorporate chemical potentials conjugated to the higher spin charges, a higher spin extension of locally flat cosmological spacetimes becomes naturally included within the set. It is shown that their thermodynamic properties can be successfully obtained exclusively in terms of gauge fields and the topology of the Euclidean manifold, which is shown to be the one of a solid torus, but with reversed orientation as compared with the one of black holes. It is also worth highlighting that regularity of the fields can be ensured through a procedure that does not require an explicit matrix representation of the entire gauge group. In few words, we show that the temporal components of generalized dreibeins can be consistently gauged away, which partially fixes the chemical potentials, so that the remaining conditions can just be obtained by requiring the holonomy of the generalized spin connection along a thermal circle to be trivial. The extension of the generalized asymptotically flat behaviour to the case of spins $s \geq 2$ is also discussed.

Keywords: Space-Time Symmetries, Classical Theories of Gravity, Conformal and W Symmetry, Gauge-gravity correspondence

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1 Introduction

Higher spin gravity in three-dimensional spacetimes has become the source of a great deal of activity; see e.g., [1–45]. Reviews about this subject can be found in refs. [46–50]. In the case of negative cosmological constant, the theory can be naturally formulated in terms of a Chern-Simons action [51–53], so that in the simplest case, the gauge group is given by two copies of $SL(3,\mathbb{R})$, and it describes nonpropagating spin-3 fields nonminimally coupled to AdS_3 gravity. In this context, black hole solutions have been recently found in [54–58]. The asymptotic structure of the theory was first explored in [59, 60], where it was found that the asymptotic symmetries are generated by two copies of the W_3 algebra with the same central extension as in the case of pure gravity on AdS [61]. This set of boundary conditions can be extended so as to incorporate chemical potentials in a way that is compatible with the asymptotic W_3 symmetries, and consequently, it contains higher spin black holes that generalize the BTZ solution [62, 63] to include spin-3 charges [57, 58]. As explained in section 3, there is a suitable gauge choice that allows to perform the vanishing cosmological constant limit in a straightforward way, so that the higher spin black hole solution in [57, 58] reduces to a higher spin extension of locally flat cosmological spacetimes [64–68]. It is found that the latter class of solutions does not fit within the set of asymptotic conditions describing asymptotically flat spacetimes in the context of higher spin gravity, independently proposed in [69, 70]. Hence, one of the purposes of this article is to extend these asymptotic conditions from scratch, so as to include the higher spin extension of locally flat cosmological spacetimes, without spoiling the original asymptotic symmetries, generated by a higher spin extension of the BMS_3 algebra with central charge. This is explicitly carried out along the lines of [57, 58], in section 2. We next show in section 3 that the special gauge choice aforementioned, actually allows to recover the whole asymptotic structure from the one proposed in [58] in the vanishing cosmological constant limit. Since the generalized asymptotic conditions incorporate the chemical potentials conjugated to the higher spin charges, the thermodynamic properties of the higher spin extension of the cosmological spacetimes can be readily analyzed. This is performed in section 4, where we start warming up with the pure gravity case, and then we show how to switch on the higher spin charges and their corresponding chemical potentials. It is worth highlighting that the thermodynamic properties can be suitably analyzed without the need of an explicit matrix representation of the entire gauge group. Finally, section 5 is devoted to final remarks, including the extension to fields of spin $s \geq 2$.

It must be pointed out that many of our results overlap with the ones recently found by Gary, Grumiller, Riegler and Rosseel [71]. Both approaches were carried out independently, and so they turn out to be radically different. Nonetheless, as it is discussed in section 5, it is reassuring to verify that our results agree in the cases that were considered in [71].

2 Generalized asymptotically flat behaviour

Higher spin gravity in three spacetime dimensions is remarkably simpler than its higher-dimensional counterparts [72, 73]. Indeed, in this case the theory can be consistently truncated in order to describe the dynamics of a finite number of fields with spin $s = 2, 3, \ldots, N$ [51–53]. Furthermore, unlike the case of higher dimensions, the three-dimensional theory also admits a suitable formulation with vanishing cosmological constant. Afterwards, for the sake of simplicity, we will focus in the case of $s = 2,\ 3$, so that we leave the extension to fields of spins $s \geq 2$ to be depicted in section 5.

The theory is described by a Chern-Simons action, given by

$$I_{CS}[A] = \frac{k}{4\pi}\int \langle AdA + \frac{2}{3}A^3\rangle\,, \tag{2.1}$$

where the gauge field $A = A_\mu dx^\mu$ reads (see e.g., [60])

$$A = \omega^a J_a + e^a P_a + W^{ab} J_{ab} + E^{ab} P_{ab}\,, \tag{2.2}$$

and the set $\{J_a, P_a, J_{ab}, P_{ab}\}$ that spans the gauge group, is such that the generators P_{ab}, J_{ab} are assumed to be symmetric and traceless. In the case of vanishing cosmological constant, as explained in [60], the corresponding Lie algebra turns out to be a generalization of the Poincaré algebra, whose nonvanishing commutators read

$$[J_a, J_b] = \epsilon_{abc}J^c\,; \qquad\qquad [J_a, P_b] = \epsilon_{abc}P^c\,,$$

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$$[J_a, J_{bc}] = \epsilon^m{}_{a(b}J_{c)m}\,; \qquad [J_a, P_{bc}] = \epsilon^m{}_{a(b}P_{c)m}\,; \qquad [P_a, J_{bc}] = \epsilon^m{}_{a(b}P_{c)m}\,, \tag{2.3}$$
$$[J_{ab}, J_{cd}] = -\eta_{(a|(c}\epsilon_{d)|b)m}J^m\,; \quad [J_{ab}, P_{cd}] = -\eta_{(a|(c}\epsilon_{d)|b)m}P^m\,,$$

which is naturally recovered through an Inönü-Wigner contraction of two copies of $sl(3,\mathbb{R})$. In (2.1), the level is determined by the Newton constant according to $k = \frac{1}{4G}$, and the nondegenerate invariant bilinear product, is such that the only nonvanishing components of the bracket are given by

$$\langle P_a J_b\rangle = \eta_{ab}\,, \qquad \langle P_{ab} J_{cd}\rangle = \eta_{ac}\eta_{bd} + \eta_{ad}\eta_{cb} - \frac{2}{3}\eta_{ab}\eta_{cd}\,. \tag{2.4}$$

The field equations are then solved by locally flat connections, fulfilling $F = dA + A^2 = 0$.

A consistent set of boundary conditions for this theory was proposed independently in [69, 70], whose asymptotic symmetries were shown to be spanned by a higher spin extension of the BMS_3 algebra with an appropriate central extension. In [70], it was also shown that there is a suitable gauge choice that allows to successfully recover the whole asymptotic structure from the one proposed independently in [59, 60] in the vanishing cosmological constant limit.

In refs. [69, 70] it was also argued, along different lines, that it would be worth exploring whether the asymptotic conditions could be extended in a consistent way with the asymptotic BMS_3 symmetry. Indeed, soon after it was shown in [57, 58] that this task can always be achieved in a systematic way and for a generic setting, where some explicit examples were constructed in the case of negative cosmological constant.

In what follows, we explain how the asymptotic conditions presented in [69, 70] can be generalized along the lines of [57, 58], so as to include chemical potentials without spoiling the original asymptotic BMS_3 symmetry.

Let us then start considering the asymptotic form of the gauge fields in [70] at a fixed time slice ($u = u_0$). It is useful to express it with the gauge choice of [74], which allows to completely capture the radial dependence through a group element of the form $h(r) = e^{-rP_0}$, so that

$$A = h^{-1}a_{(0)}h + h^{-1}dh\,, \tag{2.5}$$

where $a_{(0)} = a_\varphi d\varphi$, with[1]

$$a_\varphi = J_1 + \frac{2\pi}{k}(\mathcal{J}P_0 + \mathcal{P}J_0) + \frac{\pi}{k}(\mathcal{V}P_{00} + \mathcal{W}J_{00})\,. \tag{2.6}$$

The asymptotic form of the connection is then maintained under gauge transformations of the form $\delta A = d\Omega + [A, \Omega]$, with $\Omega = h^{-1}\eta_{(0)}h$, where $\eta_{(0)} = \eta_{(0)}\,(T, Y, Z, X)$ depends on four arbitrary functions of u_0, φ, and reads

$$\eta_{(0)} = \frac{2\pi}{k}\left(Y\mathcal{J} + T\mathcal{P} + 2Z\mathcal{V} + 2Z\mathcal{W} - \frac{k}{2\pi}T''\right)P_0 + TP_1 - T'P_2$$

[1]Our conventions agree with the ones in [70], being such that a non-diagonal Minkowski tangent space metric is assumed, whose only nonvanishing components are $\eta_{01} = \eta_{10} = \eta_{22} = 1$, and the Levi-Civita symbol fulfils $\epsilon_{012} = 1$. Nonetheless, the fields used in [70], here denoted with tilde, relate with the ones in this work according to $\mathcal{P} = \frac{k}{4\pi}\tilde{\mathcal{M}}$, $\mathcal{J} = \frac{k}{2\pi}\tilde{\mathcal{N}}$, $\mathcal{W} = \frac{k}{\pi}\tilde{\mathcal{W}}$, $\mathcal{V} = \frac{k}{\pi}\tilde{\mathcal{Q}}$. For later purposes, it is useful to introduce the subscript (0) in order to denote objects defined in the case of vanishing cosmological constant Λ, e.g., $a_{(0)} = a_{(\Lambda=0)}$.

$$
\begin{aligned}
&+\frac{2\pi}{k}\left(Y\mathcal{P}+2Z\mathcal{W}-\frac{k}{2\pi}\mu''\right)J_0+YJ_1-Y'J_2+\frac{\pi}{k}\Bigg[Y\mathcal{V}+T\mathcal{W}-\frac{8}{3}\left(X''\mathcal{P}+Z''\mathcal{J}\right)\\
&-\frac{7}{3}\left(Z'\mathcal{J}'+X'\mathcal{P}'\right)+\frac{2}{3}Z\left(\frac{12}{k}\mathcal{J}\mathcal{P}-\mathcal{J}''\right)+\frac{2}{3}X\left(\frac{6\pi}{k}\mathcal{P}^2-\mathcal{P}''\right)+\frac{k}{6\pi}X''''\Bigg]P_{00} \qquad (2.7)\\
&+\frac{4\pi}{k}\left(X\mathcal{P}+Z\mathcal{J}-\frac{k}{4\pi}X''\right)P_{01}-\frac{4\pi}{3k}\left(Z\mathcal{J}'+X\mathcal{P}'+\frac{5}{2}\left(X'\mathcal{P}+Z'\mathcal{J}\right)-\frac{k}{4\pi}X'''\right)P_{02}\\
&+XP_{11}-X'P_{12}+\frac{\pi}{k}\left[Y\mathcal{W}+\frac{2}{3}Z\left(\frac{6\pi}{k}\mathcal{P}^2-\mathcal{P}''\right)-\frac{8}{3}Z''\mathcal{P}-\frac{7}{3}Z'\mathcal{P}'+\frac{k}{6\pi}Z''''\right]J_{00}\\
&+\left(\frac{4\pi}{k}Z\mathcal{P}-Z''\right)J_{01}-\frac{4\pi}{3k}\left(Z\mathcal{P}'+\frac{5}{2}Z'\mathcal{P}-\frac{k}{4\pi}Z'''\right)J_{02}+ZJ_{11}-Z'J_{12}\,,
\end{aligned}
$$

provided the fields $\mathcal{P}$, $\mathcal{J}$, $\mathcal{W}$ and $\mathcal{V}$ transform according to

$$
\begin{aligned}
\delta_{(0)}\mathcal{J}&=2Y'\mathcal{J}+Y\mathcal{J}'+T\mathcal{P}'+2T'\mathcal{P}-\frac{k}{2\pi}T'''+2Z\mathcal{V}'+3Z'\mathcal{V}+2\mathcal{W}'X+3\mathcal{W}X'\,,\\
\delta_{(0)}\mathcal{P}&=2Y'\mathcal{P}+Y\mathcal{P}'-\frac{k}{2\pi}Y'''+2Z\mathcal{W}'+3Z'\mathcal{W}\,, \qquad (2.8)\\
\delta_{(0)}\mathcal{W}&=3Y'\mathcal{W}+Y\mathcal{W}'-\frac{2}{3}Z\left(\mathcal{P}''-\frac{8\pi}{k}\mathcal{P}^2\right)'-3Z'\left(\mathcal{P}''-\frac{32\pi}{9k}\mathcal{P}^2\right)\\
&\quad-5Z''\mathcal{P}'-\frac{10}{3}Z'''\mathcal{P}+\frac{k}{6\pi}Z^{(5)}\,,\\
\delta_{(0)}\mathcal{V}&=3Y'\mathcal{V}+Y\mathcal{V}'+T\mathcal{W}'+3T'\mathcal{W}-\frac{2}{3}Z\left(\mathcal{J}''-\frac{16\pi}{k}\mathcal{J}\mathcal{P}\right)'\\
&\quad-3Z'\left(\mathcal{J}''-\frac{64\pi}{9k}\mathcal{J}\mathcal{P}\right)-5Z''\mathcal{J}'-\frac{10}{3}Z'''\mathcal{J}-\frac{2}{3}X\left(\mathcal{P}''-\frac{8\pi}{k}\mathcal{P}^2\right)'\\
&\quad-3X'\left(\mathcal{P}''-\frac{32\pi}{9k}\mathcal{P}^2\right)-5\mathcal{P}'X''-\frac{10}{3}\mathcal{P}X'''+\frac{k}{6\pi}X^{(5)}\,.
\end{aligned}
$$

Here prime denotes derivative with respect to φ. Therefore, as explained in [57, 58], since the time evolution of the dynamical fields is generated by a gauge transformation with parameter A_u, the asymptotic symmetries will be preserved along time provided the Lagrange multiplier is of the allowed form; i.e., $A_u = h^{-1}a_u h$, with

$$
a_u=\eta_{(0)}\left(\xi,\mu,\vartheta,\varrho\right)\,, \qquad (2.9)
$$

where the "chemical potentials" ξ, μ, ϑ, ϱ, stand for arbitrary functions of time and the angle that are fixed at the boundary.

Consistency of preserving the asymptotic form of a_u under the allowed gauge transformations then implies that the field equations have to be fulfilled at the asymptotic region, i.e.,

$$
\begin{aligned}
\dot{\mathcal{J}}&=2\mu'\mathcal{J}+\mu\mathcal{J}'+\xi\mathcal{P}'+2\xi'\mathcal{P}-\frac{k}{2\pi}\xi'''+2\vartheta\mathcal{V}'+3\vartheta'\mathcal{V}+2\mathcal{W}'\varrho+3\mathcal{W}\varrho'\,,\\
\dot{\mathcal{P}}&=2\mu'\mathcal{P}+\mu\mathcal{P}'-\frac{k}{2\pi}\epsilon'''+2\vartheta\mathcal{W}'+3\vartheta'\mathcal{W}\,,\\
\dot{\mathcal{W}}&=3\mu'\mathcal{W}+\mu\mathcal{W}'-\frac{2}{3}\vartheta\left(\mathcal{P}''-\frac{8\pi}{k}\mathcal{P}^2\right)'-3\vartheta'\left(\mathcal{P}''-\frac{32\pi}{9k}\mathcal{P}^2\right)
\end{aligned}
$$

$$- 5\vartheta''\mathcal{P}' - \frac{10}{3}\vartheta'''\mathcal{P} + \frac{k}{6\pi}\vartheta^{(5)} \tag{2.10}$$

$$\dot{\mathcal{V}} = 3\mu'\mathcal{V} + \mu\mathcal{V}' + \xi\mathcal{W}' + 3\xi'\mathcal{W} - \frac{2}{3}\vartheta\left(\mathcal{J}'' - \frac{16\pi}{k}\mathcal{J}\mathcal{P}\right)'$$
$$- 3\vartheta'\left(\mathcal{J}'' - \frac{64\pi}{9k}\mathcal{J}\mathcal{P}\right) - 5\vartheta''\mathcal{J}' - \frac{10}{3}\vartheta'''\mathcal{J} - \frac{2}{3}\varrho\left(\mathcal{P}'' - \frac{8\pi}{k}\mathcal{P}^2\right)'$$
$$- 3\varrho'\left(\mathcal{P}'' - \frac{32\pi}{9k}\mathcal{P}^2\right) - 5\mathcal{P}'\varrho'' - \frac{10}{3}\mathcal{P}\varrho''' + \frac{k}{6\pi}\varrho^{(5)},$$

while the parameters of the asymptotic symmetries must satisfy the following "deformed chirality conditions", given by

$$\begin{aligned}
\dot{Y} &= Y'\mu - Y\mu' + Z''\vartheta' - Z'\vartheta'' - \frac{2}{3}\left(Z'''\vartheta - Z\vartheta'''\right) + \frac{32\pi}{3k}\mathcal{P}\left(Z'\vartheta - Z\vartheta'\right),\\
\dot{T} &= Y'\xi - Y\xi' + Z''\varrho' - Z'\varrho'' + T'\mu - T\mu' + \vartheta'X'' - \vartheta''X'\\
&\quad + \frac{2}{3}\left(Z\varrho''' - Z'''\varrho + \vartheta'''X - \vartheta X'''\right)\\
&\quad + \frac{32\pi}{3k}\left[\mathcal{J}\left(Z'\vartheta - Z\vartheta'\right) + \mathcal{P}\left(Z'\varrho - Z\varrho' + \vartheta X' - \vartheta' X\right)\right],\\
\dot{Z} &= 2Y'\vartheta - Y\vartheta' + Z'\mu - 2Z\mu',\\
\dot{X} &= 2Y'\varrho - Y\varrho' + Z'\xi - 2Z\xi' - 2\mu'X + \mu X' - T\vartheta' + 2T'\vartheta,
\end{aligned} \tag{2.11}$$

where dot corresponds to the derivative along u.

In sum, the extended asymptotic behaviour is described by gauge fields of the form (2.5), with

$$a_{(0)} = a_\varphi d\varphi + a_u du, \tag{2.12}$$

where a_φ and a_u are given by eqs. (2.6) and (2.9), respectively.

As explained in [57, 58], the canonical generators depend only on a_φ, and never on the chemical potentials, so that their expression is precisely the same as the one obtained in [70], i.e.,

$$Q_{(0)}(T, Y, Z, X) = -\int \left(T\mathcal{P} + Y\mathcal{J} + Z\mathcal{V} + X\mathcal{W}\right) d\varphi. \tag{2.13}$$

Hence, by construction, the asymptotic symmetries are still generated by the higher spin extension of the centrally-extended BMS_3 algebra, whose mode expansion is explicitly written in eq. (3.29) below.

As an ending remark of this section, according to (2.10), it is fairly clear that configurations for which the fields $\mathcal{P}$, $\mathcal{J}$, $\mathcal{V}$, $\mathcal{W}$, as well as their corresponding chemical potentials ξ, μ, ϑ, ϱ, are constant, solve the field equations. This class of solutions then explicitly reads

$$a_{(0)} = \left[J_1 + \frac{2\pi}{k}(\mathcal{J}P_0 + \mathcal{P}J_0) + \frac{\pi}{k}(\mathcal{V}P_{00} + \mathcal{W}J_{00})\right] d\varphi + a_u(\xi, \mu, \vartheta, \varrho)\, du, \tag{2.14}$$

with

$$a_u(\xi, \mu, \vartheta, \varrho) = \frac{2\pi}{k}\left(\mu\mathcal{J} + \xi\mathcal{P} + 2\vartheta\mathcal{V} + 2\varrho\mathcal{W}\right)P_0 + \xi P_1 + \frac{2\pi}{k}\left(\mu\mathcal{P} + 2\vartheta\mathcal{W}\right)J_0 + \mu J_1$$

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$$+\frac{\pi}{k}\left(\mu\mathcal{V}+\xi\mathcal{W}+\frac{8}{k}\vartheta\mathcal{J}\mathcal{P}+\frac{4\pi}{k}\varrho\mathcal{P}^2\right)P_{00}+\frac{4\pi}{k}\left(\varrho\mathcal{P}+\vartheta\mathcal{J}\right)P_{01}+\varrho P_{11} \qquad (2.15)$$
$$+\frac{\pi}{k}\left(\mu\mathcal{W}+\frac{4\pi}{k}\vartheta\mathcal{P}^2\right)J_{00}+\frac{4\pi}{k}\vartheta\mathcal{P}J_{01}+\vartheta J_{11}\,,$$

and provides the searched for higher spin extension of the locally flat cosmological spacetimes [64–68]. As it follows from (2.13), the solutions not only carry mass and angular momentum, determined by the spin-2 charges $\mathcal{P}$, $\mathcal{J}$, respectively, but they are also endowed with global charges of spin 3, determined through $\mathcal{W}$, $\mathcal{V}$. These higher spin charges are of electric and magnetic type, i.e., they are even and odd under parity, respectively.

The analysis of different classes of cosmological spacetimes endowed with higher spin fields has also been discussed in [75–78].

3 Recovering the asymptotically flat structure from a vanishing cosmological constant limit

In this section we explain how the extended asymptotically flat structure described above can be recovered from a suitable vanishing cosmological constant limit of the asymptotic conditions recently constructed in [57, 58], for the case that includes black holes endowed with global higher spin charges. By means of a suitable modification of the Lagrange multipliers, the latter set enlarges the one independently proposed in [59, 60], so as to accommodate chemical potentials in a way that it is consistent with the extended conformal symmetry, spanned by two copies of the W_3 algebra.

3.1 Extended asymptotic behaviour and higher spin black holes with negative cosmological constant

In the case of negative cosmological constant, $\Lambda=-\frac{1}{\ell^2}$, the theory is described by two independent $sl\left(3,\mathbb{R}\right)$ gauge fields, A^+ and A^-, so that the action reads

$$I=I_{CS}\left[A^+\right]-I_{CS}\left[A^-\right]\,. \qquad (3.1)$$

The level is now given by $\kappa=k\ell$, and the bracket corresponds to a quarter of the trace in the fundamental representation of $sl\left(3,\mathbb{R}\right)$, generated by L_i and W_m, with $i=-1,0,1$, and $m=-2,-1,\ldots,+2$.

As explained in [57, 58], the radial dependence can be consistently gauged away, i.e., $A^\pm=g^{-1}a^\pm g+g^{-1}dg$, with $g=g(r)$, so that the asymptotic form of the gauge fields is described through

$$a^\pm=\left(L^\pm_{\pm1}-\frac{2\pi}{\kappa}\mathcal{L}^\pm L^\pm_{\mp1}-\frac{\pi}{2\kappa}\mathcal{W}^\pm W^\pm_{\mp2}\right)d\varphi+\lambda^\pm\left(\xi_\pm,\eta_\pm\right)dt\,, \qquad (3.2)$$

where $\lambda^\pm\left(\xi_\pm,\eta_\pm\right)$ is given by

$$\lambda^\pm\left(\xi_\pm,\eta_\pm\right)=\pm\left[\xi_\pm L^\pm_{\pm1}+\eta_\pm W^\pm_{\pm2}\mp\xi'_\pm L^\pm_0\mp\eta'_\pm W^\pm_{\pm1}+\frac{1}{2}\left(\xi''_\pm-\frac{4\pi}{\kappa}\xi_\pm\mathcal{L}^\pm+\frac{8\pi}{\kappa}\mathcal{W}^\pm\eta_\pm\right)L^\pm_{\mp1}\right.$$
$$-\left(\frac{\pi}{2\kappa}\mathcal{W}^\pm\xi_\pm+\frac{7\pi}{6k}\mathcal{L}^{\pm\prime}\eta'_\pm+\frac{\pi}{3\kappa}\eta_\pm\mathcal{L}^{\pm\prime\prime}+\frac{4\pi}{3\kappa}\mathcal{L}^\pm\eta''_\pm-\frac{4\pi^2}{\kappa^2}(\mathcal{L}^\pm)^2\eta_\pm-\frac{1}{24}\eta''''_\pm\right)W^\pm_{\mp2}$$

$$+\frac{1}{2}\left(\eta_\pm''-\frac{8\pi}{\kappa}\mathcal{L}^\pm\eta_\pm\right)W_0^\pm\mp\frac{1}{6}\left(\eta_\pm'''-\frac{8\pi}{\kappa}\eta_\pm\mathcal{L}^{\pm\prime}-\frac{20\pi}{\kappa}\mathcal{L}^\pm\eta_\pm'\right)W_{\mp1}^\pm\Bigg]\,. \tag{3.3}$$

Here $\mathcal{L}^\pm$, $\mathcal{W}^\pm$, $\xi_\pm$, $\eta_\pm$ stand for arbitrary functions of time and the angular coordinate t, φ. Note that the "chemical potentials" $\xi_\pm$, $\eta_\pm$, appear only in the components of the gauge fields along time, and they are assumed to be fixed at infinity, i.e. $\delta\xi_\pm=\delta\eta_\pm=0$.

The asymptotic behaviour of the dynamical fields $a_\varphi^\pm$ is preserved under gauge transformations generated by $\lambda^\pm(\varepsilon_\pm,\chi_\pm)$, where the parameters $\varepsilon_\pm$, $\chi_\pm$ are independent functions of t, φ, provided the fields $\mathcal{L}^\pm$, $\mathcal{W}^\pm$ transform as

$$\delta\mathcal{L}^\pm=\pm2\mathcal{L}^\pm\varepsilon_\pm'\pm\varepsilon_\pm\mathcal{L}^{\pm\prime}\mp\frac{\kappa}{4\pi}\varepsilon_\pm'''\mp2\chi_\pm\mathcal{W}^{\pm\prime}\mp3\mathcal{W}^\pm\chi_\pm'\,, \tag{3.4}$$

$$\begin{aligned}\delta\mathcal{W}^\pm&=\pm3\mathcal{W}^\pm\varepsilon_\pm'\pm\varepsilon_\pm\mathcal{W}^{\pm\prime}\pm\frac{2}{3}\chi_\pm\left(\mathcal{L}^{\pm\prime\prime\prime}-\frac{16\pi}{\kappa}\left(\mathcal{L}^\pm\right)^{2\prime}\right)\pm3\left(\mathcal{L}^{\pm\prime\prime}-\frac{64\pi}{9\kappa}\left(\mathcal{L}^\pm\right)^2\right)\chi_\pm'\\&\quad\pm5\chi_\pm''\mathcal{L}^{\pm\prime}\pm\frac{10}{3}\mathcal{L}^\pm\chi_\pm'''\mp\frac{\kappa}{12\pi}\chi_\pm^{(5)}\,.\end{aligned} \tag{3.5}$$

It is then simple to verify that the fall-off of the Lagrange multipliers $a_t^\pm$ is maintained under the asymptotic symmetries, provided the field equations are fulfilled at the asymptotic region, i.e.,

$$\dot{\mathcal{L}}^\pm=\pm2\mathcal{L}^\pm\xi_\pm'\pm\xi_\pm\mathcal{L}^{\pm\prime}\mp\frac{\kappa}{4\pi}\xi_\pm'''\mp2\eta_\pm\mathcal{W}^{\pm\prime}\mp3\mathcal{W}^\pm\eta_\pm'\,, \tag{3.6}$$

$$\begin{aligned}\dot{\mathcal{W}}^\pm&=\pm3\mathcal{W}^\pm\xi_\pm'\pm\xi_\pm\mathcal{W}^{\pm\prime}\pm\frac{2}{3}\eta_\pm\left(\mathcal{L}^{\pm\prime\prime\prime}-\frac{16\pi}{\kappa}\left(\mathcal{L}^\pm\right)^{2\prime}\right)\pm3\left(\mathcal{L}^{\pm\prime\prime}-\frac{64\pi}{9\kappa}\left(\mathcal{L}^\pm\right)^2\right)\eta_\pm'\\&\quad\pm5\eta_\pm''\mathcal{L}^{\pm\prime}\pm\frac{10}{3}\mathcal{L}^\pm\eta_\pm'''\mp\frac{\kappa}{12\pi}\eta_\pm^{(5)}\,,\end{aligned} \tag{3.7}$$

while the parameters have to satisfy the following "deformed chirality conditions":

$$\begin{aligned}\dot{\varepsilon_\pm}&=\pm\left(\varepsilon_\pm\xi_\pm'-\xi_\pm\varepsilon_\pm'\right)\pm\left(\chi_\pm'\eta_\pm''-\chi_\pm''\eta_\pm'\right)\pm\frac{2}{3}\left(\chi_\pm'''\eta_\pm-\chi_\pm\eta_\pm'''\right)\pm\frac{2^6\pi}{3\kappa}\left(\chi_\pm\eta_\pm'-\eta_\pm\chi_\pm'\right)\mathcal{L}^\pm\,,\\ \dot{\chi_\pm}&=\mp\left(\chi_\pm'\xi_\pm-2\chi_\pm\xi_\pm'\right)\pm\left(\varepsilon_\pm\eta_\pm'-2\varepsilon_\pm'\eta_\pm\right)\,.\end{aligned} \tag{3.8}$$

Hence, by construction, the canonical generators associated to the asymptotic symmetries do not depend on the chemical potentials, and are given by

$$Q_\pm\left(\varepsilon_\pm,\chi_\pm\right)=\mp\int\left(\varepsilon_\pm\mathcal{L}_\pm-\chi_\pm\mathcal{W}_\pm\right)d\varphi\,, \tag{3.9}$$

so that their Poisson brackets span two copies of the W_3 algebra with the standard central extension, $c=\frac{3\ell}{2G}$ [61].

As explained in [57, 58], configurations with constant fields $\mathcal{L}^\pm$, $\mathcal{W}^\pm$, and chemical potentials $\xi_\pm$, $\eta_\pm$, given by

$$a^\pm=\left(L_{\pm1}^\pm-\frac{2\pi}{\kappa}\mathcal{L}^\pm L_{\mp1}^\pm-\frac{\pi}{2\kappa}\mathcal{W}^\pm W_{\mp2}^\pm\right)d\varphi+\lambda^\pm\left(\xi_\pm,\eta_\pm\right)dt\,, \tag{3.10}$$

with

$$\lambda^\pm\left(\xi_\pm,\eta_\pm\right)=\pm\Bigg[\xi_\pm L_{\pm1}^\pm+\eta_\pm W_{\pm2}^\pm-\frac{2\pi}{\kappa}\left(\xi_\pm\mathcal{L}^\pm-2\mathcal{W}^\pm\eta_\pm\right)L_{\mp1}^\pm$$

$$- \left(\frac{\pi}{2\kappa} \mathcal{W}^{\pm} \xi_{\pm} - \frac{4\pi^2}{\kappa^2} (\mathcal{L}^{\pm})^2 \eta_{\pm} \right) W^{\pm}_{\mp 2} - \frac{4\pi}{\kappa} \left(\mathcal{L}^{\pm} \eta_{\pm} \right) W^{\pm}_{0} \Bigg] \,, \tag{3.11}$$

manifestly solve the field equations (3.6), (3.7), and describe black holes solutions carrying not only mass and angular momentum, but also nontrivial spin-3 charges of electric and magnetic type.

3.2 A suitable gauge choice to obtain the extended asymptotic behaviour in the vanishing cosmological constant limit

As explained in [79], even in the case of pure gravity, the limiting process that allows to recover the asymptotically flat behaviour of the metric from the Brown-Henneaux boundary conditions, in a way that it is consistent with the asymptotic BMS_3 symmetry [80, 81], turns out to be a very subtle one. Hence, in order to show how the whole extended asymptotic structure described in section 2 can be obtained from the one in 3.1 in the vanishing cosmological constant limit, we follow a similar strategy as the one implemented in [70] and [74], for the cases of higher spin gravity (without chemical potentials) and supergravity, respectively.

The procedure consists in finding a suitable gauge choice that allows to take the limit in a straightforward way. As explained in [58], the searched for gauge choice must be "permissible", in the sense that it should not interfere with the asymptotic symmetry algebra. Although not strictly necessary, one of the simplest possibilities is looking for a gauge choice that does not depend on the global charges, since this ensures that the allowed gauge transformations commute with the variation of the canonical generators.

Let us then consider the extended asymptotic conditions for the theory with negative cosmological constant, described by the connections $a^{\pm}$ in eq. (3.2). Our goal can then be achieved expressing the fall-off of the entire gauge field with the following permissible gauge choice:

$$a_{(\Lambda)} := a^{+} + g^{-1} a^{-} g \,, \tag{3.12}$$

where g stands for a constant group element, given by

$$g = e^{\frac{\pi}{2}\left(L_1^{-} + L_{-1}^{-}\right)} \,. \tag{3.13}$$

It is worth pointing out that this gauge choice is even simpler than the ones performed in [70] and [74], since it only affects one of the copies of the connection. Indeed, the effect of this gauge transformation on a^{-} just amounts to modify its components according to $L_i \to (-1)^{i+1} L_{-i}$, and $W_m \to (-1)^m W_{-m}$.

It is then useful to perform the following change of basis

$$\begin{aligned}
&J_0^{\pm} = -\frac{1}{2} L_{-1}^{\pm}, \qquad J_2^{\pm} = L_0^{\pm}, \qquad J_1^{\pm} = L_1^{\pm}, \\
&T_{00}^{\pm} = -\frac{1}{4} W_{-2}^{\pm} \,, \quad T_{02}^{\pm} = \frac{1}{2} W_{-1}^{\pm} \,, \quad T_{22}^{\pm} = -W_0^{\pm} \,, \quad T_{12}^{\pm} = -W_1^{\pm} \,, \quad T_{11}^{\pm} = -W_2^{\pm} \,,
\end{aligned} \tag{3.14}$$

being equivalent to the one in [70], up to an automorphism, so that the generators T_{ab} become traceless. This change of basis is then followed by

$$J_a^{\pm} = \frac{J_a \pm \ell P_a}{2} \,; \qquad T_{ab}^{\pm} = \frac{J_{ab} \pm \ell P_{ab}}{2} \,, \tag{3.15}$$

so that the $sl(3,\mathbb{R}) \oplus sl(3,\mathbb{R})$ generators are now described by the set $\{J_a, P_a, J_{ab}, P_{ab}\}$, and the algebra reads [60]

$$
\begin{aligned}
&[J_a, J_b] = \epsilon_{abc} J^c \ ; \ [J_a, P_b] = \epsilon_{abc} P^c \ ; \ [P_a, P_b] = -\Lambda \epsilon_{abc} J^c \,, \\
&[J_a, J_{bc}] = \epsilon^m{}_{a(b} J_{c)m} \ ; \ [J_a, P_{bc}] = \epsilon^m{}_{a(b} P_{c)m} \ ; \ [P_a, J_{bc}] = \epsilon^m{}_{a(b} P_{c)m} \ ; \ [P_a, P_{bc}] = -\Lambda \epsilon^m{}_{a(b} J_{c)m} \,, \\
&[J_{ab}, J_{cd}] = -\eta_{(a|(c}\epsilon_{d)|b)m} J^m \ ; \ [J_{ab}, P_{cd}] = -\eta_{(a|(c}\epsilon_{d)|b)m} P^m \ ; \ [P_{ab}, P_{cd}] = \Lambda \eta_{(a|(c}\epsilon_{d)|b)m} J^m \,.
\end{aligned}
\tag{3.16}
$$

It is also natural and convenient to redefine the fields and chemical potentials according to

$$
\mathcal{L}^{\pm} = \frac{\ell \mathcal{P} \pm \mathcal{J}}{2} \ ; \ \mathcal{W}^{\pm} = \frac{\ell \mathcal{W} \pm \mathcal{V}}{2} \,, \tag{3.17}
$$

and

$$
\xi_{\pm} = \frac{\xi}{\ell} \pm \mu \ ; \ \eta_{\pm} = -\left(\frac{\varrho}{\ell} \pm \vartheta\right) \,, \tag{3.18}
$$

respectively; as well as renaming the time coordinate as $t = u$. The asymptotic form of the gauge field (3.12) then reduces to

$$
a_{(\Lambda)} = a_{(0)} - \frac{2\pi\Lambda}{k} \Xi \; du \,, \tag{3.19}
$$

where $a_{(0)}$ acquires the same form as in eq. (2.12), and

$$
\begin{aligned}
\Xi :=& \left(\xi \mathcal{J} + 2\varrho \mathcal{V}\right) J_0 + 2\varrho \mathcal{J} J_{01} - \frac{2}{3}\left(\varrho \mathcal{J}' + \frac{5}{2}\varrho' \mathcal{J}\right) J_{02} \\
&+ \frac{1}{2}\left(\xi \mathcal{V} + \frac{4\pi}{k}\vartheta \mathcal{J}^2 + \frac{8\pi}{k}\varrho \mathcal{J}\mathcal{P} - \frac{7}{3}\varrho' \mathcal{J}' - \frac{2}{3}\varrho \mathcal{J}'' - \frac{8}{3}\varrho'' \mathcal{J}\right) J_{00} + \frac{2\pi}{k}\varrho \mathcal{J}^2 P_{00} \,.
\end{aligned}
\tag{3.20}
$$

At this step, it must be emphasized that the generators in eq. (3.19) fulfill the algebra (3.16), with $\Lambda \neq 0$. Consistency then requires that the gauge parameters have to be redefined accordingly with the chemical potentials, i.e.,

$$
\epsilon_{\pm} = \frac{T}{\ell} \pm Y \ ; \ \chi_{\pm} = -\left(\frac{X}{\ell} \pm Z\right) \,, \tag{3.21}
$$

so that the connection (3.19) is preserved by gauge transformations spanned by

$$
\eta_{(\Lambda)} = \eta_{(0)} - \frac{2\pi\Lambda}{k} \Xi(T, Y, X, Z) \; du \,, \tag{3.22}
$$

where $\eta_{(0)}$ has the same form as in eq. (2.7), provided the fields transform as

$$
\begin{aligned}
\delta_{(\Lambda)} \mathcal{J} &= \delta_{(0)} \mathcal{J} \,, \\
\delta_{(\Lambda)} \mathcal{P} &= \delta_{(0)} \mathcal{P} - \Lambda \left[T \mathcal{J}' + 2\mathcal{J} T' + 2X \mathcal{V}' + 3\mathcal{V} X'\right] \,, \\
\delta_{(\Lambda)} \mathcal{W} &= \delta_{(0)} \mathcal{W} - \Lambda \Bigg[T \mathcal{V}' + 3T' \mathcal{V} - 3X' \mathcal{J}'' - 5X'' \mathcal{J}' - \frac{2}{3} X \mathcal{J}''' - \frac{10}{3} \mathcal{J} X''' \\
&\qquad + \frac{16\pi}{3k}\left(Z\left(\mathcal{J}^2\right)' + 2Z' \mathcal{J}^2 + 2X\left(\mathcal{P}\mathcal{J}\right)' + 4X' \mathcal{P}\mathcal{J}\right)\Bigg] \,, \\
\delta_{(\Lambda)} \mathcal{V} &= \delta_{(0)} \mathcal{V} - \frac{16\pi}{3k}\Lambda \left[X\left(\mathcal{J}^2\right)' + 2X' \mathcal{J}^2\right] \,,
\end{aligned}
\tag{3.23}
$$

where $\delta_{(0)}\mathcal{J}$, $\delta_{(0)}\mathcal{P}$, $\delta_{(0)}\mathcal{W}$, $\delta_{(0)}\mathcal{V}$ are given by eq. (2.8).

The field equations (3.6), (3.7), as well as the deformed chirality conditions in (3.8) now read

$$\begin{aligned}
\dot{\mathcal{J}} &= 2\mu'\mathcal{J} + \mu\mathcal{J}' + \xi\mathcal{P}' + 2\xi'\mathcal{P} - \frac{k}{2\pi}\xi''' + 2\vartheta\mathcal{V}' + 3\vartheta'\mathcal{V} + 2\mathcal{W}'\varrho + 3\mathcal{W}\varrho' \,, \\
\dot{\mathcal{P}} &= 2\mu'\mathcal{P} + \mu\mathcal{P}' - \frac{k}{2\pi}\epsilon''' + 2\vartheta\mathcal{W}' + 3\vartheta'\mathcal{W} - \Lambda\left[\xi\mathcal{J}' + 2\mathcal{J}\xi' + 2\varrho\mathcal{V}' + 3\mathcal{V}\varrho'\right] \,, \\
\dot{\mathcal{W}} &= 3\mu'\mathcal{W} + \mu\mathcal{W}' - \frac{2}{3}\vartheta\left(\mathcal{P}'' - \frac{8\pi}{k}\mathcal{P}^2\right)' - 3\vartheta'\left(\mathcal{P}'' - \frac{32\pi}{9k}\mathcal{P}^2\right) - 5\vartheta''\mathcal{P}' - \frac{10}{3}\vartheta'''\mathcal{P} + \frac{k}{6\pi}\vartheta^{(5)} \\
&\quad - \Lambda\Bigg[\xi\mathcal{V}' + 3\xi'\mathcal{V} - 3\varrho'\mathcal{J}'' - 5\varrho''\mathcal{J}' - \frac{2}{3}\varrho\mathcal{J}''' - \frac{10}{3}\mathcal{J}\varrho''' \\
&\quad + \frac{16\pi}{3k}\left(\vartheta\left(\mathcal{J}^2\right)' + 2\vartheta'\mathcal{J}^2 + 2\varrho\left(\mathcal{P}\mathcal{J}\right)' + 4\varrho'\mathcal{P}\mathcal{J}\right)\Bigg] \,, \\
\dot{\mathcal{V}} &= 3\mu'\mathcal{V} + \mu\mathcal{V}' + \xi\mathcal{W}' + 3\xi'\mathcal{W} - \frac{2}{3}\vartheta\left(\mathcal{J}'' - \frac{16\pi}{k}\mathcal{J}\mathcal{P}\right)' - 3\vartheta'\left(\mathcal{J}'' - \frac{64\pi}{9k}\mathcal{J}\mathcal{P}\right) - 5\vartheta''\mathcal{J}' \\
&\quad - \frac{10}{3}\vartheta'''\mathcal{J} - \frac{2}{3}\varrho\left(\mathcal{P}'' - \frac{8\pi}{k}\mathcal{P}^2\right)' - 3\varrho'\left(\mathcal{P}'' - \frac{32\pi}{9k}\mathcal{P}^2\right) - 5\mathcal{P}'\varrho'' - \frac{10}{3}\mathcal{P}\varrho''' + \frac{k}{6\pi}\varrho^{(5)} \\
&\quad - \frac{16\pi}{3k}\Lambda\left[\varrho\left(\mathcal{J}^2\right)' + 2\varrho'\mathcal{J}^2\right] \,,
\end{aligned} \tag{3.24}$$

and

$$\begin{aligned}
\dot{Y} &= Y'\mu - Y\mu' + Z''\vartheta' - Z'\vartheta'' - \frac{2}{3}\left(Z'''\vartheta - Z\vartheta'''\right) + \frac{32\pi}{3k}\mathcal{P}\left(Z'\vartheta - Z\vartheta'\right) - \Lambda\Bigg[T'\xi - T\xi' \\
&\quad - \varrho''X' + \varrho'X'' + \frac{2}{3}\left(\varrho'''X - \varrho X'''\right) + \frac{32\pi}{3k}\left(\mathcal{J}\left(Z'\varrho - Z\varrho' - \vartheta'X + \vartheta X'\right) + \mathcal{P}\left(\varrho X' - \varrho' X\right)\right)\Bigg] \,, \\
\dot{T} &= Y'\xi - Y\xi' + Z''\varrho' - Z'\varrho'' + T'\mu - T\mu' + \vartheta'X'' - \vartheta''X' + \frac{2}{3}\left(Z\varrho''' - Z'''\varrho + \vartheta'''X - \vartheta X'''\right) \\
&\quad + \frac{32\pi}{3k}\left(\mathcal{J}\left(Z'\vartheta - Z\vartheta'\right) + \mathcal{P}\left(Z'\varrho - Z\varrho' + \vartheta X' - \vartheta' X\right)\right) - \frac{32\pi}{3k}\Lambda\left[\mathcal{J}\left(\varrho X' - \varrho' X\right)\right] \,, \\
\dot{Z} &= 2Y'\vartheta - Y\vartheta' + Z'\mu - 2Z\mu' + \Lambda\left[T\varrho' - 2T'\varrho - \xi X' + 2\xi' X\right] \,, \\
\dot{X} &= 2Y'\varrho - Y\varrho' + Z'\xi - 2Z\xi' - 2\mu' X + \mu X' - T\vartheta' + 2T'\vartheta \,,
\end{aligned} \tag{3.25}$$

respectively.

Note that since the gauge choice in (3.12), (3.13) is permissible, the global charges do not change, i.e.,

$$Q_{(\Lambda)}\left(T, Y, Z, X\right) = Q_{+}\left(\varepsilon_{+}, \chi_{+}\right) - Q_{-}\left(\varepsilon_{-}, \chi_{-}\right) \,,$$

which by virtue of the redefinitions in eqs. (3.17), (3.21), they now read

$$Q_{(\Lambda)}\left(T, Y, Z, X\right) = Q_{(0)}\left(T, Y, Z, X\right) \,, \tag{3.26}$$

whose expression remarkably agrees with the one for $\Lambda = 0$ in eq. (2.13). Nonetheless, it must be emphasized that here the fields transform according to (3.23), instead of (2.8). Therefore, once expanded in Fourier modes according to $X = \frac{1}{2\pi}\sum_m X_m e^{im\theta}$, the algebra of the canonical generators, given by two copies of W_3, now reads

$$i\{\mathcal{J}_n, \mathcal{J}_m\} = (n-m)\mathcal{J}_{n+m} \,; \qquad i\{\mathcal{P}_n, \mathcal{P}_m\} = -\Lambda(n-m)\mathcal{J}_{n+m} \,,$$

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$$
\begin{aligned}
i\{\mathcal{J}_n, \mathcal{P}_m\} &= (n-m)\mathcal{P}_{n+m} + kn^3\delta_{m+n}\,, \\
i\{\mathcal{P}_n, \mathcal{W}_m\} &= -\Lambda(2n-m)\mathcal{V}_{n+m}\,; \qquad i\{\mathcal{P}_n, \mathcal{V}_m\} = (2n-m)\mathcal{W}_{n+m}\,, \\
i\{\mathcal{J}_n, \mathcal{W}_m\} &= (2n-m)\mathcal{W}_{n+m}\,; \qquad i\{\mathcal{J}_n, \mathcal{V}_m\} = (2n-m)\mathcal{V}_{n+m}\,, \\
i\{\mathcal{W}_n, \mathcal{W}_m\} &= -\frac{\Lambda}{3}(n-m)(2n^2+2m^2-mn)\mathcal{J}_{m+n} - \frac{16\Lambda}{3k}(n-m)\sum_j \mathcal{P}_j\mathcal{J}_{n+m-j}\,, \\
i\{\mathcal{W}_n, \mathcal{V}_m\} &= \frac{1}{3}(n-m)(2n^2+2m^2-mn)\mathcal{P}_{m+n} + \frac{8}{3k}(n-m)\Omega_{m+n} + \frac{k}{3}n^5\delta_{m+n}\,, \\
i\{\mathcal{V}_n, \mathcal{V}_m\} &= \frac{1}{3}(n-m)(2n^2+2m^2-mn)\mathcal{J}_{m+n} + \frac{16}{3k}(n-m)\sum_j \mathcal{P}_j\mathcal{J}_{n+m-j}\,,
\end{aligned}
\tag{3.27}
$$

where

$$
\Omega_n = \sum_m \left(\mathcal{P}_{n-m}\mathcal{P}_m - \Lambda\mathcal{J}_{n-m}\mathcal{J}_m\right)\,. \tag{3.28}
$$

3.2.1 Taking the $\Lambda \to 0$ limit

The limiting process that allows to recover the whole structure of the vanishing cosmological constant case, can then be taken in a very transparent way. Indeed, when $\Lambda = -\frac{1}{\ell^2} \to 0$, the generators of the $sl(3,\mathbb{R}) \oplus sl(3,\mathbb{R})$ algebra (3.16) clearly span their corresponding Inönü-Wigner contraction (2.3). Hence, according to (3.19), the gauge fields with the special gauge choice in (3.12) fulfill $a_{(\Lambda)} \to a_{(0)}$, i.e., they manifestly reduce to their flat counterpart in eq. (2.12). Analogously, eq. (3.22) implies that $\eta_{(\Lambda)} \to \eta_{(0)}$, so that the gauge transformations that preserve the asymptotic form of $a_{(0)}$ in (2.7), are also recovered in the limit. This is also the case of the transformation law of the fields, since when $\Lambda \to 0$, eq. (2.8) is readily obtained from (3.23). Moreover, according to eqs. (3.24) and (3.25), the field equations and the chirality conditions in the flat case, given by (2.10), (2.11), respectively, are also recovered.

Noteworthy, as expressed by (3.26), since $Q_{(\Lambda)} = Q_{(0)}$, the expression for the canonical generators is automatically obtained without the need of taking the limit.

Finally, by virtue of (3.27), it is also clear that the higher spin extension of the conformal symmetry, spanned by two copies of the W_3 algebra reduces to the higher spin extension of the BMS_3 algebra, given by

$$
\begin{aligned}
i\{\mathcal{J}_n, \mathcal{J}_m\} &= (n-m)\mathcal{J}_{n+m}\,; \qquad i\{\mathcal{P}_n, \mathcal{P}_m\} = 0\,, \\
i\{\mathcal{J}_n, \mathcal{P}_m\} &= (n-m)\mathcal{P}_{n+m} + kn^3\delta_{m+n}\,, \\
i\{\mathcal{P}_n, \mathcal{W}_m\} &= 0\,; \qquad i\{\mathcal{P}_n, \mathcal{V}_m\} = (2n-m)\mathcal{W}_{n+m}\,, \\
i\{\mathcal{J}_n, \mathcal{W}_m\} &= (2n-m)\mathcal{W}_{n+m}\,; \qquad i\{\mathcal{J}_n, \mathcal{V}_m\} = (2n-m)\mathcal{V}_{n+m}\,, \\
i\{\mathcal{W}_n, \mathcal{W}_m\} &= 0 \\
i\{\mathcal{W}_n, \mathcal{V}_m\} &= \frac{1}{3}(n-m)(2n^2+2m^2-mn)\mathcal{P}_{m+n} + \frac{8}{3k}(n-m)\Omega_{m+n} + \frac{k}{3}n^5\delta_{m+n}\,, \\
i\{\mathcal{V}_n, \mathcal{V}_m\} &= \frac{1}{3}(n-m)(2n^2+2m^2-mn)\mathcal{J}_{m+n} + \frac{16}{3k}(n-m)\sum_j \mathcal{P}_j\mathcal{J}_{n+m-j}\,,
\end{aligned}
\tag{3.29}
$$

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where

$$\Omega_n = \sum_m \mathcal{P}_{n-m}\mathcal{P}_m \,. \tag{3.30}$$

As an ending remark of this section, it is apparent that, since it has been shown that the whole asymptotic structure fulfills $a_{(\Lambda)} \to a_{(0)}$ in the vanishing Λ limit, the higher spin black hole solution (3.10) reduces to the higher spin extension of locally flat cosmological spacetimes in eq. (2.14).

4 Higher spin extension of locally flat cosmological spacetimes and thermodynamics

As explained in section 2, the generalized asymptotic conditions (2.12) naturally accommodate a higher spin extension of cosmological spacetimes, given by (2.14). The solution is explicitly described not only in terms of their global spin-2 and spin-3 charges, but also by their corresponding chemical potentials, which are strictly necessary in order to have a regular Euclidean configuration. In this case, from the metric formalism, it can be inferred that the topology of the three-dimensional manifold turns out to be the one of a solid torus, but with a reversed orientation as compared with the case of black holes [82]. This is explained in the appendix (see figure 1). Note also that, as explained in [57, 58], since all the chemical potentials are manifestly incorporated along the temporal components of the gauge fields, the analysis can be carried out for a fixed range of the angular coordinates of the torus, i.e., we assume that $0 < \tau \leq 1$, and $0 < \varphi \leq 2\pi$. This is particularly useful in the case of higher spin gravity, since the torus clearly has not enough room to accommodate all the chemical potentials in the range of coordinates.

For the class of theories under discussion, the correct expression for the entropy was first found in [11, 20] following the canonical approach. This result has been further developed in [21, 58]. According to the conventions in [58], the entropy can then be obtained from the following expression

$$\begin{aligned} S &= \frac{k}{2\pi}\left[\int_{r_+} d\tau d\varphi \,\langle A_\tau A_\varphi\rangle\right]_{\text{on-shell}} \\ &= k\left[\langle a_\tau a_\varphi\rangle\right]_{\text{on-shell}} \,. \end{aligned} \tag{4.1}$$

In the microcanonical ensemble, this is the boundary term that is needed so that the action acquires a bona fide extremum. The field equations then have to hold everywhere, which implies that the fields have to be regular at the horizon. This procedure then ensures that the first law is also fulfilled either in the canonical or in the grand canonical ensembles.

It is worth highlighting that neither the Poincaré algebra nor their higher spin extensions, as in eq. (3.16), admit a suitable standard matrix representation from which the Casimir operators, and so the invariant bilinear form (2.4) that is required to construct the action, can be recovered from the trace of a product of the generators. Consequently, regularity of the Euclidean solution, which is guaranteed by requiring the holonomy along the thermal circle to be trivial, cannot be straightforwardly implemented through its diagonalization.

In the next subsection, we describe a general procedure that allows to implement the regularity condition without the need of an explicit matrix representation of the entire gauge group, but only of its Lorentz-like subgroup.

In few words, we show that the temporal components of generalized dreibeins e_τ, can be consistently gauged away, which partially fixes the chemical potentials; so that the remaining conditions can be obtained by requiring the holonomy of the generalized spin connection ω along a thermal circle to be trivial.

The procedure aforementioned then allows to carry out the analysis of the thermodynamic properties in a direct way. As a warming up exercise, this is first performed in the case of pure gravity, and then we show how the analysis extends once the higher spin charges and their corresponding chemical potentials are switched on.

4.1 Procedure to implement the regularity conditions in a generic form

When one deals with locally flat connections defined on a solid torus, since the thermal circles $\mathcal{C}$ are contractible, regularity of the fields $a = g^{-1}dg$, implies the triviality of their holonomies along them, i.e.,

$$H_{\mathcal{C}} = P\exp\left[\int_{\mathcal{C}} a_\mu dx^\mu\right] = \exp\left[\int_0^1 a_\tau d\tau\right] = g^{-1}(\tau)g(\tau+1) = I_c\,, \tag{4.2}$$

where I_c stands for a suitable element of the center of the group. If the gauge group admits an appropriate matrix representation, the regularity conditions can then be directly implemented through the diagonalization of $H_{\mathcal{C}}$. Alternatively, according to (4.2), one could obtain the explicit form of the gauge group element g, so that regularity implies that g is well defined along the cycle. Hence, in a suitable patch around the origin, the "regularizing gauge transformation", generated by g^{-1}, makes the temporal component of the gauge fields to vanish, i.e., $a_\tau = 0$.

Note that in pure gravity, as well as in higher spin gravity, the gauge group always possesses the following structure (see, e.g., [60])

$$[J,J]\sim J\,;\qquad [J,P]\sim P\,;\qquad [P,P]\sim -\Lambda\ J\,, \tag{4.3}$$

where J stands for the Lorentz-like generators, and according to the value of Λ, the generators P correspond to the extended translations, or (A)dS boosts. The connection is then generically of the form

$$a = \omega J + eP\,,$$

being locally flat, i.e., $a = g^{-1}dg$, where the group element g, by virtue of (4.3), can always be written as

$$g = g_P\cdot g_J\,,$$

with $g_P := e^{\lambda P}$, and $g_J := e^{\Theta J}$.

Therefore, the regularity condition of the fields can always be implemented in a "hybrid way" as follows:

(i) Finding the group element g_P that allows to gauge away the temporal components of the gauge field along P, so that one can consistently set $e_\tau = 0$. This allows to express the chemical potentials of electric type in terms of the magnetic type ones and the global charges.

(ii) The remaining conditions can then be implemented through the diagonalization of the holonomy matrix associated to the spin connection along the thermal circle; i.e., without the need of finding the explicit form of g_J.

Note that in the case of a finite number of fields with spin $s = 1, 2, \ldots, N$, since the Lorentz-like group is given by $\mathrm{SL}(N, \mathbb{R})$, the holonomy associated to ω_τ always admits an irreducible matrix representation that allows to diagonalize it.

It is worth pointing out that if the radial dependence were brought back, it is clear that the regularity conditions had to be imposed at the horizon. The procedure explained above certainly can always be applied in any case. Indeed, it is simple to verify that the regularity conditions for the gauge fields that describe black holes in the case of $\Lambda < 0$, which fix the chemical potentials in terms of the global charges, are successfully reproduced in this way. It should then be emphasized that this procedure becomes particularly useful when the gauge group does not admit a suitable matrix representation, so that the Casimir operators, and hence the invariant bilinear form (2.4), cannot be recovered from the trace of a product of the generators, as it is the case of $\Lambda = 0$.

4.2 Warming up with pure gravity

For the sake of simplicity, let us first consider the case of pure gravity with vanishing cosmological constant, so that the gauge group is spanned by the Poincaré algebra, whose Lorentz subalgebra is given by $sl(2, \mathbb{R})$. Hence, the field configuration that describes cosmological spacetimes can be obtained from (2.14), in the case of vanishing higher spin charges and their corresponding chemical potentials; i.e., for $\mathcal{V} = \mathcal{W} = \vartheta = \varrho = 0$. The connection then reads

$$\begin{aligned} a_{(0)} = &\left(J_1 + \frac{2\pi}{k}\mathcal{J}P_0 + \frac{2\pi}{k}\mathcal{P}J_0\right) d\varphi \\ &+ \left[\mu\left(J_1 + \frac{2\pi}{k}\mathcal{P}J_0\right) + \xi P_1 + \frac{2\pi}{k}\left(\mu\mathcal{J} + \xi\mathcal{P}\right)P_0\right] du\,, \end{aligned} \tag{4.4}$$

and hence, according to (4.1) the entropy is readily found to be given by

$$S = 4\pi\left[\xi\mathcal{P} + \mu\mathcal{J}\right]_{\text{on-shell}}\,, \tag{4.5}$$

where the chemical potentials have to fulfill the regularity conditions. According to the procedure described above, the first step (i) consists in finding a suitable gauge transformation g_P that allows to consistently gauge away the temporal components of the dreibein, i.e., $e_u = 0$. It is simple to see that the required permissible group element is of the form

$$g_P = e^{\lambda_2 P_2}\,, \tag{4.6}$$

so that the gauge field now reads

$$a = \left(J_1 + \lambda_2 P_1 + \frac{2\pi}{k}\left(\mathcal{J} - \lambda_2 \mathcal{P}\right) P_0 + \frac{2\pi}{k}\mathcal{P} J_0\right) d\varphi$$
$$+ \left[\mu\left(J_1 + \frac{2\pi}{k}\mathcal{P} J_0\right) + \left(\mu\lambda_2 + \xi\right) P_1 + \frac{2\pi}{k}\left(\mu\mathcal{J} - \left(\mu\lambda_2 - \xi\right)\mathcal{P}\right) P_0\right] du\,.$$

Hence, the dreibein component e_u^1 vanishes if

$$\lambda_2 = -\frac{\xi}{\mu}\,,$$

while the remaining component e_u^0 also does provided the following condition is fulfilled:

$$\mu = -2\xi\frac{\mathcal{P}}{\mathcal{J}}\,.$$

In this gauge, the connection is then explicitly given by

$$a = \left(J_1 - \frac{\xi}{\mu}P_1 + \frac{2\pi}{k}\left(\mathcal{J} + \frac{\xi}{\mu}\mathcal{P}\right) P_0 + \frac{2\pi}{k}\mathcal{P} J_0\right) d\varphi - 2\xi\frac{\mathcal{P}}{\mathcal{J}}\left(J_1 + \frac{2\pi}{k}\mathcal{P} J_0\right) du\,.$$

It is worth to remark that the regularizing group element g_P in (4.6) is non singular and globally well-defined.

The remaining step (ii) amounts to require the holonomy of the spin connection along the thermal circle to be trivial. In the fundamental representation of $sl(2,\mathbb{R})$, this condition reduces to

$$tr\left[\left(\omega_\tau\right)^2\right] + 2\pi^2 = 2\pi^2 - \frac{8\pi}{k}\frac{\xi^2\mathcal{P}^3}{\mathcal{J}^2} = 0\,,$$

being solved by

$$\xi^2 = \frac{\pi k \mathcal{J}^2}{4\mathcal{P}^3}\,. \tag{4.7}$$

Note that since we are dealing with a cosmological horizon, the orientation of the solid torus is reversed as compared with the one of the black hole, so that the chemical potential ξ corresponds to the minus branch of (4.7). This goes by hand with the positivity of the Hawking temperature, since $\xi = -\frac{1}{T}$.

In sum, the regularity conditions imply that the chemical potentials become fixed in terms of the global charges according to

$$\mu = sgn(\mathcal{J})\sqrt{\frac{\pi k}{\mathcal{P}}}\,, \qquad \xi = -\frac{\sqrt{\pi k}\,|\mathcal{J}|}{2\mathcal{P}^{3/2}}\,,$$

which allows to express the entropy in terms of the global charges as

$$S = 2\pi\sqrt{\frac{\pi k}{\mathcal{P}}}\,|\mathcal{J}|\;. \tag{4.8}$$

This result agrees with the one for General Relativity, i.e., $S = \frac{A}{4G}$ (see appendix), which in the metric formalism, was explicitly carried out in [67, 68].

4.3 Switching on higher spin charges and chemical potentials

Here we show that the thermodynamic analysis of the higher spin extension of the cosmological spacetimes (2.14) proceeds as explained above in a straightforward way. Indeed, in this case the entropy (4.1) evaluates as

$$S = 2\pi \left[2\xi\mathcal{P} + 2\mu\mathcal{J} + 3\varrho\mathcal{W} + 3\vartheta\mathcal{V}\right]_{\text{on-shell}} , \tag{4.9}$$

where the chemical potentials have to satisfy the regularity conditions. In order to solve them, let us apply the first step (i), which consists in finding the gauge transformation g_P that allows to gauge away the components of the dreibein along time. The permissible globally well-defined gauge transformation is found to be given by

$$g_P = e^{\hat{\lambda}} , \tag{4.10}$$

with

$$\hat{\lambda} = \frac{(32\pi\mathcal{J}\mathcal{P}^2 - 9k\mathcal{V}\mathcal{W})P_2 + 10\pi\mathcal{P}(2\mathcal{P}\mathcal{V} - 3\mathcal{J}\mathcal{W})P_{02} + (6k\mathcal{P}\mathcal{V} - 9k\mathcal{J}\mathcal{W})P_{12}}{64\pi\mathcal{P}^3 - 27k\mathcal{W}^2} , \tag{4.11}$$

so that the temporal components of the dreibein vanish provided

$$\xi = \frac{32\pi\mathcal{P}(3\mathcal{J}\mathcal{W} - 2\mathcal{P}\mathcal{V})\vartheta + \left(9k\mathcal{V}\mathcal{W} - 32\pi\mathcal{J}\mathcal{P}^2\right)\mu}{64\pi\mathcal{P}^3 - 27k\mathcal{W}^2} , \tag{4.12}$$

$$\varrho = \frac{\left(18k\mathcal{V}\mathcal{W} - 64\pi\mathcal{J}\mathcal{P}^2\right)\vartheta + (9k\mathcal{J}\mathcal{W} - 6k\mathcal{P}\mathcal{V})\mu}{64\pi\mathcal{P}^3 - 27k\mathcal{W}^2} . \tag{4.13}$$

The entropy (4.9) then simplifies as

$$S = 2\pi \left[\mu\mathcal{J} + \vartheta\mathcal{V}\right]_{\text{on-shell}} . \tag{4.14}$$

Step (ii) is then implemented through requiring the holonomy of the spin connection along the thermal circle to be trivial. Since the Lorentz-like group is now given by $sl(3,\mathbb{R})$, the remaining conditions reduce to

$$tr\left[(\omega_\tau)^2\right] + 8\pi^2 = \frac{8\pi}{k}\mathcal{P}\mu^2 + \frac{24\pi}{k}\mathcal{W}\vartheta\mu + \frac{128\pi^2}{3k^2}\mathcal{P}^2\vartheta^2 - 8\pi^2 = 0 , \tag{4.15}$$

$$tr\left[(\omega_\tau)^3\right] = \frac{\pi}{k}\mathcal{W}\mu^3 + \frac{32\pi^2}{3k^2}\mathcal{P}^2\vartheta\mu^2 + \frac{16\pi^2}{k^2}\mathcal{P}\mathcal{W}\vartheta^2\mu + \frac{16\pi^2}{k^2}\mathcal{W}^2\vartheta^3 - \frac{512\pi^3}{27k^3}\mathcal{P}^3\vartheta^3 = 0 , \tag{4.16}$$

which do not depend on $\mathcal{J}$, $\mathcal{V}$.

Conditions (4.15), (4.16) admit different branches of solutions. In order to make contact with the cosmological configurations in the case of pure gravity, it is convenient to consider the following classes of solutions

$$\mu = \pm\sqrt{\frac{\pi k}{\mathcal{P}}}\cos\left(\frac{2\Phi}{3}\right)\sec\left(\Phi\right) , \tag{4.17}$$

$$\vartheta = \frac{\sqrt{3}k}{4\mathcal{P}}\sin\left(\frac{\Phi}{3}\right)\sec\left(\Phi\right) , \tag{4.18}$$

with

$$\sin(\Phi) = \mp\frac{3}{8}\sqrt{\frac{3k}{\pi\mathcal{P}^3}}\mathcal{W}\,, \tag{4.19}$$

so that consistency implies that the sign in (4.17) coincides with the one of the angular momentum $\mathcal{J}$. Hence, the entropy can be expressed in terms of the global charges according to

$$S = 2\pi\sqrt{\frac{\pi k}{\mathcal{P}}}\sec\left(\Phi\right)\left[\left|\mathcal{J}\right|\cos\left(\frac{2\Phi}{3}\right) + \sqrt{\frac{3k}{\pi\mathcal{P}}}\frac{\mathcal{V}}{4}\sin\left(\frac{\Phi}{3}\right)\right]. \tag{4.20}$$

Note that the branch that is continuously connected with the cosmological spacetime of General Relativity ($\Phi = 0$) corresponds to $-\frac{\pi}{2} < \Phi < \frac{\pi}{2}$. The advantage of writing the entropy in terms of the "angular variable" Φ, is that it also holds for different branches.

As a final remark, in addition to $\mathcal{P} > 0$, the bound that guarantees that the entropy is a real function, directly comes from (4.19), which is given by

$$\left|\mathcal{W}\right| \leq \frac{8}{3\sqrt{3}}\sqrt{\frac{\pi}{k}}\mathcal{P}^{\frac{3}{2}}\,.$$

When this bound saturates, the configuration is "extremal" in the sense that the holonomy of the generalized spin connection along the thermal circle is no longer trivial, because there is a change in the topology ($\mathbb{R} \times S^1 \times S^1$). Note that in the branch that is connected with the pure gravity solution, positivity of the entropy implies that the angular momentum has to be bounded from below according to

$$\left|\mathcal{J}\right| > -\sqrt{\frac{3k}{\pi\mathcal{P}}}\frac{\mathcal{V}\sin\left(\frac{\Phi}{3}\right)}{4\cos\left(\frac{2\Phi}{3}\right)}\,,$$

so that the bound becomes nontrivial provided the sign of the spin-3 charge of electric type $\mathcal{W}$ is the opposite of its magnetic-like counterpart $\mathcal{V}$. This is the analogue of the condition that guarantees the existence of an event horizon in the case of pure gravity (see eq. (A.2)).

5 Final remarks

The extension of the generalized asymptotically flat behaviour to the case of spins $s \geq 2$ can be directly performed along the lines of [57, 58]. Thus, in order to incorporate the chemical potentials, one begins with the asymptotic behaviour at a fixed time slice $u = u_0$. The radial dependence can also be gauged away by a group element of the form $h(r) = e^{-rP_0}$, and hence, as described in [70], the asymptotic form of the spacelike connection is given by

$$a_\varphi = J_1 + \frac{2\pi}{k}(\mathcal{J}P_0 + \mathcal{P}J_0) + \frac{\pi}{k}(\mathcal{V}_3 P_{00} + \mathcal{W}_3 J_{00}) + \frac{\pi}{k}(\mathcal{V}_4 P_{000} + \mathcal{W}_4 J_{000}) + \ldots \tag{5.1}$$

where the spin-s generators $P_{a_1\cdots a_{s-1}}$, $J_{a_1\cdots a_{s-1}}$, are assumed to be fully symmetric and traceless, and the fields $\mathcal{M}$, $\mathcal{J}$, $\mathcal{W}_3$, $\mathcal{V}_3$, $\mathcal{W}_4$, $\mathcal{V}_4$, $\ldots$, stand for arbitrary functions of u_0, φ. It was also shown in [70] that the asymptotic behaviour (5.1) can be consistently recovered from the vanishing Λ limit of its AdS_3 analogue [59, 60, 83], after a suitable permissible gauge choice.

The asymptotic form of the connection is then maintained under gauge transformations of the form $\delta a = d\eta + [a, \eta]$, where $\eta = \eta\,(T, Y, Z_3, X_3, Z_4, X_4, \ldots)$ depends on arbitrary parameters of u_0, φ, provided the fields transform in a suitable way. Consequently, since the dynamical fields evolve in time through a gauge transformation generated by a_u, the asymptotic symmetries will be preserved along time evolution provided the Lagrange multiplier belongs to the allowed family, i.e.,

$$a_u = \eta\,(\xi, \mu, \vartheta_3, \varrho_3, \vartheta_4, \varrho_4, \ldots)\ , \tag{5.2}$$

where $\xi, \mu, \vartheta_3, \varrho_3, \vartheta_4, \varrho_4, \ldots$ stand for arbitrary functions of time and the angular coordinates that are assumed to be fixed at the boundary, and describe the chemical potentials conjugated to the corresponding global charges. Preserving the asymptotic form of a_u then implies that the field equations hold at the asymptotic region, and also provides a precise set of conditions for the parameters that generate the asymptotic symmetries.

The asymptotically flat behaviour in the case of spins $s \geq 2$, is then described by gauge fields of the form

$$a = a_\varphi d\varphi + a_u du\,, \tag{5.3}$$

with a_φ and a_u given by eqs. (5.1) and (5.2), respectively.

By construction, since the surface integrals that describe the canonical generators depend only on a_φ, and not on the chemical potentials, their expression coincides with the one that can be obtained from [70],

$$Q\,(T, Y, Z_3, X_3, Z_4, X_4, \ldots) = -\int \left(T\mathcal{P} + Y\mathcal{J} + Z_3\mathcal{V}_3 + X_3\mathcal{W}_3 + Z_4\mathcal{V}_4 + X_4\mathcal{W}_4 + \cdots\right) d\varphi\,, \tag{5.4}$$

and hence, the algebra of the asymptotic symmetries is still generated by the corresponding higher spin extension of the centrally-extended BMS_3 algebra.

The locally flat cosmologies can then be readily extended to the case that includes chemical potentials and higher spin charges of spin $s \geq 2$. Indeed, this is the case when the fields $\mathcal{M}$, $\mathcal{J}$, $\mathcal{W}_3$, $\mathcal{V}_3$, $\mathcal{W}_4$, $\mathcal{V}_4$, $\ldots$, and the arbitrary functions $\xi, \mu, \vartheta_3, \varrho_3, \vartheta_4, \varrho_4, \ldots$ are constant.

It is also worth pointing out that a different possible extension of our results could be carried out along a different front. For instance, note that the flat analogue of the well-known result that allows to describe AdS_3 gravity with Brown-Henneaux boundary conditions in terms of a Liouville theory at the boundary [84], has recently been constructed in [85]; and it has also been shown that the dual theory at null infinity can be consistently recovered from the vanishing Λ limit of its AdS_3 counterpart [86]. Thus, following these lines, and assuming that the asymptotically flat behaviour of gravity coupled to spin-3 fields is described as in [69, 70], the higher spin extension of the dual theory at null infinity was recently shown to correspond to a flat analogue of Toda theory [87]. It would then be interesting to explore how the dual theory becomes modified once the generalized asymptotically flat behaviour, described in section 2 is taken into account, as well as carrying out the extension to fields of spin $s \geq 2$.

As a final remark, we would like to make a comparison with the results that have been recently reported by Gary, Grumiller, Riegler and Rosseel in [71]. Since the generalized asymptotically flat behaviour in the case of spin-3 gravity also follows the lines of [57, 58], we naturally agree. Nonetheless, in order to make the link with the vanishing Λ limit, the path they follow makes use of a prescription introduced by Krishnan, Raju and Roy in [88]. The prescription requires the use of certain 6×6 matrix representation constructed out of Grassmann variables, along with the introduction of different notions of "twisted" and "hatted" traces that allow to recover the metric and the spin-3 field, as well as the invariant bilinear tensor that defines the bracket, respectively. Besides, the regularity conditions of the fields are also obtained in a completely different approach. Indeed, in order to carry out the computation, they use another (9×9) matrix representation, followed by a prescription that partially relies on the vanishing Λ limit. Therefore, taking into account that we have followed radically different approaches, it is very reassuring to check that our results for the entropy of the higher spin extension of the cosmological spacetimes agree in the cases that were considered in [71]. This can be explicitly seen as follows: if one restricts our entropy formula (4.20) to the branch that is connected to the pure gravity case, once the global charges are mapped according to

$$\mathcal{P} = \frac{k}{4\pi}\hat{\mathcal{M}}\,; \qquad \mathcal{J} = \frac{k}{2\pi}\hat{\mathcal{L}}\,; \qquad \mathcal{W} = \frac{2k}{\pi}\hat{\mathcal{V}}\,; \qquad \mathcal{V} = \frac{4k}{\pi}\hat{\mathcal{U}}\,,$$

so that the variables $\hat{\mathcal{R}}$ and $\hat{\mathcal{P}}$ become

$$\frac{\hat{\mathcal{R}}-1}{\hat{\mathcal{R}}^{3/2}} = \frac{1}{4}\sqrt{\frac{k}{\pi}}\frac{\mathcal{W}}{\mathcal{P}^{3/2}}\,; \qquad \hat{\mathcal{P}} = \frac{1}{16}\sqrt{\frac{k}{\pi}}\frac{\mathcal{V}}{\mathcal{J}\sqrt{\mathcal{P}}}\,,$$

it reduces to the corresponding expression given in [71]. Here we have used a hat in order to distinguish their variables from ours.

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A Contact with the cosmological spacetime metric

In order to recover the cosmological spacetime metric in the case of pure gravity with $\Lambda = 0$, one has to restore the radial dependence of the gauge fields. It is useful to consider the following gauge choice:

$$g_\rho = e^{\rho P_2}\,,$$

so that the full gauge field now reads

$$A = \omega^a J_a + e^a P_a = g_\rho^{-1} a_{(0)} g_\rho + g_\rho^{-1} d g_\rho \,,$$

where $a_{(0)}$ is given by (4.4). The connection then reduces to

$$\begin{aligned} A = & \left(J_1 + \rho P_1 + \frac{2\pi}{k} \left(\mathcal{J} - \rho \mathcal{P} \right) P_0 + \frac{2\pi}{k} \mathcal{P} J_0 \right) d\varphi \\ & + \left[\mu \left(J_1 + \frac{2\pi}{k} \mathcal{P} J_0 \right) + \left(\mu\rho + \xi \right) P_1 + \frac{2\pi}{k} \left(\mu \mathcal{J} - \left(\mu\rho - \xi \right) \mathcal{P} \right) P_0 \right] dt + P_2 d\rho \,, \end{aligned}$$

and hence, the spacetime metric

$$ds^2 = \eta_{ab} e^a_\mu e^b_\nu dx^\mu dx^\nu \,,$$

is directly obtained in Schwarzschild-like coordinates,

$$ds^2 = -\frac{4\pi}{k} \left(\frac{\pi \mathcal{J}^2}{k r^2} - \mathcal{P} \right) \xi^2 dt^2 + \frac{k}{4\pi} \left(\frac{\pi \mathcal{J}^2}{k r^2} - \mathcal{P} \right)^{-1} dr^2 + r^2 \left[\left(\mu + \frac{2\pi \mathcal{J} \xi}{k r^2} \right) dt + d\varphi \right]^2 , \tag{A.1}$$

where

$$\rho = \frac{\mathcal{J} + \sqrt{\mathcal{J}^2 - \frac{k}{\pi} \mathcal{P} r^2}}{2\mathcal{P}} \,,$$

which possesses a cosmological horizon located at

$$r = r_c = |\mathcal{J}| \sqrt{\frac{\pi}{k \mathcal{P}}} > 0 \,. \tag{A.2}$$

The Euclidean continuation of the cosmological spacetime metric is recovered through $t \to -i\tau$, followed by

$$\mathcal{P} = -\mathcal{P}_E \,; \quad \mathcal{J} = i \mathcal{J}_E \,; \quad \xi = \xi_E \,; \quad \mu = i \mu_E \ ,$$

so that the Euclidean metric reads

$$ds^2 = \frac{4\pi}{k} \left(\mathcal{P}_E - \frac{\pi \mathcal{J}_E^2}{k r^2} \right) \xi^2 d\tau^2 + \frac{k}{4\pi} \left(\mathcal{P}_E - \frac{\pi \mathcal{J}_E^2}{k r^2} \right)^{-1} dr^2 + r^2 \left[\left(\mu_E + \frac{2\pi \mathcal{J}_E \xi}{k r^2} \right) d\tau + d\varphi \right]^2 , \tag{A.3}$$

This class of spaces was first discussed in [64–66], and its thermodynamic properties have been thoroughly analyzed in [67, 68].

It is worth emphasizing that here we have included the chemical potentials explicitly in the metric, so that the range of the coordinates is assumed to be fixed according to $0 < \tau \leq 1$, and $0 < \varphi \leq 2\pi$. It is then simple to verify that the Euclidean metric (A.3) is regular at the horizon provided

$$\mu_E = \sqrt{\frac{\pi k}{\mathcal{P}}} \,, \tag{A.4}$$

$$\xi_E = -\frac{\sqrt{\pi k}\, |\mathcal{J}|}{2 \mathcal{P}^{3/2}} \,, \tag{A.5}$$

while for at $r \to \infty$, asymptotically approaches to a conical defect.

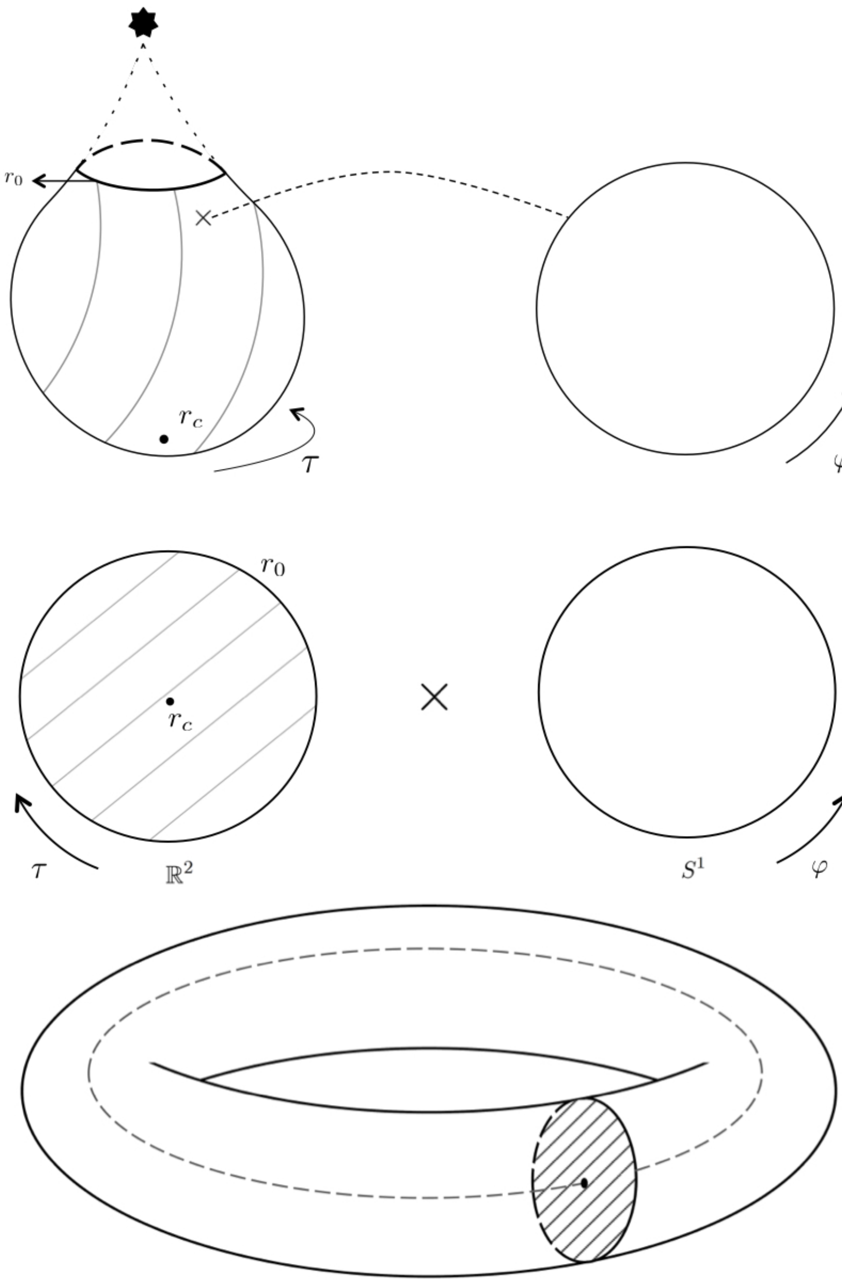

Figure 1. The sequence shows that the topology of the Euclidean cosmological spacetime coincides with the one of a black hole; i.e., it corresponds to $\mathbb{R}^2 \times S^1$ (solid torus), but with reversed orientation (compare with figure 1 of [58]). The cosmological horizon r_c is located at the "south pole" of the $r - \tau$ surface, which asymptotically approaches to a conical defect at the tip of the drop, so that a regulator at $r = r_0$ has to be introduced. Noncontractible cycles then run along the circle S^1, being parametrized by the angle φ.

The topology of the Euclidean manifold can then be directly inferred from the metric (A.3), which as shown in figure 1, corresponds to the one of a solid torus ($\mathbb{R}^2 \times S^1$), but with reversed orientation as compared with the one of the black hole. Note that the cosmological horizon r_c is located at the "south pole" of the $r - \tau$ surface, and hence the relationship between the "chemical potential" ξ and the Hawking temperature is given by $\xi = -\frac{1}{T}$, which explains the use of the minus branch in (A.5).

It is simple to verify that the entropy $S = \frac{A}{4G}$ agrees with the one obtained exclusively in terms of the gauge fields in eq. (4.8).

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