Highway Bridge Vibration Studies

John T. Gaunt
Research Engineer

C. Douglas Sutton
Research Engineer
Joint Highway Research Project
School of Civil Engineering, Purdue University

INTRODUCTION

The goal of the bridge engineer is to design economical structures which are safe, durable, and serviceable. Determining the dynamic response of bridges has been the topic of numerous studies in recent years. Much attention has been focused on maximum dynamic displacements and moments and on the distribution of loads to the floor system—information necessary to design for adequate strength. Relatively little concern has been given to the comfort of persons crossing the bridges. Transportation agencies do, however, occasionally receive comments and complaints from maintenance workers, pedestrians, and passengers in halted vehicles about the vibration of bridges.

Although people are subjected to the vibration of many structures, there is seldom any direct provision in design codes to ensure user comfort. The AASHTO Specifications have traditionally imposed restrictions upon girder span-depth ratios and upon live load deflections in the hope that these limits will provide satisfactory dynamic performance. The human body, however, is primarily sensitive to accelerations rather than to displacements so that the code requirements may not necessarily achieve the desired results. In addition, the code deflection limits may tend to hinder the economical use of modern high strength steel.

This report primarily summarizes research described in greater detail in three JHRP reports by Aramraks, Kropp, and Shahabadi. The general objectives of the project have been to obtain a better understanding of the dynamic performance of highway bridges and of the vibrations sensed by bridge users in order to aid in the development and utilization of a dynamic-based design criterion which could more effectively ensure user comfort. Specific tasks have included:

1) Determination of reasonable dynamic criteria for user sensitivity to vibrations,

2) Identification through analytical studies of the parameters of the bridge-vehicle system which are most significant in their effect upon the dynamic response of the bridge,

3) Measurement and analysis of the dynamic performance of common types of highway bridges under actual traffic in the field,

4) Comparison of analytical predictions with field measurements, and

5) Investigation of a proposed dynamic-based design criterion for controlling bridge vibrations.

HUMAN SENSITIVITY TO VIBRATIONS

Human reactions to vibrations are both physiological and psychological; low frequency, large amplitude vibrations, for example, are associated with sea sickness. On the other hand, when a person feels the traffic-induced vibration of a bridge, his reaction may be primarily psychological. He may associate this unexpected motion with poor design and possible collapse. Buildings and bridges are not suppose to move!

A literature search was carried out to identify what constitutes an objectionable level of vibration for pedestrians on bridges. Wright and Green's report contains an extensive bibliography on the subject. Experiments have been of two types: (1) people subjected to the vibration of actual structures in the field, and (2) people subject to controlled "shake table" vibrations in a laboratory. In these latter experiments tables are usually excited in simple harmonic motion of various amplitudes and frequencies. Results of these tests are presented in the form of "sensitivity curves," which delineate levels of vibration perception in the amplitude-frequency domain. The scopes of these curves, when plotted on a log-log scale, indicate whether sensitivity is related to velocity, acceleration, or jerk.

One early study was carried out by Reiher and Meister. They subjected some ten people, aged 20 to 37 years, to vertical sinusoidal vibration without damping for about five minutes. Their results, shown in Figure 1, indicate that lower sensitivity levels for steady state harmonic motion depend on velocity. Higher sensitivity levels appear to be more nearly related to accelerations. Lenzen, in a later study of the vibration of steel joist-concrete slab floors, suggested using the Reiher and Meister curves with the tolerance limits increased by a factor of 10 if the amplitude decays to less than 10% of its initial magnitude in 5 to 12 cycles.

More recently Wiss and Parmlee investigated the effect of damping upon sensitivity to floor vibrations. Like Reiher and Meister, they found sensitivity to be proportional to the product of maximum displacement and frequency. For a given sensitivity rating they found that the amplitude-frequency product could be approximately twice as much when the damping was increased from 0% to 3% of critical.

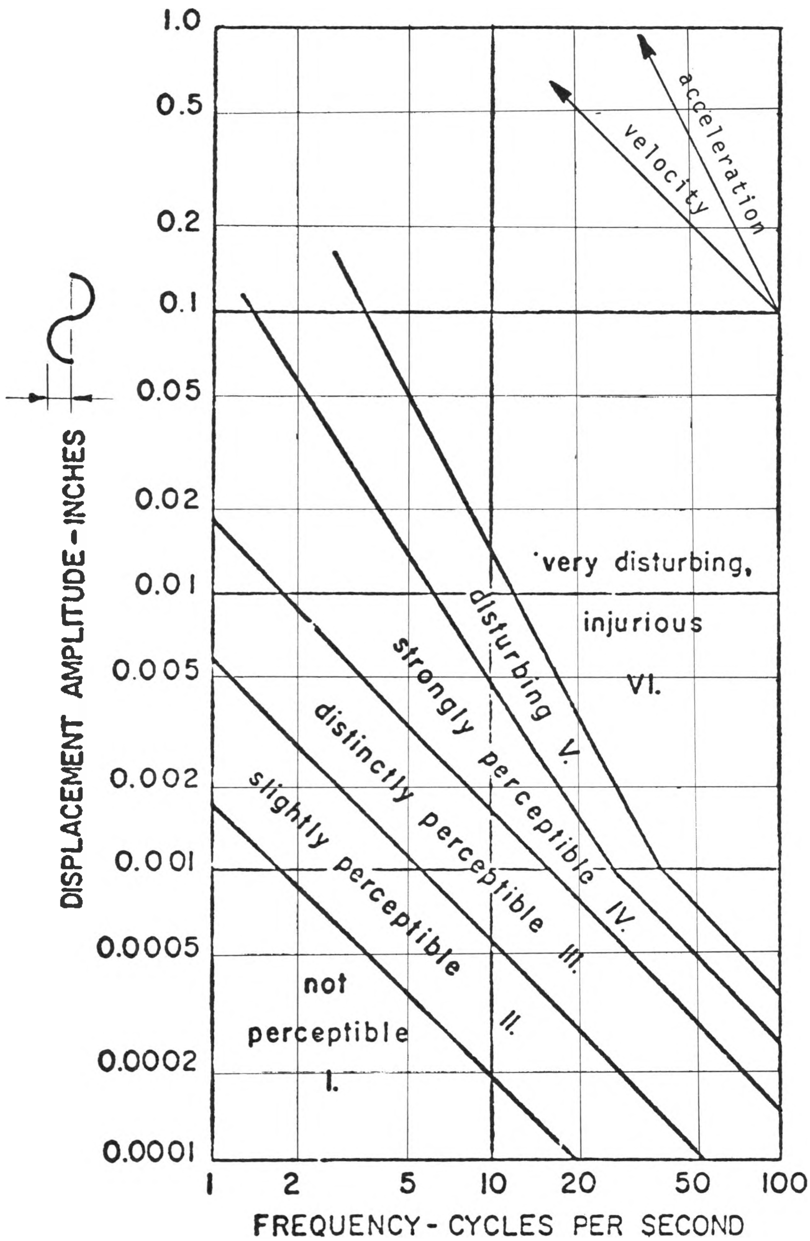

Figure 1. Domains of Various Strengths of Sensations for Standing Persons Subject to Vertical Vibration, After Reiher and Meister.

Experimental data from several investigators have been summarized by Goldman as shown in Figure 2. In an attempt to select a single

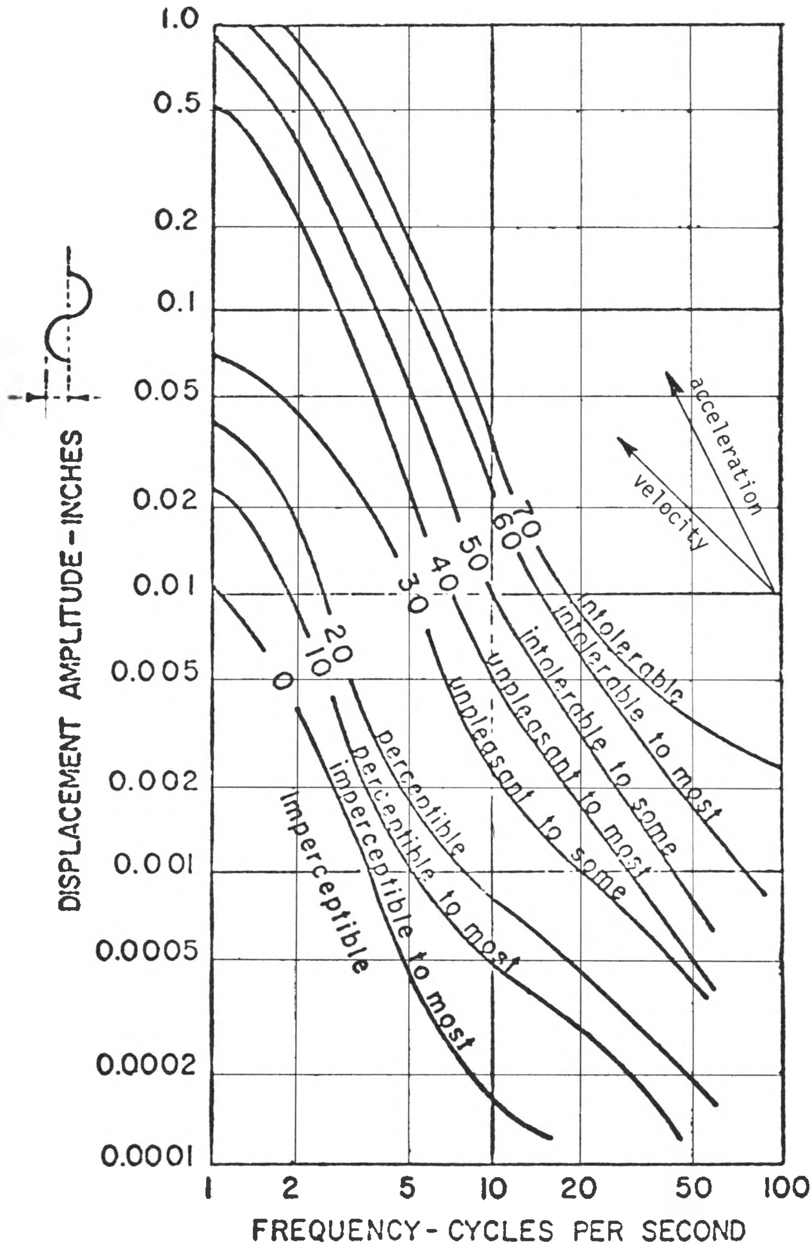

Figure 2. Contours of Equal Sensitivity to Vibration, After Goldman.

quantitative measure of vibrations, Wright and Walker observed that in the frequency range of interest for bridges (1 to 20 Hz), Goldman's curves are essentially constant acceleration lines. They have proposed a peak acceleration limit of 100 in./sec², which corresponds to Goldman's "unpleasant to some" curve with a tenfold increasefor short duration vibrations. This magnitude is definitely perceptible but is said to be within the tolerable range if pedestrians are aware that some motion is to be expected.

A review of the literature has shown that there is no single parameter which can completely represent the shadings of human sensitivity to vibration. Within the relevant frequency range for highway bridges, arguments can be made for using either a velocity or an acceleration limit as a serviceability criterion. This research project has focused primarily on accelerations. The 100 in./sec² acceleration limit proposed by Wright and Walker seems reasonable although somewhat higher than sugested by others, probably because of the somewhat arbitrary tenfold increase in magnitude taken to account for the shorter duration of the large amplitude vibrations.

ANALYTICAL STUDIES

Method of Analysis for Simple Span Bridges

The method of analysis for the dynamic response of simple span multi-girder bridges used in this project was developed earlier by Oran and Veletsos at the University of Illinois. Their computer program was modified somewhat to provide more acceleration information.

For the analysis the bridge is represented as a plate continuous over flexible beams. Both flexural and torsional stiffness of the beams are considered. The mass of the slab is assumed to be uniformly distributed, and the mass per unit length of each beam is assumed to be constant. The vehicle is represented by a single axle, two-wheel loading consisting of a sprung mass and two equal unsprung masses. The two identical springs are assumed to be linear elastic. Damping has been neglected for both the vehicle and the bridge.

The major steps of the analysis are: (1) determination of the instantaneous values of the interacting forces between the vehicle and the bridge itself and (2) evaluation of the deflections and moments produced by these forces. The dynamic deflection configuration of the bridge is represented by a Fourier series with time dependent coefficients. The equations of motion are formulated by application of LaGrange's equation and solved by numerical integration.

Acceleration Studies for Simple Span Bridges

Because of the strong relation between acceleration and vibration perception, the investigation focused on the variation of maximum bridge accelerations with several significant parameters of the bridge-vehicle system. The study was restricted to steel beam bridges with reinforced concrete decks. Standard designs with 4 to 8 parallel beams and spans to 70 ft were considered. Fundamental bending frequencies ranged from 4 to 16 Hz. The vehicle was represented by a 72 kip sprung mass traveling across the span at 60 mph.

Factors considered included: (a) bridge parameters, such as span and stiffness, (b) vehicle parameters, such as velocity and transverse position of the wheels, and (c) construction parameters, such as roadway roughness. It was determined that the maximum accelerations, which usually occur at midspan of the edge beams, decrease as the span length increases. Although static deflections are inversely proportional to bridge stiffness, maximum accelerations were found to increase only slightly when lighter A572 steel beams were substituted for the A36 beams of the basic design.

Over a vehicle speed range of 20 to 70 mph, maximum bridge accelerations were almost directly proportional to speed. By varying the transverse position of the vehicle on the bridge, it was shown that edge beam (curb) accelerations are greatest when the vehicle travels along the edge of the roadway and decrease when the vehicle travels near the center line. In contrast, center beam accelerations increase as the vehicle moves towards the center line and are slightly larger than edge beam accelerations when the vehicle straddles the center line. For most situations, however, edge beam accelerations are the largest.

Several previous test reports have indicated that surface roughness can significantly affect bridge vibrations. The computer program used to analyze simple span bridges could represent surface roughness as a constant amplitude sine wave. By varying the number of half sine waves it was possible to approach a resonant condition where the time required for the vehicle to cross one roughness wave corresponded roughly to the fundamental frequency of the bridge. Maximum accelerations with a periodic deck roughness were as much as five times as great as those for the same bridge with a smooth deck.

Analysis of Continuous Bridges

A general theory for the dynamic analysis of continuous bridges was developed by Huang and Veletsos. A computer program developed by Huang was used for a parametric study of two- and three-span symmetric beam bridges.

For this analysis the bridge is modeled as a single continuous beam with lumped masses. Viscous damping of the bridge is considered by locating dashpots at the mass coordinate points. Since the bridge is idealized as a single beam, torsional vibration modes and the rolling of the vehicle cannot be considered. However, a more sophisticated vehicle model is used. A tractor-trailer is represented by a three-axle load unit consisting of two interconnected masses. Each axle has springs and a friction device to simulate the suspension system.

The equations of motion for the vehicle and for each mass point of the bridge form a set of simultaneous, second-order differential equations which are solved by a numerical integration scheme. Evaluation of the interacting forces between the bridge and the vehicle is a major intermediate step.

Acceleration Studies for Continuous Bridges

Both two- and three-span symmetric continuous steel girder bridges with concrete decks were studied. The accuracy of the analysis depends on the number of lumped masses chosen as well as the size of the integration steps. Good stability of the solution was obtained lumping the masses of the bridge at the quarter and midpoints of each span and by dividing the time required for the vehicle to cross the bridge into 2000 integration steps.

As with the simple span bridges, maximum accelerations decreased with span length and increased only slightly when the stiffness of the beam was reduced, indicating again that vibration control is not directly related to deflection control. Maximum accelerations for two-span bridges were about 50% higher than those of three-span bridges of equal span length. The highest accelerations occurred in simple span bridges.

For two- and three-span bridges with spans in the 60 ft range, the largest accelerations occurred with a trailer axle spacing of about 40% of the span. Maximum accelerations again increased with vehicle speed. A comparison of accelerations was also made for one-, two-, and three-axle vehicle models of the same weight. Maximum accelerations produced by the two- and three-axle vehicle models were about the same, but they were about two-thirds of the maximums produced by the single-axle vehicle.

If the vehicle was oscillating somewhat as it entered the bridge, due to approach pavement roughness or a discontinuity at the abutment, maximum accelerations were increased by as much as 50%. As with the simple-span bridges, very large peak accelerations could be generated by adjusting the deck surface roughness so that the frequency of oscillation of the interacting forces was close to the fundamental frequency of the bridge.

EXPERIMENTAL STUDY

Although the dynamic response of bridges has been the subject of several analytical investigations in recent years, only a few experimental studies have been reported. The objective of this phase of the research was to measure and document certain cynamic response characteristics of typical highway bridges. The principal activities involved field testing of a number of bridges and reduction and analysis of the collected data.

Test Program

Dynamic response information was collected on-site in analog form on magnetic tape for some 62 representative beam-type bridges throughout the state of Indiana. Categories included composite and noncomposite simple span and continuous steel beam and plate-girder bridges as well as simple span and continuous reinforced concrete girder bridges and prestressed concrete I-beam bridges. The number spans varied from one to four, the span lengths from 27 ft to 129 ft, the deck width from 24 ft to 51 ft, and the year of construction from 1929 to 1972. Test variables included speed of the vehicle crossing the bridge, type and weight of vehicle, and transverse location of the vehicle on the bridge.

Although a main objective of the testing program was to determine responses for a broad range of bridge structures under normal traffic, it was also necessary to utilize a reference test vehicle with known characteristics in order to compare analytical predictions with field measurements. Moreover, for certain bridges with low truck traffic, using the test vehicle was the only practical way to obtain significant dynamic response records. The reference vehicle used was a school bus owned by the Indiana State Highway Commission Research and Training Center. This bus, a 1969 International with dual rear wheels and a 23-ft wheel base, was loaded so that the gross vehicle weight was 21,000 lb.

Instrumentation and Testing

Acceleration was the quantity of primary interest because of its relation to human sensitivity to vibration. Accelerometers were attached to both curbs at or near midspan of each span to make it possible to identify and analyze both flexural and torsional modes of vibration for each bridge. In addition, one taut wire cantilever beam deflection transducer was attached at the accelerometer location in the first span on the traffic lane side of each bridge. The output signals from the accelerometers and the deflection gage were recorded in analog form on magnetic tape using one to three 4-channel tape recorders, depending on the number of accelerometers used.

Personnel from the Indiana State Highway Commission Research and Training Center carried out the data collection. A typical bridge-testing crew consisted of two engineers and four assistants. All equipment and instruments were transported to the bridge site in a mobile laboratory, which was equipped with a portable AC generator. During the tests, personnel were stationed off the bridge and hidden from view when possible so as not to influence the normal flow of traffic.

One person stationed near the bridge in position to view the approaching traffic was in voice communication with the operator of the recording equipment in the mobile lab. Vehicle crossing records were collected only when, in the judgement of the vehicle spotter, the bridge was relatively quiescent immediately prior to vehicle entry and there was at least a 15-second gap before the entry of another vehicle so that free vibration could aldo be recorded. The spotter entered coded vehicle-type identification data by means of a hand held encoder.

Data Reduction and Analysis

More than 13,000 deflection and acceleration records corresponding to over 2200 vehicle crossings were actually collected; however because of cost and time constraints only 900 vehicle crossing records have been analyzed. Of these approximately 65% were for trucks, 30% were for the test vehicle, and 5% were for various light vehicles.

The first step in data reduction involved converting the analog (continuous) response records into digital form for processing. The analog records were first filtered to remove high frequency noise and then sampled at 5 millisecond intervals. The analysis of this considerable volume of data was limited to determination of maximum values of deflection, velocity, acceleration, and jerk, as well as equivalent viscous damping and frequency content of each record. These peak values have been adopted as indices of overall bridge performance.

Since velocity and jerk were not measured directly by transducers, it was necessary to obtain these quantities indirectly. To accomplish this, algorithms were developed to differentiate the digitized acceleration and deflection files and to integrate the acceleration files. Unfortunately, direct differentiation of a raw data file significantly magnifies the inherent random noise in the digitized data. To obtain satisfactory results by differentiation it was necessary first to smooth the data files through the use of "smoothing polynomials."

To obtain satisfactory results by integrating acceleration files it was necessary to apply suitable corrections to eliminate so-called baseline errors which arise because the reference axis of zero acceleration is generally unknown due to indeterminate initial conditions. Good comparisons were then found between twice-differentiated deflection records and corresponding accelerometer records and between deflection records and corresponding twice-integrated accelerometer records.

Table 1. Summary of Test Results for Heaviest Vehicle Class

Bridges	Span Length(s) (ft)	Absolute/Mean Maximum Deflection (in.)	Absolute/Mean Maximum Velocity (in./sec)	Absolute/Mean Maximum Acceleration (in./sec^2)	Absolute/Mean Maximum Jerk (in./sec^3)
Category 1					
SB-A-1	60	−0.060	1.38	89	7760
through		0.021	0.87	61	5080
SB-A-5					
SB-C-1	72	−0.058	1.16	50	2320
		0.031	0.67	33	1490
Category 2					
CSB-A-1	47.5-57-47.5	+0.143	1.55	105	7790
through		0.027	0.82	53	5110
CSB-A-4					
CSB-B-1	60-72-60	−0.075	1.40	123	4580
through		0.041	0.97	58	2900
CSB-B-4					
CSB-C-1	68-85-68	−0.137	1.84	83	5910
		0.082	1.20	52	2740
Category 4					
KCSB-C-1	76-76	−0.228	NA	133	NA
through		0.099	NA	74	NA
KCSB-C-4					
Category 7					
RCB-A-1	34-34-34	−0.014	0.43	45	4300
through		0.007	0.29	24	3150
RCG-A-3					
RCB-B-1	36-36-36	−0.025	0.21	59	1440
through		0.009	0.15	25	1150
RCG-B-4					
Category 9					
CRCS-A-1	27-36-27	−0.008	0.15	30	1280
through		0.003	0.10	16	700
CRCS-A-6					
Category 10					
PCIB-A-1	70-72-72-70	−0.066	0.89	41	3280
and		0.037	0.61	31	2180
PCIB-A-2					

Summary of Test Results

Major results of the test program are shown in Table 1. All maxima shown were produced by heavy five-axle tractor-trailers. The mean maximum values were determined as the mean of the maxima, without regard to sign for all heavy vehicle crossings for that particular bridge. Maxima for deflection, velocity, and jerk are at the point where the deflection gage was located. The maximum for acceleration is the largest value which was measured by any accelerometer on the bridge.

Steel bridges exhibited generally higher responses (acceleration levels were about twice as large) than reinforced or prestressed concrete bridges. Levels of acceleration recorded were generally well within acceptable ranges. There were only five instances in the entire testing program where a single vehicle crossing produced an acceleration greater than 100 in/sec^2. The highest mean maximum acceleration (for all recorded heavy vehicle crossings of a bridge) of only 74 in/sec^2 was exhibited by the two-span continuous-composite steel bridges.

Equivalent viscous damping ratios for free vibration of the test bridges were generally in the range of 1% to 2%. Fundamental flexural frequencies calculated using properties of the bridge cross sections compared favorably with measured values. Spectral analysis of the acceleration records disclosed that several frequencies, in addition to the fundamental one, were excited by each vehicle crossing, indicating the complex nature of the actual vibrations.

DESIGN IMPLICATIONS

In 1977 the AASHTO Bridge Specifications were revised, allowing the designer the discretion of exceeding the recommended live load deflection limit. No specific guidance is given other than a reference to Wright and Walker's report "Criteria for the Deflection of Steel Bridges" which recommends that the estimated maximum acceleration should not exceed 100 in/sec^2. The maximum acceleration is estimated simply as the product of the maximum dynamic displacement (about 30% of the static deflection) and the square of the fundamental bending frequency of the bridge.

Another similar approach to vibration control is contained in the new Ontario Highway Bridge Design Code. A deflection limit is given as a function of the fundamental bending frequency of the bridge of correlate response with human sensitivity. Three vibration levels, depending on the degree of pedestrian usage, are specified in the form of design curves, which correspond roughly to constant velocity lines for simple harmonic motion. Both methods, simple for designers to use, give similar deflection limits in the middle range of bridge fundamental frequencies.

Wright and Walker's method was used to estimate the maximum acceleration for nine of the test bridges. Static deflections were calculated for a 0.7 wheel load distribution factor, assuming an HS20 truck traveling at 55 mph. Although the three simple-span bridges were designed and built as noncomposite structures, properties of the composite section were used to calculate deflections and natural frequencies because frequencies calculated in this way compared favorably with those measured in the field. All of the two-span bridges were composite construction.

A summary of bridge properties, estimated accelerations, and peak accelerations measured in the field under actual traffic is shown in Table 2. In general, estimated values do indicate correct trends. The accelerations of the simple-span bridges were properly predicted to be larger than those of the two-span bridges, primarily because of the higher natural frequencies.

Experimentally-determined fundamental-bending frequencies, maximum deflections, and maximum accelerations for 14 series of steel beam and girder-test bridges are shown in Table 3. For each bridge the product of the maximum displacement and the square of the fundamental circular frequency is shown in the last column. Comparisons with the maximum accelerations are surprisingly good.

CONCLUSIONS AND RECOMMENDATIONS

1) Analytical studies have shown roadway roughness to be a significant factor influencing bridge deck accelerations. Rougher decks do cause higher accelerations. Although a roughness index might be incorporated into a simple acceleration estimation formula, the condition of the deck surface varies with time. The best recommendation for vibration control is to build and maintain smooth roadways and smooth transitions from the approach slab to the bridge deck.

2) Moderate success has been achieved in checking field measurements analytically. Without precise knowledge of the intial conditions of the vehicle and the roadway roughness, a computer program based upon a perfect model of the system cannot yield a precise dynamic response history for a vehicle crossing. Also the models underlying the computer programs used in this study lacked some of the sophistication necessary for more accurate results. For example, in modeling a two-span bridge as a single continuous beam, rolling of the vehicle and torsional response of the bridge are lost. Of course, the computer program did properly establish trends, identify significant parameters, and predict peak responses.

3) The human body is sensitive to motion. Both velocity and acceleration criteria recently have been proposed and could be used successfully to limit bridge vibrations to levels which are not objectionable to pedestrians, maintenance workers, cyclists, etc. Endorsement of the Wright and Walker recommendations for vibration control by AASHTO is a needed improvement in bridge design practice. Considering the complexities of computing and actual dynamic response history of a

bridge, the simple displacement times frequency squared expression yields a reasonable and practical estimate of peak acceleration.

4) It has been possible to measure the dynamic response of typical simple-span and continuous-beam bridges and to reduce the data to obtain vibrational characteristics. Good comparisons were obtained between twice-differentiated displacements and measured accelerations and between twice-integrated accelerations and measured displacements. The relatively low levels of acceleration measured for steel and concrete beam bridges seem to indicate that more flexible designs would still give vibration levels which would not be objectionable. The next logical step in this research would be to design and build a bridge more flexible than permitted by previous AASHTO rules, using the Wright and Walker guidelines for vibration control, and to monitor its dynamic performance under actual traffic and an instrumented control vehicle.

Table 2. Comparison of Measured Peak Accelerations With Those Predicted by Wright and Walker's Equation

Bridge Indentification	Span (ft)	EI Beam (K-in.^{2})	Static* Defl (in.)	Fundamental Frequency (Hz)	Peak Accelerations Predicted* in./sec^2	Peak Accelerations Measured in./sec^2
Single Span						
SB-A-1	60	$.604 \times 10^9$	0.284	6.39	116	89
SB-B-1	55	$.490 \times 10^9$	0.263	6.95	128	57
SB-C-1	72	$.636 \times 10^9$	0.485	4.82	118	50
Two Span						
KCSB-B-1	103.5	$.703 \times 10^9$	0.965	2.19	60	33
KCSB-C-2	76	$.746 \times 10^9$	0.351	3.73	56	133
KCSB-D-2	96	1.898×10^9	0.285	2.62	24	42
KCSG-A-1	122.5	2.564×10^9	0.452	2.27	27	30
KCSG-B-1	128	2.860×10^9	0.294	2.20	26	23
KCPG-A-1	129	3.018×10^9	0.448	2.36	28	27

*Based on HS20 Truck Traveling 55 mph.

Table 3. Measured Fundamental Frequencies and Maximum Measured Accelerations and Deflections for the Bridges in the Study.

Bridge Type	Fundamental Bending Frequency (Hz)	Maximum Deflection (in.)	Maximum Acceleration (in/sec^2)	Estimated Acceleration $(2\pi f)^2 D$
Single Span				
SB-A-1 thru SB-A-5	7.62	.060	89	137
SB-B-1 thru SB-B-3	6.77	.047	57	85
SB-C-1	4.88	.058	50	54
Two Span				
KCSB-A-1 and KCSB-A-2	2.73	.277	34	81
KCSB-B-1	2.15	.124	33	22
KCPG-P-1 and KCPG-B-2	2.25	.154	36	32
KCSB-C-1 thru KCSB-C-4	3.81	.228	133	130
KCSB-D-1 and KCSB-D-2	2.83	.123	42	38
KCSB-A-1 and KCSG-A-2	2.34	.131	30	28
KCSB-B-1 and KCSG-B-2	2.22	.117	23	22
KCPG-A-1 thru KCPG-A-3	2.44	.112	27	26
Three Span				
CSB-A-1 thru CSB-A-4	7.54	.049	105	109
CSB-B-1 thru CSB-B-4	5.27	.075	124	82
CSB-C-1	3.91	.137	83	82

REFERENCES

1. Standard Specifications for Highway Bridges, American Association of State Highway and Transportation Officials, Twelfth Edition, 1977.
2. Aramraks, T., "Highway Bridge Vibration Studies," Joint Highway Research Project Report JHRP-75-2, Engineering Experiment Station, Purdue University, 1975.
3. Kropp, P., "Experimental Study of the Dynamic Response of Highway Bridges," Joint Highway Research Project Report JHRP-77-5, Engineering Experiment Station, Purdue University, 1977.
4. Shahabadi, A., "Bridge Vibration Studies," Joint Highway Research Project Report JHRP-77-17, Engineering Experiment Station, Purdue University, 1977.
5. Wright, D.T. and Green, R., "Human Sensitivity to Vibrations," Report No. 7, Department of Civil Engineering, Queen's University, Kingston, Ontario, February, 1959.
6. Reiher, H. and Meister, J.J., "The Effect of Vibration on People," (in German), Forschung auf dem Gebite des Ingenieurwesens, v2, p. 381, 1931. (Translation: Report No. F-TS-616-RE H.Q. Air Material Command, Wright Field, Ohio, 1946).
7. Lenzen, K.H., "Vibration of Steel-Concrete Slab Floor," *AISC Engineering Journal*, Vol. 3, No. 3, July, 1966, pp. 133-136.
8. Parmalee, R.A. and Wiss, J.F., "Human Perception of Transient Vibrations," Journal of the Structural Division, ASCE, April 1974.

9. Goldman, D.E., "A Review of the Subjective Response to Vertical Motion of the Human Body in the Frequency Range 1 to 70 Cycles Per Second," Naval Medical Research Institute, Report No. 1, Project MM004001, March 16, 1948.
10. Wright, R.N. and Walker, W.H., "Criteria for the Deflection of Steel Bridges," A.I.S.I. Steel Research for Construction, Bulletin No. 19, November 1971.
11. Oran, C. and Veletsos, A.S., "Analysis of Static and Dynamic Responses of Simple-Span, Multigirder Bridges," University of Illinois at Urbana-Champaign, Civil Engineering Studies, Structural Research Series 221, July, 1961.
12. Veletsos, A.S. and Huang, T., "Analysis of Dynamic Response of Highway Bridges," *Journal of Engineering Mechanics Division*, ASCE, Vol. 96, No. EM5, October 1970.
13. Huang, T., "Dynamic Response of Three Span Continuous Highway Bridges," University of Illinois at Urbana-Champaign, Ph.D. Thesis, 1960.
14. Casaletto, J.J. and Rice, J.R., "Smoothing and Estimating Derivatives of Equispaced Data," Purdue University Computer Sciences Department, CSD TR 35, May, 1969.
15. "The Determination of True Ground Motion by Integration of Strong-Motion Records: A Symposium," Bulletin of the Seismological Society of American, Vol. 33, No. 1, January, 1943.
16. Nigam, N.C. and Jennings, P.C., "Digital Calculation of Response Spectra from Strong-Motion Earthquake Records," California Institute of Technology, Earthquake Engineering Research Laboratory, June 1968.
17. "Strong Motion Earthquake Accelerograms, Digitized and Plotted Data," California Institute of Technology, Earthquake Engineering Research Laboratory, Report No. EERL 71-50, September 1971.
18. Ontario Highway Bridge Design Code, Ontario Ministry of Transportation and Communications, First Edition, 1978.