## Hilbert Space Methods in Signal Processing

This lively and accessible book describes the theory and applications of Hilbert spaces, and also presents the history of the subject to reveal the ideas behind theorems and the human struggle that led to them.

The authors begin by establishing the concept of "countably infinite," which is central to the proper understanding of separable Hilbert spaces. Fundamental ideas such as convergence, completeness, and dense sets are first demonstrated through simple familiar examples and then formalized. Having addressed fundamental topics in Hilbert spaces, the authors then go on to cover the theory of bounded, compact, and integral operators at an advanced but accessible level. Finally, the theory is put into action, considering signal processing on the unit sphere, as well as reproducing kernel Hilbert spaces. The text is interspersed with historical comments about central figures in the development of the theory, which helps to bring the subject to life.

**Rodney A. Kennedy** is a Professor in the Research School of Engineering and the Head of the Applied Signal Processing research group at the Australian National University, Canberra. He has won a number of prizes in engineering and mathematics, including UNSW University and ATERB Medals. He has supervised more than 40 Ph.D. students and co-authored approximately 300 research papers. He is a Fellow of the IEEE.

**Parastoo Sadeghi** is a Fellow in the Research School of Engineering, at the Australian National University, Canberra. She has published around 90 refereed journal and conference papers, and received two IEEE Region 10 paper awards. She is a Senior Member of the IEEE.

"With the engineering research community in mind, the authors present a thoughtfully constructed, in-depth treatment of Hilbert spaces that includes a detailed coverage of signals-and-systems on the 2-sphere and a fresh perspective on reproducing kernel Hilbert spaces. This book provides a friendly, witty, and thorough introduction to this mathematically rich field and will likely become a mainstay of the engineering research literature."

*Phil Schniter, The Ohio State University*

"A book of this mathematical sophistication shouldn't be this fun to read – or teach from! Guilty pleasure aside, the treatment on Hilbert spaces and operator theory is remarkable in its lucidity and completeness – several other textbooks' worth of material. More than half of the book consists of new insights into spherical data analysis cast in a general framework that will make any of us working in this and adjacent research areas reach for this book to properly understand what it is that we have done."

*Frederik J. Simons, Princeton University*

# Hilbert Space Methods in Signal Processing

RODNEY A. KENNEDY
Australian National University, Canberra

PARASTOO SADEGHI
Australian National University, Canberra

CAMBRIDGE UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9781107010031



First published 2013

Printed and bound in the United Kingdom by the MPG Books Group

*A catalogue record for this publication is available from the British Library*

ISBN 978 1 107 01003 1 Hardback

**For our parents**
**Joan and William**
**Akram and Mostafa, and for our children**
**Lachlan, Kelan, and Aiden**

Cambridge University Press
978-1-107-01003-1 - Hilbert Space Methods in Signal Processing
Rodney A. Kennedy and Parastoo Sadeghi
Frontmatter
More information

# Contents

Cambridge University Press
978-1-107-01003-1 - Hilbert Space Methods in Signal Processing
Rodney A. Kennedy and Parastoo Sadeghi
Frontmatter
More information

# Preface

This is our book on the theory of Hilbert spaces, its methods and usefulness in signal processing research. It is pitched at a graduate student level, but relies only on undergraduate background material. There are many fine books on Hilbert spaces and our intention is not to generate another book to stick on the pile or to be used to level a desk. So from the onset, we have sought to synthesize the book with special goals in mind.

The needs and concerns of researchers in engineering differ from those of the pure sciences. It is difficult to put the finger on what distinguishes the engineering approach that we take. In the end, if a potential use emerges from any result, however abstract, then an engineer would tend to attach greater value to that result. This may serve to distinguish the emphasis given by a mathematician who may be interested in the proof of a foundational concept that links deeply with other areas of mathematics or is part of a long-standing human intellectual endeavor — not that engineering, in comparison, concerns less intellectual pursuits. As an example, Carleson in 1966 proved a conjecture by Luzin in 1915 concerning the almost-everywhere convergence of Fourier series of continuous functions. Carleson's theorem, as it is called, has its roots in the questions Fourier asked himself, in French presumably, about the nature of convergence of the series named after him. As a result it is important for mathematics, but less clear for engineers.

However, there is an important observation to be made here in that from the time of Fourier's first results in 1807 to Carleson's results in 1966, it was more than 150 years and from Fourier to today it is more than 200 years. In these long intervals of time, a lot of very bright people have been thinking about and refining ideas. So to learn any new topic, such as Hilbert spaces, is rather unnatural because it hides the human struggle to understand. Most mathematical treatments have the technical material laid out in a very concise and logical form. As a result the material comes across as a bit dry, at least to many people wanting or needing to learn. It does need livening up. So this book makes an attempt to inject a bit of life into the learning process. There are some mildly risqué characterizations of the founders of the theory and the style of writing is intentionally light and more reflective of the lecturing style than a written discourse. Of these founders we have immense admiration.

As said above, learning is a human endeavor and the material we are learning is the culmination of centuries of efforts of significant people who will be remembered and revered in future centuries, much more than celebrities and world leaders of today. Therefore it is of considerable interest to know why the ideas took so long and what were the stumbling blocks on the way. So one of the special goals of the book is to become comfortable with ideas and get a feeling for how to think about Hilbert spaces in the right way. Armed with the right elementary notions, self-study and taking on more advanced material and extensions are possible. An example is the theory on sets and infinity that Cantor laid down starting in 1873. This led to a proper understanding of the important constructions in Fourier series and generalizations to Hilbert spaces. So in this book we do spend some time on the nature of infinity, which might seem a very non-engineering concern. We defend this because countable infinity is a pillar of the theory, at least to better understand in what sense a Fourier series can represent a function, to understand separability of a space, to understand what breaks when we combine an infinite number of very smooth functions and end up with something unexpected. Cantor's work is important because of the influence it had on Hilbert and subsequent developments. In addition, we have been intrigued by Cantor's personal struggles and confrontations with other mathematicians, which contributed to him spending some time in an asylum. And this tends to be true of a number of the central figures that we meet on our journey, such as Cauchy, Bessel, Schmidt and Hilbert. They have an otherworldliness to their personalities and tend to have amusing anecdotes supporting their eccentricities.

Another goal in the book is to not let rigor dominate the material and look to reveal the nub of each result. Once an idea or concept is formulated, then the more technical results can follow, but often we direct the reader to two beautiful books more suited to mathematicians, the elegant (Helmberg, 1969) and the sublime classic (Riesz and Sz.-Nagy, 1990). Mathematician Weyl Hermann (1885–1955) was quoted as saying:

> "My work always tried to unite the truth with the beautiful, but when I had to choose one or the other, I usually chose the beautiful."

This reveals a lot about Weyl — a student of Hilbert, his successor at Göttingen, and a leading mathematician of the twentieth century – and what really motivated his work and his fallibility. We are not in his league, nor aspire to be, but we have a similar weakness for revealing intuitive and direct demonstrations for results even when the approach might lack absolute rigor or skip some technicalities.

The book comes in three parts as revealed in the table of contents. As an alternative to the contents, in engineering terms, Part I is mostly about signals, Part II is about systems and Part III puts the theory into action. Part III reflects material closer to our research interests, but distinguishes itself from the standard time domain signals and systems which are a feature of many engineering texts. We provide quite a lengthy treatment of signals and systems where the domain is the 2-sphere where the power of Hilbert spaces lays bare strong analogies with time domain signal processing. The final chapter of the book, Chapter 10, provides an accessible, somewhat original, treatment on reproducing kernel Hilbert spaces (RKHS) which draws on many theoretical aspects developed in all other chapters.

Some of the research results presented in Part III, especially in Chapter 8 and Chapter 9, have stemmed from joint work with our former and current PhD students. This research was supported under Australian Research Council's Discovery Projects funding scheme (project number DP1094350). We are also grateful to Zubair Khalid for proofreading many parts of the book and providing useful feedback and for some simulation results in Chapter 8. We acknowledge the support and encouragement of our colleagues and students at the Australian National University. Last, but not least, special thanks go to Andrew Hore, our cartoonist/illustrator, `http://funnyworksoz.com/`, who did a wonderful job with the illustration of characters that you see throughout the book. We do hope that, like us, you enjoy these lively and detailed illustrations.