

## HILBERT-TYPE INEQUALITIES INVOLVING DIFFERENTIAL OPERATORS, THE BEST CONSTANTS, AND APPLICATIONS

VANDANJAV ADIYASUREN, TSERENDORJ BATBOLD AND MARIO KRNIĆ

*Abstract.* Motivated by some recent results, in this article we derive several Hilbert-type inequalities with a differential operator, regarding a general homogeneous kernel. Moreover, we show that the constants appearing on the right-hand sides of these inequalities are the best possible. The general results are then applied to some particular examples of homogeneous kernels and compared with previously known from the literature.

*Mathematics subject classification (2010):* Primary 26D10, 26D15, Secondary 33B15.

*Keywords and phrases:* Hilbert-type inequality, Hardy inequality, differential operator, homogeneous function, Gamma function.

### REFERENCES

- [1] M. ABRAMOWITZ, I. A. STEGUN, *Handbook of mathematical functions with formulas, graphs, and mathematical tables*, National Bureau of Standards, Applied Math. Series 55, 4th printing, Washington 1965.
- [2] L. E. AZAR, *Two new forms of Hilbert-type integral inequality*, *Math. Inequal. Appl.*, **17**, 3 (2014), 937–946.
- [3] A. ČIŽMEŠIJA, M. KRNIĆ, J. PEČARIĆ, *General Hilbert's inequality with non-conjugate parameters*, *Math. Inequal. Appl.* **11** (2008), 237–269.
- [4] R. L. GRAHAM, D. E. KNUTH, D. PATASHNIK, *Concrete mathematics – A Foundation for Computer Science*, Addison-Wesley Publishing Company, 1989.
- [5] G. H. HARDY, *Note on some points in the integral calculus*, *LXIV*, *Messenger Math.* **57** (1928), 12–16.
- [6] G. H. HARDY, J. E. LITTLEWOOD, G. PÓLYA, *Inequalities*, 2<sup>nd</sup> edition, Cambridge University Press, Cambridge, 1967.
- [7] M. KRNIĆ, J. PEČARIĆ, *General Hilbert's and Hardy's inequalities*, *Math. Inequal. Appl.* **8** (2005), 29–52.
- [8] M. KRNIĆ, G. MINGZHE, J. PEČARIĆ, G. XUEMEI, *On the best constant in Hilbert's inequality*, *Math. Inequal. Appl.* **8** (2005), 317–329.
- [9] A. KUFNER, L. MALIGRANDA, L. E. PERSSON, *The Hardy inequality – About its history and some related results*, Vydavatelský servis, Pilsen, 2007.
- [10] I. PERIĆ, P. VUKOVIĆ, *Hardy-Hilbert's inequalities with a general homogeneous kernel*, *Math. Inequal. Appl.* **12** (2009), 525–536.