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HISTORICAL EVOLUTION OF VORTEX-LATTICE METHODS

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Good morning. In this short talk I will give a review of the beginnings and some orientation of the vortex-lattice method. The vortex-lattice method is a discrete vortex colocation method for obtaining numerical solutions to the loading integral equation relating normal velocity and wing loading. It is a branch of computer fluid dynamics which in turn is mathematically descended from finite-difference concepts. Finite-difference concepts had been applied to the development of calculus which dates it a relatively long time ago. For our subject the beginning is much more current. Here for orientation we will follow the historical course of the vortex-lattice method in conjunction with its field of computational fluid dynamics. An outline of the concurrent development of computer fluid dynamics and vortex-lattice methods is as follows:

- L.F. RICHARDSON (1910)
- L. PRANDTL (1918, 1921)
- H. LIEPMANN (1918)
- R. COURANT, K. FRIEDRICHS, AND H. LEWY (1928) ELLIPTIC AND HYPERBOLIC EQUATIONS
- A. THOM (1928) FIRST NUMERICAL SOLUTION OF VISCOUS FLUID-DYNAMICS PROBLEM
- 1/4 - 3/4 RULE CHORD CONCEPT (1937)
- G. H. SHORTLEY, AND R. WELLER (1938)
- LOS ALAMOS SCIENTIFIC LABORATORY (WORLD WAR II)
- V.M. FALKNER (1943) FIRST USE OF NAME VORTEX-LATTICE THEORY
- R. V. SOUTHWELL (1946)
- C.M. TYLER, JR. (1949)
- D. N. DeG. ALLEN, AND S.C.R. DENNIS (1951)
- D. N. DeG. ALLEN, AND R.V. SOUTHWELL (1955)
- F. H. HARLOW, AND J. E. FROMM (1965)
- AERODYNAMIC ANALYSIS REQUIRING ADVANCED COMPUTERS, NASA SP347 (1975)

Since many mathematical models of fluid dynamics can be expressed as partial differential equations then, historically, computer fluid dynamics can be said to start with L. F. Richardson's paper. Some consider this paper as the foundation of modern numerical analysis of partial differential equations. He applied his methods to the engineering problem of determining stresses in a masonry dam. In 1918 Prandtl formulated the lifting-line theory. The chord loading is concentrated into a single load vortex, thus it is a one panel chord-wise vortex lattice with flow conditions satisfied at the load line. In 1938 Prandtl proposed an explicit finite-difference method for solving boundary-layer equations. Liepmann showed how to improve the convergence rate of

Richardson's procedure. In later years Liepmann's method was found very compatible with electronic computers and has been further developed. The classic paper of Courant, Friedrichs, and Levy has become a guide for practical fluid flow computational solutions. A. Thomm did early computational work in fluid flow, two-dimensional and flow past circular cylinders.

The 1/4-3/4 rule has a fundamental role in vortex-lattice methods. This concept first appeared in a paper by E. Pistolesi in 1937. He in effect did a single panel vortex-lattice solution for a two-dimensional wing and found that with the load vortex at the 1/4 chord line and downwash or normal wash point (no-flow through condition) at 3/4 chord, the section lift and moment for constant angle of attack is exactly that of thin wing theory. And lift is predicted exactly for wing with parabolic camber. This rule was first applied to wings of finite aspect ratio by W. Mutterperl (1941) and J. Weissinger (1942) and very often since by others. P.A. Byrd (Ing.-Arch. 19, 321-323, 1951) expanded Pistolesi's work for sections divided into more than one panel on the chord and with the 1/4-3/4 rule applied for each panel found that lift and moment are predicted exactly. In later years this chordwise rule received further mathematical attention. Shortley and Weller developed block relaxation-a developed version of Liepmann's method. It was this work from Ohio State University I had used in a graduate course at Washington State in 1943 to numerically solve the Laplace equation for determining the stress pattern in a twisted grooved rod. Work at the Los Alamos Scientific Laboratory has contributed much to the advancement of computer fluid dynamics. This includes the work of J. von Neumann, J. Fromm, and F. Harlow. From Los Alamos a graphics fluid dynamics motion picture was circulated in this country in the 1960's. It showed a computer fluid dynamics flow prediction of a dam bursting and the water cascading down a gorge. V. Falkner covered the wing with a grid of straight horseshoe vortices. Wing surface loadings were predicted. In one report he uses the title, "The Solution of Lifting Plane Problems by Vortex Lattice Theory," A.R.C.R.& M. 2591, 1947, which is a first use of this name. Faulkner's method and variations were tried extensively throughout the industry during the 1950's. However, the calculation effect was large which limited the number of panels then accuracy became questionable for some configuration designs. The vortex-lattice method had to await computer capability. Southwell improved the relaxation procedure by scanning the mesh for larger residuals for new values calculation. This scanning procedure is not so suitable for electronic computers. Tyler, in a Ph.D. dissertation, and Allen and Dennis developed relaxation method solutions for computing wing lifting surface loading. Using Southwell's relaxation method, Allen and Southwell did a solution for the viscous imcompressible flow over a cylinder. The year 1965 is considered by some as a modern start to computer or computational fluid dynamics. Harlow and Fromm provided stimulus and awareness in a Scientific American paper entitled, "Computer Experiments in Fluid Dynamics" which includes the concept of numerical simulation. It has been observed that the percentage of published scientific engineering numerical methods papers to total papers has increased twenty fold in the decade of 1963 to 1973. The year 1965 can be considered as the start of the computational vortex-lattice method. It has had a many fold growth in applications and development during the last decade. It was certainly influenced by the stimulus and awareness of the potential of the scientific computer occurring throughout the field of computational fluid dynamics. In the mid 1960's four independent papers appeared on vortex-lattice methods, respectively by Rubbert, Dulmovits, Hedman, and Belotserkovskii. These were extensions of Faulkner's method and adapted to electronic computers. For the reported work of the 1960's and 70's reference can be made to the bibliography list of this workshop. The state of the art in general computational fluid dynamics is demonstrated in the volumes of NASA SP-347 which is the result of a March 4-6, 1975 NASA conference at Langley.

Computer capacity is developing rapidly. Computational speed has been increased by a factor of 2.5 each year. The application of the vortex-lattice method is being made to increasingly complex configuration designs such as multi-planes, nonplanar wings, interference, and wing tip. It is a powerful tool as an aid in parameter study and optimization. Currently attention is being directed toward further improving the vortex-lattice representation by lattice arrangement, panel geometry, and by better mathematical modeling of the flow in the panel region. These have been referred to as advanced panel methods. However, in some of these developments the simplicity of an elemental vortex representation is lessened and leads to greater mathematical model complexity of the panel flow, but computational efficiency may be increased. In summary, this is computationally a new technology field only about 10 years old. It is computer oriented with numerical simulation of the physical laws governing the problem. It is a supplement to the two disciplines of theory and experiment. It can logically be extended to find answers of complex flow impractical to measure experimentally. In this workshop we will learn of many unique utilizations of the vortex-lattice method, of lattice analytical advancements, and the power and nature of this new discipline. Thank you.

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