

HÖLDER-TYPE SPACES, SINGULAR OPERATORS, AND FIXED POINT THEOREMS

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Abstract. In this note, we give a sufficient condition for the existence of Hölder-type solutions to a class of fractional initial value problems involving Caputo derivatives. Since imposing (classical or general) global Lipschitz conditions on the nonlinear operators involved leads to degeneracy phenomena, the main emphasis is put on local Lipschitz conditions or fixed point principles of Schauder and Darbo type. To this end, we study continuity and boundedness conditions for linear Riemann-Liouville operators and nonlinear Nemytskij operators in Hölder spaces of integral type which have much better properties than classical Hölder spaces.

Key Words and Phrases: Initial value problem, Caputo derivative, singular integral equation, Riemann-Liouville operator, Nemytskij operator, integral-type Hölder space, Schauder fixed point theorem, Darbo fixed point theorem.

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