# Holographic model for antiferromagnetic quantum phase transition induced by magnetic field

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We propose a gravity dual of antiferromagnetic quantum phase transition induced by magnetic field and study the critical behavior around the quantum critical point. It turns out that the boundary critical theory is a strong coupling theory with dynamic exponent $z = 2$ and that the hyperscaling law is violated and logarithmic corrections appear near the quantum critical point. Some novel scaling relations are predicated, which can be tested by experiment data in the future. We also make some comparison with experimental data on low-dimensional magnets $BiCoPO_5$ and pyrochlores $Er_{2-2x}Y_{2x}Ti_2O_7$.

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## I. INTRODUCTION

Quantum phase transition (QPT) and the behavior of quantum systems in the vicinity of the corresponding quantum critical point (QCP) have attracted a lot of attention both on the theory and experiment sides recently [1–3]. In contrast to their classical counterparts induced by thermal fluctuations arising at finite temperature $T > 0$, QPTs happen at zero temperature and are governed by quantum fluctuations associated with the Heisenberg uncertainty and are driven by a certain control parameter rather than temperature, e.g., composition, magnetic field or pressure, etc. In condensed matter physics, such a quantum criticality is considered to play an important role in some interesting phenomena [4,5].

One of intensively discussed QPTs is ordered-disordered QPT in antiferromagnetic materials induced by magnetic field (see for example, Refs. [6–10]), especially in the heavy-fermion systems, since they can be tuned continuously from an antiferromagnetic (AF) state to a paramagnetic metallic state by varying a single parameter [4]. In these materials, QPT naturally belongs to the phenomenon involving strongly correlated many-body systems [11–13]. However, the complete theoretical descriptions valid in all the energy (or temperature) regions are still lacking. To study and characterize strongly coupled quantum critical systems, some new methods are called for.

Thanks to the feature of the weak/strong duality, the AdS/CFT correspondence provides a powerful approach to study such strongly coupled systems. This duality relates a weak coupling gravitational theory in a $(d+1)$-dimensional asymptotically anti-de Sitter (AdS) space-time to a $d$-dimensional strong coupling conformal field theory in the AdS boundary [14–16]. In recent years, we have indeed witnessed that the duality has been extensively applied into condensed matter physics and some significant progresses have been made [17–20]. In Ref. [21] we realized the ferromagnetic/paramagnetic phase transition in a holographic setup, and in Ref. [22] the holographic antiferromagnetic/paramagnetic phase transition was studied. We showed that the antiferromagnetic transition temperature $T_N$ is indeed suppressed by an external magnetic field and tends to zero when the magnetic field reaches its critical value $B_c$. In this way the antiferromagnetic QPT induced by magnetic field is realized. However, it was shown that the model proposed in Ref. [21] contains a vector ghost; very recently, a modified model was proposed [23], which is shown not only ghost free but also causal well defined, while it keeps the main results in the original model qualitatively. Here we will elaborate in some detail this QPT and study the corresponding critical properties in this new model.

## II. HOLOGRAPHIC MODEL

To describe the spontaneous staggered magnetization which breaks the time reversal symmetry, we introduce two real antisymmetric tensor fields coupled with U(1) Maxwell field strength [22]. Based on the discussions in Refs. [22,23], we take the bulk action as follows,

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\bigg[R + \frac{6}{L^2} - F^{\mu\nu}F_{\mu\nu} - \lambda^2(L_1 + L_2 + L_{12})\bigg], \tag{1}$$

where

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$$L_{12} = \frac{k}{2} M^{(1)\mu\nu} M^{(2)}_{\mu\nu},$$
$$L_{(a)} = \frac{1}{12} (dM^{(a)})^{\mu\nu\tau} (dM^{(a)})_{\mu\nu\tau} + \frac{m^2}{4} M^{(a)\mu\nu} M^{(a)}_{\mu\nu} + \frac{1}{2} M^{(a)\mu\nu} F_{\mu\nu} + J V(M^{(a)}_{\mu\nu}),$$
$$V(M^{(a)}_{\mu\nu}) = ({}^{*}M^{(a)}{}_{\mu\nu} M^{(a)\mu\nu})^2, \qquad a = 1, 2. \tag{2}$$

Here $^{*}$ is the Hodge star dual operator, and $dM$ denotes the exterior derivative of $M$. $L$ is the radius of AdS space, $2\kappa^2 = 16\pi G$ with $G$ the Newtonian gravitational constant; $k$, $m^2$, and $J$ are all model parameters with $J < 0$, $\lambda^2$ characterizes the backreaction of the two polarization fields $M^{(a)}_{\mu\nu}$ to the background geometry; and $L_{12}$ describes the interaction between two polarization fields. Note that, by rescaling the polarization fields and the parameter $J$, $\lambda^2$ can also be viewed as the coupling strength between the polarization fields and the background Maxwell field. In the AdS/CFT duality, the model parameters $m$ and $k$ are related to the dual operator dimension in the boundary field theory, $J$ is related to the self-coupling coefficient of the magnetic moment, and $k$ describes the interaction between two kinds of magnetic moments. The reason to introduce two antisymmetric fields for describing the antiferromagnetism was elaborated in Ref. [22]. Note that the form of $V(M^{(a)}_{\mu\nu})$ is not unique; we choose this form as it can lead to spontaneous symmetry breaking (see Fig. 1 in Ref. [23]) and to simplify the equations of motion of the model. Compared with the original model for antiferromagnetism in Ref. [22], the key change is to replace the covariant derivative of the polarization field $M$ by the exterior derivative. This change can avoid the problems such as ghost and causal violation, while keeping the significant results in the original model qualitatively, and in addition this model has a potential origin in string/M theory [23].

In the probe limit of $\lambda \to 0$, we can neglect the backreaction of the two polarization fields on the background geometry. The background we will consider is a dyonic Reissner–Nordström–AdS black brane solution of the Einstein–Maxwell theory with a negative cosmological constant, and the metric reads [24]

$$ds^2 = r^2(-f(r)dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2 f(r)},$$
$$f(r) = 1 - \frac{1 + \mu^2 + B^2}{r^3} + \frac{\mu^2 + B^2}{r^4}. \tag{3}$$

Here both the black brane horizon $r_h$ and AdS radius $L$ have been set to be unity. The temperature of the black brane is

$$T = (3 - \mu^2 - B^2)/4\pi. \tag{4}$$

For the solution (3), the corresponding gauge potential is $A_\mu = \mu(1 - 1/r)dt + Bxdy$. Here $\mu$ is the chemical potential, and $B$ can be viewed as the external magnetic field of the dual boundary field theory.

We consider a self-consistent ansatz for the tensor fields with nonvanishing components $M^{(a)}_{tr}, M^{(a)}_{xy}$ $(a = 1, 2)$ and define

$$\alpha = (M^{(1)}_{xy} + M^{(2)}_{xy})/2, \qquad \beta = (M^{(1)}_{xy} - M^{(2)}_{xy})/2. \tag{5}$$

By this definition, the antiferromagnetic order parameter, i.e., the staggered magnetization, can be expressed as [22,23]

$$N^{\dagger}/\lambda^2 = -\int dr \beta / r^2. \tag{6}$$

Then the antiferromagnetism phase corresponds to the case when $N^{\dagger} \neq 0$. In our model, it just corresponds to the case of $\beta \neq 0$, while $\alpha = 0$.

With this ansatz, it is found that the equations for $M^{(a)}_{tr}$ are algebraic ones and can be solved directly [25]. Therefore, we pay our main attention to $\alpha$ and $\beta$. At the horizon, the regular initial conditions should be imposed. The behavior of the solutions of equations in the UV region (near the AdS boundary) depends on the value of $m^2 + k$. When $m^2 + k = 0$, the asymptotic solutions will have a logarithmic term; we will not consider this case here. Instead when $m^2 + k \neq 0$, we have the asymptotic solution as [25]

$$\alpha_{\rm UV} = \alpha_+ r^{(1+\delta_1)/2} + \alpha_- r^{(1-\delta_1)/2} - \frac{B}{m^2 + k},$$
$$\beta_{\rm UV} = \beta_+ r^{(1+\delta_2)/2} + \beta_- r^{(1-\delta_2)/2},$$
$$\delta_1 = \sqrt{1 + 4k + 4m^2}, \qquad \delta_2 = \sqrt{1 - 4k + 4m^2}, \tag{7}$$

where $\alpha_\pm$ and $\beta_\pm$ are all finite constants. To make the system condense into the antiferromagnetic phase, as in Ref. [22], we require that the term associated with the magnetic field $B$ in Eq. (7) should be the leading term. When $B = 0$, we require that the condensation for $\beta$ appears spontaneously. Based on these considerations, we find the parameters have to satisfy $m^2 > k > 0$ and

$$J_c^+(k, m^2) < J < J_c^-(k, m^2), \tag{8}$$

with $J_c^{\pm}(k, m^2) = -(m^2 + k)^2 (m^2 + 3/2 \pm k)/12$ and $\alpha_+ = \beta_+ = 0$ according to the AdS/CFT dictionary (for details please see Ref. [25]).

## III. QCP, ENERGY GAP, AND SPECTRUM

Let us first consider the influence of the external magnetic field $B$ on the antiferromagnetic critical temperature $T_N$. Near the critical temperature, the staggered magnetization is very small; i.e., $\beta$ is a small quantity. In that case we can neglect the nonlinear terms of $\beta$ and obtain the equations for $\alpha$ and $\beta$,

$$\alpha'' + \frac{f'\alpha'}{f} - \frac{m^2_{\alpha\mathrm{eff}}}{r^2 f}\alpha = \frac{B}{r^2 f}, \qquad \beta'' + \frac{f'\beta'}{f} - \frac{m^2_{\beta\mathrm{eff}}}{r^2 f}\beta = 0. \tag{9}$$

Here $m^2_{\alpha\mathrm{eff}}$ and $m^2_{\beta\mathrm{eff}}$ are two functions of $\alpha$ [25]. Without loss of generality, we can set $\beta(r_h) = 1$. With increasing the magnetic field $B$ from zero, the effective mass square of $\beta$ increases, so that the critical temperature $T_N$ decreases. The critical temperature is plotted as a function of the external magnetic field in Fig. 1. When $T_N$ is decreased to zero, an $\mathrm{AdS}_2$ geometry emerges near the horizon [25]. The existence of a stable IR fixed point in the emergent $\mathrm{AdS}_2$ region demands

$$B = B_c \equiv -m^2_{\alpha\mathrm{eff}}|_{\alpha=\alpha_c}, \qquad m^2_{\beta\mathrm{eff}}|_{\alpha=\alpha_c} = 0 \tag{10}$$

at the horizon $r = r_h = 1$. Then we can see that in the case of $T = 0$, when $|B| < |B_c|$, $\beta$ is unstable near the horizon, and the condensation happens so that the staggered magnetization is no longer vanishing. When $|B| > |B_c|$, however, $\beta$ is stable at the horizon, and the staggered magnetization is zero. Therefore, a QPT occurs at $|B| = |B_c|$, and the system is quantum disorder when $|B| > |B_c|$.

To investigate the magnetic fluctuations in the vicinity of QCP, we need to consider the perturbations of two polarization fields. To make the system be self-consistent at the linear order, the perturbations for all components of the polarization fields have to be considered,

$$\delta M^{(a)}_{\mu\nu} = \epsilon C^{(a)}_{\mu\nu} e^{-i(\omega t + qx)}, \qquad (\mu,\nu) \neq (r,y),(t,x)$$
$$\delta M^{(a)}_{\mu\nu} = i\epsilon C^{(a)}_{\mu\nu} e^{-i(\omega t + qx)}, \qquad (\mu,\nu) = (r,y),(t,x). \tag{11}$$

Putting these perturbations into the equation of motions and compute to the first order for $\epsilon$, we can get their equations of the perturbations (the details for perturbational equations can be found in Ref. [25]). In general, because of the nonlinear potential, all the components of two polarization fields couple with each other. Let $\tilde{\beta} = (C^{(1)}_{xy} - C^{(2)}_{xy})/2$. In the paramagnetic magnetic phase ($T > T_N$ or $B > B_c$), when $q, \omega \to 0$, the equations for $\tilde{\beta}$ decouple from others [25]. By imposing the ingoing condition at the horizon, it has the following asymptotic solution in the UV region,

$$\tilde{\beta} \simeq \tilde{\beta}_+ r^{(1+\delta_2)/2} + \tilde{\beta}_- r^{(1-\delta_2)/2}. \tag{12}$$

According to the dictionary of AdS/CFT, up to a positive constant, the retarded Green's function for $\tilde{\beta}$ reads

$$G_{\beta\beta} = \tilde{\beta}_-/\tilde{\beta}_+. \tag{13}$$

Using the retarded Green's function, we can define the spectrum function as $P(\omega,\vec{q}) = \mathrm{Im}G(\omega,\vec{q})/\pi$. When we turn on a small momentum $\vec{q}$, the energy of a long-life quasiparticle, which corresponds to the peak of $P(\omega,\vec{q})$, can be given by following dispersion relation:

$$\omega_* = \Delta + \epsilon_{\vec{q}}, \qquad \epsilon_{\vec{q}=0} = 0. \tag{14}$$

Here $\Delta$ is the energy gap of quasiparticle excitation. In the vicinity of QCP, for the case of $\omega = 0$, the retarded Green's function usually has the form of $G \sim 1/(q^2 + 1/\xi^2)$, where $\xi$ is called correlation length. At QCP, in general, the energy gap vanishes. Thus, we have $\omega_* = \epsilon_{\vec{q}}$. In addition, for a small frequency and wave vector, we can define the dynamic exponent $z$ as $\omega_* \sim q^z$.

## IV. NUMERICAL RESULTS

As the equations involved here are nonlinear, we have to solve them numerically. The different parameters which satisfy the restrictions (8) give similar results, so we here just take $m^2 = 1$, $k = 7/8$ as typical examples in Figs. 1–3. In Figs. 2 and 3 and the left plot of Fig. 1, we take $J = -0.67$. In the right plot of Fig. 1, we take $J = -0.71$.

We can see from Fig. 1 that the Néel temperature $T_N$ is suppressed by external magnetic field. There is a critical magnetic field for given parameters, at which $T_N$ is zero and QPT occurs. After being rescaled, different parameters give similar behaviors with some slight differences. The physical picture of this QPT can be understood as follows. When magnetic field reaches its critical value, there magnetic spins become partially aligned along the direction of the magnetic field. Therefore, the system requires less thermal energy to destroy the remaining magnetic spins order.

The holographic model can give some interesting scaling relations near the QCP. For small $B$, numerical results show that $T_N - T_{N0} \propto B^2$, where $T_{N0}$ denotes the critical temperature in the case without an external magnetic field. When the magnetic field is close to $B_c$, we find that Néel temperature is fitted well by the following relation:

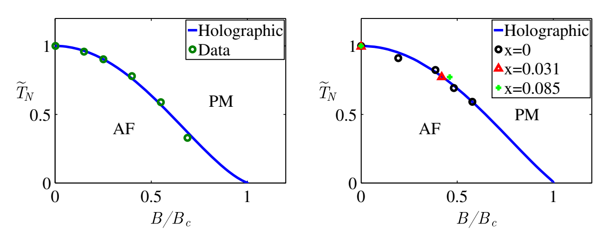


FIG. 1 (color online). The antiferromagnetic critical temperature $T_N$ vs the external magnetic field $B$ compared with experimental data. The critical temperature $T_N$ is calculated with the aid of the solution of Eqs. (9). At each fixed value of magnetic field $B$, the temperature is determined at which the AF condensate starts to appear. *Left*: Comparison with experimental data from $\mathrm{BiCoPO_5}$. *Right*: Comparison with experimental data for pyrochlore compounds: $\mathrm{Er_{2-2x}Y_{2x}Ti_2O_7}$. The experimental data are from Refs. [26,27] and rescaled.

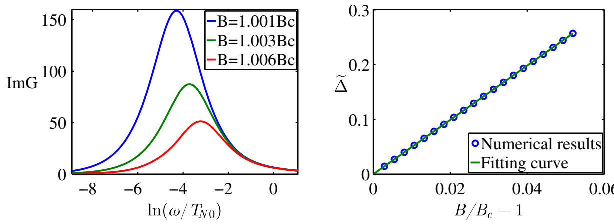


FIG. 2 (color online). *Left:* The antiferromagnetic spectrum function in the case with a different magnetic field when $B > B_c$. *Right*: The gap energy vs the external magnetic field when $B/B_c - 1 \to 0^+$.

$$\tilde{T}_N / \ln \tilde{T}_N \propto (1 - B/B_c), \quad (15)$$

where $\tilde{T}_N = T_N/T_{N0}$. We will analytically present the relation (15) by considering the emergent geometry $AdS_2$ in the IR limit [25].

When magnetic field $B$ is larger than the critical value $B_c$, the antiferromagnetic phase disappears even at zero temperature. In this case, the system comes into the quantum disordered phase at zero temperature, in which there is a gapped magnetic excitation. In the left plot of Fig. 2, we show Im$G$ with respect to the frequency of the antiferromagnetic excitation in the case with a different magnetic field. In the case of $0 < B/B_c - 1 \ll 1$, there is a distinct peak which gives the energy gap for the excitation. With an increasing magnetic field, the peak moves toward higher energy and becomes more and more indistinct. This means that the gap increases but the lifetime decreases when the magnetic field increases. At the critical magnetic field $B = B_c$, we see $\omega_* = 0$, which corresponds to a gapless long-lifetime antiferromagnetic excitation. In the region of $B/B_c - 1 \to 0^+$, we find the energy gap is fitted well by the following equation (see the right plot of Fig. 2):

$$\tilde{\Delta} \propto (B/B_c - 1), \quad \text{with} \quad \tilde{\Delta} = \Delta/T_{N0}. \quad (16)$$

In the left plot of Fig. 3, we plot the inverse Green's function $G^{-1}_{\beta\beta}(q)$ in the case of $\omega = 0$ and $|1 - B/B_c| = 0.01$. We can see that it obeys the behavior of $G^{-1} \sim q^2 + 1/\xi^2$ as we expected before. Thus, the Green's function can give the correlation length by fitting

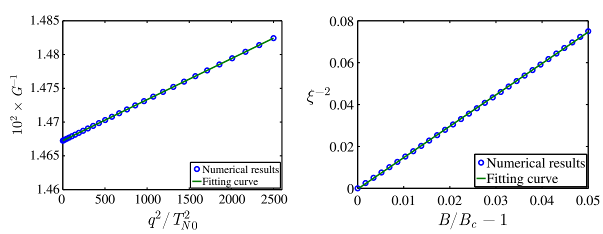


FIG. 3 (color online). *Left:* $G^{-1}$ as a function of $q^2$ when $\omega = 0$ in the case of $|1 - B/B_c| = 0.01$. The solid line is the fitting curve by $G^{-1} \propto q^2 + 1/\xi^2$. *Right*: The correlation length $\xi$ vs the magnetic field when $|B/B_c - 1| \to 0$.

the curve of $G^{-1}$ as a function of $q^2$, which is shown in the right plot of Fig. 3. We see that the correlation length $\xi$ as a function of the tuning parameter $B$ obeys the following relation:

$$\xi \propto |B/B_c - 1|^{-\nu}, \quad \text{with} \quad \nu \simeq 1/2. \quad (17)$$

As for the dynamical exponent $z$, in antiferromagnetic metals, $z = 2$ [5]. In our holographic model, the dynamical exponent can be calculated by using a similar numeric method. The results indeed show that $z \simeq 2$. This numerical result can be confirmed by the emergent $AdS_2$ geometry in the IR region [25] and agrees with the result from a different holographic model proposed in Ref. [28]. Furthermore, from the energy gap (16), we see that this energy gap satisfies the universal scaling relation $\Delta \sim |B - B_c|^{z\nu}$, which further indicates $z = 2$ in this model.

## V. DISCUSSION

The relation we found in this paper of the Néel temperature with respect to the magnetic field in the vicinity of QCP is quite nontrivial. Note that the relation (15) is not the usual power-law behavior or square-root form, but it is an expected result in the 2D QPTs in the strong coupling case [5]. This nontrivial coincidence strongly implies a connection between these two different theories. In the antiferromagnetic metal, magnetic ordered is dominated by itinerant electrons and dynamical exponent $z$ is 2 near the QCP. Since our dual boundary theory is a 2D theory, the effective dimension is thus $d_{\text{eff}} = d + z = 4$, which is just the upper critical dimension of the Hertz field theory [2,29]. In this case, the hyperscaling is violated, and logarithmic correction behavior appears. In fact, the $d = z = 2$ quantum critical theory is in general not a weak coupling theory at any $T > 0$. Instead, a strongly coupled effective classical model emerges that can be used to determine the critical dynamics [30]. Our results show that it can be described well by AdS/CFT correspondence, and this provides a new example of the applicability of the gravity/gauge duality in condensed matter theory.

It is quite interesting to apply this holographic model to realistic materials. Since the holographic model is independent of the microscopic details of the materials and their interactions, it should be suitable for a class of materials. Two potential AF-QPT materials are $BiCoPO_5$ with critical magnetic field $B_c \simeq 15.3$ T (which is obtained by fitting a power-law relation [26]) and $Er_{2-2x}Y_{2x}Ti_2O_7$ with $B_c \simeq 1.5$ T for $x = 0$ [27,31]. In Fig. 1 we present the experimental data for these two materials and the holographic results, where we choose the model parameters so that they can give out the best fitting. The holographic model gives $B_c \simeq 16.2$ and 1.45 T. While the experiment data show that the energy gap for $Er_{2-2x}Y_{2x}Ti_2O_7$ when $B \geq B_c$ obeys the linear relationship (16) with slope 4.2 (see Fig. 8 in Ref. [27]), our holographic model gives the slope 5.0 with the chosen model parameters. These two slopes are in the same order. Note that these two kinds of materials have different microscopic structures and complex interactions; it is remarkable that the simple holographic model can give a self-consistent description for those two materials. In addition, it is also worth mentioning here that the doping at the magnetic site ($x$ up to 0.085) has a very little influence on the critical behavior of $Er_{2-2x}Y_{2x}Ti_2O_7$. This indicates that an emergent universal behavior appears in these materials from very different microscopic details and could be described by the holographic model.

As the critical behavior of a QPT induced by a magnetic field, the three scaling relations (15), (16), and (17) near the critical point are our main results from the holographic model. Besides the energy gap (16), our predications on the scaling relations (15) and (17) can also be confirmed by experiments. Unfortunately at the moment they cannot be checked by the existing experiments because the experiment data are absent when the Néel temperature is very close to zero. Of course whether the holographic model is suitable for these two materials needs more evidence. It is also very interesting to find more materials satisfying the conditions of this model and to check our predictions. We expect that this model can be confirmed or falsified experimentally soon.

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