

Lawrence Berkeley National Laboratory

Lawrence Berkeley National Laboratory

Title

Holographic Protection of Chronology in Universes of the Godel Type

Permalink

<https://escholarship.org/uc/item/3g38g267>

Author

Boyda, Edward

Publication Date

2009-10-06

Holographic Protection of Chronology in Universes Gödel Type

Edward K. Boyda, Surya Ganguli, Petr Horava and Uday Varadarajan

*Berkeley Center for Theoretical Physics and Department of Physics
University of California, Berkeley, CA 94720-7300*

and

*Theoretical Physics Group, Lawrence Berkeley National Laboratory
Berkeley, CA 94720-8162, USA*

This work was supported in part by the Director, Office of Science, Office of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor The Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or The Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or The Regents of the University of California.

Holographic Protection of Chronology in Universes of the Gödel Type

Edward K. Boyda, Surya Ganguli, Petr Hořava and Uday Varadarajan

*Berkeley Center for Theoretical Physics and Department of Physics
University of California, Berkeley, CA 94720-7300*

and

*Theoretical Physics Group, Lawrence Berkeley National Laboratory
Berkeley, CA 94720-8162, USA*

ekboyda, sganguli, horava, udayv@socrates.berkeley.edu

We analyze the structure of supersymmetric Gödel-like cosmological solutions of string theory. Just as the original four-dimensional Gödel universe, these solutions represent rotating, topologically trivial cosmologies with a homogeneous metric and closed timelike curves. First we focus on “phenomenological” aspects of holography, and identify the preferred holographic screens associated with inertial comoving observers in Gödel universes. We find that holography can serve as a chronology protection agency: The closed timelike curves are either hidden behind the holographic screen, or broken by it into causal pieces. In fact, holography in Gödel universes has many features in common with de Sitter space, suggesting that Gödel universes could represent a supersymmetric laboratory for addressing the conceptual puzzles of de Sitter holography. Then we initiate the investigation of “microscopic” aspects of holography of Gödel universes in string theory. We show that Gödel universes are T-dual to pp-waves, and use this fact to generate new Gödel-like solutions of string and M-theory by T-dualizing known supersymmetric pp-wave solutions.

December 2002

1. Introduction

Many long-standing conceptual questions of quantum gravity, and even of classical general relativity, are finding their answers in string theory. Among the most notable examples are various classes of supersymmetric timelike singularities, or the microscopic explanation of Bekenstein-Hawking entropy for a class of configurations controllable by space-time supersymmetry. On the other hand, many puzzles of quantum gravity still remain unanswered. In particular, the role of time in cosmological, and other time-dependent, solutions of string theory still defies any systematic understanding.

While many crucial questions of quantum gravity are associated with high space-time curvature or with cosmological horizons, some puzzles become apparent already in spacetimes with very mild curvature, no horizons, and even trivial topology. How can the low-energy classical relativity fail to represent a good approximation to quantum gravity for small curvature and in the absence of horizons? Arguments leading to the holographic principle [1] indicate that general relativity misrepresents the true degrees of freedom of quantum gravity, by obscuring the fact that they are secretly holographic. In those instances where string theory has been successful in resolving puzzles of quantum gravity, it has done so by identifying the correct microscopic degrees of freedom, which frequently are poorly reflected by the naive (super)gravity approximation. In this paper we investigate an example in which holography suggests a very specific dramatic modification of the degrees of freedom in quantum gravity already at very mild curvatures, in a homogeneous and highly supersymmetric cosmological background.

Historically, microscopic holography in string theory has been relatively easier to understand for solutions with a “canonical” preferred holographic screen which is observer-independent, and typically located at asymptotic infinity. Holography in *AdS* spaces is a prime example of this. On the other hand, cosmological backgrounds in string theory require an understanding of holography in more complicated environments, which may not exhibit canonical, observer-independent preferred screens at conformal infinity. Here, the prime example is given by de Sitter space: When viewed from the perspective of an inertial observer living in the static patch, the preferred holographic screen in de Sitter space is most naturally placed at the cosmological horizon. This leads to the fascinating idea of observer-dependent holographic screens, associated with a finite number of degrees of freedom accessible to the observer (for more details, see e.g. [2-6]; see also [7,8] for a complementary point of view on de Sitter holography that uses other preferred screens, not associated with an inertial observer).

Of course, string theory promises to be a unified theory of gravity and quantum mechanics, but it is at present unclear how it manages to reconcile the general relativistic concept of time (notoriously difficult because of spacetime diffeomorphism invariance) with the quantum mechanical role of time as an evolutionary Hamiltonian parameter. Again,

this problem becomes somewhat trivialized in the presence of supersymmetry, but persists in all but the most trivial time-dependent backgrounds of string theory.

In this paper, we analyze a class of supersymmetric solutions of string theory and M-theory, which – at least in the classical supergravity approximation – are described by geometries with no global time function. In particular, we focus our attention on string theory analogs of Gödel’s universe. Gödel’s original solution [9] is a homogeneous rotating cosmological solution of Einstein’s equations with pressureless matter and negative cosmological constant, which played an important role in the conceptual development of general relativity. Recently, a supersymmetric generalization of Gödel’s universe has been discussed in a remarkable paper by Gauntlett *et al.* [10], who classified all supersymmetric solutions of five-dimensional supergravity with eight supercharges, and found a maximally supersymmetric Gödel-like solution that can be lifted to a solution of M-theory with twenty Killing spinors. The existence of this solution was also noticed previously by Tseytlin, see Footnote 26 of [11]. It is worth stressing that the Gödel universe of M-theory is time-orientable: There is an invariant notion of future and past lightcones, at each point in spacetime. Also, there is a global time *coordinate* t , and in fact $\partial/\partial t$ is an everywhere time-like Killing vector (in effect, making supersymmetry possible). However, t is not a global time *function*: The surfaces of constant t are not everywhere spacelike.¹ Actually, the solution cannot be foliated by everywhere-spacelike surfaces at all – the classical Cauchy problem is always ill-defined in this spacetime. It is hard to imagine how such an apparently pathological behavior of global time could be compatible with the conventional role of time in the Hamiltonian picture of quantum mechanics. Indeed, this solution turns out to have classical pathologies: Just as Gödel’s original solution, the supersymmetric Gödel metric allows closed timelike curves, seemingly suggesting either the possibility of time travel (cf. [13]) or at least grave causality problems.

These classical pathologies could imply that the Gödel solution, despite its high degree of supersymmetry, stays inconsistent even in full string or M-theory. There are of course pathological solutions of Einstein’s equations whose problems do not get resolved in string theory, with the negative-mass Schwarzschild black hole being one example.

However, there are reasons why one might feel reluctant to discard this solution as manifestly unphysical, despite the sicknesses of the classical metric: This solution is homogeneous, its curvature can be kept small everywhere (in particular, there are no singularities and no horizons), and the solution is highly supersymmetric. It is also impossible to eliminate the closed timelike curves by going to a universal cover – indeed, the Gödel solution is already topologically trivial.²

¹ See, e.g., [12] for a detailed discussion of the distinction between a global time coordinate and a global time function.

² This should be contrasted with the case of solutions with “trivial” (in the sense of Carter

We feel that any solution should be presumed consistent until proven otherwise, and this will be our attitude towards the Gödel solution in this paper. Our aim will be to analyze holographic properties of the supersymmetric Gödel solution in string theory. The solution is remarkably simple, and as we will see in Section 5, turns out to be related by duality to the solvable supersymmetric pp-wave backgrounds much studied recently. However, before we attempt the analysis of “microscopic” holography in string theory, we will first adopt a more “phenomenological” approach as pioneered by Bousso [15] (see [3,16] for reviews), and analyze the structure of preferred holographic screens implied by the covariant prescription [15] for their identification in classical (super)gravity solutions. This “phenomenological” analysis leads to valuable hints, indicating how the problem of closed timelike curves may be resolved in the Gödel universe. Indeed, we will claim that the apparent pathologies of the semiclassical supergravity solution can be resolved when holography is properly taken into account. Semiclassical general relativity without holography is not a good approximation of this solution, despite its small curvature, absence of horizons, and trivial spacetime topology.

Notice also that homogeneity of the Gödel solution makes things at least superficially worse: It implies that there are closed timelike curves through every point in spacetime. However, these closed timelike curves are also in a sense (to be explained below) topologically “large.” Our analysis of the structure of holographic screens in this geometry reveals an intricate system of observer-dependent preferred holographic screens, which always carve out a causal part of spacetime, and effectively screen all the closed timelike curves and hide any violations of causality from the inertial observer. In fact, the causal structure of the part of spacetime carved out by the screen is precisely that of an AdS space, cut off at some finite radial distance.

The preferred holographic screens in the Gödel universe are very much like the screens associated with the inertial observers in the static patch of de Sitter space. First of all, they are associated with the selection of an observer (and therefore represent “movable,” non-canonical screens, not located at conformal infinity). Moreover, they are compact, implying a finite covariant bound on entropy and – in the strong version of the holographic principle – a finite number of degrees of freedom associated with any inertial observer. Thus, the Gödel universe should serve as a useful supersymmetric laboratory for addressing some of the conceptual puzzling issues of de Sitter holography.

The results of our “phenomenological” analysis of holography also reveal the importance, for cosmological spacetimes, of a local description of physics as associated with an observer inside the universe. It is not sensible to pretend that the observer stays at asymp-

[14]) closed timelike curves, such as those in the flat Minkowski spacetime with time compactified on \mathbf{S}^1 , where the closed timelike curves can be eliminated by lifting the solution to its universal cover.

otic infinity, and observes only elements of the traditionally defined S-matrix (or some suitable analogs thereof). Clearly, this only stresses the need for a conceptual framework defining more environmentally-friendly, “cosmological” observables as associated with cosmological observers in string theory.

The structure of the paper is as follows. In Section 2, we set the stage by reviewing and analyzing Gödel’s cosmological solution $\mathcal{G}_3 \times \mathbf{R}$ of Einstein’s gravity in four space-time dimensions. Despite its simplicity, this solution already exhibits all the crucial issues of our argument. We apply Bousso’s prescription for the covariant holographic screens, and find screens that are observer-dependent, compact, and causality-preserving. In addition, we establish connection with holography in *AdS* spaces: Gödel’s solution can be viewed as a member of a two-parameter moduli space of homogeneous solutions of Einstein’s equations with trivial spacetime topology, with $AdS_3 \times \mathbf{R}$ also in this moduli space. We show that under the corresponding deformation the observer-dependent preferred holographic screens of Gödel’s universe recede to infinity and become the canonical holographic boundary of $AdS_3 \times \mathbf{R}$. In Section 3 we move on to the supersymmetric Gödel universe of M-theory, which can be written as $\mathcal{G}_5 \times \mathbf{R}^6$. First we analyze the \mathcal{G}_5 part of the geometry as a solution of minimal $d = 5$ supergravity, study in detail the structure of geodesics in this solution and use it to determine the preferred holographic screens, and show how chronology can be protected by holography. Then we extend our analysis to the full $\mathcal{G}_5 \times \mathbf{R}^6$ Gödel geometry in M-theory. Section 4 points out remarkable analogies between holography in the supersymmetric Gödel universe and holography in de Sitter space. In Section 5, we embark on the analysis of “microscopic” duality of Gödel universes in string theory. First, we compactify the M-theory solution on \mathbf{S}^1 to obtain a Gödel solution of Type IIA superstring theory, and show that upon further \mathbf{S}^1 compactification the Type IIA Gödel universe is T-dual to a supersymmetric Type IIB pp-wave, which can be obtained as the Penrose limit of the intersecting D3-D3 system. We point out that this Gödel/pp-wave T-duality is a much more general phenomenon, and can be used to construct new Gödel universes in string and M-theory by T-dualizing known pp-waves. The relation to pp-waves is just one aspect of the remarkable degree of solvability of Gödel solutions in string theory. We intend to present a more detailed analysis of “microscopic” aspects of holography in the Gödel universes of string and M-theory elsewhere [17]. In Appendix A we summarize some geometric properties of the supersymmetric Gödel solutions.

2. Holography in Gödel’s Four-Dimensional Universe

2.1. Gödel’s solution

In 1949, on the occasion of Albert Einstein’s 70th birthday, Kurt Gödel presented a rotating cosmological solution [9] of Einstein’s equations with negative cosmological

constant and pressureless matter; this solution is topologically trivial and homogeneous but exhibits closed timelike curves. Our exposition of Gödel's solution follows [9,18].

The spacetime manifold of this solution has the trivial topology of \mathbf{R}^4 , which we will cover by a global coordinate system (τ, x, y, z) . The metric factorizes into a direct sum of the (trivial) metric dz^2 on \mathbf{R} and a nontrivial metric on \mathbf{R}^3 ,

$$ds_4^2 = ds_3^2 + dz^2, \quad (2.1)$$

where

$$ds_3^2 = -d\tau^2 + dx^2 - \frac{1}{2}e^{4\Omega x} dy^2 - 2e^{2\Omega x} d\tau dy. \quad (2.2)$$

This class of solutions is characterized by a rotation parameter Ω . We will refer to the manifold \mathbf{R}^3 equipped with the non-trivial part (2.2) of Gödel's metric as \mathcal{G}_3 . Thus, in our notation, Gödel's universe is $\mathcal{G}_3 \times \mathbf{R}$. The metric on \mathcal{G}_3 has a four-dimensional group of isometries. The geometry exhibits dragging of inertial frames, associated with rotation. The full four-dimensional geometry solves Einstein's equations, with the value of the cosmological constant and the density of pressureless matter both determined by the rotation parameter Ω ,

$$\rho = \frac{\Omega^2}{2\pi G_N}, \quad \Lambda = -2\Omega^2. \quad (2.3)$$

Historically, this solution was instrumental in the discussion of whether or not classical general relativity satisfies Mach's principle (see, e.g., [19], Sect. 12.4).

While the homogeneity of Gödel's universe is (almost) manifest in the coordinate system used in (2.2), the rotational symmetry of ds_3^2 around any point in space becomes more obvious in cylindrical coordinates (t, r, ϕ) , in which the metric takes the following form,

$$ds_3^2 = -dt^2 + dr^2 - \frac{1}{\Omega^2} (\sinh^4(\Omega r) - \sinh^2(\Omega r)) d\phi^2 - \frac{2\sqrt{2}}{\Omega} \sinh^2(\Omega r) dt d\phi. \quad (2.4)$$

Indeed, $\partial/\partial\phi$ is a Killing vector, of norm squared

$$\left| \frac{\partial}{\partial\phi} \right|^2 = \frac{1}{\Omega^2} (1 - \sinh^2(\Omega r)) \sinh^2(\Omega r). \quad (2.5)$$

The orbits of this Killing vector are closed, and become closed timelike curves for $r > r_0$,

$$r_0 = \frac{1}{\Omega} \operatorname{arcsinh}(1) \equiv \frac{1}{\Omega} \ln(1 + \sqrt{2}). \quad (2.6)$$

We will call the surface of $r = r_0$ the *velocity-of-light surface*; the null geodesics emitted from the origin in this coordinate system reach the velocity-of-light surface in a finite

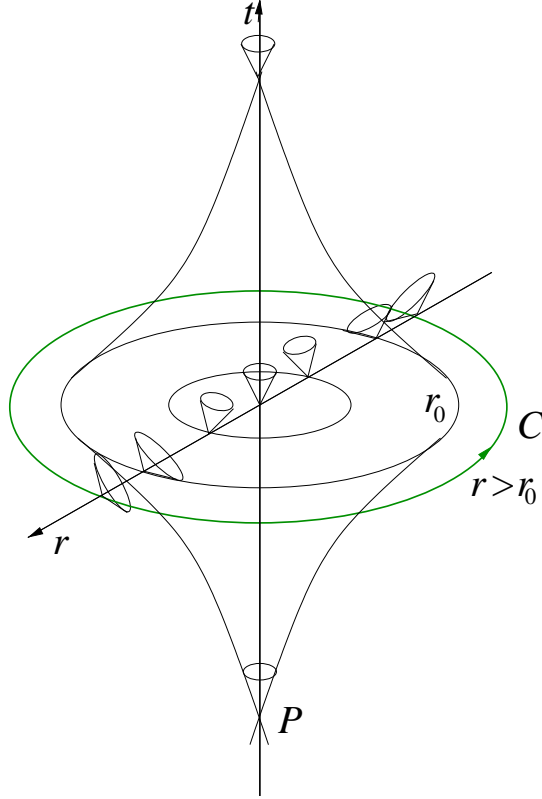


Fig. 1: The geometry of the three-dimensional part \mathcal{G}_3 of Gödel's universe, with the flat fourth dimension z suppressed. Null geodesics emitted from the origin P follow a spiral trajectory, reach the velocity-of-light surface at the critical distance r_0 , and spiral back to the origin in finite affine parameter. The curve C of constant $r > r_0$ tangent to $\partial/\partial\phi$ is an example of a closed timelike curve. A more detailed version of this picture appears in Hawking and Ellis [18].

affine parameter, and then spiral back to the origin where they refocus, again in finite affine parameter.

The homogeneity of the solution implies that there are closed timelike curves through every point in spacetime. Note that in a well-defined sense all the closed timelike curves are topologically “large”: In order to complete a closed timelike trajectory starting at any point P , one has to travel outside of the velocity-of-light surface (as defined by an observer at P) before being able to return to P along a causal curve. This fact will play an important role in our argument for the holographic resolution of the problem of closed timelike curves below. Notice also that none of the closed timelike curves is a geodesic, and that the closed timelike curves cannot be trivially eliminated by a lift to the universal cover: The manifold is already topologically trivial.

Gödel's universe represents a solution with a good timelike Killing vector (indeed, $\partial/\partial t$ is Killing and everywhere timelike), which however cannot be used to define a universal

time function: The slices of the foliation by surfaces of constant t are not everywhere spacelike. The classical Cauchy problem is always globally ill-defined for this geometry.

2.2. Preferred holographic screens in Gödel's universe

We now apply Bousso's phenomenological framework for holography [15,3,16] to Gödel's universe. We identify its preferred holographic screens, associated with particular observers as follows:

Consider a geodesic observer comoving with the distribution of dust in Gödel's universe (and placed at the origin $r = 0$ of our coordinate system without loss of generality). Imagine that the observer sends out lightrays in all directions from the origin at some fixed time, say $t = 0$. These lightrays will at first expand – i.e., the surfaces that they reach in some fixed affine parameter λ will grow in area, at least for small enough values of λ . The preferred holographic screen will be reached when we reach the surface of maximal area (or maximal geodesic expansion).

Alternatively, one can follow *incoming* lightrays into their past, until reaching the surface where the geodesics no longer expand. This is again the location of the preferred screen \mathcal{B} . The preferred screen \mathcal{B} can then be used to impose a covariant bound on the entropy inside the region of space surrounded by \mathcal{B} [15], which should not exceed one-fourth of the area of \mathcal{B} in Planck units.

We will first analyze the three-dimensional part \mathcal{G}_3 of Gödel's solution, which contains much of the nontrivial geometry. Even though all the geodesics of Gödel's universe are known [20], one can in fact use the symmetries of \mathcal{G}_3 to determine the location of the screen without any explicit knowledge of the geodesic curves. Since \mathcal{G}_3 is rotationally invariant in ϕ , all the null geodesics emitted from the origin will reach the same radial distance $r(\lambda)$ within the same affine parameter (assuming that we use a rotationally invariant normalization of λ for geodesics emitted in different directions from the origin), and also for the same global time coordinate t . Thus, to determine the surface of maximal geodesic expansion, we can just evaluate the area of the surfaces of constant r and t (in our case of course one-dimensional),

$$A = \frac{2\pi}{\Omega} \sinh(\Omega r) \sqrt{1 - \sinh^2(\Omega r)}, \quad (2.7)$$

and maximize it as a function of r . This very simple calculation yields a preferred screen \mathcal{H} that is isomorphic to a cylinder of constant $r = r_{\mathcal{H}}$ and any t , with

$$r_{\mathcal{H}} = \frac{1}{\Omega} \operatorname{arcsinh} \left(\frac{1}{\sqrt{2}} \right). \quad (2.8)$$

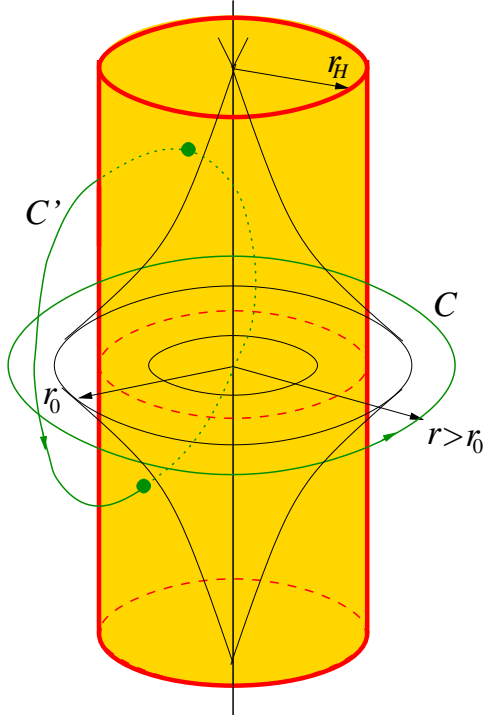


Fig. 2: The geometry of our preferred holographic screen in Gödel's universe, as defined by the inertial observer following the comoving geodesic at the origin of spatial coordinates. The translationally invariant dimension z is again suppressed. Two closed timelike curves are indicated: One, C , at constant value of $t = 0$ and $r > r_0$ is outside of the preferred screen, while another, C' , passes through the origin at $t = 0$ and intersects the screen in two, causally connected points.

Of course, this screen is observer-dependent, in this case associated with the comoving inertial observer located at the origin for all values of t . Other comoving inertial observers would see different but isomorphic screens, in a pattern similar to the structure of cosmological horizons associated with inertial observers in de Sitter space.

One can take advantage of the rotational symmetry of the solution, and visualize the location of the preferred screen using a spacetime diagram of the type introduced by Bousso [15] (see Figure 3). This diagram suppresses the dimension of rotational symmetry ϕ , and its points represent (in our case one-dimensional) orbits of the rotation group, i.e., surfaces of constant r and t . For each such surface, one can define the total of four lightsheets: Two oriented forward in time, and two oriented backward. In generic points of the diagram, two of these lightsheets will be non-expanding. At each point of the Bousso diagram one can draw a wedge pointing in the direction of non-expanding lightsheets. These wedges then point in the direction of the preferred holographic screen.

One can directly verify that our preferred holographic screen satisfies the defining

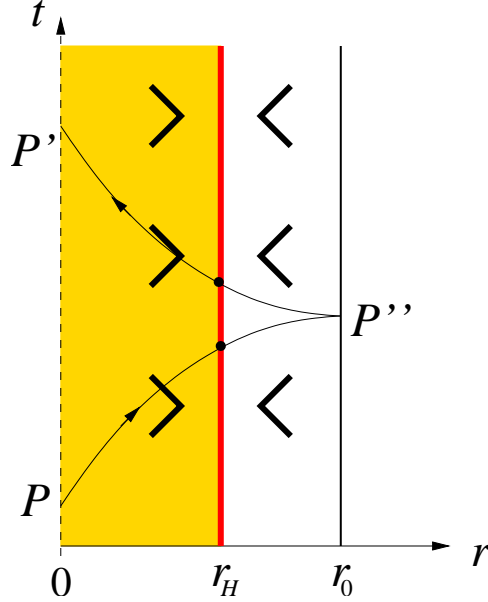


Fig. 3: The Bousso diagram for the \mathcal{G}_3 part of the Gödel universe metric, with the angular coordinate ϕ suppressed, and the structure of non-expanding lightsheets indicated by the bold wedges. The preferred holographic screen is at the finite value $r_{\mathcal{H}}$ of the radial coordinate r , strictly smaller than the location of the velocity-of-light surface at r_0 . A null geodesic sent from P would reach the velocity-of-light surface at P'' in a finite affine parameter, and refocus again at the spatial origin in P' .

property

$$\theta = 0, \quad (2.9)$$

where θ is the expansion parameter defined for a spacelike codimension-two surface B (in any spacetime with coordinates x^μ) as

$$\theta = h^{\mu\nu} D_\mu \zeta_\nu, \quad (2.10)$$

with ζ_μ the light-like covector orthogonal to B (smoothly but arbitrarily extended to some neighborhood of B), D_μ is the covariant derivative, and $h_{\mu\nu}$ is the induced metric on B . The most convenient way of identifying the surface of $\theta = 0$ in Gödel's universe is to use as ζ the vector tangent to the congruence of null geodesics emitted by the observer at the origin. An explicit calculation confirms in this case that θ is proportional to $\partial_r g_{\phi\phi}$, and therefore vanishes at the surface of $r = r_{\mathcal{H}}$.

The metric induced on the preferred holographic screen \mathcal{H} is of signature $(-+)$, everywhere nonsingular:

$$ds_{\mathcal{H}}^2 = -dt^2 + \frac{1}{4\Omega^2} d\phi^2 + \frac{\sqrt{2}}{\Omega} d\phi dt \quad (2.11)$$

with $0 \leq \phi \leq 2\pi$. The preferred holographic screen carves out a cylindrical compact region of spacetime (which we will call the *holographic region*) in the \mathcal{G}_3 part of Gödel's universe,

centered on the comoving inertial observer at the origin. This region contains no closed timelike curves, as can be easily demonstrated by noticing that the causal structure of the holographic region is identical to that of a cylindrical portion of (the universal cover of) AdS_3 . The closed timelike curves of the full \mathcal{G}_3 geometry fall into two categories: Either they stay completely outside of the holographic region, or they enter it and leave it again after traveling a causal trajectory within the holographic region.

Preferred screens in $\mathcal{G}_3 \times \mathbf{R}$

The full Gödel universe is of the direct product form $\mathcal{G}_3 \times \mathbf{R}$. The presence of the extra, translationally-invariant dimension parametrized by z actually implies a richer structure of preferred screens than the one we just found in the \mathcal{G}_3 factor. This is in fact a preview of what we will find in the next section in the case of supersymmetric Gödel solutions in M-theory and string theory: Those solutions typically also contain extra flat dimensions.

First of all, there is one preferred screen that can be easily identified: The three-dimensional surface $\mathcal{H} \times \mathbf{R}$, where \mathcal{H} is the preferred screen associated with the observer at the spatial origin in \mathcal{G}_3 , and \mathbf{R} is the extra coordinate z , clearly satisfies the zero-expansion condition (2.9). Thus, by definition, this surface $\mathcal{H} \times \mathbf{R}$ is a preferred screen. This screen is observer-dependent, and the observer associated with it can be thought of either as a string wrapped around z or as a more traditional observer “delocalized” along z , each localized at the origin of coordinates in \mathcal{G}_3 . Unless we compactify z on \mathbf{S}^1 , the overall area of this translationally-invariant screen is of course infinite, but the screen still has a finite “area density” per unit distance along z .

Alternatively, one can ask what is the preferred screen associated with an localized inertial observer in $\mathcal{G}_3 \times \mathbf{R}$. If one follows null geodesics emitted from (or converging onto) a point in $\mathcal{G}_3 \times \mathbf{R}$ where the the observer is located, one finds that the surface of maximal geodesic expansion is at a finite distance from the observer in all space directions including z . This compact, translationally-noninvariant screen is completely contained within the velocity-of-light surface as defined by the observer.

For either of these two classes of screens in $\mathcal{G}_3 \times \mathbf{R}$, all closed timelike curves are again either hidden outside of the screen or broken by it into causal observable pieces.

Covariant entropy bounds and screen complementarity

The existence of preferred screens, and the structure of the Bousso diagram for Gödel’s universe imply a holographic entropy bound on the amount of entropy through any spatial slice of the compact holographic region associated with each screen. This entropy is limited by one fourth of the area of the screen measured in Planck units. Our screen is neither at conformal infinity, nor located at a horizon. The closest analog would be the preferred holographic screen located at the equator of the Einstein static universe. Just as in that

case, the holographic screen of Gödel’s universe can be used to bound the entropy in either direction normal to the screen. In particular, the lightrays that start at the screen and travel in the direction of larger values of r refocus at the velocity-of-light surface, and then travel back again to the screen. This is rather reminiscent of the behavior of lightrays in Einstein’s static universe: lightrays emitted from one pole of the spatial sphere reach the screen at the equator and travel to the other hemisphere, refocus at the opposite pole, and travel back to the screen and then to the point they were originally emitted from.

The strong version of the holographic principle suggests that the compact holographic screen implies a finite bound on the number of degrees of freedom effectively accessible to the inertial observer. The good causal structure of the holographic region associated with that observer may suggest that the quantum mechanics of this finite number of degrees of freedom could be well-defined, and screened from the acausal behavior outside of the velocity-of-light surface by a screen complementarity principle.

Of course, one may find the very definition of entropy in spacetimes with closed timelike curves somewhat problematic. However, in the case of Gödel’s universe all that matters for our argument is the region strictly below the velocity-of-light surface. One can in principle imagine cutting Gödel’s solution off at some finite r larger than $r_{\mathcal{H}}$ but smaller than r_0 , and replacing the outside with some causal geometry. The covariant entropy bound can then be safely applied to the holographic region, without any possible conceptual difficulties with the definition of entropy in the presence of closed timelike curves.

The intricate structure of compact preferred screens associated with the observers in Gödel’s universe suggests that holography may be the correct, causal way of thinking about this geometry without modifying it. However, one is forced to replace the naive “metaobserver” perspective of the geometry by a system of local observers, each of which sees a causal region screened from the rest of the naive classical geometry by the preferred holographic screen. Each individual observer would only have access to a finite amount of degrees of freedom associated with the corresponding holographic region. Within this finite number of degrees of freedom, causality and quantum mechanics would be protected.

In this paper we will not discuss non-inertial observers attempting to travel along closed timelike curves. In the spirit of Hawking’s original chronology protection conjecture [21], one may expect a large backreaction from the geometry that can protect the solution from such observers.

2.3. Gödel’s universe as deformed AdS_3 and holography

It is useful to embed our discussion of Gödel’s universe into a broader framework. Consider all spacetime-homogeneous metrics of the Gödel type. It has been shown [22] that this family of metrics is parametrized by two parameters, Ω and m , with the metric

given by

$$ds^2 = - \left(dt + \frac{4\sqrt{2}\Omega}{m^2} \sinh^2 \left(\frac{mr}{2} \right) d\phi \right)^2 + \frac{1}{m^2} \sinh^2(mr) d\phi^2 + dr^2 + dz^2, \quad (2.12)$$

with $\Omega \in \mathbf{R}$ and $m^2 \in \mathbf{R}$. For $m^2 = 4\Omega^2$, we recover Gödel's metric (2.2). On the other hand, for $m^2 = 8\Omega^2$ we get the direct-product metric on $AdS_3 \times \mathbf{R}$ [23]. Notice also that the metric simplifies in the limit of $m \rightarrow 0$ keeping Ω fixed; this metric has been analyzed by Som and Raychaudhuri [24], and is in fact a closer analog of the string theory Gödel universe than Gödel's solution itself.

Since all the solutions in (2.12) are rotationally invariant, we can easily identify the preferred screens for this entire family of metrics using the same symmetry argument as in Gödel's universe itself. The holographic screens \mathcal{H} of the non-trivial three-dimensional part of (2.12) are now located at

$$r_{\mathcal{H}} = \frac{2}{m} \operatorname{arcsinh} \left(\left(\frac{16\Omega^2}{m^2} - 2 \right)^{-1/2} \right). \quad (2.13)$$

Thus, for $m^2 < 8\Omega^2$, the screen is at a finite value of $r_{\mathcal{H}}$, and as we approach the $AdS_3 \times \mathbf{R}$ limit it recedes to infinity and becomes the canonical holographic screen of AdS_3 . This connection with AdS_3 leads to a particularly intriguing way of thinking about holography of this family of solutions in terms of breaking conformal invariance on the holographic screen of AdS_3 once we move away from the AdS_3 limit.

Clearly, our observation that preferred holographic screens can either screen closed timelike curves or break them up into causal pieces is not restricted to homogeneous space-times. An example of the same phenomenon in an inhomogeneous solution can be easily found: Consider the classic cylindrically symmetric inhomogeneous solution with closed timelike curves found in 1937 by van Stockum, [25], which in the cylindrical coordinates takes the form

$$ds^2 = -dt^2 - 2\Omega r^2 d\phi dt + r^2(1 - \Omega^2 r^2) d\phi^2 + e^{-\Omega^2 r^2} (dz^2 + dr^2). \quad (2.14)$$

It is straightforward to show that the preferred holographic screen as defined by the inertial observer located at the origin is again compact and shields the closed timelike curves from the observer, just as in the case of the homogeneous Gödel universe.

3. Holography in the Supersymmetric Gödel Universe

The Gödel solution of M-theory found in [10] has a direct product form $\mathcal{G}_5 \times \mathbf{R}^6$, where the non-trivial five-dimensional part \mathcal{G}_5 represents a maximally supersymmetric solution of

minimal supergravity in five dimensions. The underlying spacetime of \mathcal{G}_5 is topologically trivial, isomorphic to \mathbf{R}^5 . Again, just as in the case of Gödel's four-dimensional solution, much of the nontrivial structure of the solution is carried in this five-dimensional factor \mathcal{G}_5 , which plays a role analogous to \mathcal{G}_3 of the previous section. We will therefore study holography of this five-dimensional solution first.

3.1. Holography in the Gödel universe of $N = 1$ $d = 5$ supergravity

The five-dimensional Gödel geometry \mathcal{G}_5 is a maximally supersymmetric, topologically trivial, homogeneous solution of minimal five-dimensional supergravity [10]. We introduce generic coordinates X^μ , $\mu = 0, \dots, 4$ on \mathbf{R}^5 , but we will soon specialize to several specific coordinate systems. The minimal $d = 5$ supergravity contains an Abelian gauge field A_μ whose field strength $F_{\mu\nu}$ we normalize such that the Lagrangian has the following form,

$$\mathcal{L}_5 = \frac{1}{2\kappa_5^2} \int d^5 X \left(R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \dots \right), \quad (3.1)$$

where the “...” stand for a Chern-Simons self-interaction of the gauge field and for fermionic terms.

The Gödel solution takes the form of a fibration over the flat Euclidean \mathbf{R}^4 with fibers isomorphic to \mathbf{R} and with a simple twist, which in a Cartesian coordinate system t, x_i , $i = 1, \dots, 4$, can be written as

$$ds^2 = -(dt + \beta\omega)^2 + \sum_{i=1}^4 dx_i^2, \quad (3.2)$$

$$F = 2\sqrt{3}\beta J,$$

with the twist one-form ω given by

$$\omega = x_1 dx_2 - x_2 dx_1 + x_3 dx_4 - x_4 dx_3 \equiv J_{ij} x_i dx_j, \quad (3.3)$$

and $J_{12} = -J_{21} = J_{34} = -J_{43} = 1$ a preferred Kähler form on \mathbf{R}^4 . In (3.2), β is a constant rotation parameter, of mass dimension one. Without any substantial loss of generality, we will assume β to be positive.

As remarked in [10], this solution is homogeneous, and in fact has a nine-dimensional group of bosonic isometries. The Killing vectors are given by

$$\begin{aligned} P_0 &= \partial_t, \\ P_i &= \partial_i - \beta J_{ij} x_j \partial_t, \\ L &= x_1 \partial_2 - x_2 \partial_1 + x_3 \partial_4 - x_4 \partial_3, \\ R_1 &= x_1 \partial_2 - x_2 \partial_1 - x_3 \partial_4 + x_4 \partial_3, \\ R_2 &= x_1 \partial_3 - x_3 \partial_1 + x_2 \partial_4 - x_4 \partial_2, \\ R_3 &= x_1 \partial_4 - x_4 \partial_1 + x_3 \partial_2 - x_2 \partial_3, \end{aligned} \quad (3.4)$$

where $\partial_i = \partial/\partial x_i$. The commutation relations of this bosonic symmetry algebra are

$$\begin{aligned} [R_\alpha, R_\beta] &= 2\epsilon_{\alpha\beta\gamma} R_\gamma, & [L, R_\alpha] &= 0, \\ [P_i, P_j] &= 2\beta J_{ij} P_0. \end{aligned} \tag{3.5}$$

Here $\alpha, \beta, \dots = 1, 2, 3$ go over a basis of anti-selfdual two-tensors in \mathbf{R}^4 . R_α and L act on the momenta P_i as rotations. Thus, we find that the symmetry algebra of the Gödel universe \mathcal{G}_5 is given by the semidirect product $H(2) \ltimes (SU(2) \times U(1))$, where $H(2)$ is the Heisenberg algebra with five generators.³

While the translation symmetries P_i of the solution are almost manifest in the cartesian coordinates t, x_i , the rotation symmetries are rather obscure. It is therefore convenient to introduce a new coordinate system. First, we introduce a pair of polar coordinates, one in each of the two main planes of rotation,

$$\begin{aligned} x_1 &= r_1 \cos \phi_1, & x_3 &= r_2 \cos \phi_2, \\ x_2 &= r_1 \sin \phi_1, & x_4 &= r_2 \sin \phi_2. \end{aligned} \tag{3.6}$$

In these ‘‘bipolar’’ coordinates, the metric becomes

$$\begin{aligned} ds^2 &= -dt^2 - 2\beta(r_1^2 d\phi_1 + r_2^2 d\phi_2)dt + dr_1^2 + dr_2^2 - 2\beta^2 r_1^2 r_2^2 d\phi_1 d\phi_2 \\ &\quad + r_1^2(1 - \beta^2 r_1^2) d\phi_1^2 + r_2^2(1 - \beta^2 r_2^2) d\phi_2^2. \end{aligned} \tag{3.7}$$

The non-Abelian part of the rotation symmetries becomes manifest in spherical coordinates $(r, \phi_1, \phi_2, \vartheta)$, with $\vartheta \in [0, \pi/2)$,

$$\begin{aligned} x_1 + ix_2 &= r e^{i\phi_1} \cos \vartheta, \\ x_3 + ix_4 &= r e^{i\phi_2} \sin \vartheta, \end{aligned} \tag{3.8}$$

which bring the metric to the following form,

$$ds^2 = - \left(dt + \frac{\beta r^2}{2} \sigma_3 \right)^2 + dr^2 + r^2 d\Omega_3^2. \tag{3.9}$$

³ As we will see in Section 4, the remarkable similarity between this symmetry algebra and a pp-wave symmetry algebra is not a coincidence: When lifted to string theory, the Gödel solution is actually T-dual to a supersymmetric pp-wave! Notice, however, that in the symmetry algebra of \mathcal{G}_5 , the central extension generator P_0 of the Heisenberg algebra is represented by a timelike Killing vector, while in the pp-wave it would be null. One can actually show by a direct calculation that the five-dimensional Gödel universe (or the string theory lifts thereof to be studied below) does not admit any covariantly constant null vectors, which proves that it is not ‘‘secretly’’ a pp-wave in unusual coordinates.

Here $d\Omega_3^2$ is the standard unit-volume metric on \mathbf{S}^3 , and σ_3 is one of the right-invariant one-forms on $SU(2)$,

$$\sigma_3 = 2(\cos^2 \vartheta d\phi_1 + \sin^2 \vartheta d\phi_2). \quad (3.10)$$

It is clear from this expression for the metric that even though the solution does not exhibit the full $SO(4) \sim SU(2) \times SU(2)$ rotation symmetry in \mathbf{R}^4 , the non-zero rotation parameter β keeps the right $SU(2)$ (together with a $U(1)$ subgroup of the left $SU(2)$) unbroken.

It was also noted in [10] that the Gödel universe \mathcal{G}_5 preserves all eight supersymmetries of minimal $d = 5$ supergravity. Thus, the bosonic symmetry algebra (3.5) will extend to a superalgebra with eight supercharges Q . It is natural to split Q into two four-component spinors, Q^\pm . In this notation, the (anti)commutation relations of the full symmetry superalgebra can be written as follows,

$$\begin{aligned} [P_0, Q^\pm] &= 0, & [R, Q^\pm] &= \Gamma_R Q^\pm, \\ [P_i, Q^+] &= 0, & [P_i, Q^-] &= \beta J_{ij} \Gamma^j Q^+, \\ \{\bar{Q}^+, Q^+\} &= \Gamma^0 P_0, & \{\bar{Q}^-, Q^+\} &= \Gamma^i P_i, \\ \{\bar{Q}^-, Q^-\} &= \Gamma^0 (P_0 + 2\beta L), \end{aligned} \quad (3.11)$$

together with (3.5). In (3.11), R denotes any of the rotation generators R_α or L , and Γ_R is a shorthand for the generator of conventional rotations associated with $R \in SO(4)$, in the corresponding spinor representation of $SO(4)$.

Once we examine the structure of preferred holographic screens in the next subsection, it will be interesting to see how these screens are compatible with the structure of the supersymmetry algebra (3.5), (3.11).

3.2. Preferred holographic screens

Consider an inertial, comoving observer located at an arbitrary point in space, which we place without any loss of generality at the origin of cartesian coordinates $x_i = 0$. Since we are focusing on the perspective of an observer at the origin, it will be convenient to use either the ‘‘bipolar’’ or the spherical coordinates.

The symmetry arguments that allowed us to identify the preferred screen in Gödel’s universe \mathcal{G}_3 without actually calculating the geodesics can in fact be extended to the supersymmetric solution \mathcal{G}_5 as well. Despite the fact that the full $SO(4)$ rotation symmetry of \mathbf{R}^4 is broken to an $SU(2) \times U(1)$ subgroup, the unbroken group still acts transitively on the three-spheres of constant r . Indeed, one can think of the \mathbf{S}^3 at constant r as a copy of $SU(2)$, on which the full $SO(4)$ rotations would act by the left action of one $SU(2)$ and the right action of the other $SU(2)$. In the Gödel solution, the metric on the \mathbf{S}^3 of constant radius is that of a squashed three-sphere, which still leaves the (transitive) right action by $SU(2)$ unbroken. This unbroken $SU(2)$ is sufficient to reduce our analysis of the location

of preferred screens to the maximization of the area of the surfaces \mathbf{S}^3 of constant r as a function of r (at constant t), precisely as in the simpler case of \mathcal{G}_3 studied in the previous section. Without knowing the precise structure of the null geodesics emitted at some time $t < 0$ in all directions from the origin, the symmetries imply that these geodesics will reach the \mathbf{S}^3 of some fixed radius r at $t = 0$.

Thus, in order to find the preferred holographic screens associated with the inertial comoving observer at the origin, we only need to maximize the volume of the \mathbf{S}^3 at fixed r , as a function of r . The induced metric on the \mathbf{S}^3 of radius r at constant t is given by

$$ds_{\text{ind}}^2 = r^2 d\vartheta^2 + r^2 \cos^2 \vartheta (1 - \beta^2 r^2 \cos^2 \vartheta) d\phi_1^2 + r^2 \sin^2 \vartheta (1 - \beta^2 r^2 \sin^2 \vartheta) d\phi_2^2 - 2\beta^2 r^4 \cos^2 \vartheta \sin^2 \vartheta d\phi_1 d\phi_2, \quad (3.12)$$

implying that the induced area of this surface is given by

$$\mathcal{A}(r) = \int_{\mathbf{S}^3} \sqrt{h_{\text{ind}}} = 2\pi^2 r^3 \sqrt{1 - \beta^2 r^2}, \quad (3.13)$$

where h_{ind} is the determinant of the induced metric (3.12). We conclude that the preferred holographic screen is located at radial distance r (call it $r_{\mathcal{H}}$) where the area (3.13) is maximized,

$$r_{\mathcal{H}} = \frac{\sqrt{3}}{2\beta}. \quad (3.14)$$

The screen carries a Lorentz-signature induced metric,

$$ds_{\mathcal{H}}^2 = -dt^2 - \frac{3}{2\beta} (\cos^2 \vartheta d\phi_1 + \sin^2 \vartheta d\phi_2) dt + \frac{3}{4\beta^2} \left[d\vartheta^2 + \cos^2 \vartheta d\phi_1^2 + \sin^2 \vartheta d\phi_2^2 - \frac{3}{4} (\cos^2 \vartheta d\phi_1 + \sin^2 \vartheta d\phi_2)^2 \right], \quad (3.15)$$

with each spacelike slice of constant t isomorphic to the squashed three-sphere of radius $r_{\mathcal{H}}$ and squashing parameter $3/4$. The screen metric (3.15) seems to exhibit dragging of frames, but this is an artifact of a coordinate choice. Upon introducing new angular coordinates by $\bar{\phi}_1 = \phi_1 - 4\beta t$, $\bar{\phi}_2 = \phi_2 - 4\beta t$, (3.15) becomes

$$ds_{\mathcal{H}}^2 = -4dt^2 + \frac{3}{4\beta^2} \left[d\vartheta^2 + \cos^2 \vartheta d\bar{\phi}_1^2 + \sin^2 \vartheta d\bar{\phi}_2^2 - \frac{3}{4} (\cos^2 \vartheta d\bar{\phi}_1 + \sin^2 \vartheta d\bar{\phi}_2)^2 \right]. \quad (3.16)$$

This phenomenon is analogous to the behavior of horizons in rotating black holes in five dimensions [26].

The screen and its location in the Gödel universe can be visualized exactly as in Fig. 2, with ϕ now collectively denoting the coordinates on the squashed three-sphere. Again, the

preferred screen cuts out a compact region of space with the observer inside, which we will refer to as the holographic region.

The compact preferred holographic screen also implies a finite bound on the entropy that flows through a space-like section of the holographic region. This entropy has to be smaller than one fourth of the area of the screen in Planck units,

$$S \leq \frac{2\pi^3 r_{\mathcal{H}}^3}{\kappa_5^2}. \quad (3.17)$$

(Notice that our κ_5 is related to the 5d Newton constant by $8\pi G_N = \kappa_5^2$.)

It is interesting to analyze the symmetries preserved by the screen. While all the rotation symmetries $SU(2) \times U(1)$ as well as the time translation symmetry are left unbroken, all the space translations are broken by the screen. Similarly, the structure of the supersymmetry algebra reveals that one half of the supercharges (namely Q^-) will be broken by the screen, while the remaining half of supersymmetry represented by Q^+ (and associated with Killing spinors which are simply constant) is compatible with the presence of the screen. Thus, the screen can preserve as much as 1/2 of the full supersymmetry of the Gödel solution, leaving an unbroken symmetry which coincides with the symmetry left unbroken by the choice of the inertial comoving observer. Once we lift the solution to M-theory, we can also think of the preferred comoving observer as a massless particle moving with the speed of light along the extra dimension and preserving 1/2 of supersymmetry. Thus, the symmetries of the observer seem compatible with the symmetries that can be left unbroken by her preferred holographic screen.

In order to verify that this simplified argument for identifying the preferred screens, which relies on the large symmetry of the solution, coincides with the conventional local definition of the screen [15] as the surface of vanishing expansion parameter $\theta = 0$ of the null geodesics emitted from (or, by the time reflection symmetry, sent towards) the origin in space, we must first analyze the structure of geodesic motion in the Gödel spacetime. This analysis will also refine our understanding of the Gödel universe geometry.

3.3. Geodesics in the Gödel universe \mathcal{G}_5

In this subsection we will find all the geodesics in the Gödel universe.

First, one can use the symmetries of the solution to simplify the analysis. By homogeneity, it will be sufficient to consider geodesics through the origin P of our coordinate system, $P \equiv \{t = x_m = 0\}$. In any case, for the identification of the preferred screens we are primarily interested in null geodesics emitted from the origin.⁴

⁴ Moreover, since the $SU(2)$ part of the symmetry group acts transitively on the celestial sphere at P , one could rotate the initial momentum vector along the geodesic to lay entirely in

We will write the tangent vector to the geodesic as

$$\xi = \dot{t} \frac{\partial}{\partial t} + \dot{r}_1 \frac{\partial}{\partial r_1} + \dot{\phi}_1 \frac{\partial}{\partial \phi_1} + \dot{r}_2 \frac{\partial}{\partial r_2} + \dot{\phi}_2 \frac{\partial}{\partial \phi_2}, \quad (3.18)$$

where $\dot{} \equiv d/d\lambda$ denotes the derivative with respect to an affine parameter λ along the geodesic.

The large amount of symmetry of the Gödel universe allows us to explicitly solve for all the geodesics without any restrictions. First of all, the following integrals of motion will be useful,

$$\begin{aligned} (\xi, \xi) &= -M^2, & (\xi, \partial_t) &= -E, \\ (\xi, \partial_{\phi_1}) &= L_1, & (\xi, \partial_{\phi_2}) &= L_2. \end{aligned} \quad (3.19)$$

Here L_1, L_2 are the angular momenta in the two preferred planes of rotation. The \pm sign of M^2 corresponds to timelike and spacelike geodesics, with E the energy of the particle in the timelike case. In the null case $M^2 = 0$ we will find it convenient to rescale the affine parameter λ along the geodesic so as to set $E = 1$.

The integrals of motion (3.19) imply

$$\begin{aligned} \dot{\phi}_1 &= \beta E + \frac{L_1}{r_1^2}, & \dot{\phi}_2 &= \beta E + \frac{L_2}{r_2^2}, \\ \dot{t} &= (1 - \beta^2 r_1^2 - \beta^2 r_2^2) E - \beta(L_1 + L_2), \end{aligned} \quad (3.20)$$

as well as

$$(\dot{r}_1)^2 + (\dot{r}_2)^2 - (1 - \beta^2 r_1^2 - \beta^2 r_2^2) E^2 + 2\beta E(L_1 + L_2) + \frac{L_1^2}{r_1^2} + \frac{L_2^2}{r_2^2} = -M^2. \quad (3.21)$$

In order to identify the holographic screen we need the null geodesics going through the origin. Note that for non-zero values of the angular momenta L_1 or L_2 , the effective potential for r_1 and r_2 precludes the geodesics from reaching the origin $r_1 = r_2 = 0$. Thus, all the geodesics passing through the origin will have $L_1 = L_2 = 0$, and we focus on those now.⁵ In order to separate \dot{r}_1 from \dot{r}_2 we need one more integral of motion. Consider

$$\begin{aligned} (\xi, R_3) &\equiv (\sin \phi_1 \sin \phi_2 + \cos \phi_1 \cos \phi_2) \left(\frac{r_2}{r_1} L_1 + \frac{r_1}{r_2} L_2 \right) \\ &+ (\sin \phi_1 \cos \phi_2 - \cos \phi_1 \sin \phi_2) (r_2 \dot{r}_1 - r_1 \dot{r}_2). \end{aligned} \quad (3.22)$$

the $x_3 = x_4 = 0$ plane. By angular momentum conservation, corresponding to the two Killing vectors $\partial/\partial\phi_1$ and $\partial/\partial\phi_2$, the geodesic would then stay in the $x_3 = x_4 = 0$ plane throughout its history.

⁵ Of course, all the geodesics with nonzero angular momenta can be easily obtained from those with zero angular momenta by the action of the large isometry group of the Gödel metric.

At zero angular momentum, (3.22) has to vanish, implying that the angle ϑ between r_1 and r_2 is another integral of motion. Thus, the equations of motion for the geodesics that pass through the origin of space simplify to

$$(\dot{r})^2 + \beta^2 r^2 E^2 = E^2 - M^2, \quad (3.23)$$

plus (3.20) with L_i set to zero. These can be easily solved, yielding

$$\begin{aligned} r_1 &= \frac{1}{\beta} \sqrt{1 - M^2} \sin(\beta\lambda) \cos \vartheta, \\ r_2 &= \frac{1}{\beta} \sqrt{1 - M^2} \sin(\beta\lambda) \sin \vartheta, \\ t &= \frac{1}{2}(1 + M^2)\lambda + \frac{1}{4\beta}(1 - M^2) \sin(2\beta\lambda) + t_0, \\ \phi_1 &= \beta\lambda + \phi_1^{(0)}, \\ \phi_2 &= \beta\lambda + \phi_2^{(0)}. \end{aligned} \quad (3.24)$$

with $\vartheta \in [0, \pi/2)$ and $\phi_1^{(0)}, \phi_2^{(0)} \in [0, 2\pi)$ all constants. We have rescaled the affine parameter λ so as to set E equal to one. For null geodesics, $M^2 = 0$, while for the timelike geodesics $M^2 \in [0, 1]$ as a result of our rescaling of λ . Notice that the comoving time t at the origin (the coordinate corresponding to the Killing vector ∂_t) is *not* a good affine parameter along the null geodesics passing through the origin. Instead, either one of the two main rotation angles ϕ_1, ϕ_2 plays the role of a natural affine parameter (as long as β is nonzero of course).

Even though the spherical coordinate system is not smooth at the origin, it is easy to verify – by switching to the original Cartesian coordinate system – that the system of null geodesics (3.24) represents the complete system of all geodesics passing through the origin. Indeed, the tangent vector to this congruence at $\lambda = 0$ is given in the Cartesian coordinates by

$$\begin{aligned} \xi|_{\lambda=0} &= \frac{\partial}{\partial t} + \cos \vartheta \cos \phi_1^{(0)} \frac{\partial}{\partial x_1} + \cos \vartheta \sin \phi_1^{(0)} \frac{\partial}{\partial x_2} \\ &\quad + \sin \vartheta \cos \phi_2^{(0)} \frac{\partial}{\partial x_3} + \sin \vartheta \sin \phi_2^{(0)} \frac{\partial}{\partial x_4}, \end{aligned} \quad (3.25)$$

demonstrating that the constants $\vartheta, \phi_1^{(0)}$ and $\phi_2^{(0)}$ are indeed parametrizing the entire celestial sphere at the origin.

Thus, we see an interesting refocusing behavior of all geodesics in the Gödel universe:

They start moving from the origin towards larger values of r , which at first means larger proper-radius spheres, but then at affine parameter

$$\lambda = \frac{\pi}{2\beta} \quad (3.26)$$

they reach the velocity-of-light surface, located at the largest value r_0 of the radial coordinate r that is accessible by geodesic motion from the origin,

$$r_0 = \frac{1}{\beta}. \quad (3.27)$$

By that time, both ϕ_1 and ϕ_2 change exactly by $\pi/2$. Then it takes another

$$\Delta\lambda = \frac{\pi}{2\beta} \quad (3.28)$$

to complete one period of oscillation and refocus at the origin. The amount of global comoving time coordinate elapsed during the completion of one oscillation cycle equals

$$\Delta t = \frac{\pi}{2\beta}. \quad (3.29)$$

Note that the lightray arrives with its momentum equal to the initial-value momentum; thus, the lightray traveled a full circle in the (x_1, x_2) plane. The same holds true for the (x_3, x_4) plane.

During one refocusing cycle, the proper area of the three-sphere reached by the geodesics reaches a maximum twice, precisely when they reach the preferred screen – first on their way out towards the velocity-of-light surface (where the proper area of the \mathbf{S}^3 goes to zero) and then again on their way back to the origin. In fact, they reach the holographic screen for the first time at affine parameter

$$\lambda = \frac{\pi}{3\beta}, \quad (3.30)$$

one third into the oscillation cycle.

Since any given geodesic moves around a circle in each of the preferred planes of rotation, it is instructive to use the translation symmetries of the solution, and transform (3.24) into the frame associated with the observer at the center of this circular motion. The Killing vectors (3.4) can be easily integrated to give finite translations. For example, we find that a finite translation by a along x_2 is accompanied by an x_1 -dependent translation in t ,

$$x'_2 = x_2 + a, \quad x'_1 = x_1, \quad t' = t + \beta x_1 a. \quad (3.31)$$

When one transforms (3.24) to the primed coordinates associated with the center of the circular motion of a geodesic, the x_1 -dependent time translation (3.31) eliminates the $\sin(2\beta\lambda)$ term in the expression for t as a function of the affine parameter in (3.24). Thus, t becomes a good affine parameter precisely for the class of geodesics that circle around the origin at fixed constant r .

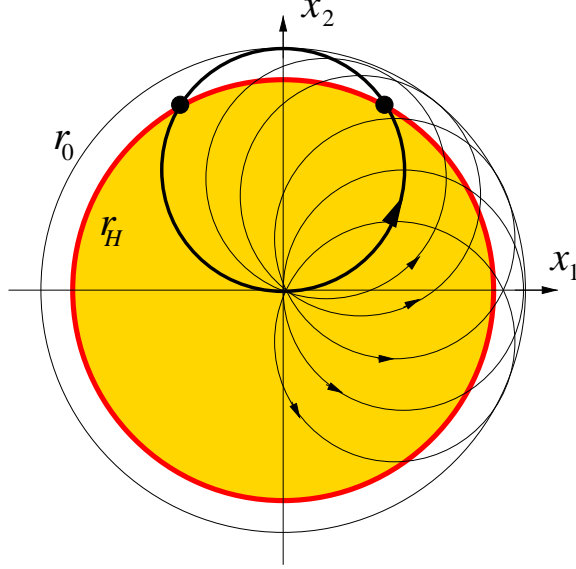


Fig. 4: The behavior of null geodesics emitted from an arbitrary point P in the Gödel universe, with the initial momentum in the (x_1, x_2) plane, and with several such geodesics indicated. Each geodesic travels along a circular trajectory, reaches the velocity-of-light surface and returns back to P , penetrating the preferred screen exactly twice during each rotation cycle.

So far, we were mainly concentrating on null geodesics emanating from the origin. The analysis is easily extended to timelike geodesics, which turn out to exhibit a similar cyclic behavior. However, they only reach up to a certain critical distance r_M strictly smaller than the distance r_0 of the velocity-of-light surface,

$$r_M = \sqrt{1 - M^2} r_0. \quad (3.32)$$

In terms of the global comoving time coordinate t , the timelike geodesics sent from the origin take longer to refocus at the origin than null geodesics, the refocusing time being

$$\Delta t(M) = \frac{(1 + M^2)\pi}{2\beta}. \quad (3.33)$$

The geodesic expansion θ

We are now in a position to verify that the holographic screen is indeed located at $r_{\mathcal{H}} = \sqrt{3}/2\beta$ by a direct analysis of the geodesics in the Gödel metric. Recall that according to Bousso's prescription [15], the screen is determined as the surface \mathcal{B} where the geodesic expansion θ vanishes, leading to the “equation of motion” for the preferred holographic screen,

$$\theta = 0, \quad (3.34)$$

with $\theta \equiv h^{\mu\nu} D_\mu \xi_\nu$ defined as the contraction of the covariant derivative $D_\mu \xi_\nu$ of the null covector ξ_μ with respect to the induced metric $h_{\mu\nu}$ on \mathcal{B} .

The null geodesics (3.24) define a congruence whose associated tangent vector is

$$\xi = (1 - \beta^2 r^2) \frac{\partial}{\partial t} + \beta \left(\frac{\partial}{\partial \phi_1} + \frac{\partial}{\partial \phi_2} \right) + \sqrt{1 - \beta^2 r^2} \frac{\partial}{\partial r}. \quad (3.35)$$

Its covector dual (which we denote by the same letter ξ) has a rather simple form,

$$\xi \equiv \xi_\mu dX^\mu = -dt + \sqrt{1 - \beta^2 r^2} dr. \quad (3.36)$$

We can now evaluate the covariant derivative $D_\mu \xi_\nu$ and contract it against the induced metric $h^{\mu\nu}$, to obtain the geodesic expansion θ . After some straightforward algebra,

$$\theta = \frac{3 - 4\beta^2 r^2}{r\sqrt{1 - \beta^2 r^2}}. \quad (3.37)$$

Thus, θ vanishes precisely at $r = r_{\mathcal{H}} \equiv \sqrt{3}/2\beta$, in accord with our anticipation in (3.14). Notice also that θ diverges at the origin and at the velocity-of-light surface, confirming that those are indeed caustics of the geodesic motion.

3.4. The Gödel universe of M-theory

The lift of the five-dimensional Gödel universe \mathcal{G}_5 to M-theory involves adding six flat dimensions \mathbf{R}^6 , which we parametrize by coordinates y_a , $a = 1, \dots, 6$. Together, t, x_i and y_a form a coordinate system X^M on \mathbf{R}^{11} , with $M = 0, \dots, 10$. The action of eleven-dimensional supergravity has the form

$$\mathcal{L}_{11} = \frac{1}{2\kappa^2} \int d^{11}X \left(R - \frac{1}{48} G_{MNPQ} G^{MNPQ} + \dots \right), \quad (3.38)$$

where “...” stand for the Chern-Simons term plus fermionic terms. The eleven-dimensional Gödel solution is then given by

$$ds^2 = -(dt + \beta\omega)^2 + \sum_{i=1}^4 dx_i^2 + \sum_{a=1}^6 dy_a^2, \quad (3.39)$$

$$G_{ijab} = 2\beta J_{ij} K_{ab},$$

with all the other non-zero components of G_{MNPQ} related to (3.39) by permutations of indices, and the Kähler form K on the \mathbf{R}^6 factor defined by $K_{12} = -K_{21} = K_{34} = -K_{43} = K_{56} = -K_{65} = 1$.

Consider again the congruence of all null geodesics emitted from the origin in space, where our comoving observer is located. The longitudinal momenta K^a along y_a are conserved, leading to the following congruence of null geodesics:

$$\begin{aligned}
r_1 &= \frac{1}{\beta} \sqrt{1 - K^2} \sin(\beta\lambda) \cos \vartheta, \\
r_2 &= \frac{1}{\beta} \sqrt{1 - K^2} \sin(\beta\lambda) \sin \vartheta, \\
t &= \frac{1}{2}(1 + K^2)\lambda + \frac{1}{4\beta}(1 - K^2) \sin(2\beta\lambda) + t_0, \\
\phi_1 &= \beta\lambda + \phi_1^{(0)}, \\
\phi_2 &= \beta\lambda + \phi_2^{(0)}, \\
y^a &= K^a \lambda.
\end{aligned} \tag{3.40}$$

Just as in the case of four-dimensional Gödel's solution $\mathcal{G}_3 \times \mathbf{R}$ discussed in the previous section, one can use geodesics in the supersymmetric Gödel solution $\mathcal{G}_5 \times \mathbf{R}^6$ of M-theory to define several different classes of preferred screens. First of all, there is a preferred screen which is a direct product of \mathbf{R}^6 and the screen that we found at $r = r_{\mathcal{H}}$ in \mathcal{G}_5 . This screen is translationally invariant along all the extra dimensions y_a , and clearly satisfies the $\theta = 0$ condition trivially. It is observer-dependent, and should be associated with an observer localized at a point in \mathcal{G}_5 but otherwise delocalized along \mathbf{R}^6 , or with the maximal expansion of lightrays sent with zero momentum K^a from the origin in \mathcal{G}_5 and arbitrary y_a .

In addition, observers localized in a point P both in \mathcal{G}_5 and in \mathbf{R}^6 will naturally see a compact screen in all directions. The precise location of this compact screen can be found by considering the full congruence (3.40) of geodesics emitted from P . One can in principle calculate the expansion parameter θ and find the preferred compact screen as the surface of maximal expansion.

Using the affine parameter λ and the total momentum $K^2 \equiv K_a K^a$ along \mathbf{R}^6 as coordinates, the shape of the screen is determined from the $\theta = 0$ condition by a rather complicated implicit function of λ and K^2 ,

$$\begin{aligned}
0 &= \frac{1}{2\lambda} \sin^{-1}(\beta\lambda) \left[(1 - K^2)\beta\lambda \cos(\beta\lambda) + K^2 \sin(\beta\lambda) \right]^{-1} \\
&\quad \times \left[5K^2 + 2(1 - K^2)\beta^2\lambda^2 + (-5K^2 + 4(1 - K^2)\beta^2\lambda^2) \cos(2\beta\lambda) \right. \\
&\quad \left. + 2(3 - K^2)\beta\lambda \sin(2\beta\lambda) \right].
\end{aligned} \tag{3.41}$$

This screen is compact in all space dimensions, and exhibits $SO(6) \times SU(2) \times U(1)$ rotation invariance, with $SO(6)$ acting on y_a and $SU(2) \times U(1)$ on x_i .

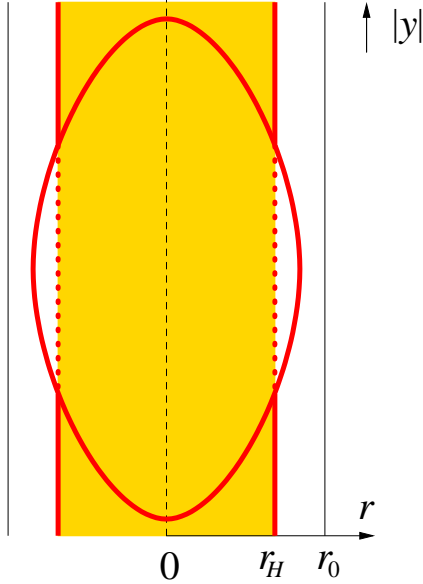


Fig. 5: The two types of preferred screens in the M-theory Gödel $\mathcal{G}_5 \times \mathbf{R}^6$. The translationally-invariant screen is located at $r_{\mathcal{H}}$ in \mathcal{G}_5 for all values of $|y|$, and can be associated with an extended observer delocalized or wrapped along y_a . The screen associated with a localized observer is compact in all space directions, and extends beyond $r_{\mathcal{H}}$, closer to the velocity-of-light surface r_0 .

There are several interesting points about this compact screen. First of all, along $y_a = 0$ this screen extends in the r directions beyond the location $r_{\mathcal{H}}$ of the translationally invariant screen. This is in fact intuitively clear: once we add the flat dimensions y_a , the tendency of the geodesics to expand in the y_a dimensions competes against the refocusing behavior of the geodesics in the r direction of \mathcal{G}_5 , effectively slowing down the process of reaching the surface of maximal area, which now happens for a slightly larger value of r . Notice also that the entire compact screen still fits nicely within the velocity-of-light surface as defined by our observer. Therefore, closed timelike curves are again shielded from the observer by this screen.

4. Analogies with Holography in de Sitter Space

Holography in de Sitter space is difficult due to the absence of a solvable model or an explicit embedding of de Sitter into string theory. As we have seen in the previous sections, holography in the Gödel universes exhibits notable analogies with holography in de Sitter space.

There are two important classes of preferred holographic screens in de Sitter [15]: First, the future and past infinity are global, observer-independent screens of Euclidean signature. An attempt to formulate holography using these screens [7] has led to the

conjectured dS/CFT correspondence [8]. However, it is difficult to associate these global screens with an observer inside de Sitter: Distinct points at future infinity in de Sitter are outside of each other's causal influence, and any operational definition of measurable correlations seems to require a metaobserver.

The second class of screens is more suitable for the description of physics as seen by an observer inside de Sitter [2,3,6]: The preferred screen of a given observer is located at his or her cosmological horizon. Since the area of this observer-dependent screen is finite, the strong version of the holographic principle implies a finite number of degrees of freedom in the quantum mechanics associated with that observer. The finiteness of the number of degrees of freedom accessible to any given observer leads to various conceptual puzzles, such as the recently discussed question of time recurrences [27]. Observers following different trajectories have access to different holographic regions, perhaps suggesting a quantum mechanical description of de Sitter space as a web of infinitely many Hilbert spaces (each associated with an observer and grasping a finite number of degrees of freedom) with a complicated system of maps between them (reflecting the exchange of data between causally connected observers, and the horizon complementarity principle).

Given the conceptual complexity of de Sitter holography, it would be very helpful to have an explicit simple solvable model exhibiting similar properties. We believe that the supersymmetric Gödel universes may provide such a model. Indeed, preferred screens appearing in Gödel holography share many properties with the second type of preferred screens in de Sitter space:

- Both represent an example of homogeneous geometries with screens that are only defined when an observer has been selected. Observers following different worldlines will see different holographic screens.
- The underlying spacetime geometry is homogeneous, but this homogeneity is broken by the selection of the observer, and consequently by the location of the observer-dependent holographic screen, implying that the screen breaks spontaneously some of the symmetries of the naive vacuum. This picture of observer-dependent holography stresses the importance of a local, environmentally-friendly definition of cosmological observables.
- The finite proper area of the holographic screen implies a finite bound on the entropy that flows through the compact holographic region of space associated with the observer. In addition, the strong version of the holographic principle suggests that the observer has only access to a finite number of degrees of freedom. Since the volume of space accessible to the observer is effectively finite, the system has effectively been put in a finite box. Some of the conceptual difficulties with a possible stringy realization of de Sitter space are connected to the fact that it is very difficult to confine strings in a finite box.

There are also some qualitative differences between Gödel and de Sitter holography worth pointing out:

- In the Gödel universe, the preferred screens are timelike, just as the canonical global screen in AdS space. On the other hand, the observer-dependent preferred screens in de Sitter space are null.
- The Gödel universe is supersymmetric.

In order to decide whether holography in the Gödel universe can be used as a supersymmetric laboratory for exploring conceptual questions arising in de Sitter holography (or more generally, holography in cosmological spacetimes), one needs a more microscopic understanding of the Gödel universes in string and M-theory.

5. T-Duality of Gödel Universes

One can compactify one of the flat directions \mathbf{R}^6 (say y_6) of the M-theory Gödel solution on \mathbf{S}^1 with constant radius \mathcal{R} and obtain the following Type IIA Gödel background,

$$\begin{aligned}
 ds^2 &= -(dt + \beta\omega)^2 + \sum_{i=1}^4 (dx_i)^2 + \sum_{a=1}^5 (dy_a)^2, \\
 H_{ij5} &= 2\beta J_{ij}, \\
 F_{ijab} &= 2\beta J_{ij} K_{ab},
 \end{aligned}
 \tag{5.1}$$

where now in Type IIA theory $a, b \dots = 1, \dots, 5$. The dilaton is constant, implying that the string coupling g_s can be kept small everywhere, and the Gödel solution is a solution of weakly coupled Type IIA superstring theory. Now, we can T-dualize along various dimensions.

5.1. T-duality to a supersymmetric Type IIB pp-wave

The H -field of the Type IIA Gödel solution (5.1) extends along y_5 , the dimension that was paired up in M-theory with the extra dimension y_6 . It turns out that T-duality along this dimension is particularly interesting. We first rename $y_5 \equiv z$, and use the gauge in which

$$B_{iz} = \beta J_{ij} x^j. \tag{5.2}$$

Due to the absence of $g_{z\mu}$ cross-terms in the metric, no B -field will be generated after T-duality, and one gets

$$ds_{\text{IIB}}^2 = -dt^2 - 2\beta\omega (dt + dz) + \sum_{i=1}^4 dx_i^2 + \sum_{a=1}^4 dy_a^2 + dz^2. \tag{5.3}$$

To see that this Type IIB solution is in fact a supersymmetric pp-wave, it will be convenient to change the coordinates as follows. First, define lightcone coordinates $u = t+z$, $v = t-z$, and also switch from the Cartesian coordinates x_i to the “bipolar” coordinates given in (3.6). Then, we perform a u -dependent rotation in each of the two preferred planes of rotation,

$$\tilde{\phi}_i = \phi_i - \beta u. \quad (5.4)$$

Upon introducing new Cartesian coordinates \tilde{x}_i ,

$$\begin{aligned} \tilde{x}_1 + i\tilde{x}_2 &= r_1 e^{i\tilde{\phi}_1}, \\ \tilde{x}_3 + i\tilde{x}_4 &= r_2 e^{i\tilde{\phi}_2}, \end{aligned} \quad (5.5)$$

the Type IIB metric (5.3) T-dual to the Gödel universe becomes

$$ds_{\text{IIB}}^2 = -du dv - \beta^2 \left(\sum_{i=1}^4 \tilde{x}_i^2 \right) du^2 + \sum_{i=1}^4 d\tilde{x}_i^2 + \sum_{a=1}^4 dy_a^2. \quad (5.6)$$

This metric has the standard form of a supersymmetric pp-wave, with the Gödel rotation parameter β precisely equal to the conventionally normalized μ parameter of the pp-wave. One can also easily T-dualize the Ramond-Ramond fields: The self-dual Type IIB five-form of the Type IIB solution is given by $F_5 \sim du \wedge \tilde{J} \wedge K$, where $\tilde{J} = \sum_{i,j=1}^4 J_{ij} d\tilde{x}_i \wedge d\tilde{x}_j$ and $K = \sum_{a,b=1}^4 K_{ab} dy_a \wedge dy_b$. This Type IIB solution is in fact the supersymmetric pp-wave resulting from the Penrose limit of the near-horizon $AdS_3 \times \mathbf{S}^3 \times T^4$ geometry of a system of intersecting D3-branes, and was first found in [28].

5.2. Gödel/pp-wave T-duality

We have shown that the Type IIA Gödel universe is T-dual to a Type IIB pp-wave. One can turn this observation around, and ask whether other known pp-waves can also be T-dual to new Gödel-like universes. We indeed find a rich picture of Gödel/pp-wave duality which goes beyond the scope of the pp-wave T-dualities discussed in the literature (see, e.g., [29]).

Before generalizing the result of the previous subsection to a broader class of Gödel/pp-wave pairs, it is instructive to first clarify which Killing dimension of the Type IIB pp-wave is being compactified on \mathbf{S}^1 and T-dualized to produce the Type IIA Gödel universe. Consider first the Killing vector

$$\xi_0 = \frac{\partial}{\partial u} - \frac{\partial}{\partial v} \quad (5.7)$$

of the Type IIB pp-wave background. This vector is space-like at the origin, but becomes time-like at some critical radial distance. One can remedy this problem by augmenting ξ_0 with a rotation in each of the two preferred planes

$$\xi = \frac{\partial}{\partial u} - \frac{\partial}{\partial v} + \beta \left(\frac{\partial}{\partial \tilde{\phi}_1} + \frac{\partial}{\partial \tilde{\phi}_2} \right). \quad (5.8)$$

This Killing vector ξ is everywhere spacelike, with the space-like rotation off-setting the effect of the du^2 terms in the metric to keep this modified Killing vector spacelike. Moreover, the norm of ξ is

$$|\xi|^2 = 1. \quad (5.9)$$

Consequently, if we compactify the orbit of ξ on a circle of fixed radius R and T-dualize, the dilaton field of the resulting solution will stay constant. This T-duality is precisely the inverse of the IIA \rightarrow IIB T-duality that maps the Gödel solution to the pp-wave. Note that closed timelike curves are introduced even though the orbifold action is generated by an everywhere-spacelike Killing vector.

5.3. New supersymmetric Gödel universes in string and M-theory

These observations lead to a very simple and general prescription for constructing a large class of Gödel/pp-wave T-dual pairs. Start with any pp-wave in which an analog of the Killing vector ξ of (5.8) (and satisfying (5.9) if we want constant g_s) can be identified. Compactification on \mathbf{S}^1 along this Killing direction followed by T-duality produces a Gödel like solution of the T-dual string theory.

As an example of this, we present a new supersymmetric Gödel universe of Type IIA theory, as the T-dual of the maximally supersymmetric Type IIB pp-wave [30]. Using the obvious generalization of (5.8) that now involves four independent rotations in four independent two-planes of the pp-wave, we obtain a Type IIA geometry with a constant H_3 and F_4 . This Type IIA solution can be lifted to an M-theory solution of \mathbf{R}^{11} topology. Its metric factorizes to a product of a non-trivial metric on a \mathcal{G}_9 factor and the flat metric on \mathbf{R}^2 ,

$$ds^2 = -(dt + \beta\varpi)^2 + \sum_{I=1}^8 (dx_I)^2 + \sum_{A=1}^2 (dy_A)^2, \quad (5.10)$$

$$\varpi = J_{IJ} x_I dx_J,$$

and the four-form strength can be written as

$$\begin{aligned} G_{ijkl} &= 4\beta\epsilon_{ijkl}, & G_{ijAB} &= -2\beta J_{ij} K_{AB}, \\ G_{mnpq} &= -4\beta\epsilon_{mnpq}, & G_{mnAB} &= -2\beta J_{mn} K_{AB}, \end{aligned} \quad (5.11)$$

where $i, \dots = 1, \dots, 4$ and $m, \dots = 5, \dots, 8$, while the indices $I, \dots = 1, \dots, 8$ and $A, B = 1, 2$; all the non-zero components of the Kähler forms J_{IJ} and K_{AB} are now given by $J_{12} = -J_{21} = J_{34} = -J_{43} = J_{56} = -J_{65} = J_{78} = -J_{87} = 1$ and $K_{12} = -K_{21} = 1$.

This new supersymmetric Gödel solution $\mathcal{G}_9 \times \mathbf{R}^2$ of M-theory exhibits exactly the same qualitative holographic features as the $\mathcal{G}_5 \times \mathbf{R}^6$ solution. In particular one finds compact closed timelike curves that are topologically large, and the analysis of geodesics reveals the same qualitative structure of holographic screens.

6. Discussion

Following a phenomenological approach to holography, we have identified preferred holographic screens as seen by inertial observers in a class of homogeneous universes of the Gödel type, with closed timelike curves. The structure of holographic screens change dramatically the question of causality, by hiding all closed timelike curves or breaking them into causal pieces. It is tempting to suspect that holography serves as the chronology protection agency, and in combination with a version of the complementarity principle can lead to a consistent quantum mechanical description of this universe. We also noticed close analogies with the structure of holographic screens in de Sitter space, which can make the Gödel universes an interesting supersymmetric laboratory for exploring de Sitter holography. This phenomenological identification of natural screens does not tell us, however, whether the holographic dual is given by some self-consistent quantum mechanics, or whether the pathology of closed timelike curves is just translated into some inconsistency of the holographic dual. These and similar questions require a microscopic understanding of holography in Gödel universes in string or M-theory. We have found evidence that the Gödel-like cosmologies represent a remarkable and highly solvable class of solutions of string theory, and are in fact T-dual to solvable supersymmetric pp-wave solutions. Further investigation of microscopic aspects of Gödel universes and their holography in string and M-theory is in progress [17].

Acknowledgement

We wish to thank Raphael Bousso, Lisa Dyson, Ori Ganor, Eric Gimon, Mukund Rangamani, Harvey Reall, and Radu Tatar for useful discussions. This work was supported by the Berkeley Center for Theoretical Physics, by the NSF Grant PHY-0098840, and by DOE Grant DE-AC03-76SF00098. This paper is an outcome of the bonus class project assigned by one of us (P.H.) in his string theory course at UC Berkeley in September 2002.

Note Added

As was pointed out to us after the completion of this work, the T-duality relation between the 5d Gödel universe (5.1) and the geometry of (5.3) can also be obtained from a T-duality relation found a week earlier by Herdeiro in [32], by taking the limit of zero charges $P = Q = Q_{KK} = 0$ in Eqn. (4.11) of [32].⁶ However, it was not realized in [32] that the T-dual of the Gödel universe is a supersymmetric Hpp-wave; instead, this T-dual was conjecturally interpreted in [32] as a rotating background.

⁶ We thank Harvey Reall for pointing this out to us.

Appendix A. Geometry of the Gödel Universes

In this appendix we collect various aspects of the Riemannian geometry of the Gödel universes \mathcal{G}_5 and \mathcal{G}_9 that play a central role in the paper.

We are using the +++ conventions of MTW [31]; in particular, our metric is of the “mostly plus” signature.

The five-dimensional Gödel universe

In the original Cartesian coordinates t, x_i it is natural to introduce a vielbein

$$e^0 = dt + \beta\omega, \quad e^i = dx_i, \quad i = 1, \dots, 4, \quad (\text{A.1})$$

so that the metric on \mathcal{G}_5 can be written simply as

$$g_{\mu\nu} = -e_\mu^0 e_\nu^0 + \sum_i e_\mu^i e_\nu^i. \quad (\text{A.2})$$

In this vielbein, the spin connection one-forms are

$$\begin{aligned} \Omega_{ij} &= \beta J_{ij} dt + \beta^2 J_{ij} J_{k\ell} x_k dx_\ell, \\ \Omega_{i0} &= -\Omega_{0i} = \beta J_{ij} dx_j. \end{aligned} \quad (\text{A.3})$$

These simple expressions for the spin connection can be used to easily extract the form of the Ricci tensor in the Cartesian coordinates,

$$R_{\mu\nu} dX^\mu dX^\nu = 4\beta^2 dt^2 + 8\beta^3 J_{ij} x_i dt dx_j + 2\beta^2 (\delta_{ij} - 2\beta^2 J_{ik} J_{j\ell} x_k x_\ell) dx_i dx_j. \quad (\text{A.4})$$

The scalar curvature is constant,

$$R = 4\beta^2, \quad (\text{A.5})$$

as is indeed implied by the homogeneity of the solution. The Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ has the pressureless fluid form,

$$\begin{aligned} G_{\mu\nu} dX^\mu dX^\nu &= 6\beta^2 dt^2 + 12\beta^3 J_{ij} x_i dx_j dt + 6\beta^4 J_{ik} J_{j\ell} x_k x_\ell dx_i dx_j \\ &= 6\beta^2 u_\mu u_\nu dX^\mu dX^\nu, \end{aligned} \quad (\text{A.6})$$

with

$$u_\mu dX^\mu = -dt - \beta J_{ij} x_i dx_j \quad (\text{A.7})$$

the covariant dual of the timelike Killing vector $\partial/\partial t$. This is matched by the energy-momentum tensor of the constant gauge field strength $F \sim J$, which is also of the pressureless fluid form.

For the calculation of the geodesic expansion parameter θ in the body of the paper, it is also useful to know the non-zero Christoffel symbols in the ‘‘bipolar’’ coordinates $(r_1, \phi_1, r_2, \phi_2)$,

$$\begin{aligned}
\Gamma_{r_1 t}^t &= \beta^2 r_1, & \Gamma_{\phi_1 t}^{r_1} &= \beta r_1, & \Gamma_{r_1 t}^{\phi_1} &= -\frac{\beta}{r_1}, \\
\Gamma_{\phi_1 r_1}^t &= \beta^3 r_1^3, & \Gamma_{\phi_1 \phi_1}^{r_1} &= -r_1(1 - 2\beta^2 r_1^2), & \Gamma_{\phi_1 r_1}^{\phi_1} &= \frac{1}{r_1} - \beta^2 r_1, \\
\Gamma_{\phi_2 r_1}^t &= \beta^3 r_1 r_2^2, & \Gamma_{\phi_1 \phi_2}^{r_1} &= \beta^2 r_1 r_2^2, & \Gamma_{\phi_2 r_1}^{\phi_1} &= -\frac{\beta^2 r_2^2}{r_1}, \\
\Gamma_{r_2 t}^t &= \beta^2 r_2, & \Gamma_{\phi_2 t}^{r_2} &= \beta r_2, & \Gamma_{r_2 t}^{\phi_2} &= -\frac{\beta}{r_2}, \\
\Gamma_{\phi_2 r_2}^t &= \beta^3 r_2^3, & \Gamma_{\phi_2 \phi_2}^{r_2} &= -r_2(1 - 2\beta^2 r_2^2), & \Gamma_{\phi_2 r_2}^{\phi_2} &= \frac{1}{r_2} - \beta^2 r_2, \\
\Gamma_{\phi_1 r_2}^t &= \beta^3 r_2 r_1^2, & \Gamma_{\phi_1 \phi_2}^{r_2} &= \beta^2 r_1^2 r_2, & \Gamma_{\phi_1 r_2}^{\phi_2} &= -\frac{\beta^2 r_1^2}{r_2}.
\end{aligned} \tag{A.8}$$

The nine-dimensional Gödel universe

This solution, discussed in Section 5, is T-dual to the maximally supersymmetric Type IIB pp-wave.

We again introduce the natural vielbein in which the metric is of the form (A.2),

$$e^0 = dt + \beta\omega, \quad e^I = dx_I, \quad i = 1, \dots, 8. \tag{A.9}$$

In this basis, the spin connection one-forms are given by

$$\begin{aligned}
\Omega_{IJ} &= \beta J_{IJ} dt + \beta^2 J_{IJ} J_{KL} x_K dx_L, \\
\Omega_{I0} &= -\Omega_{0I} = \beta J_{IJ} dx_J,
\end{aligned} \tag{A.10}$$

with the Ricci tensor

$$\begin{aligned}
R_{MN} dX^M dX^N &= 8\beta^2 dt^2 + 16\beta^3 J_{IJ} x_I dx_J dt \\
&\quad + (2\beta^2 \delta_{IJ} + 8\beta^4 J_{IK} x_K J_{JL} x_L) dx_I dx_J,
\end{aligned} \tag{A.11}$$

the scalar curvature

$$R = 8\beta^2, \tag{A.12}$$

and the Einstein tensor

$$\begin{aligned}
(R_{MN} - \frac{1}{2} R g_{MN}) dX^M dx^N &= 12\beta^2 dt^2 + 24\beta^3 J_{IJ} x_I dx_J dt \\
&\quad - (2\beta^2 \delta_{IJ} - 12\beta^4 J_{IK} x_K J_{JL} x_L) dx_I dx_J.
\end{aligned} \tag{A.13}$$

Notice that unlike in the case of the five-dimensional Gödel solution, the Einstein tensor of the nine-dimensional Gödel universe is no longer of the pressureless fluid form.

References

- [1] G. 't Hooft, "Dimensional Reduction in Quantum Gravity," gr-qc/9310026
L. Susskind, "The World as a Hologram," *J. Math. Phys.* **36** (1995) 6377,
hep-th/9409089.
- [2] T. Banks, "Cosmological Breaking of Supersymmetry or Little Lambda Goes Back to the Future II," hep-th/0007146
see also T. Banks and W. Fischler, "M-Theory Observables for Cosmological Space-Times," hep-th/0102077
W. Fischler, unpublished (2000)
W. Fischler, "Taking de Sitter Seriously," talk given at *Role of Scaling Laws in Physics and Biology (Celebrating the 60th Birthday of Geoffrey West)*, Santa Fe, December 2000 (unpublished).
- [3] L. Susskind, "Twenty Years of Debate with Stephen," hep-th/0204027.
- [4] L. Dyson, J. Lindesay and L. Susskind, "Is There Really a de Sitter/CFT Duality," *JHEP* **0208** (2002) 045.
- [5] R. Bousso, "Positive Vacuum Energy and the N-Bound," *JHEP* **0011** (2000) 038,
hep-th/0010252.
- [6] P. Hořava, talk at Strings 2001 at Mumbai, India (January 2001)
V. Balasubramanian, P. Hořava and D. Minic, "Deconstructing de Sitter," *JHEP* **0105** (2001) 043, hep-th/0103171.
- [7] E. Witten, talk at Strings 2001 in Mumbai, India (January 2001)
E. Witten, "Quantum Gravity in de Sitter Space," hep-th/0106109.
- [8] A. Strominger, "The dS/CFT Correspondence," *JHEP* **0110** (2001) 034, hep-th/0106113
see also M. Spradlin, A. Strominger and A. Volovich, "Les Houches Lectures on de Sitter Space," hep-th/0110007.
- [9] K. Gödel, "An Example of a New Type of Cosmological Solutions of Einstein's Field Equations of Gravitation," *Rev. Mod. Phys.* **21** (1949) 447.
- [10] J.P. Gauntlett, J.B Gutowski, C.M. Hull, S. Pakis and H.S. Reall, "All Supersymmetric Solutions of Minimal Supergravity in Five Dimensions," hep-th/0209114.
- [11] A.A. Tseytlin, "Extreme Dyonic Black Holes in String Theory," *Mod. Phys. Lett.* **A11** (1996) 689, hep-th/9601177.
- [12] G.W. Gibbons and C.A.R. Herdeiro, "Supersymmetric Rotating Black Holes and Causality Violation," *Class. Q. Grav.* **16** (1999) 3619, hep-th/9906098
C.A.R. Herdeiro, "Special Properties of Five Dimensional BPS Rotating Black Holes," *Nucl. Phys.* **B582** (2000) 363, hep-th/0003063.
- [13] M. Visser, *Lorentzian Wormholes: From Einstein to Hawking* (AIP Press, 1995).

- [14] B. Carter, “Global Structure of the Kerr Family of Gravitational Fields,” *Phys. Rev.* **174** (1968) 1559.
- [15] R. Bousso, “Holography in General Space-times,” *JHEP* **9906** (1999) 028, hep-th/9906022.
- [16] D. Bigatti and L. Susskind, “TASI Lectures on the Holographic Principle,” hep-th/0002044
R. Bousso, “The Holographic Principle,” *Rev. Mod. Phys.* **74** (2002) 825, hep-th/0203101.
- [17] E.K. Boyda, S. Ganguli, P. Hořava and U. Varadarajan, work in progress.
- [18] S.W. Hawking and G.F.R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, Cambridge, 1973).
- [19] R. Adler, M. Bazin and M. Schiffer, *Introduction to General Relativity* (Mc Graw-Hill, 1965).
- [20] S. Chandrasekhar and J.P. Wright, “The Geodesics in Gödel’s Universe,” *Proc. Nat. Acad. Sci.* **47** (1961) 341.
- [21] S.W. Hawking, “The Chronology Protection Conjecture,” *Phys. Rev.* **D46** (1992) 603.
- [22] A.K. Raychaudhuri and S.N.G. Thakurta, “Homogeneous Space-times of the Gödel Type,” *Phys. Rev.* **D22** (1980) 802
M.J. Rebouças and J. Tiomno, “Homogeneity of Riemannian Space-times of Gödel Type,” *Phys. Rev.* **D28** (1982) 1251.
- [23] M. Rooman and Ph. Spindel, “Gödel Metric as a Squashed Anti-de Sitter Geometry,” *Class. Q. Grav.* **15** (1998) 3241, gr-qc/9804027.
- [24] M.M. Som and A.K. Raychaudhuri, “Cylindrically Symmetric Charged Dust Distributions in Rigid Rotation in General Relativity,” *Proc. Roy. Soc. London* **A304** (1968) 81.
- [25] W.J. van Stockum, “Gravitational Field of a Distribution of Particles Rotating About an Axis of Symmetry,” *Proc. Roy. Soc. Edin.* **57** (1937) 135
F.J. Tipler, “Rotating Cylinders and the Possibility of Global Causality Violations,” *Phys. Rev.* **D9** (1974) 2203
P. Klepáč and J. Horský, “Charged Perfect Fluid and Scalar Field Coupled to Gravity,” *Czech. J. Phys.* **51** (2001) 1177, gr-qc/0109043.
- [26] J.P. Gauntlett, R.C. Myers and P.K. Townsend, “Black Holes of $D = 5$ Supergravity,” *Class. Q. Grav.* **16** (1999) 1, hep-th/9810204
P.K. Townsend, “Surprises with Angular Momentum,” hep-th/0211008.
- [27] L. Dyson, M. Kleban and L. Susskind, “Disturbing Implications of a Cosmological Constant,” hep-th/0208013
T. Banks, W. Fischler and S. Paban, “Recurring Nightmares? : Measurement Theory in de Sitter Space,” hep-th/0210160.

- [28] M. Cvetič, H. Lü and C.N. Pope, “Penrose Limits, PP-Waves and Deformed M2-Branes,” hep-th/0203082.
- [29] R. Güven, “Plane Wave Limits and T-Duality,” *Phys. Lett.* **B482** (2000) 255, hep-th/0005061
J. Michelson, “(Twisted) Toroidal Compactification of pp-Waves,” *Phys. Rev.* **D66** (2002) 066002, hep-th/0203140.
- [30] M. Blau, J. Figueroa-O’Farrill, C. Hull and G. Papadopoulos, “A New Maximally Supersymmetric Background of IIB Superstring Theory,” *JHEP* **0201** (2002) 047, hep-th/0110242.
- [31] C.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation* (Freeman, 1973).
- [32] C.A.R. Herdeiro, “Spinning Deformations of the D1-D5 System and a Geometric Resolution of Closed Timelike Curves,” hep-th/0212002.