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Holographic Q-lattices

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ABSTRACT: We introduce a new framework for constructing black hole solutions that are holographically dual to strongly coupled field theories with explicitly broken translation invariance. Using a classical gravitational theory with a continuous global symmetry leads to constructions that involve solving ODEs instead of PDEs. We study in detail $D = 4$ Einstein-Maxwell theory coupled to a complex scalar field with a simple mass term. We construct black holes dual to metallic phases which exhibit a Drude-type peak in the optical conductivity, but there is no evidence of an intermediate scaling that has been reported in other holographic lattice constructions. We also construct black holes dual to insulating phases which exhibit a suppression of spectral weight at low frequencies. We show that the model also admits a novel $AdS_3 \times \mathbb{R}$ solution.

KEYWORDS: Gauge-gravity correspondence, Black Holes in String Theory, AdS-CFT Correspondence, Holography and condensed matter physics (AdS/CMT)

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1 Introduction

It is a remarkable fact that many phenomena observed in condensed matter systems are now known to have gravitational analogues via the AdS/CFT correspondence. One area of focus, where there has been significant recent progress, concerns the holographic description of physics associated with a “lattice”. More specifically, there are now several different constructions of black hole solutions that are holographically dual to strongly coupled systems which explicitly break translation invariance using a spatially periodic deformation [1–6].

One motivation for constructing such black holes arises in the context of studying the optical conductivity of strongly coupled systems at finite charge density. In the absence of a lattice the translation invariance of the system implies that there is a delta function peak at zero frequency, implying that the system is an ideal conductor. To extract more realistic metallic behaviour one can investigate the impact of a lattice. The first construction of electrically charged black holes describing holographic lattices was made in $D = 4$ Einstein-Maxwell theory coupled to a real scalar field [1]. For the specific black holes that were constructed, it was shown that the system is in a metallic phase with the delta function peak smeared out into a Drude-type peak.¹ This observed low frequency behaviour is consistent with the general analysis of conductivities that was made earlier in [12] (see also [15]).

Moving away from the low-frequency regime, with the scale set by the chemical potential, a particularly striking conclusion of [1] was that the optical conductivity appears

¹Drude-type physics has also been discussed in a holographic context in, for example, [7–17].

to exhibit a power-law behaviour at intermediate frequencies. More precisely the optical conductivity was seen to have the form

$$|\sigma(\omega)| = B\omega^{-2/3} + C, \tag{1.1}$$

where B, C are frequency independent constants, and furthermore, the same behaviour was also seen for other lattices and other spacetime dimensions in [2, 3, 5]. Since an intermediate scaling of the optical conductivity for the high T_c cuprates is seen with the same scaling exponent $-2/3$, albeit with $C = 0$ and a frequency independent phase (e.g. [18, 19]), it is important to analyse this result in more detail. In fact for the holographic lattice that we construct in this paper we will not see such scaling behaviour. We will discuss the connection between our results and [1–3, 5] at the end of the paper.

A more recent motivation for studying holographic lattices is that it provides a framework for investigating metal-insulator transitions within a holographic context [4]. This is particularly interesting because there are many perplexing systems, such as the cuprates, where such transitions are observed and holographic techniques may provide important new insights. The strategy of [4] is to construct black holes dual to holographic lattices that flow in the IR to metallic ground states and then to vary the strength and/or the periodicity of the lattice aiming to induce a transition to a new insulating phase. In [4] this was achieved using $D = 5$ electrically charged black holes dual to helical lattices. Furthermore, new zero temperature insulating ground states that break translation invariance were also found in [4].

An important technical issue that arises in constructing black holes dual to lattices is that, in general, they require solving partial differential equations. For example, the holographic lattices that were constructed in [1–3, 5] break translation invariance in one of the spatial dimensions and lead to a problem in PDEs in two variables; the one spatial direction as well as a radial direction. For the general setup where the translation invariance is broken in all of the spatial directions, time independent black holes in D spacetime dimensions will typically depend on $D - 2$ spatial variables as well as a radial variable, leading to PDEs in $D - 1$ variables. For $D = 4, 5$ solving such PDEs numerically is an involved exercise. An interesting exception is the construction of the $D = 5$ black holes dual to helical lattices [4], where a Bianchi VII₀ symmetry was utilised to construct black holes by solving ODEs only.

In this paper we introduce a new framework for constructing holographic lattices that also involves just solving ODEs. The key idea is to break the translation invariance by exploiting a continuous global symmetry of the bulk classical gravitational theory. A simple theory that can be used to illustrate the idea, which is also the theory we will focus on in the paper, consists of Einstein-Maxwell theory coupled to a complex scalar field, ϕ . The field ϕ is neutral with respect to the Maxwell field, and the model is taken to have a global U(1) symmetry in addition to the U(1) gauge-symmetry associated with the Maxwell field. For example, the Lagrangian density involving ϕ can take the form

$$\mathcal{L}(\phi) = \sqrt{-g} [-|\partial\phi|^2 - V(|\phi|)], \tag{1.2}$$

leading to the following contribution to the bulk stress-tensor

$$T_{\mu\nu}(\phi) = \partial_{(\mu}\phi\partial_{\nu)}\phi^* - \frac{1}{2}g_{\mu\nu} [|\partial\phi|^2 + V(|\phi|)] . \quad (1.3)$$

The breaking of the translation invariance in, say, the x_1 direction can be achieved using the ansatz $\phi = e^{ikx_1}\varphi(r)$ and it is clear from the form of the stress tensor given in (1.3) that this can be combined with an ansatz for the metric and Maxwell fields that is dependent on the radial variable only.² This construction shares some similarities with the construction of Q-balls [21], which exploits a global symmetry and a time dependent phase to construct spherically symmetric solitons, and so we call them holographic Q-lattices.

It is worth noting that this particular Q-lattice, involving a single complex scalar field, can be viewed as arising from two real scalar fields, with the same mass, each with a periodic spatial dependence in the same direction that is shifted by an amount $\pi/2k$. In this sense it can be viewed as a simple generalisation of the lattice studied in [2]. More generally, this lattice construction can easily be extended to study the breaking of translation invariance in additional spatial directions by considering a model with a larger global symmetry. For example, one can use a model with additional complex scalar fields and with additional global U(1) symmetries. One can also have larger global symmetry groups and/or use higher rank tensor fields instead of scalars. Such lattices will be studied in detail elsewhere.

The plan of the rest of the paper, including some of the key results, are as follows. In section 2 we study $D = 4$ Einstein-Maxwell theory coupled to a complex scalar field with a simple mass term. We construct Q-lattice black holes that describe metallic phases which at zero temperature approach $AdS_2 \times \mathbb{R}^2$ in the far IR. We numerically calculate the low temperature behaviour of the DC resistivity and extract the scaling behaviour that is predicted from [12] using the memory matrix formalism. This comprises the first³ numerical confirmation of [12] for fully back reacted black holes and complements the recent analytic results of [20] in the context of perturbative lattices. We also construct black holes that describe insulating phases, realising the first holographic metal-insulator transition for $d = 3$ field theories. At low temperatures there is a transfer of spectral weight in the insulating phase and the real part of the optical conductivity develops a mid frequency hump. Some details of the conductivity calculation is presented in section 3, which includes some new technical material. Interestingly, the model that we analyse also admits an $AdS_3 \times \mathbb{R}$ solution which we discuss in an appendix. We conclude with some final comments in section 4, including a discussion of the absence of intermediary scaling in the optical conductivity.

2 Black hole solutions

We shall consider $D = 4$ Einstein-Maxwell theory coupled to a complex field ϕ with action given by

$$S = \int d^4x \sqrt{-g} \left[R + 6 - \frac{1}{4}F^2 - |\partial\phi|^2 - m^2|\phi|^2 \right] , \quad (2.1)$$

²In the process of writing up this work, this possibility was also pointed out in a footnote in [20].

³We will comment on the results of [1] in section 4.

where $F = dA$. We have set $16\pi G = 1$ and also fixed the scale of the cosmological constant for convenience. The equations of motion can be written

$$\begin{aligned}
 R_{\mu\nu} &= g_{\mu\nu} \left(-3 + \frac{m^2}{2} |\phi|^2 \right) + \partial_{(\mu} \phi \partial_{\nu)} \phi^* + \frac{1}{2} \left(F_{\mu\nu}^2 - \frac{1}{4} g_{\mu\nu} F^2 \right), \\
 \nabla_{\mu} F^{\mu\nu} &= 0, \quad (\nabla^2 - m^2) \phi = 0,
 \end{aligned}
 \tag{2.2}$$

and admit an AdS_4 vacuum solution, with unit radius, which is dual to a $d = 3$ CFT. The CFT has two global abelian symmetries. The first arises from the gauge symmetry in the bulk and there is a corresponding conserved current which is dual to the bulk-gauge field A . The second arises from the global symmetry in the bulk, associated with multiplying ϕ by a constant phase, and there is not a corresponding conserved current⁴ in the CFT. The CFT also has a complex scalar operator with scaling dimension $\Delta = 3/2 \pm (9/4 + m^2)^{1/2}$ dual to the scalar field ϕ . We want this to be a relevant operator in a unitary CFT and hence we take $-9/4 \leq m^2 < 0$.

The CFT at finite temperature T and chemical potential μ can be holographically described by the standard electrically charged AdS-RN black solution given by

$$\begin{aligned}
 ds^2 &= -U dt^2 - U^{-1} dr^2 + r^2 (dx_1^2 + dx_2^2), \\
 A &= \mu \left(1 - \frac{r_+}{r} \right) dt,
 \end{aligned}
 \tag{2.3}$$

with $\phi = 0$ and $U = r^2 - (r_+^2 + \frac{\mu^2}{4}) \frac{r_+}{r} + \frac{\mu^2 r_+^2}{4r^2}$. The temperature is given by $T = (12r_+^2 - \mu^2)/16\pi r_+$ and at $T = 0$ it approaches the following $AdS_2 \times \mathbb{R}^2$ solution as $r \rightarrow r_+$:

$$\begin{aligned}
 ds^2 &= \frac{1}{6} ds^2(AdS_2) + dx_1^2 + dx_2^2, \\
 F &= \frac{1}{\sqrt{3}} Vol(AdS_2),
 \end{aligned}
 \tag{2.4}$$

where $ds^2(AdS_d)$ denotes the standard unit radius metric on AdS_d .

For the mass window $-9/4 \leq m^2 < -3/2$ the scalar field ϕ violates the AdS_2 BF bound and hence the AdS-RN black hole solution will become unstable at some temperature, leading to a different $T = 0$ ground state. In order to exclude this possibility, for most of the paper we will consider

$$m^2 = -\frac{3}{2} \quad \leftrightarrow \quad \Delta = \frac{3 + \sqrt{3}}{2}.
 \tag{2.5}$$

At the end of the paper we will comment on the case $m^2 = -2$ and $\Delta = 2$.

2.1 Black hole ansatz for the holographic Q -lattice

We are interested in describing the $d = 3$ CFT with chemical potential μ and an explicit breaking of translation invariance in one of the spatial directions, which we take to be x_1 .

⁴A discussion of such global symmetries arising in a different holographic context appears in [22].

The ansatz we shall consider is given by

$$\begin{aligned}
 ds^2 &= -U dt^2 + U^{-1} dr^2 + e^{2V_1} dx_1^2 + e^{2V_2} dx_2^2, \\
 A &= a dt, \\
 \phi &= e^{ikx_1} \varphi,
 \end{aligned} \tag{2.6}$$

where U, V_1, V_2, a and φ are functions of the radial co-ordinate only and k is a constant. Substituting this ansatz into (2.2) we find that the equations of motion can be equivalently recast as four second order ODEs for V_1, V_2, a, φ and one first order ODE for U . It is useful to note that this ansatz is invariant under the scaling $t \rightarrow ct, x_i \rightarrow cx_i, r \rightarrow c^{-1}r$ and $U \rightarrow c^{-2}U, e^{V_i} \rightarrow c^{-1}e^{V_i}, a \rightarrow c^{-1}a, k \rightarrow c^{-1}k$.

We will impose the following boundary conditions on the ODEs. We demand that we have a regular solution at the black hole event horizon at $r = r_+$, which leads to an expansion depending on six independent constants $r_+, V_{1+}, V_{2+}, V_{22}, a_+$ and φ_+ . Specifically as $r \rightarrow r_+$ we have

$$\begin{aligned}
 U &= 4\pi T(r - r_+) + \dots, \\
 V_1 &= V_{1+} + \left(1 - \frac{4e^{-2V_{1+}}\varphi_+^2 k^2}{12 - a_+^2 - 2\varphi_+^2 m^2}\right) V_{22}(r - r_+) \dots, \\
 V_2 &= V_{2+} + V_{22}(r - r_+) \dots, \\
 a &= a_+(r - r_+) + \left(-1 + \frac{2e^{-2V_{1+}}\varphi_+^2 k^2}{12 - a_+^2 - 2\varphi_+^2 m^2}\right) a_+ V_{22}(r - r_+)^2 \dots, \\
 \varphi &= \varphi_+ + \frac{4(m^2 + e^{-2V_{1+}}k^2)}{12 - a_+^2 - 2\varphi_+^2 m^2} \varphi_+ V_{22}(r - r_+) \dots,
 \end{aligned} \tag{2.7}$$

where T is the temperature of the black hole given by

$$T = (4\pi)^{-1} \frac{12 - a_+^2 - 2\varphi_+^2 m^2}{4V_{22}}. \tag{2.8}$$

At the UV boundary, $r \rightarrow \infty$, we demand that we approach AdS_4 with deformations corresponding to chemical potential μ and lattice deformation parameter λ . We find that, schematically, we can develop the expansion

$$\begin{aligned}
 U &= r^2 + \dots - \frac{M}{r} + \dots, \\
 V_1 &= \log r + \dots + \frac{V_v}{r^3} + \dots, \\
 V_2 &= \log r + \dots - \frac{V_v}{r^3} + \dots, \\
 a &= \mu + \frac{q}{r} \dots, \\
 \varphi &= \frac{\lambda}{r^{3-\Delta}} + \dots + \frac{\varphi_c}{r^\Delta} + \dots.
 \end{aligned} \tag{2.9}$$

This gives a UV expansion that depends on seven parameters $M, V_v, \mu, q, \lambda, \varphi_c$ and k .

Notice that for fixed m^2 , the holographic Q-lattice is specified by three dimensionless quantities fixing the deformations in the UV: T/μ , $\lambda/\mu^{3-\Delta}$ and k/μ . We thus expect a three-parameter family of black holes. We have four second order ODEs and one first order ODE, and so a solution is specified by nine parameters. We have six parameters for the near horizon expansion plus another seven for the UV expansion. After subtracting one for the scaling symmetry that the system of ODEs possesses, we deduce that there is indeed, generically, a three-parameter family of black hole solutions. We also note that the scaling symmetry can be used to set $\mu = 1$ if one wishes.

We will choose specific values in the two-dimensional space parameterised by $\lambda/\mu^{3-\Delta}$ and k/μ , and then examine the behaviour as T/μ is lowered. In particular, we will see that there is a transition from metallic to insulating behaviour as we move in this two-dimensional space.

2.2 Black holes dual to the metallic phase

The CFT deformed by the Q-lattice will be in a metallic phase if the zero temperature limit of the black hole solutions interpolate between the lattice deformed AdS_4 in the UV and the stable $AdS_2 \times \mathbb{R}^2$ solution in the IR. Indeed this will happen when the lattice deformation in the UV becomes an irrelevant deformation of the $AdS_2 \times \mathbb{R}^2$ solution in the IR, and then the general arguments of [12], based on the memory matrix formalism, show that the $T = 0$ ground state should be metallic. In particular, at low temperatures, $T \ll \mu$, the DC resistivity is expected to scale as⁵

$$\rho \sim \left(\frac{T}{\mu}\right)^{2\Delta(k)-2}, \tag{2.10}$$

where $\Delta(k)$ is the smallest scaling dimension of the k -dependent irrelevant operators in the locally quantum critical theory arising in the IR. In addition to k , $\Delta(k)$ depends on other UV data, as we discuss below. Furthermore, there should be a Drude peak in the optical conductivity at small temperatures, which at $T = 0$ becomes a delta-function at zero frequency.

To examine when this situation can arise we now analyse perturbations about the $AdS_2 \times \mathbb{R}^2$ solution. Within our ansatz we consider

$$\begin{aligned} U &= 6r^2(1 + u_1 r^\delta), & V_1 &= v_{10}(1 + v_{11} r^\delta), & V_2 &= v_{20}(1 + v_{21} r^\delta), \\ a &= 2\sqrt{3}r(1 + a_1 r^\delta), & \phi &= e^{ikx_1} \varphi_1 r^\delta. \end{aligned} \tag{2.11}$$

The corresponding perturbations are associated with operators with scaling dimension $\Delta = -\delta$ or $\Delta = \delta + 1$ in the locally quantum critical IR theory captured by the $AdS_2 \times \mathbb{R}^2$ solution. We find after substituting into the equations of motion the exponents come in four pairs, satisfying $\delta_+ + \delta_- = -1$, with $\delta_+ = 0, 0, 1$ and a mode just involving the scalar field with $\delta_+ = \delta_\varphi$, where

$$\delta_\varphi = -\frac{1}{2} + \frac{1}{2\sqrt{3}} \sqrt{3 + 2m^2 + 2e^{-2v_{10}} k^2}. \tag{2.12}$$

⁵Note that a different, non-standard, definition of $\Delta(k)$ is used in [4, 12, 20] for this expression.

There is also another additional single mode with $\delta_+ = -1$ (corresponding to r_+ in (2.15) below). What is most significant here is that the scalar field perturbation will be an irrelevant deformation in the IR (i.e. $\delta_\varphi > 0$), provided that the lattice deformation in the IR satisfies

$$(e^{-v_{10}}k)^2 > -m^2. \tag{2.13}$$

In this case the dimension of the irrelevant operator in the locally quantum critical theory is given by $\Delta(k) = 1 + \delta_\varphi$ and we have

$$\Delta(k) = \frac{1}{2} + \frac{1}{2\sqrt{3}}\sqrt{3 + 2m^2 + 2e^{-2v_{10}}k^2}. \tag{2.14}$$

When (2.13) is satisfied we can use the two marginal modes with $\delta_+ = 0$ and the two irrelevant modes to construct domain walls interpolating between the lattice deformed AdS_4 in the UV and the $AdS_2 \times \mathbb{R}^2$ solution in the IR. Specifically, we can develop the following IR expansion

$$\begin{aligned} U &= 6(r - r_+)^2 \left(1 - \frac{4}{3v_{10}}V_+(r - r_+) + \dots \right), \\ V_1 &= v_{10}(1 + V_+(r - r_+) + \dots), \\ V_2 &= v_{20} \left(1 + \frac{v_{10}}{v_{20}}V_+(r - r_+) + \dots \right), \\ a &= \sqrt{12}(r - r_+)(1 - v_{10}V_+ \dots), \\ \varphi &= \varphi_+(r - r_+)^{\delta_\varphi} + \dots \end{aligned} \tag{2.15}$$

We have five IR parameters, $r_+, v_{10}, v_{20}, V_+, \varphi_+$ and hence when combined with the UV expansion (2.9) and taking into the scaling symmetry, we expect, generically, a two parameter family of solutions which can be labelled by $\lambda/\mu^{3-\Delta}$ and k/μ .

For the values of $\lambda/\mu^{3-\Delta}, k/\mu$ where these domain walls exist, we expect that they will arise as the zero temperature limit of lattice deformed black holes which will have, for very small T/μ , DC resistivity scaling as in (2.10) and a Drude peak in the optical conductivity for small ω/μ , of the form

$$\sigma \sim \frac{K\tau}{1 - i\omega\tau}, \tag{2.16}$$

for constant K, τ . It should be stressed that the value of $\Delta(k)$ appearing in the DC resistivity depends on the value of v_{10} which is fixed by the details of domain wall solution, including all UV data. In effect the value of v_{10} is renormalising the lattice momentum from k in the UV to $e^{-v_{10}}k$ in the IR.

One might expect that this metallic scenario unfolds for large wavelength and small Q-lattice deformations of the AdS-RN black hole i.e. $\lambda/\mu^{3-\Delta} \ll 1$ and $k/\mu \ll 1$. As an illustrative example, we have numerically constructed Q-lattice black holes in the metallic phase with $\lambda/\mu = 1/2$ and $k/\mu = 1/\sqrt{2}$. By examining the properties of these solutions

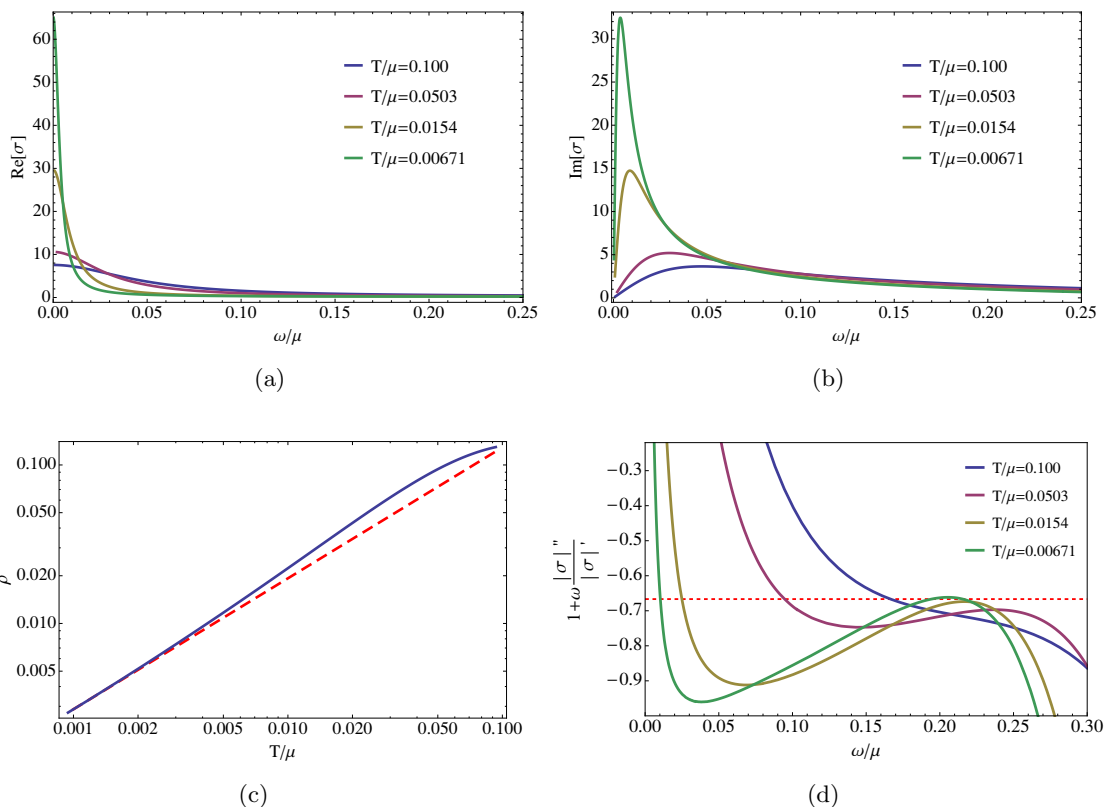


Figure 1. Black holes in the metallic phase for lattice parameters $\lambda/\mu = 1/2$ and $k/\mu = 1/\sqrt{2}$. Panels (a) and (b) shows the real and imaginary parts of the optical conductivity, $Re(\sigma)$ and $Im(\sigma)$, respectively, for four different temperatures. As the the temperature is lowered, the Drude peak becomes more pronounced. Panel (c) shows the behaviour of the DC resistivity, ρ , as a function of T/μ . The blue line is the data and the red dashed line is the scaling expected from (2.10). Panel (d) shows a plot of $1 + \omega|\sigma''|/|\sigma'|$ versus frequency; there is no evidence for an intermediate scaling of the form (1.1), which corresponds to the red dashed line.

at very low temperatures, we find that they approach domain walls interpolating between AdS_4 in the UV and $AdS_2 \times \mathbb{R}^2$ in the IR. In section 3 we describe the calculation of the optical conductivity; the results for the metallic phase black holes that we have constructed are presented in figure 1.

In figure 1(c) we see that the DC resistivity increases with temperature and hence we do indeed have a metallic phase. In figures 1(a) and 1(b) we have plotted the real and imaginary part of the optical conductivity, respectively, for four different temperatures. In particular, in 1(a) we see the Drude-type peaks appearing, which get more pronounced as the temperature is lowered. By fitting⁶ to (2.16) we obtain the values for $\tau\mu$ and K/μ given in table 1.

To observe the exact scaling behaviour $\rho \sim (T/\mu)^{2\Delta(k)-2} = (T/\mu)^{2\delta_\varphi}$, with $\Delta(k), \delta_\varphi$, as in (2.14), (2.12), as predicted by [12], is not straightforward because the scaling only

⁶For $\omega \ll T$ we make the four parameter fit: $1/\sigma = (a_1 + a_2\omega^2) - i\omega(a_3 + a_4\omega^2)$, for constants a_i , where we used $\sigma^*(\omega) = \sigma(-\omega)$, and we note that $a_1 = (K\tau)^{-1} = \rho$ and $a_3 = K^{-1}$.

T/μ	$\tau\mu$	K/μ
0.1	20	0.37
0.0503	33	0.32
0.0154	113	0.26
0.00671	272	0.24

Table 1. Parameters after fitting to the Drude behaviour (2.16) for small ω , for the black holes in the metallic phase for lattice parameters $\lambda/\mu = 1/2$ and $k/\mu = 1/\sqrt{2}$.

manifests itself when $T \ll \mu$. We have constructed the black hole solutions down to temperatures $T/\mu \sim 2.5 \times 10^{-7}$ and, as noted, we find that the black holes approach the $AdS_2 \times \mathbb{R}^2$ solution. By identifying v_{10} with V_{1+} we deduce that $k \sim 0.707$ gets renormalised to a value $e^{-v_{10}}k \sim 2.236$ and hence $\Delta(k) \sim 1.413$ corresponding to the scaling $\rho \sim (T/\mu)^{0.826}$. We have calculated the conductivity for temperatures down to $T/\mu \sim 7 \times 10^{-4}$ and from this deduced the DC resistivity. The scaling behaviour eventually manifests itself at these low temperatures as one can see from panel (c) of figure 1. Our results in 1(c) are consistent with this scaling to the order of less than 1%. This is the first direct check of the prediction of [12] for back-reacted holographic lattices.⁷ Note that for very large temperatures the resistivity should eventually approach unity, which is the constant value for the AdS-Schwarzschild black hole at zero momentum [24].

We can also investigate the possibility that there is a scaling of the form (1.1), which has been reported for other models in the range $2 \lesssim \omega\tau \lesssim 8$ [1–3, 5]. If this scaling is present then $1 + \omega|\sigma|''/|\sigma|' = -2/3$. Our results are plotted in figure 1(d) and, for example, from table 1 for $T/\mu = 0.1$ the relevant range is $0.1 \lesssim \omega/\mu \lesssim 0.4$, while for $T/\mu = 0.00671$ it is $0.0073 \lesssim \omega/\mu \lesssim 0.029$. Our results show that there is a strong temperature dependence and there is no evidence of a mid frequency scaling region. Note that $|\sigma|$ has a minimum at some value of ω and hence the function $1 + \omega|\sigma|''/|\sigma|'$ will diverge at that point and, furthermore for larger values of ω it will be positive. Finally we note that for very large ω/μ and fixed T , the conductivity should approach that of the AdS-RN black hole with $\sigma \rightarrow 1$ [24].

2.3 Black holes dual to the insulating phase

The metallic phase discussed in the last subsection arises for a given UV lattice, specified by $\lambda/\mu^{3-\Delta}$ and k/μ , whenever the $T = 0$ ground state approaches $AdS_2 \times \mathbb{R}^2$ in the far IR. In this section we will construct black holes where this does not occur and we will see that they exhibit insulating behaviour.

We focus on the specific values $\lambda/\mu^{3-\Delta} = 2$ and $k/\mu = 1/2^{3/2}$. The optical conductivity and the DC resistivities for these black holes are displayed in figure 2. The DC resistivity is increasing as we lower the temperature indicating that the system is in an insulating

⁷The recent analytic results on the scaling of the DC resistivity for perturbative lattices [20] also confirmed the prediction of [12]. Note, though, that the order in perturbations that were considered do not include back reaction of the metric and, in particular, that length scales get renormalised from the UV to the IR. Analytic results for back-reacted Q lattice black holes will appear in [23].

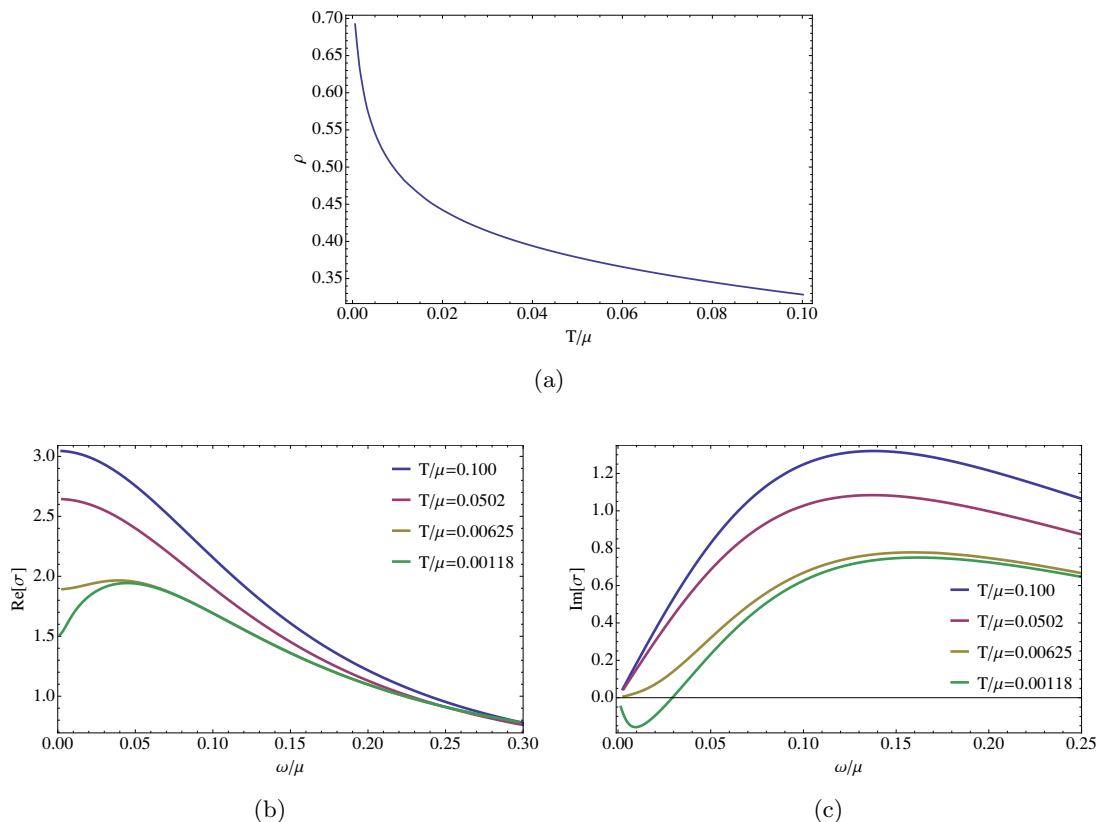


Figure 2. Black holes in the insulating phase for lattice parameters $\lambda/\mu^{3-\Delta} = 2$ and $k/\mu = 1/2^{3/2}$. Panel (a) shows the behaviour of the DC resistivity, ρ , as a function of T/μ . Panels (b) and (c) show the real and imaginary parts of the optical conductivity, $Re(\sigma)$ and $Im(\sigma)$, respectively, for four different temperatures. For very low temperatures we see in panel (b) the suppression of spectral weight for small ω and the development of a mid-frequency hump.

phase. Furthermore, for very low temperatures, for example $T/\mu \sim 0.00118$, we see that the real part of the optical conductivity reveals a suppression of spectral weight for small ω/μ , with the weight being transferred to a mid frequency hump. Very similar behaviour was seen for the helical lattice black holes dual to insulating phases in [4].

Lowering the temperature further we might expect to find the $T = 0$ ground states for this insulating phase. Actually this is not guaranteed as there are certainly situations in holography where black holes only exist down to a minimum temperature, for example [25]. For the insulating black holes with the above lattice parameters we have found an interesting feature at the low temperature $T_c/\mu \sim 2.8 \times 10^{-5}$. Specifically we find that there appears to be a kink in the entropy density versus temperature curve, with $s'(T_c) = 0$, which at first sight appears to represent a minimum temperature. However, closer detailed numerical investigation shows that there is another branch of insulating black holes at lower temperature, with broadly similar insulating behaviour. The simplest interpretation is that there is a first order transition at T_c . Assuming this to be the case, we have found that the low temperature branch exists at least down to the ultra low temperatures $T/\mu \sim 10^{-9}$.

Furthermore, we find that the entropy density is going to zero and that the solutions are becoming singular. We are particularly interested in extracting the far IR behaviour of the $T = 0$ black holes. However, in general, this is a non-trivial task unless some simplification represents itself in the numerical solutions, such as the functions approaching a power-law behaviour. We have not been able to find any evidence for such power law behaviour in the present setting.

It would be certainly interesting to explore these issues further. Note that we have considered other values for the UV lattice data, finding somewhat similar results, but a more comprehensive analysis of the behaviour for general values of $\lambda/\mu^{3-\Delta}$ and k/μ is left for future work. One point that is worth highlighting is that the model also possesses another fixed point solution that may play an important role in understanding the phase structure of the model. As we describe in the appendix there is a novel electrically neutral $AdS_3 \times \mathbb{R}$ fixed point solution with a spectrum containing modes corresponding to both irrelevant and relevant operators. The presence of the relevant operator indicates that for generic lattice data it will not be possible to construct domain wall solutions interpolating between AdS_4 in the UV and $AdS_3 \times \mathbb{R}$ in the IR. However, it is possible that a fine tuned domain wall solution exists for specific lattice data, which might correspond to an unstable RG flow providing a bifurcation between the metallic and insulating behaviours analogous to what was observed for the helical black hole lattices in [25].

3 Conductivity

In this section we explain how we calculate the conductivity for the black holes that we have constructed. Although the general idea is standard, the technical implementation in the presence of the lattice deformation warrants some discussion. We consider the following consistent linear perturbation about the black hole solutions

$$\begin{aligned} \delta g_{tx_1} &= \delta h_{tx_1}(t, r), \\ \delta A_{x_1} &= \delta a_{x_1}(t, r), \\ \delta \phi &= ie^{ikx_1} \delta \varphi(t, r), \end{aligned} \tag{3.1}$$

where $\delta h_{tx_1}, \delta a_{x_1}$ and $\delta \varphi$ are all *real* functions of (t, r) and we note the factor of i in the last line. After substituting into the equations of motion we obtain real partial differential equations. We next allow for a time dependence of the form $e^{-i\omega t}$ by writing

$$\begin{aligned} \delta h_{tx_1}(t, r) &= e^{-i\omega t} \delta h_{tx_1}(r), \\ \delta a_{x_1}(t, r) &= e^{-i\omega t} \delta a_{x_1}(r), \\ \delta \varphi(t, r) &= e^{-i\omega t} \delta \varphi(r), \end{aligned} \tag{3.2}$$

and we are lead to the following system of ODEs:

$$\begin{aligned} \delta a''_{x_1} + (U^{-2}\omega^2 - U^{-1}a'^2) \delta a_{x_1} + (U^{-1}U' - V'_1 + V'_2) \delta a'_{x_1} \\ + 2i\frac{k}{\omega}a' (\varphi'\delta\varphi - \varphi\delta\varphi') = 0, \end{aligned}$$

$$\begin{aligned}
&\delta\varphi'' + (U^{-2}\omega^2 - m^2U^{-1} - k^2U^{-1}e^{-2V_1})\delta\varphi \\
&\quad + (U^{-1}U' + V_1' + V_2')\delta\varphi' - ik\omega U^{-2}e^{-2V_1}\varphi\delta h_{tx_1} = 0, \\
&\delta h'_{tx_1} + a'\delta a_{x_1} - 2V_1'\delta h_{tx_1} - 2i\frac{k}{\omega}U(\varphi'\delta\varphi - \varphi\delta\varphi') = 0.
\end{aligned} \tag{3.3}$$

At the black hole event horizon we impose purely ingoing boundary conditions with the perturbations behaving as

$$\begin{aligned}
\delta a_{x_1} &= (r - r_+)^{-i\omega/4\pi T} \left(\delta a_{x_1}^{(+)} + \dots \right), \\
\delta\varphi &= (r - r_+)^{-i\omega/4\pi T} \left(\delta\varphi^{(+)} + \dots \right), \\
\delta h_{tx_1} &= (r - r_+)^{-i\omega/4\pi T} \left(\delta h_{tx_1}^{(+)}(r - r_+) + \dots \right),
\end{aligned} \tag{3.4}$$

where the dots refer to terms higher order in $(r - r_+)$. The regularity of this perturbation at the black horizon can be seen by using ingoing Eddington-Finkelstein coordinates (v, r) with $v = t + \log(r - r_+)^{\frac{1}{4\pi T}}$. Using the equations of motion we find that this expansion is fixed by two parameters $\delta a_{x_1}^{(+)}, \delta\varphi^{(+)}$ with

$$\delta h_{tx_1}^{(+)} = -\frac{a_+\delta a_{x_1}^{(+)} + 2k\varphi_+\delta\varphi^{(+)}}{r_+^2(1 - i\frac{\omega}{4\pi T})}. \tag{3.5}$$

In the UV we impose that as $r \rightarrow \infty$:

$$\begin{aligned}
\delta h_{tx_1} &= \delta h_{tx_1}^{(0)}r^2 + \dots, \\
\delta a_{x_1} &= \delta a_{x_1}^{(0)} + \frac{\delta a_{x_1}^{(1)}}{r} + \dots, \\
\delta\varphi &= \frac{\delta\varphi^{(0)}}{r^{3-\Delta}} + \dots + \frac{\delta\varphi^{(1)}}{r^\Delta} + \dots
\end{aligned} \tag{3.6}$$

Now we are interested in a perturbation that switches on an electric field and then we want to read off the current to obtain the conductivity. One might be tempted to set $\delta h_{tx_1}^{(0)} = \delta\varphi^{(0)} = 0$ but this over constrains the system. To see this we note that a solution to the ODEs (3.3) is specified by five parameters. From the IR and UV expansions (3.4), (3.6) we have a total of seven parameters. However, since the ODEs (3.3) are linear we can scale one of the seven parameters to unity, leaving six. This means that we need to impose just one more constraint on the parameters. This constraint can be found as follows.

To ensure that we are extracting just the current-current correlator, we can use diffeomorphisms and gauge-transformations to demand that the perturbation satisfies, as $r \rightarrow \infty$,

$$\begin{aligned}
&\frac{1}{r^2}(\delta g_{\mu\nu} + \mathcal{L}_\zeta g_{\mu\nu}) \rightarrow 0, \\
&\delta A + \mathcal{L}_\zeta A + d\Lambda \rightarrow e^{-i\omega t}\mu_{x_1}dx_1, \\
&r^{3-\Delta}(\delta\phi + \mathcal{L}_\zeta\phi) \rightarrow 0,
\end{aligned} \tag{3.7}$$

where ζ^μ and Λ are smooth and μ_{x_1} will be the source for the current. For our specific set-up we can take $\Lambda = 0$ and the only non-vanishing component of ζ^μ to be $\zeta^x = \epsilon e^{-i\omega t}$

where ϵ is a small parameter. From this we can deduce that we have $\mu_{x_1} = \delta a_{x_1}^{(0)}$ and that we should impose the condition

$$\delta\varphi^{(0)} - i\frac{k\lambda}{\omega}\delta h_{tx_1}^{(0)} = 0. \tag{3.8}$$

The optical conductivity is then given by

$$\sigma(\omega) = -\frac{i}{\omega} \frac{\delta a_{x_1}^{(1)}}{\delta a_{x_1}^{(0)}}. \tag{3.9}$$

The DC resistivity is given by $\rho = 1/\sigma(0)$. It is worth mentioning that to calculate ρ numerically, one needs to calculate the optical conductivity for $\omega \ll T$.

4 Final comments

We have studied holographic Q-lattices for Einstein-Maxwell theory coupled to a single complex scalar field in $D = 4$ space-time dimensions. We have shown that the system exhibits both metallic and insulating phases. The metallic phase is governed by the electrically charged $AdS_2 \times \mathbb{R}^2$ solution that appears in the IR region of the $T = 0$ electrically charged AdS-RN solution. We showed in detail that the phase exhibits a Drude-type peak and furthermore, at low temperatures the DC resistivity exhibits a scaling behaviour confirming the prediction of [12].

We have also constructed Q-lattice black holes in a new insulating phase down to very low temperatures. For temperatures lower than $T/\mu \sim 10^{-3}$ we see a transferral of spectral weight in the optical conductivity and the generation of a mid frequency hump. At temperatures $T/\mu \sim 2.8 \times 10^{-5}$ we have found evidence for a first order transition to another branch of insulating black holes. It would be interesting to investigate these further including trying to elucidate the ultimate IR ground states at $T = 0$ which seem to have vanishing entropy density. A possibly related issue, is to further understand the role played by the neutral $AdS_3 \times \mathbb{R}$ ground state that we have found and discussed in the appendix.

We focussed on the case where the mass of the complex scalar is given by $m^2 = -3/2$ with $\Delta = (3 + \sqrt{3})/2$ in the $d = 3$ CFT, which saturates the $AdS_2 \times \mathbb{R}^2$ BF bound, corresponding to a stable metallic phase. We have also made some numerical investigations into the case $m^2 = -2$ with $\Delta = 2$ in the $d = 3$ CFT. We have constructed black holes with conductivities exhibiting metallic and insulating behaviours much as in figure 1. However, for this case the complex scalar violates the $AdS_2 \times \mathbb{R}^2$ BF bound and hence, at least for the metallic black holes, one will find an additional new phase appearing at low temperatures.⁸ When there is no lattice deformation a possible ground state for this model was identified in [26]. It will be interesting to see how this is modified by the lattice deformation and also to investigate the impact on the insulating phase.

It is also natural to consider a more general class of models including a coupling of the scalar field to the gauge field and a more general potential than the simple mass term. We

⁸The same is true for the model considered in [1].

expect that within this more general class of models it will be possible to obtain the many novel IR ground states in explicit form [23]. It will be particularly interesting to explore interconnections with charge density waves [27] which should lead to close analogues of Mott insulating ground states. Such models can be studied in various spacetime dimensions.

For the Q-lattices that we have constructed for specific values of lattice strength λ and wave-number k , for both $m^2 = -3/2$ and $m^2 = -2$, we find no evidence that the metallic phase has an intermediate scaling of the form (1.1). How can this be reconciled with the results reported in [1–3, 5], where numerical evidence for this behaviour was found and moreover it was suggested that this might be a universal feature of holographic lattices? One possibility is that the numerical evidence found in those papers is actually misleading and in fact there is not a robust power-law behaviour for the lattices considered.

An interesting perspective is to consider the same model (2.1) that we have in this paper, but with a family of lattice deformations, labelled by α , given by

$$\phi = \sqrt{2}\lambda (\cos \alpha \cos kx_1 + i \sin \alpha \sin kx_1) \frac{1}{r^{3-\Delta}} + \dots \tag{4.1}$$

as $r \rightarrow \infty$. For $\alpha = \pi/4$ this gives the family Q-lattices that we discussed in this paper, while for $\alpha = 0$ it gives the lattices discussed in [2] (who just considered $m^2 = -2$). Notice that the strength of the lattice, λ , does not depend on α and also that for $\alpha \neq (2n+1)\pi/4$, for integer n , the metric will be co-homogeneity two and one will need to solve PDEs.

For this general family of lattices we can use the results of [12] and also of [27, 28] to deduce the scaling behaviour of the DC resistivity in the metallic phase. In addition to the scalar mode with wave-number k , with dimension (2.14) in the IR, one also needs to take into account⁹ longitudinal modes involving perturbations in A_t, A_{x_1} and $g_{tt}, g_{x_1x_1}, g_{tx_1}, g_{x_2x_2}$ and with wave-number $2k$ (corresponding to the fact that the scalar lattice sources them at least at quadratic order). From the analysis presented in [27] (in particular equation (2.17)), one can deduce that when $m^2 \leq -1/4$ the DC resistivity scaling will always be determined by the decoupled scalar mode in the IR. Interestingly for $-1/4 < m^2 < 0$, for certain windows of k , the scaling can be determined by the longitudinal modes. Note in particular, for the scalar lattice in [1] with $m^2 = -2$ and $\alpha = 0$, we are arguing that the DC resistivity scaling is actually governed by the scalar mode and not one of the longitudinal modes as was stated in [1]. Note that this work also claimed to see a numerical fit to a scaling governed by the longitudinal mode: we believe that the fitting was misleading and that continuing to lower temperatures will reveal the scaling behaviour that we are predicting.

It is also worth pointing out that we do not expect the black hole solutions will be substantially different as we vary α away from $\pi/4$, despite the fact that one is solving PDEs instead of ODEs as in this paper. While additional harmonics of the bulk fields will play a role, the higher harmonics are expected to be exponentially suppressed. In fact this was seen in the numerical work in [1]. Thus it is natural to expect that conductivity for non-zero ω is also not substantially different from what we have seen in this paper.

⁹Note that there will also be scalar modes with wave-number nk and longitudinal modes with wave-number $2nk$, for $n > 1$, but these will be more irrelevant in the IR and hence will not dominate the scaling of the DC resistivity.

All of the constructions in this paper have just involved classical gravity. It is worth recalling, however, that there are good reasons to expect that there are no global symmetries in theories of quantum gravity (e.g. [29]). One point of view is that we are just studying a sector of a larger classical theory that does not have a global symmetry. Alternatively we can view the breaking of the continuous symmetry as a higher order effect in the large N expansion. Within these contexts, or closely related ones, we think that top-down constructions should be possible.

Finally we point out that the holographic lattice constructions that we have discussed in this paper, where the translation symmetry is broken explicitly, can also be adapted to situations where the symmetry is broken spontaneously.

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A A novel $AdS_3 \times \mathbb{R}$ solution

Provided that $m^2 < 0$ (equivalently, the operator dual to ϕ in the $d = 3$ CFT dual to the AdS_4 vacuum is a relevant operator), the model (2.1) admits an electrically neutral $AdS_3 \times \mathbb{R}$ solution given by

$$\begin{aligned} ds^2 &= \frac{1}{3} ds^2(AdS_3) + dx_1^2, \\ \phi &= \frac{6}{-m^2} e^{i\sqrt{-m^2}x_1}, \end{aligned} \tag{A.1}$$

with $A = 0$.

To explore whether there are domain wall solutions which can connect this solution with AdS_4 , we investigate the spectrum for this fixed point. Within our ansatz (2.6) we can consider the perturbations given by

$$\begin{aligned} U &= 3r^2(1 + u_1 r^\delta), & V_1 &= v_{11} r^\delta, & V_2 &= \log(r) + v_{21} r^\delta, \\ a &= a_1 r^{1+\delta}, & \phi &= \left(\frac{6}{-m^2}\right)^{1/2} e^{i\sqrt{-m^2}x_1} \phi_1 r^\delta. \end{aligned} \tag{A.2}$$

These perturbations correspond to scaling dimension $\Delta = -\delta$ or $\Delta = \delta + 2$ in the $d = 2$ CFT dual to the $AdS_3 \times \mathbb{R}$ solution. We find that the exponents come in four pairs with $\delta_+ + \delta_- = -2$ and there is an unpaired mode with $\delta = -1$. The paired modes have δ_+ values given by 0, -1 and

$$\delta_1 = -1 + \frac{1}{\sqrt{3}} \sqrt{9 - 2\sqrt{3}\sqrt{3 - m^2}}, \quad \delta_2 = -1 + \frac{1}{\sqrt{3}} \sqrt{9 + 2\sqrt{3}\sqrt{3 - m^2}}. \tag{A.3}$$

We see that in the mass range $-9/4 \leq m^2 < 0$, which is relevant for trying to map onto AdS_4 in the UV, δ_1 corresponds to a relevant operator (i.e. $\delta_1 < 0$) and δ_2 corresponds to an irrelevant operator (i.e. $\delta_2 > 0$). Note that both of these deformations have $a_1 = 0$ in (A.2) and do not involve the gauge-field.

A parameter count now reveals that, generically, because of the presence of the relevant operator, there will not be domain wall solutions interpolating between the lattice deformed AdS_4 in the UV and $AdS_3 \times \mathbb{R}$ in the IR. However, there is the possibility that there is a fine-tuned domain wall solution. If this exists it might correspond to a bifurcating, unstable RG solution, separating the metallic and insulating behaviours, as in figure 2 of [4].

More generally, we expect that there are closely related models where the $AdS_3 \times \mathbb{R}$ geometry has irrelevant operators in the IR so that one can construct domain walls that interpolate from the Q-lattice deformed AdS_4 in the UV. Furthermore, changing the dimension of space-time and the number, n , of spatial directions where translation invariance is broken by the holographic Q-lattice will allow one to construct domain walls from AdS_D in the UV and various $AdS_{D-n} \times \mathbb{R}^n$ in the IR. This will be explored in detail elsewhere.

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