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## Holography for cosmology

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We propose a holographic description of four-dimensional single-scalar inflationary universes, and show how cosmological observables, such as the primordial power spectrum, are encoded in the correlation functions of a three-dimensional quantum field theory (QFT). The holographic description correctly reproduces standard inflationary predictions in the regime where a perturbative quantization of fluctuations is justified. In the opposite regime, wherein gravity is strongly coupled at early times, we propose a holographic description in terms of perturbative large  $N$  QFT. Initiating a holographic phenomenological approach, we show that models containing only two parameters,  $N$  and a dimensionful coupling constant, are capable of satisfying the current observational constraints.

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### I. INTRODUCTION

Over the past two decades, striking new observations have transformed cosmology from a qualitative to a quantitative science. A minimal set of cosmological parameters characterizing the observed universe, the concordance cosmology, have now been measured to within a few percent [1]. These observations reveal a spatially flat universe, endowed with small-amplitude primordial perturbations that are approximately Gaussian and adiabatic with a nearly scale-invariant spectrum. This data is consistent with the generic predictions of inflationary cosmology and set inflation as the leading theoretical paradigm for the initial conditions of big bang cosmology. With future observations promising an unprecedented era of precision cosmology, the constraints on cosmological parameters are expected to tighten further still, particularly as regards the inflationary sector. This presents a unique window to Planck-scale physics and a challenge for fundamental theory.

During the past decade we have also witnessed exciting new developments in fundamental theory. Holographic dualities have been proposed and developed leading to a new viewpoint for physical reality. Holography states that any quantum theory of gravity should have a description in terms of a quantum field theory (QFT) which does not contain gravity in one dimension less. It is natural to wonder how cosmology fits into the holographic framework and the main aim of this paper is to propose a concrete holographic framework for inflationary cosmology.

Apart from the conceptual advances that such a development would imply, there are also a number of more pragmatic reasons for developing such a framework. First, uncovering the structure of three-dimensional QFT in cosmological observables brings in new intuition about their

structure and may lead to more efficient computational techniques, cf. the computation of non-Gaussianities in [2]. Second, holographic dualities are strong/weak coupling dualities meaning that in the regime where the one description is weakly coupled the other is strongly coupled. This provides an arena for constructing new models with intrinsic strong-coupling gravitational dynamics at early times that have only a weakly coupled three-dimensional QFT description and are thus outside the class of model described by standard inflation. Such models may well lead to qualitatively different predictions for the cosmological observables that will be measured in the near future.

Any holographic proposal should specify what the dual QFT is and how to use it to compute cosmological observables. The holographic description we propose uses the one-to-one correspondence between cosmologies and domain-wall spacetimes discussed in [3,4] and assumes that the standard gauge/gravity duality is valid. More precisely, the steps involved are illustrated in Fig. 1. The first step is to map any given inflationary model to a domain-wall spacetime. For inflationary cosmologies that at late times approach either a de Sitter spacetime or a power-law scaling solution [5], the corresponding domain-wall solutions describe holographic renormalization group flows.

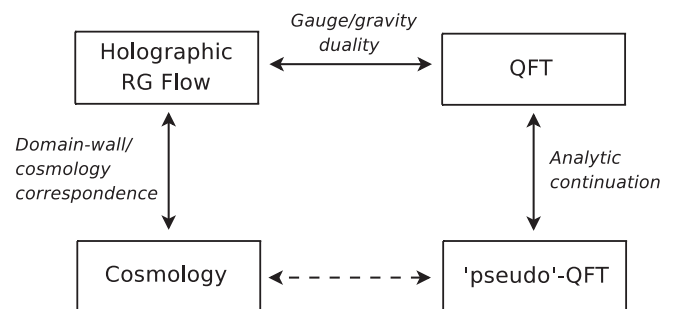


FIG. 1. The “pseudo” QFT dual to inflationary cosmology is operationally defined using the correspondence of cosmologies to domain walls and standard gauge/gravity duality.

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For these cases there is an operational gauge/gravity duality; namely, one has a dual description in terms of a three-dimensional QFT. Now, the map between cosmologies and domain walls can equivalently be expressed entirely in terms of QFT variables, and amounts to a certain analytic continuation of parameters and momenta. Applying this analytic continuation we obtain the QFT dual of the original cosmological spacetime.

We shall call the resulting theory a “pseudo” QFT because we currently only have an operational definition of this theory. Namely, we do the computations in the QFT theory dual to the corresponding domain wall and then apply the analytic continuation. Perhaps a more fundamental perspective is to consider the QFT action, with complex parameters and complex fields as the fundamental objects, and then to consider the results on different real domains as applicable to either domain walls or cosmologies. Note that the supergravity embedding of the domain-wall/cosmology correspondence discussed in [6] works in precisely this way.

The holographic description should reproduce standard inflationary results in their regime of applicability, namely, when the fluctuations around the cosmological background can be perturbatively quantized. New results should follow by applying the duality in the cases where such a perturbative quantization of fluctuations is not justified. In the remainder of this paper, we show first that the standard results are indeed reproduced, then move to construct new models which are strongly coupled at early times but have a weakly coupled large  $N$  QFT description.

## II. DOMAIN-WALL/COSMOLOGY CORRESPONDENCE

For simplicity, we focus on spatially flat universes equipped with a single minimally coupled scalar field, but the results can be extended to more general cases (e.g., nonflat, multiscalar, noncanonical kinetic terms, etc.) The linearly perturbed metric and scalar field may be written in the form

$$\begin{aligned} ds^2 &= \eta dz^2 + a^2(z)[\delta_{ij} + h_{ij}(z, \vec{x})]dx^i dx^j, \\ \Phi &= \varphi(z) + \delta\varphi(z, \vec{x}), \end{aligned} \quad (1)$$

where  $\eta = -1$  in the case of cosmology, in which case  $z$  is the time coordinate, and  $\eta = +1$  in the case of domain-wall solutions, in which case  $z$  is the radial coordinate. We take the domain wall to be Euclidean for later convenience. A Lorentzian domain wall may be obtained by continuing one of the  $x^i$  coordinates to become the time coordinate [4]. The continuation to a Euclidean domain wall is convenient, however, because the QFT vacuum state implicit in the Euclidean formulation maps to the Bunch-Davies vacuum on the cosmology side. Other choices of cosmological vacuum require considering the boundary QFT in different

states, as may be accomplished using the real-time formalism of [7].

The actions for both the cosmology and the Euclidean domain wall may be written in the combined form,

$$S = \frac{\eta}{2\kappa^2} \int d^4x \sqrt{|g|} [-R + (\partial\Phi)^2 + 2\kappa^2 V(\Phi)], \quad (2)$$

where  $\kappa^2 = 8\pi G$  and we take the scalar field  $\Phi$  to be dimensionless. For background solutions in which the evolution of the scalar field is (piecewise) *monotonic*,  $\varphi(z)$  can be inverted to give  $z(\varphi)$ , allowing the Hubble rate  $H = \dot{a}/a$  to be expressed in terms of some “fake superpotential”  $W(\varphi)$  as  $H(z) = -(1/2)W(\varphi)$ . The complete equations for the background are then

$$\frac{\dot{a}}{a} = -\frac{1}{2}W, \quad \dot{\varphi} = W', \quad 2\eta\kappa^2 V = W'^2 - \frac{3}{2}W^2, \quad (3)$$

where  $W' = dW/d\varphi$ . This first-order formalism goes back to the work of [8] (for cosmology), where it was obtained by application of the Hamilton-Jacobi method. In [4] this formalism was linked to the notion of (fake) (pseudo) supersymmetry.

The equations of motion for the perturbations are

$$\begin{aligned} 0 &= \ddot{\zeta} + (3H + \dot{\epsilon}/\epsilon)\dot{\zeta} - \eta a^{-2} q^2 \zeta, \\ 0 &= \ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} - \eta a^{-2} q^2 \gamma_{ij}, \end{aligned} \quad (4)$$

where  $\vec{q}$  is the comoving wave vector of the perturbations, and the background quantity  $\epsilon(z)$  is defined as  $\epsilon = -\dot{H}/H^2 = 2(W'/W)^2$ .  $\gamma_{ij}$  is a transverse traceless metric perturbation and  $\zeta = \psi + (H/\dot{\varphi})\delta\varphi$  is the standard gauge-invariant variable constructed from a metric perturbation,  $h_{ij} = -2\psi(z, \vec{x})\delta_{ij}$ , and the scalar perturbation  $\delta\varphi$ .

Defining now the analytically continued variables  $\bar{\kappa}$  and  $\bar{q}$  according to

$$\bar{\kappa}^2 = -\kappa^2, \quad \bar{q} = -iq, \quad (5)$$

it is easy to see that a perturbed cosmological solution written in terms of the variables  $\kappa$  and  $q$  continues to a perturbed Euclidean domain-wall solution expressed in terms of the variables  $\bar{\kappa}$  and  $\bar{q}$ .

We have thus established that the correspondence between cosmologies and domain walls holds, not only for the background solutions, but also for linear perturbations around them. This is the basis for the relation between power spectra and holographic two-point functions, to be discussed momentarily. The argument can be generalized to arbitrary order to relate non-Gaussianities to holographic higher-point functions [9].

## III. QUANTIZATION OF PERTURBATIONS

In the inflationary paradigm, cosmological perturbations originate on subhorizon scales as quantum fluctuations of the vacuum. Quantizing the perturbations in the usual

manner, one finds the scalar and tensor superhorizon power spectra

$$\begin{aligned}\Delta_S^2(q) &= \frac{q^3}{2\pi^2} \langle \zeta(q)\zeta(-q) \rangle = \frac{q^3}{2\pi^2} |\zeta_{q(0)}|^2, \\ \Delta_T^2(q) &= \frac{q^3}{2\pi^2} \langle \gamma_{ij}(q)\gamma_{ij}(-q) \rangle = \frac{2q^3}{\pi^2} |\gamma_{q(0)}|^2,\end{aligned}\quad (6)$$

where  $\gamma_{q(0)}$  and  $\zeta_{q(0)}$  are the constant late-time values of the cosmological mode functions  $\gamma_q(z)$  and  $\zeta_q(z)$ .

The mode functions are themselves solutions of the classical equations of motion (4) (setting  $\gamma_{ij} = \gamma_q e_{ij}$ , for some time-independent polarization tensor  $e_{ij}$ ). To select a unique solution for each mode function, we impose the Bunch-Davies vacuum condition  $\zeta_q, \gamma_q \sim e^{-iq\tau}$  as  $\tau \rightarrow -\infty$ , where the conformal time  $\tau = \int^z dz'/a(z')$ . The normalization of each solution (up to an overall phase) may then be fixed by imposing the canonical commutation relations for the corresponding quantum fields. This leads to the Wronskian conditions,

$$i = \zeta_q \Pi_q^{(\zeta)*} - \Pi_q^{(\zeta)} \zeta_q^*, \quad i/2 = \gamma_q \Pi_q^{(\gamma)*} - \Pi_q^{(\gamma)} \gamma_q^*, \quad (7)$$

where  $\Pi_q^{(\zeta)} = 2\epsilon a^3 \kappa^{-2} \dot{\zeta}_q$  and  $\Pi_q^{(\gamma)} = (1/4)a^3 \kappa^{-2} \dot{\gamma}_q$  are the canonical momenta associated with each mode function, and we have set  $\hbar$  to unity.

To make connection with the holographic analysis to follow, we introduce the linear response functions  $E$  and  $\Omega$  satisfying

$$\Pi_q^{(\zeta)} = \Omega \zeta_q, \quad \Pi_q^{(\gamma)} = E \gamma_q. \quad (8)$$

(These quantities are well defined since we have already selected a unique solution for each mode function.) Substituting these definitions into the Wronskian conditions, which are valid at all times, the cosmological power spectra may be reexpressed as

$$\Delta_S^2(q) = \frac{-q^3}{4\pi^2 \text{Im}\Omega_{(0)}(q)}, \quad \Delta_T^2(q) = \frac{-q^3}{2\pi^2 \text{Im}E_{(0)}(q)}, \quad (9)$$

where  $\text{Im}\Omega_{(0)}$  and  $\text{Im}E_{(0)}$  are the constant late-time values of the imaginary part of the response functions. We will see shortly how the response functions also give the two-point functions of the pseudo-QFT.

Let us now consider the corresponding domain-wall solution obtained by applying the continuation (5). The early-time behavior  $\sim e^{-iq\tau}$  of the cosmological perturbations maps to the exponentially decaying behavior  $\sim e^{\bar{q}\tau}$  in the interior of the domain wall ( $\tau \rightarrow -\infty$ ). Such regularity in the interior is a prerequisite for holography, explaining our choice of sign in the continuation of  $q$ .

The domain-wall response functions  $\bar{E}$  and  $\bar{\Omega}$  [10] are defined analogously to (8), namely,

$$\bar{\Pi}_{\bar{q}}^{(\zeta)} = -\bar{\Omega} \zeta_{\bar{q}}, \quad \bar{\Pi}_{\bar{q}}^{(\gamma)} = -\bar{E} \gamma_{\bar{q}}, \quad (10)$$

where  $\bar{\Pi}_{\bar{q}}^{(\zeta)} = 2\epsilon a^3 \bar{\kappa}^{-2} \dot{\zeta}_{\bar{q}}$  and  $\bar{\Pi}_{\bar{q}}^{(\gamma)} = (1/4)a^3 \bar{\kappa}^{-2} \dot{\gamma}_{\bar{q}}$  are the radial canonical momenta. The minus signs are inserted so that  $\bar{\Omega}(-iq) = \Omega(q)$  and  $\bar{E}(-iq) = E(q)$ . By choosing the arbitrary overall phase of the cosmological perturbations appropriately, we may ensure that the domain-wall perturbations are everywhere real. The domain-wall response functions are then purely real, while their cosmological counterparts are complex.

## IV. HOLOGRAPHIC ANALYSIS

There are two classes of domain-wall solutions for which holography is well understood.

### A. Asymptotically anti-de Sitter spacetime (AdS) domain walls

In this case the solution behaves asymptotically as

$$a(z) \sim e^z, \quad \varphi \sim 0 \quad \text{as } z \rightarrow \infty. \quad (11)$$

The boundary theory has a UV fixed point which corresponds to the bulk AdS critical point. Depending on the rate at which  $\varphi$  approaches zero as  $z \rightarrow \infty$ , the QFT is either a deformation of the conformal field theory (CFT) or else the CFT in a state in which the dual scalar operator acquires a nonvanishing vacuum expectation value (see [11] for details). Under the domain-wall/cosmology correspondence, these solutions are mapped to cosmologies that are asymptotically de Sitter at late times.

### B. Asymptotically power-law solutions

In this case the solution behaves asymptotically as

$$a(z) \sim (z/z_0)^n, \quad \varphi \sim \sqrt{2n} \log(z/z_0) \quad \text{as } z \rightarrow \infty, \quad (12)$$

where  $z_0 = n - 1$ . This case has only very recently been understood [12]. For  $n = 7$  the asymptotic geometry is the near-horizon limit of a stack of D2 brane solutions. In general, these solutions describe QFTs with a dimensionful coupling constant in the regime where the dimensionality of the coupling constant drives the dynamics. Under the domain-wall/cosmology correspondence, these solutions are mapped to cosmologies that are asymptotically power law at late times.

Holographic two-point functions are now obtained by solving the linearized equations of motion about the domain-wall solution with Dirichlet boundary conditions at infinity and imposing regularity in the interior. It will suffice to discuss the two-point function for the energy-momentum tensor. On general grounds, the two-point function takes the form

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl}, \quad (13)$$



where  $\Pi_{ijkl}$  is the three-dimensional transverse traceless projection operator defined by  $\Pi_{ijkl} = \pi_{i(k}\pi_{l)j} - (1/2)\pi_{ij}\pi_{kl}$  and  $\pi_{ij} = \delta_{ij} - \bar{q}_i\bar{q}_j/\bar{q}^2$ . The holographic computation amounts to the extracting the coefficients  $A$  and  $B$  from the asymptotics of the linearized solution. Using the radial Hamiltonian method developed in [10], one finds

$$A(\bar{q}) = 4\bar{E}_{(0)}(\bar{q}), \quad B(\bar{q}) = \frac{1}{4}\bar{\Omega}_{(0)}(\bar{q}), \quad (14)$$

where the zero subscript indicates the leading (constant) term as  $z \rightarrow \infty$ , after holographic renormalization has been performed [10,13].

## V. CONTINUATION TO THE PSEUDO-QFT

We now wish to reexpress the bulk analytic continuation (5) in terms of QFT variables. This amounts to

$$\bar{N}^2 = -N^2, \quad \bar{q} = -iq, \quad (15)$$

where the barred quantities are associated with the QFT dual to the domain wall. We thus find that the power spectrum for any inflationary cosmology that is asymptotically de Sitter or asymptotically power law can be directly computed from the two-point function of a three-dimensional QFT via the formulas

$$\Delta_S^2(q) = \frac{-q^3}{16\pi^2 \text{Im}B(-iq)}, \quad \Delta_T^2(q) = \frac{-2q^3}{\pi^2 \text{Im}A(-iq)}. \quad (16)$$

## VI. BEYOND THE WEAK GRAVITATIONAL DESCRIPTION

In the discussion so far we have assumed that the description in terms of gravity coupled to a scalar field is valid at early times, and that the perturbative quantization of fluctuations can be justified. The holographic description also allows us to obtain results when these assumptions do not hold. At early times, the theory may be strongly coupled with no useful description in terms of low-energy fields (such as the metric and the scalar field). The holographic setup allows us to extract the late-time behavior of the system, which can be expressed in terms of low-energy fields, from QFT correlators. (For further details see [13].) Here we only note that this is the counterpart of the standard fact that in gauge/gravity duality the asymptotic behavior of bulk fields near the boundary of spacetime is reconstructed by the correlators of the dual QFT [14]. This late-time behavior is precisely the information we need to compute the primordial power spectra and other cosmological observables.

Ideally, one would deduce from a string/M-theoretic construction what the dual QFT is. Instead we initiate here a holographic phenomenological approach. The dual QFT would involve scalars, fermions, and gauge fields and

it should admit a large  $N$  limit. The question is then whether one can find a theory which is compatible with current observations. In particular, one might consider either deformations of CFTs or theories with a single dimensionful parameter, as these QFTs have already featured in our discussion above.

We will discuss here super-renormalizable theories that contain one dimensionful coupling constant. A prototype example is  $SU(N)$  Yang-Mills theory coupled to a number of scalars and fermions, all in the adjoint of  $SU(N)$ . To extract predictions we need to compute the coefficients  $A$  and  $B$  of the two-point function of the energy-momentum tensor (13), analytically continue the results, and insert them in the holographic formulas for the power spectra.

First, the leading contribution to the two-point function of the energy-momentum tensor is at one loop. Since the energy-momentum tensor has dimension three,

$$A(\bar{q}) \sim \bar{N}^2 \bar{q}^3, \quad B(\bar{q}) \sim \bar{N}^2 \bar{q}^3. \quad (17)$$

A generic such model thus leads to a scale-invariant spectrum at leading order in  $N$ . To fix the parameters of these models, we may then compare with observations. Comparing the observed amplitude of the scalar power spectrum [1] with its holographic value we find  $N \sim O(10^4)$ , justifying the large  $N$  limit. To determine the coupling constant  $g_{\text{YM}}^2$  we may compare with the tilt of the spectrum. The precise formula requires a two-loop computation [15] and will be reported elsewhere [9]. One

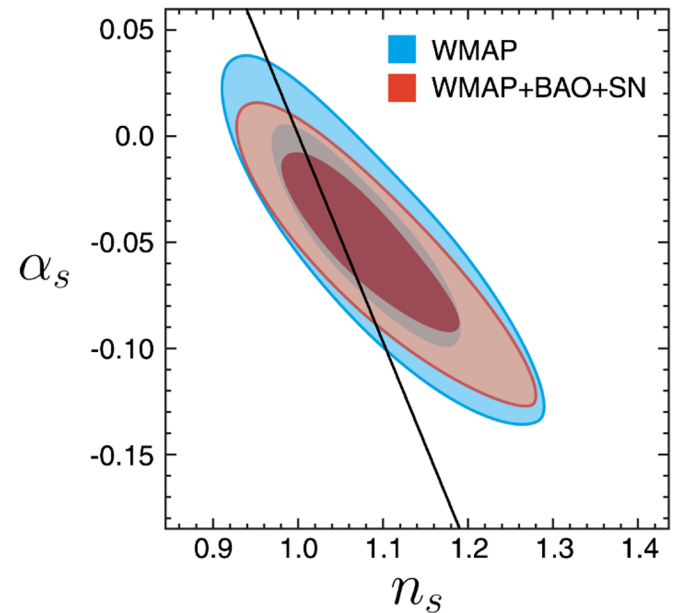


FIG. 2 (color online). The straight line is the leading order prediction of holographic models with a single dimensionful coupling constant for the correlation of the running  $\alpha_s$  and the scalar tilt  $n_s$ . The data show the 68% and 95% C.L. constraints (marginalizing over tensors) at  $q = 0.002 \text{ Mpc}^{-1}$ , and are taken from Fig. 4 of [1]. As new data appear the allowed region should shrink to a point, which is predicted to lie close to the line.

can obtain an order of magnitude estimate, however, on general grounds. The perturbative expansion depends on the effective dimensionless coupling constant  $g_{\text{eff}}^2 = g_{\text{YM}}^2 \bar{N} / \bar{q}$ , and the leading correction to the two-point function yields  $(n_s - 1) = c g_{\text{eff}}^2$ , where the constant  $c$  is of order one and depends on the details of the theory. From Table 4 of [1], one finds that  $(n_s - 1) \sim O(10^{-2})$ , thus we find that  $g_{\text{eff}}^2 \sim O(10^{-2})$  justifying the perturbative QFT treatment. Independently of the details of the theory, the scalar index runs:  $\alpha_s = dn_s/d \ln q = -(n_s - 1) + O(g_{\text{eff}}^4)$ . This prediction is qualitatively different from slow-roll inflation [for which  $\alpha_s/(n_s - 1)$  is of first-order in slow-roll [17]], yet is nonetheless consistent with the constraints on  $n_s$  and  $\alpha_s$  given in [1] for a wide range of values of  $n_s$  and  $\alpha_s$ , as illustrated in Fig. 2. The ratio of tensor to scalar power spectra can be computed from (16), yielding  $r = 32 \text{Im}B(-iq)/\text{Im}A(-iq)$ . For massless scalars and for vector fields  $A = B = (1/256)\bar{N}^2\bar{q}^3$  (for conformally coupled scalars  $B = 0$  instead), and for massless fermions  $A = (1/128)\bar{N}^2\bar{q}^3$  and  $B = 0$ . With appropriate field content, one can thus satisfy the current observational bound on  $r$ .

Once  $N$ ,  $g_{\text{YM}}^2$ , and the field content are fixed, all other cosmological observables (such as non-Gaussianities, etc.) follow uniquely from straightforward computations. We will present details of the correspondence between higher-order QFT correlation functions and non-Gaussian cosmological observables elsewhere [9]. Our results indicate, however, that the non-Gaussianity parameter  $f_{\text{NL}}^{\text{local}}$  [18] is independent of  $N$  to leading order, consistent with current observational evidence [1].

## VII. CONCLUSIONS

We have presented a concrete proposal describing holography for cosmology, and initiated a holographic phenomenological approach capable of satisfying current observational constraints. Clearly, one would like to further develop holographic phenomenology and obtain precise predictions for the cosmological observables to be measured by forthcoming experiments. We hope to report on this in the near future.

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