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HOMEWORK IN MACROECONOMICS:

HOUSEHOLD PRODUCTION AND AGGREGATE FLUCTUATIONS

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ABSTRACT

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This paper explores some macroeconomic implications of including household production in an otherwise standard real business cycle model. We calibrate the model based on microeconomic evidence and long run considerations, simulate it, and examine its statistical properties. Our finding is that introducing home production significantly improves the quantitative performance of the standard model along several dimensions. It also implies a very different interpretation of the nature of aggregate fluctuations.

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## I. Introduction

This project explores some of the implications of introducing home, or nonmarket, production into an otherwise standard model of aggregate fluctuations. This is of interest because the household sector is large, whether measured in terms of inputs or output. Data from the Michigan Time Use Survey indicate that an average married couple spends 33 percent of their discretionary time working for paid compensation, and 28 percent, only slightly less, working in the home (see Table 1 in Benhabib, Rogerson and Wright 1990a, which is based on the data summarized in Hill 1985; see also Juster and Stafford 1990 for a recent description of time use data from other countries and periods).<sup>1</sup> Studies that attempt to measure the value of household production indicate that it is also large; Eisner's (1988) recent summary of this literature suggests an estimate of home produced output relative to measured GNP in the range of 20 - 50 percent.

These facts lead us to conclude that home production is an empirically significant entity at the aggregate level. In light of this, why is it conspicuously absent from most models of aggregate economic activity? One possible conjecture is that the behavior of the home sector is approximately independent of the market. However, the evidence indicates that individuals employed in the market sector spend much less time working in the home than unemployed individuals and also that employed agents with higher wages substitute out of home and into market production (see Benhabib, Rogerson

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<sup>1</sup> Discretionary time includes market work, homework, and leisure, all three of which are measured directly in the time use survey data. Equivalently, discretionary time is total time minus personal care, which is mainly sleep, and a few other miscellaneous activities.

and Wright 1990a, Tables 2 and 3, and Rios-Rull 1990). This suggests that the home sector is not only large, but that there is a good deal of substitutability between it and the market, and leads us to believe that household production may be an important missing element in existing models of the aggregate economy.<sup>2</sup>

Our plan is to incorporate a home production sector into a simple real business cycle model of the type pioneered by Kydland and Prescott (1982), by assuming that households have access to home production functions that use time and capital to produce a nontradable consumption good. We expect, *a priori*, this will make a difference, for the following reasons. When individuals are able to substitute between market and nonmarket production over time, volatility in market activity can arise due to relative productivity differentials between the two sectors, and not just absolute productivity shocks, as is the case in one sector models. Furthermore, the size of the fluctuations induced by productivity shocks will depend on the degree to which agents are willing to substitute between home and market commodities (both time and goods) at a given date, and not just the degree to which they are willing to substitute between these commodities at different dates, as is the case in the standard real business cycle model.

To facilitate comparison, we stay as close as possible to the specification of the stochastic growth model described in Hansen (1985),

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<sup>2</sup> This has previously been argued by Becker (1988), although he stresses *family behavior*, while we abstract from issues such as marriage, fertility, and so on, in this paper. Rios-Rull (1990) and Greenwood and Hercowitz (1990) have also recently emphasized the importance of home production for macroeconomics.

Prescott (1986), or Plosser (1989), for example.<sup>3</sup> It has been established that this framework does fairly well at accounting for certain salient aspects of the postwar U.S. data. Using functional forms and parameter values that conform to microeconomic studies and long run observations, it accounts for a sizeable fraction of observed fluctuations in macroeconomic variables at cyclical frequencies, given reasonable estimates of the process of technological change. Further, the model is consistent with phenomena such as consumption being less volatile than output and investment more volatile than output, as observed not only in the postwar U.S. economy, but also across many countries and time periods (see, e.g., Bacus and Kehoe 1989). Nevertheless, it is apparent that this model does not do as well along some dimensions as it does others.

We intend to focus on the following six problems with the standard real business cycle model. Compared to the data, in the model:

1. output fluctuates too little;
2. relative to output, labor hours fluctuate too little;
3. relative to output, consumption fluctuates too little;
4. relative to output, investment fluctuates too much;
5. productivity's correlation with output or hours is far too high;
6. labor hours used to produce consumption goods are out of phase.

Most of these have been recognized before (an exception is problem 6; we will argue, however, that it is the key to understanding the others). Also,

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<sup>3</sup> Our message is meant to be independent of whether or not market failures, taxes, money, information problems, frictional unemployment, or other complications are empirically important; nevertheless, we stick to a very basic version of the model in order to isolate the impact of home production.

various extensions of the basic framework are known to help resolve some of these problems. We will demonstrate that introducing home production improves the performance of the model along all of these dimensions simultaneously.

The paper is organized as follows. In Section II, we review the basic home production framework. In Section III, we introduce home production into the stochastic growth model, and discuss our choice of functional forms and parameter values. In Section IV, we simulate numerical solutions to the model and compare its cyclical properties to those of the standard model and to the actual data. In Section V, we present our conclusions.

## II. The Basic Home Production Model

Before studying the dynamic model, it is instructive to begin with a review of the static theory of home production.<sup>4</sup> Consider an individual with a utility function,  $U = U(c_m, c_n, h_m, h_n)$ , defined over four objects: consumption of a market good  $c_m$ , consumption of a home produced or nonmarket good  $c_n$ , hours of work in the market sector  $h_m$ , and hours of work in the home or nonmarket sector  $h_n$ , who solves the following problem:

$$\begin{aligned} & \max U(c_m, c_n, h_m, h_n) \\ & \text{st } c_m \leq x + wh_m, \quad c_n \leq g(h_n) \\ & \quad c_j \geq 0, \quad h_j \geq 0, \quad h_m + h_n \leq 1 \end{aligned}$$

Here,  $w$  denotes the real wage and  $x$  denotes exogenous income. What makes this a model with home production is the constraint  $c_n \leq g(h_n)$ , where  $g(\cdot)$  is the home production function. We assume  $U_1 > 0$ ,  $U_2 > 0$ ,  $U_3 < 0$ ,  $U_4 < 0$ , and  $g' > 0$ . Also,  $g$  is concave and  $U$  is strictly concave.

Substituting the home production constraint at equality into the utility function reduces the problem to

$$\max U[c_m, g(h_n), h_m, h_n] \text{ st } c_m \leq x + wh_m, \quad (2.1)$$

(nonnegativity constraints on all variables, including leisure,  $L = 1 - h_m - h_n$ , are assumed to be nonbinding from now on). Define the home work function  $h_n$

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<sup>4</sup> The fundamental reference is Becker (1965), although our particular specification is closer to Gronau (1977, 1985).

$= h(c_m, h_m)$  to be the unique value of  $h_n$  that maximizes the objective function in (2.1), for given values of the market variables  $(c_m, h_m)$ , and define the home consumption function by  $c_n = c(c_m, h_m) = g \circ h(c_m, h_m)$ . If we then define

$$V(c_m, h_m) = U[c_m, c(c_m, h_m), h_m, h(c_m, h_m)], \quad (2.2)$$

previous assumptions imply that  $V(\cdot)$  is continuous, monotonic, and strictly concave (Benhabib, Rogerson and Wright 1990a, Theorem 1), and so is a well-behaved *reduced form* utility function over market quantities only.

It follows that problem (2.1) generates the same values of  $c_m$  and  $h_m$  as the problem that does not explicitly contain home production,

$$\max V(c_m, h_m) \text{ st } c_m \leq x + wh_m. \quad (2.3)$$

The point is that for any model with home production there is a model without home production, but with different preferences, that generates the same outcome for market quantities, and so there is a sense in which models with home production are observationally equivalent to those without. Yet it is precisely because preferences would have to be different, if home production was not explicitly included, that we find it a useful concept for understanding and interpreting economic phenomena.

As an example, consider the case where technology shocks are added to the market and home production functions, with  $s_m$  and  $s_n$  being the two shocks. Following the procedure outlined above, it will now appear in the reduced form that preferences are stochastic, even though true underlying preferences are deterministic. In this example, when  $s_m$  is relatively high, labor will flow into the market resulting in a positive correlation between

productivity and  $h_m$ ; conversely, when  $s_n$  is relatively high, labor will flow the other way, raising productivity as  $h_m$  falls, and therefore causing a negative correlation between market hours and productivity. With both shocks present, it becomes possible to reconcile the lack of a strong empirical correlation between employment and productivity with theories based on technology shocks, as we show in our real business cycle model in Section IV.<sup>5</sup>

We emphasize that many implications of introducing home production do not depend on it being stochastic. One example is that leisure in the reduced form utility function can behave quite differently from leisure in the underlying utility function and, in particular, the wealth effect is changed. Thus, whether or not it is stochastic, introducing home production

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<sup>5</sup> Clearly, one could also explain the employment - productivity observations by adding a preference shock to a model with a market productivity shock, as in Christiano and Eichenbaum (1988). As already explained, home production does not allow us to generate outcomes that cannot be generated without it; it simply provides an alternative interpretation. As Becker (1972, p. 5) writes, "The assumption of stable preferences provides a stable foundation for generating predictions about responses to various changes, and prevents the analyst from succumbing to the temptation of simply postulating the required shift in preferences to 'explain' all apparent contradictions to his predictions." However, he notes that "preferences that are assumed to be stable do not refer to market goods and services, like oranges, automobiles, or medical care, but to the underlying objects of choice that are produced by each household using market goods and services, their own time, and other inputs." Similarly, Juster and Stafford (1990, p. 22) write, "While it is common for economists to assume stable preferences, it is less common to assume stationary technology. From our perspective, technology is subject to change just as readily in the household sector as in the industrial sector."



has an impact on labor supply elasticities, as shown in Section IV. In fact, once home production is introduced, the reduced form utility function can imply leisure is an inferior good, even if the underlying utility function implies it is a normal good. Hence, the inclusion of home production not only affects labor supply quantitatively, it also impinges on a variety of qualitative results in macro and labor economics that are known to depend critically on this wealth effect.<sup>6</sup>

To close this section, we present a parametric example that illustrates some of the effects in the preceding discussion. Begin by letting

$$U = \ln(C) + A \cdot \ln(1-h_m-h_n) + B h_m, \quad (2.4)$$

where C is the composite consumption good:

$$C = \left( a_m c_m^e + a_n c_n^e \right)^{1/e} \quad (2.5)$$

Assume  $A > B \geq 0$  and  $e \leq 1$ . The elasticity of substitution between market and nonmarket consumption is  $1/(1-e)$ . If  $B = 0$ , then market and nonmarket work are perfect substitutes, whereas if  $B > 0$ , then working in the market is preferred to working around the house. For the time being, we also

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<sup>6</sup> For instance, in indivisible labor models without home production, laid off workers are involuntarily unemployed if and only if leisure is inferior; but with home production, involuntary unemployment can occur if leisure is normal (see Benhabib, Rogerson and Wright 1990a). Similarly, in implicit contract models with asymmetric information, without home production there is underemployment if and only if leisure is inferior, but with home production underemployment can occur if leisure is normal (see Nosal, Rogerson and Wright 1990).

assume that home production is linear,  $c_n = s_n h_n$ , which permits closed form solutions to be obtained.

Consider first the case  $e = 0$ , so that (2.5) defines a Cobb - Douglas function. Assuming an interior solution, the homework function is

$$h_n = h(c_m, h_m) = \left[ \frac{a_n}{A+a_n} \right] (1-h_m). \quad (2.6)$$

Substituting this into (2.4), the reduced form utility function becomes

$$V = a_m \ln(c_m) + (A+a_n) \ln(1-h_m) + B h_m. \quad (2.7)$$

In this case, home production changes nothing, in the sense that if we had ignored it, or simply set  $c_n$  and  $h_n$  equal to constants in (2.4), then except for the constants in (2.7) nothing would have changed. To get any substantive effects from introducing home production, in this example, it is necessary to assume  $e \neq 0$ .

Consider next the case where  $c_m$  and  $c_n$  are perfect substitutes,  $e = 1$ , and for ease of notation set  $a_n = a_m = 1$ . Assuming an interior solution, the homework function is now

$$h_n = h(c_m, h_m) = \frac{s_n (1-h_m) - A c_m}{s_n (1+A)} \quad (2.8)$$

and the reduced form utility function becomes

$$V(c_m, h_m) = (1+A) \cdot \ln[c_m + s_n (1-h_m)] + B h_m. \quad (2.9)$$

If  $B = 0$ , then (2.9) implies the wealth effect on leisure is identically zero; if  $B > 0$ , then leisure is actually an inferior good in (2.9), even

though it is normal in the underlying utility function. Further, in the reduced form described by (2.9), the marginal disutility of work varies with  $s_n$ . In this case, home production qualitatively changes the specification.

### III. The Dynamic Framework

Our starting point is the real business cycle model. There is a representative agent, with preferences over stochastic sequences of consumption and labor hours  $\{c_t, h_t\}$  described by

$$w = E \sum_{t=0}^{\infty} \beta^t u(c_t, h_t), \quad (3.1)$$

where  $E$  denotes the expectation and  $\beta \in (0,1)$  is the discount factor. The agent has one unit of time to divide between leisure and labor each period. Labor and capital are used to produce output according to a constant returns to scale technology,  $y_t = s_t \Gamma^t f_t(h_t, k_t)$ , where  $s_t$  is stochastic and  $\Gamma \geq 1$  is a deterministic growth term. Capital evolves according to  $k_{t+1} = (1-\delta)k_t + i_t$ , where  $i_t$  is investment and  $\delta \in (0,1)$  is the depreciation rate, while  $s_t$  evolves according to an autoregressive process. Feasibility requires  $c_t + i_t \leq y_t$  for all  $t$ .

The first step is to extend this framework to include household variables. Preferences are now given by

$$W = E \sum_{t=0}^{\infty} \beta^t U(c_{mt}, c_{nt}, h_{mt}, h_{nt}), \quad (3.2)$$

where the arguments of  $U(\cdot)$  at each date are as defined in the previous section. The market technology is now written  $y_t = s_{mt} \Gamma^t f_t(h_{mt}, k_{mt})$ , where  $h_{mt}$  and  $k_{mt}$  are hours and capital in market production, while the nonmarket technology is  $c_{nt} = s_{nt} \Gamma^t g_t(h_{nt}, k_{nt})$ , where  $h_{nt}$  and  $k_{nt}$  are hours and capital in home production. The two technologies display constant returns to scale and common exogenous growth. Feasibility requires  $c_{mt} + i_t \leq y_t$ , plus  $k_{mt} + k_{nt} \leq k_t$ , where  $k_t$  is now the total capital stock, for all  $t$ .

Notice that capital is produced exclusively in the market sector, even though it is an input to both market and home production. Total capital evolves according to  $k_{t+1} = (1-\delta)k_t + i_t$ , while  $s_{mt}$  and  $s_{nt}$  evolve according to a process that will be specified in detail later.<sup>7</sup>

It is standard in this literature to impose the condition that market hours should be constant along a balanced growth path (i.e., when  $s_{mt}$  and  $s_{nt}$  are constant). As discussed in King, Plosser and Rebello (1987), in models that do not explicitly include home production, this implies that the utility function must be of the form

$$u(c,h) = \ln(c) + v(h), \text{ or } u(c,h) = \frac{c^{1-r}}{1-r} v(h) \quad (3.3)$$

where  $r > 0$ ,  $r \neq 1$ , and  $v(\cdot)$  is a concave function. As the properties of the model do not change significantly as  $r$  varies over a reasonable range (see Hansen 1988), the logarithmic case is often studied, and it is often further assumed that  $v(h) = B \cdot \ln(1-h)$ . With a Cobb - Douglas production function and  $s_t = \rho s_{t-1} + \varepsilon_t$ , where  $\rho \in (0,1)$  and  $\varepsilon_t$  is i.i.d., this is

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<sup>7</sup> We assume capital is freely mobile between the home and market sectors. An alternative would be to assume capital in a given sector cannot be transformed once it is in place. Theoretically, these two cases are polar extremes; but from a practical perspective, the difference is not substantial here. By choosing to not replace worn out capital in one sector and putting all new investment in the other, the economy can reallocate a considerable amount of capital across sectors without actually moving the stuff that is in place. In the simulations conducted in this paper, only infrequently does any capital actually move between sectors, and then, the amount that does move is quite small (rarely more than one half of one percent of the stock in the declining sector).

exactly the specification in Hansen's (1985) base case (without indivisible labor). In the remainder of the paper, we refer to this specification as *the standard model*.

In the home production model, one specification that delivers constant hours along a balanced growth path is

$$U = \frac{[C^b L^{1-b}]^{1-r} - 1}{1-r} \quad (3.4)$$

where  $C = [ac_m^e + (1-a)c_n^e]^{1/e}$  and  $L = 1-h_m-h_n$ . Following Hansen (1985), we set  $r = 1$ , which allows us to write

$$U = (b/e) \ln[ac_m^e + (1-a)c_n^e] + (1-b) \ln(1-h_m-h_n). \quad (3.5)$$

We also assume Cobb - Douglas production functions in both sectors,  $f(h_m, k_m) = k_m^\theta h_m^{1-\theta}$  and  $g(h_n, k_n) = k_n^\eta h_n^{1-\eta}$ . Our goal is to see how this model differs quantitatively from the model without home production. First, however, we show how to nest the standard model within our framework.

There is a general observational equivalence result for dynamic economies, similar to the static result in the previous section (see Benhabib, Rogerson and Wright 1990a), and in some special cases, we can solve for the reduced form explicitly. With  $e = 0$ , (3.5) becomes

$$U = ab \cdot \ln(c_m) + (1-a)b \cdot \ln(c_n) + (1-b) \ln(1-h_m-h_n). \quad (3.6)$$

If we also set  $\eta = 0$ , so that  $c_n = s_n r^t h_n$ , we can solve for the homework function  $h_n = \phi \cdot (1-h_m)$ , where  $\phi = (1-a)b/(1-ab)$ . If we insert this into (3.6) and simplify, the reduced form utility function is

$$V = \ln(c_m) + B \cdot \ln(1-h_m), \quad (3.7)$$

which is identical to the utility function in the standard model without home production. Hence, the home production economy with  $e = \eta = 0$  generates exactly the same time paths for  $y_t$ ,  $c_{mt}$ , etc., as the standard model.

We now turn to the parameter values we use in the experiments reported in the next section. We set the discount rate to  $\beta = .99$ . The parameters  $a$  and  $b$  are chosen (see Appendix A for details) so that the steady state of the model yields values for market work and homework that correspond to the numbers discussed in the Introduction,  $h_m = .33$  and  $h_n = .28$ .<sup>8</sup> The remaining preference parameter is  $e$ . Eichenbaum and Hansen (1990) uses aggregate data to estimate a model in which individuals value both the services of market consumption goods and the flow of services from consumer durables, where the latter can be thought of as the output of a home production process that uses durables as its only input. Although their results are sensitive to various assumptions, for one set of findings they report there is "very little evidence against the hypothesis that the services from durable and non-durable goods are perfect substitutes"

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<sup>8</sup> Greenwood and Hercowitz (1990), in a closely related model, interpret all nonmarket time as home production. Following Gronau (1977), we prefer to divide discretionary time into three components – market work, home work, and leisure – rather than two. We think it is useful, in principle, to differentiate between time spent in household production activities, which generates disutility, and time spent in leisure activities, which does not. Furthermore, in practice, we do have good cross sectional measurements of all three components in the time use data.

(p. 63), which would suggest setting  $e = 1$ .

Cross sectional data can also provide some information. Consider a static model, in which agent  $i$  has preferences described by

$$U_i = \ln \left[ a_i c_{mi}^e + (1-a_i) c_{ni}^e \right] + v_i (1-h_{mi} - h_{ni}), \quad (3.8)$$

where  $a_i$  differs across the population according to some distribution. Suppose all agents have the same home production technology,  $c_{ni} = B \cdot h_{ni}$ , but each agent faces a potentially different market wage,  $w_i$ . Then the first order condition for agent  $i$  implies

$$\ln(h_{mi}/h_{ni}) = \frac{1}{e-1} \ln(B) - \frac{e}{e-1} \ln(w_i) + \frac{1}{e-1} \ln[(1-a_i)/a_i]. \quad (3.9)$$

This leads to a regression equation of the form

$$\ln(h_{mi}/h_{ni}) = \alpha_0 + \alpha_1 \ln(w_i) + \xi_i. \quad (3.10)$$

The interpretation of the above model is that it represents the average time allocation decision as a function of the long run wage. Using the pooled data from the Panel Study of Income Dynamics described in Rios-Rull (1990), we estimate the above equation and derive a value of  $e = 0.6$ , somewhat lower than the value implied by Eichenbaum and Hansen's (1990) results.<sup>9</sup> The two methods are polar extremes - the first uses aggregate

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<sup>9</sup> We consider this estimate highly preliminary, for a variety of reasons. For one thing, his sample selection criterion severely under reports low wage workers (presumably with very low ratios of market to home hours). As a rough correction, we either adjusted home hours for the two lowest wage groups so that their total work is the same as the other groups, or we



data and abstracts from the time allocation decision, whereas the second uses micro data and abstracts from savings or capital. While we emphasize the need for further empirical work along these lines, for the time being the mean value of these two numbers will be used:  $e = .8$ .

Concerning technology, we set  $\Gamma = 1$ ,  $\delta = .025$ , and  $\theta = .36$ , as in most of the relevant literature. Capital's share in the home production function is set to  $\eta = .08$ , producing a steady state  $c_n/y$  ratio of .26.<sup>10</sup> At the same time, this yields a steady state  $k_n/k_m$  ratio of .14. If one wanted to define household capital to include housing plus consumer durables, this ratio is certainly too small; but in this framework, it is not clear that housing should be interpreted as part of  $k_n$ . This model is intended to capture the household's decision to combine its labor with machines, like stoves or washing machines, to cook or do laundry versus purchasing meals or cleaning services in the market. Houses do not need to be combined with labor, at least not in the same way that stoves or washing machines do, in order to be useful. Because much of the time spent in one's home is leisure, sleep, and so on, rather than homework *per se*, it seems appropriate that  $k_n$  does not include the housing stock in our framework.<sup>11</sup>

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simply ignored the lowest wage groups. Either method results in a point estimate of about  $e = .6$ .

<sup>10</sup> Recall that the range of estimates surveyed in Eisner (1988) is .20 - .50. We prefer to be at the low end of this range, since we have abstracted from taxation on market activity.

<sup>11</sup> A steady state  $k_n/k_m$  ratio of .14 matches the data if we define  $k_n$  to include household equipment and furniture but not houses. Alternatively, Greenwood and Hercowitz (1990) include housing plus durables in  $k_n$ . Combined with their assumption that  $h_n$  includes all time not sleeping or

It remains to describe the shocks. The usual assumption is that  $s_{mt+1} = .95s_{mt} + \varepsilon_{mt}$ , where  $\varepsilon_{mt}$  is normal with a standard deviation of around  $\sigma_m = .007$ . In the absence of evidence concerning the home technology, we simply assume it follows a similar process,  $s_{nt+1} = .95s_{nt} + \varepsilon_{nt}$ , where the standard deviation of  $\varepsilon_{nt}$  is  $\sigma_n$ . We have experimented with setting  $\sigma_n = 0$  (so that the home technology is nonstochastic), and also with  $\sigma_n = .007$  (so that it fluctuates as much as the market technology). Only the latter simulations are reported below, but the basic message turns out to not depend heavily on  $\sigma_n$  (see Benhabib, Rogerson and Wright 1990b, Appendix B). This is because for most of our results it is *relative* productivity variation that matters, and there will be relative productivity variation even if  $s_n$  is constant as long as  $s_m$  fluctuates. However, one exception to this is that the behavior of average productivity does depend critically on  $\sigma_n$ , as will be discussed below.<sup>12</sup>

This leaves one parameter, the correlation between the two shocks,  $\gamma = \text{corr}(\varepsilon_m, \varepsilon_n)$ . Our view is that  $\gamma$  is certainly positive, but that it is also certainly less than one (because sometimes technological innovations affect productivity mainly in the market, like microcomputers, and sometimes they affect productivity mainly in the home, like microwave ovens). Smaller values of  $\gamma$  imply more frequent relative productivity differentials between the two sectors and, therefore, more frequent opportunities for short run

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working for paid compensation, their model implies  $c_n/y$  is 2.9 in steady state, compared to our value of 1/4. In any case, including a realistic tax system in our model would increase considerably both  $c_n/y$  and  $k_n/k_m$ .

<sup>12</sup> It is also the case that the results reported below are not sensitive to the serial correlation properties of the home shock.

substitution between the market and the home. Intuitively, then, the smaller is  $\gamma$ , the greater is the extent to which home production should affect the cyclical behavior of the system. We set  $\gamma = 2/3$  for the simulations reported below, although the basic results are not affected too much if we choose  $\gamma = 1/2$  or  $3/4$ , for example (again, see Benhabib, Rogerson and Wright 1990b, Appendix B).

#### IV. Results

In this section, we compare certain statistics from simulations of the model with those from the data. All series are filtered using the Hodrick - Prescott technique before computing any statistics. Table 1a summarizes the behavior of the U.S. quarterly data for the period 1954.1 - 1988.2, in terms of the following variables:  $y$  = output,  $c_m$  = market consumption,  $i$  = investment,  $h_m$  = market hours,  $k$  = capital, and  $w$  = average productivity (defined by  $w = y/h_m$ ; we use the  $w$  notation because a Cobb - Douglas technology implies that the average product is proportional to the marginal product, which equals the real wage in equilibrium). The table provides the percent standard deviation of output, and for each additional series  $x$ , it provides the percent standard deviation of  $x$  relative to the percent standard deviation of  $y$ , and the correlation of  $x$  with  $y$ .<sup>13</sup>

Table 1b provides a summary of the properties of the standard model without home production using the parameter values described in the previous section; these numbers are averages over 50 simulations, each of length 143 (the number of quarters in our data). The results are the same as those reported by Hansen (1985) for his base economy, except that some new statistics have been added; in particular, we disaggregate total market hours into those used to produce consumption goods,  $h_c$ , and those used to produce investment goods,  $h_i$ , where  $h_c = (c/y)h_m$  and  $h_i = (i/y)h_m$ . Many

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<sup>13</sup> Note that our output series is constructed as  $y = c_m + i$ . Also note that our consumption data correspond to expenditures on nondurables and services, while purchases of consumer durables have been included in investment. This results in a standard deviation of  $i$  relative to  $y$  of 2.82, lower than the number typically reported; e.g., Kydland and Prescott (1991) report 3.17.

authors have commented on how well this model captures certain aspects of the data. Nevertheless, we wish to draw attention to several dimensions along which there appears to be room for improvement.

First, the model economy is not as volatile as the actual economy: the model has a standard deviation of output equal to only 1.29 percent, compared to 1.74 in the data. Second, independent of the overall volatility of the model, consumption is not volatile enough and investment is too volatile *relative to* output. Because output is the sum of consumption and investment and all three are highly correlated, the standard deviation of  $y$  is essentially a weighted average of the standard deviations of  $c_m$  and  $i$ ; hence, insufficient volatility in consumption and excess volatility in investment tend to go together. A further difficulty is that total market hours do not fluctuate enough in the model. Further still, observe that although the correlations between output and most of the other variables are reasonable, the correlation between output and productivity is significantly off target. These are problems 1 - 5 listed in the Introduction.

These problems are fairly well known, and various embellishments of the basic framework have been shown to help each of them in isolation. There is, however, a feature of the model that has not been commented on before, but which is closely related. This is that the model implies an almost perfect negative correlation between output and the hours allocated to the production of consumption goods:  $\text{corr}(y_t, h_{ct}) = -.98$ . Although it is not shown in the table,  $h_{ct}$  is also highly negatively correlated with  $h_t$  and, *a fortiori*,  $h_{it}$ . These predictions fly in the face of conventional wisdom concerning actual business cycles, which is that various sectors tend to move up and down together.<sup>14</sup> Furthermore, we now show that these predictions

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<sup>14</sup> Economists often define the cycle as the recurrent comovement of the

will come out of any specification of the standard model that yields constant hours along a balanced growth path and implies that labor's share of income is constant (as it is with a Cobb - Douglas technology).

To see this, recall that the class of preferences that delivers constant hours along a balanced growth path is described by (3.3). Consider the case where  $u = \log(c_t) + v(h_t)$  (the other case is similar). At each point in time, the efficiency condition equating the marginal rate of substitution with the marginal product of labor in the production of consumption goods is  $c_t v'(h_t) = \text{MPL}_t$ , or

$$h_{ct} v'(h_t) = \frac{\text{MPL}_t \cdot h_{ct}}{c_t} \quad (4.1)$$

The right hand side is labor's share of output in the consumption sector, which is constant by assumption. Hence, as long as  $v(\cdot)$  is (strictly) concave, an increase in total hours  $h_t$  must be accompanied by a (strict) decline in  $h_{ct}$ . Since  $h_{ct}$  and  $h_t$  move in opposite directions, so do  $h_{ct}$  and  $h_{it}$ .<sup>15</sup>

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*outputs* of various market sectors (see Lucas 1976, e.g.); but we doubt if anyone would argue empirically that employment hours in different sectors move out of phase. Without attempting to catalogue various sectors as consumption or investment, there is no major sector of the U.S. economy that is known to have countercyclical employment (see Murphy, Schleifer and Vishny 1989). Using data provided by Donna Costello, from five countries each disaggregated into five sectors, we examined the correlations with output of each sector's hours, and the cross correlations between sectoral hours. Almost all of these correlations were positive, and none were significantly negative.

<sup>15</sup> We point out that these results are actually reinforced if labor's share

The intuition is quite simple. Any specification that implies hours do not change along a balanced growth path also implies individuals never supply more labor in order to produce more output for immediate consumption. In particular, in a model that is otherwise standard except that it has no capital, employment is *constant*, and consumption fluctuates one-for-one with the technology shock. Even though agents have the opportunity to work harder when productivity is high and increase consumption even more, they choose not to work harder if the only reward is increased contemporaneous consumption. When capital accumulation is reintroduced, labor does vary with productivity due to intertemporal substitution opportunities; but individuals still do not work more to increase current consumption. In fact, since consumption now moves less than one-for-one with output, individuals spend less time in the production of consumption goods when productivity is high.

We therefore have the following characterization of business cycles in the standard model: good times are periods when resources flow from the production of consumption goods to the production of investment goods. We think that it is useful to focus on this because it sheds light on some of

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is countercyclical rather than constant (it is slightly countercyclical in the data). Also, our argument does not restrict the technologies in the investment and consumption sectors to be the same or subject to the same shock, and so moving to a more general two sector model will not change things. The best one can do within the standard model is to make  $v(\cdot)$  linear, in which case hours in the consumption sector will be *constant* over the cycle. The indivisible labor economy studied by Rogerson (1984, 1988) and Hansen (1985) is equivalent to an economy where  $v(\cdot)$  is linear. Hence, in that economy, although  $h_{ct}$  and  $h_{it}$  do not move together at least they do not move in opposite directions over the cycle.

the other problems with the standard model. The fact that consumption is too smooth and investment too volatile relative to output is easy to understand given that labor is being moved out of the production of consumption goods and into the production of investment goods as the cycle moves from trough to peak. Similarly, the fact that total hours are too smooth relative to output is easy to understand given that  $h_{ct}$  is countercyclical; if  $h_{ct}$  did not decrease whenever  $h_{it}$  rose, the sum would be more volatile. Furthermore, if  $h_{ct}$  could be increased during upswings without decreasing  $h_{it}$ , total output would also be more volatile.

Hence, several discrepancies between theory and data can be traced back to  $h_{ct}$  being out of phase with  $y$ . What is needed is a mechanism that leads to hours in the consumption sector responding positively to an increase in market productivity, and the addition of home production provides exactly this. In addition to the standard motive for increasing labor hours when market productivity is high (capital accumulation), in the home production economy, there is an additional motive to simultaneously substitute market for home produced consumption. The latter effect involves the transfer of hours from the home into the market consumption sector during upswings in the business cycle and, if it is large enough, could thereby produce a procyclical pattern to  $h_{ct}$ . Depending on the exact parameterization, the addition of a household sector may imply that upswings in aggregate market activity correspond to periods when labor flows from the home into all market sectors, rather than periods when labor flows from the consumption into the investment sector.

Table 1c reports the results for simulations of the home production economy using the parameter values discussed above. Compared to the standard model, the volatility of investment relative to output has



decreased, while that of consumption has increased. Additionally, the variability of market hours relative to output is greater than in the standard model. Output is also more volatile. All of these improvements can be interpreted in light of the fact that  $h_{ct}$  is procyclical in the home production economy. Although the correlation between  $y$  and  $h_c$  is not large in Table 1c, as long as it is even slightly procyclical, the model improves along all of these dimensions. Figure 1 plots the responses of hours to a one percent shock in  $s_m$ , in the economies with and without home production. This shows clearly how  $h_c$  and  $h_i$  are out of phase in the standard model, while they move together in the home production model, with  $h_n$  taking up the slack.<sup>16</sup>

One prediction of the standard model that seems to be closer to the actual data is the volatility of productivity; but it misses so badly on the correlation between  $w_t$  and  $y_t$  that getting the standard deviation right seems to be of little consolation. Furthermore, the standard model also predicts the correlation between  $w_t$  and  $h_{mt}$  is .99, and many commentators criticize the real business cycle paradigm for this. In the aggregate U.S. data,  $w_t$  and  $h_{mt}$  are in fact negatively correlated, as shown in Figure 2a (which plots percentage deviations after filtering). For comparison, the data generated by the standard model are plotted in Figure 2b. To say these pictures are different would be an understatement. Now, it may be argued

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<sup>16</sup> Although market consumption is more volatile here than in the standard model, it is really the composite good C that consumers care about, and that is actually quite smooth. Similarly, hours in home production act like a buffer against volatility in market labor, so that leisure L is also quite smooth. Hence, although market activity in the home production economy is more volatile, agents in the model actually don't mind.

that there are problems with the data, and correcting for measurement error suggests that the true correlation may actually be positive, perhaps even as high as .44 (see Christiano and Eichenbaum 1988, Table A.3). But even under the most favorable assumptions, it is certainly not .99.

The feature of the standard model responsible for this inconsistency with the data is that it is driven by a single shock to technology, which implies a very tight relation between productivity and output or productivity and hours. Loosely speaking, shocks to the market technology shift labor demand and trace out a stable labor supply curve. The home production economy with only a single shock to the market technology – i.e., with  $\sigma_n = 0$  – also traces out a stable labor supply curve, as shown in Figure 2c. Notice, however, that this curve is much more elastic than the one in Figure 2b. In contrast to the standard model, which relies exclusively on intertemporal substitution, the home production model also includes intratemporal substitution between market work and homework at a given date. This makes the labor supply response in the home production economy more similar to that in the data, but still does not generate the cloud seen in Figure 2a.

By including innovations to the home technology that are less than perfectly correlated with those in the market, we add a second shock. When both shocks are present, the net effect is as depicted in Figure 2d. The correlation between  $h_{mt}$  and  $w_t$  in this case is .49, which is much better than the standard model, although perhaps still high. However, this statistic is sensitive to the relative size of the shocks. Increasing  $\sigma_n$  from .007 to .010, and keeping  $\sigma_m$  as well as all of the other parameters the same, the correlation between  $h_{mt}$  and  $w_t$  is reduced to .08, which is well within the acceptable range. This change also implies that the standard

deviation of productivity relative to output and the correlation between productivity and output become very close to the data, although market consumption becomes somewhat too volatile (see Benhabib, Rogerson and Wright 1990b, Table 2d). In any case, it is clear that there is no problem, in theory, accounting for the productivity - hours observations using models driven exclusively by technology shocks.<sup>17</sup>

To close this section, we compare our model with the closely related model of Greenwood and Hercowitz (1990).<sup>18</sup> The focus of their paper is the behavior of investment in market and nonmarket capital over the cycle, which we denote  $i_m$  and  $i_n$ . Although not reported in Table 1, our model obviously generates predictions for these variables; in particular, the correlation between  $y$  and  $i_m$  is .23 and the correlation between  $y$  and  $i_n$  is .35. Hence,  $i_m$  and  $i_n$  are both procyclical. However, our model is *not* consistent with one feature of the data emphasized by Greenwood and Hercowitz: the correlation between  $i_m$  and  $i_n$  in our simulations is negative (about -.75), while in the U.S. economy it is positive (about .75 in the quarterly data, if we measure  $i_n$  by purchases of consumer durables). This is not really a surprise, since Greenwood and Hercowitz argue in some detail that any specification with a Cobb - Douglas home technology tends to make investment

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<sup>17</sup> Similar shocks to labor supply could be generated by assuming that preferences vary over time, which is the solution proposed by Christiano and Eichenbaum (1988).

<sup>18</sup> There are many technical differences in the two models; for instance, they calibrate to annual data, they assume the technology shock follows a random walk, they assume all nonmarket time is an input into home production instead of allowing for leisure, and their model includes taxation. These are not overly important for the following discussion.

in the two types of capital negatively correlated.

Greenwood and Hercowitz also discuss how one can get around this, by assuming a home production function of the form

$$c_n = \left[ \eta k_n^\lambda + (1-\eta)(s_n h_n)^\lambda \right]^{1/\lambda} \quad (4.2)$$

where Cobb - Douglas is the case of  $\lambda = 0$ . With  $\lambda < 0$ , they show that  $i_m$  and  $i_n$  can become positively correlated in their model, at least when they assume the two shocks  $s_m$  and  $s_n$  are perfectly correlated. With negative values of  $\lambda$ , we can also get  $i_m$  and  $i_n$  to be positively correlated in our model. However, there are two qualifications. First, the result is not very robust, in the sense that the correlation between  $s_m$  and  $s_n$  must be very close to 1. Second, for parameter values that yield a positive correlation between  $i_m$  and  $i_n$ , we found that several of the previously noted improvements in the model were diminished and, in particular,  $h_c$  was countercyclical. Constructing a model that simultaneously accounts for hours and the two investments remains a topic for additional research.

## V. Conclusion

The results in the previous section show that the existence of a household sector can have a large effect on the behavior of aggregate market variables. It is natural to inquire how sensitive these results are to the particular values of the parameters that we chose, especially since some of them are not especially well measured. Obviously, parameter values will matter somewhat; as discussed earlier, if  $e = \eta = 0$  then the home production economy exactly reproduces the statistics of the standard model.<sup>19</sup> We have experimented with changing all of the parameter values in a neighborhood of those discussed above. For example, changing  $e$  or  $\gamma$  changes agents' willingness or incentive to substitute between the market and nonmarket sectors; if  $e$  gets too low or  $\gamma$  too high, we approach the standard model, while if  $e$  gets too high or  $\gamma$  too low, the effects in Table 1c become exaggerated. Nevertheless, including home production improves the performance of the model along several dimensions over a sizable region of parameter space (see Benhabib, Rogerson and Wright 1990b, Appendix B, for details).

The fact that the household sector is large is incontrovertible. Models without home production implicitly make the assumption that the willingness and/or the incentive of individuals to substitute between market and nonmarket activity is small, which does not seem to be the conclusion one would want to draw from the evidence. The fact that available evidence

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<sup>19</sup> Even with  $\eta$  set at our preferred value of .08, when  $e = 0$  the two models are remarkably similar. Hence, one way to interpret the standard model is that it contains a home sector implicitly, but assumes that  $e$  is close to zero.

on some important variables is imperfect leads us to conclude that future research ought to subject parameters such as the elasticity of substitution and technological progress in the nonmarket sector to the same level of analysis that has been afforded variables like the coefficient of risk aversion and the Solow residual in the market sector. If theory predicted that the choice of these parameters was of minor importance, then their values would not be of much interest to macroeconomists; but this is not what theory predicts.

## Appendix A

Here we analyze the deterministic steady state, and demonstrate how the parameters  $a$  and  $b$  are chosen. Begin by setting the shocks to their unconditional means,  $s_m = s_n = 1$ , and substituting the constraints into the utility function, to yield the objective function

$$\sum \beta^t U \left[ f(h_{mt}, k_{mt}) - k_{mt+1} - k_{nt+1} + (1-\delta)(k_{mt} + k_{nt}), g(h_{nt}, k_{nt}), h_{mt}, h_{nt} \right].$$

The first order conditions for maximizing this objective are

$$U_1(t)f_1(t) + U_3(t) = 0 \tag{A.1}$$

$$U_2(t)g_1(t) + U_4(t) = 0 \tag{A.2}$$

$$U_1(t)f_2(t) + (1-\delta)U_1(t) = \beta^{-1}U_1(t+1) \tag{A.3}$$

$$U_2(t)g_2(t) + (1-\delta)U_1(t) = \beta^{-1}U_1(t+1) \tag{A.4}$$

where the notation  $F(t)$  indicates that a function  $F(\cdot)$  is being evaluated at arguments as of date  $t$ . In the steady state, of course, these arguments do not depend on time.

We are given values for the parameters  $\beta$ ,  $\delta$ ,  $\theta$  and  $\eta$ , plus the steady state time allocation  $h_m^*$  and  $h_n^*$ . Using the functional forms described in the text, (A.3) immediately implies  $\theta(k_m/h_m)^{\theta-1} = \beta^{-1-1+\delta}$ , and this can be solved for  $k_m^*$ . The first order conditions also imply the following relation between the capital labor ratios in the market and household,

$$k_m/h_m = \frac{\eta(1-\theta)}{\theta(1-\eta)} k_n/h_n,$$

which can be solved for  $k_n^*$  given the solution for  $k_m^*$ . Now  $c_n^* = g(h_n^*, k_n^*)$  and  $c_m^* = f(h_m^*, k_m^*) - i^*$ , where  $i^* = \delta(k_m^* + k_n^*)$ . Notice that we have solved for the steady state allocation  $(c_m^*, c_n^*, h_m^*, h_n^*)$  without using the instantaneous utility function at all. The strategy now is to determine the parameters  $a$  and  $b$  of this function so that this solution satisfies the marginal conditions (A.1) and (A.2). For the preference structure in the text, there is a unique such  $a$  and  $b$ . Note that the elasticity parameter  $e$  affects the implied values of  $a$  and  $b$ , but none of the observable variables, while the risk aversion parameter  $r$  does not affect the steady state at all.



## References

- Bacus, D. and P. Kehoe (1989) "International Evidence on the Historical Properties of Business Cycles," Federal Reserve Bank of Minneapolis Working Paper 402R.
- Becker, G.S. (1965) "A Theory of the Allocation of Time," *Economic Journal* 75, 493-517.
- Becker, G.S. (1988) "Family Economics and Macro Behavior," *American Economic Review* 78 (March), 1-13.
- Becker, G.S. (1972) *The Economic Approach to Human Behavior*.
- Benhabib, J., R. Rogerson and R. Wright (1990a) "Homework in Macroeconomics I: Basic Theory," manuscript.
- Benhabib, J., R. Rogerson and R. Wright (1990b) "Homework in Macroeconomics II: Aggregate Fluctuations," manuscript.
- Christiano, L. and Eichenbaum, M. (1988) "Is Theory Really Ahead of Measurement? Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations," Federal Reserve Bank of Minneapolis Working Paper 412 and NBER Working Paper 2700.
- Eichenbaum, M. and L.P. Hansen (1990) "Estimating Models with Intertemporal Substitution Using Aggregate Data," *Journal of Business and Economic Statistics* 8, 53-69.
- Eisner, R. (1988) "Extended Accounts for National Income and Product," *Journal of Economic Literature* 26, 1611-1684.
- Greenwood, J. and Hercowitz, Z. (1990) "The Allocation of Capital and Time Over the Business Cycle," manuscript.
- Gronau, R. (1977) "Leisure, Home Production and Work - The Theory of the Allocation of Time Revisited," *Journal of Political Economy* 85, 1099-1123.
- Gronau, R. (1985) "Home Production - A Survey," *Handbook of Labor Economics*, vol. 1, edited by O. Ashenfelter and R. Layard, Amsterdam: North-Holland.
- Hansen, G. (1985) "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics* 16, 309-325.
- Hansen, G., (1988) "Growth and Fluctuations," manuscript.
- Hill, M. (1985) "Patterns of Time Use," in *Time, Goods and Well-Being*, edited by F.T. Juster and F.P. Stafford, University of Michigan.
- Juster, F.T. and F.P. Stafford (1990) "The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement," Working Paper, Social Research Center, Institute for Social Research, University of Michigan.

- King, R., C. Plosser and S. Rebello (1987) "Production, Growth and Cycles: Technical Appendix," manuscript, University of Rochester.
- Kydland, F. and E. Prescott (1982) "Time to Build and Aggregate Fluctuations," *Econometrica* 50 (November), 1345-1370.
- Kydland, F. and E. Prescott (1991) "Hours and Employment Variation in Business Cycle Theory," *Economic Theory*, in press.
- Lucas, R. (1976) "Understanding Business Cycles" *Carnegie-Rochester Conference Series in Public Policy* 5, 7-29.
- Murphy, K., A. Schliefer and R. Vishny (1989) "Building Blocks of Market Clearing Macroeconomic Models," in O. Blanchard and S. Fisher, editors, *NBER Macroeconomics Annual*, 1989, MIT Press Cambridge.
- Nosal, E., R. Rogerson and R. Wright (1990) "A Note on Labor Contracts with Asymmetric Information and Household Production," manuscript.
- Plosser, C. (1989) "Understanding Real Business Cycles," *Journal of Economic Perspectives* 3, 51-77.
- Prescott, E. (1986) "Theory Ahead of Business Cycle Measurement," *Federal Reserve Bank of Minneapolis Quarterly Review* Fall 1986, 9-22.
- Rios-Rull, J.-V. (1990) "Working in the Market, Home Production and the Acquisition of Skills: A General Equilibrium Approach," manuscript.
- Rogerson, R. (1984) "Topics in the Theory of Labor Markets," unpublished Ph.D. dissertation, University of Minnesota.
- Rogerson, R. (1988) "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics* 21 (January), 3-17.

Table 1

a) U.S. Data:  $\text{std}(y) = 1.74$

x =	$c_m$	i	w	$k_m$	$h_m$
$\frac{\text{std}(x)}{\text{std}(y)}$	.49	2.82	.52	.38	.86
cor(x,y)	.76	.96	.51	.28	.86

b) Standard Model:  $\text{std}(y) = 1.29$

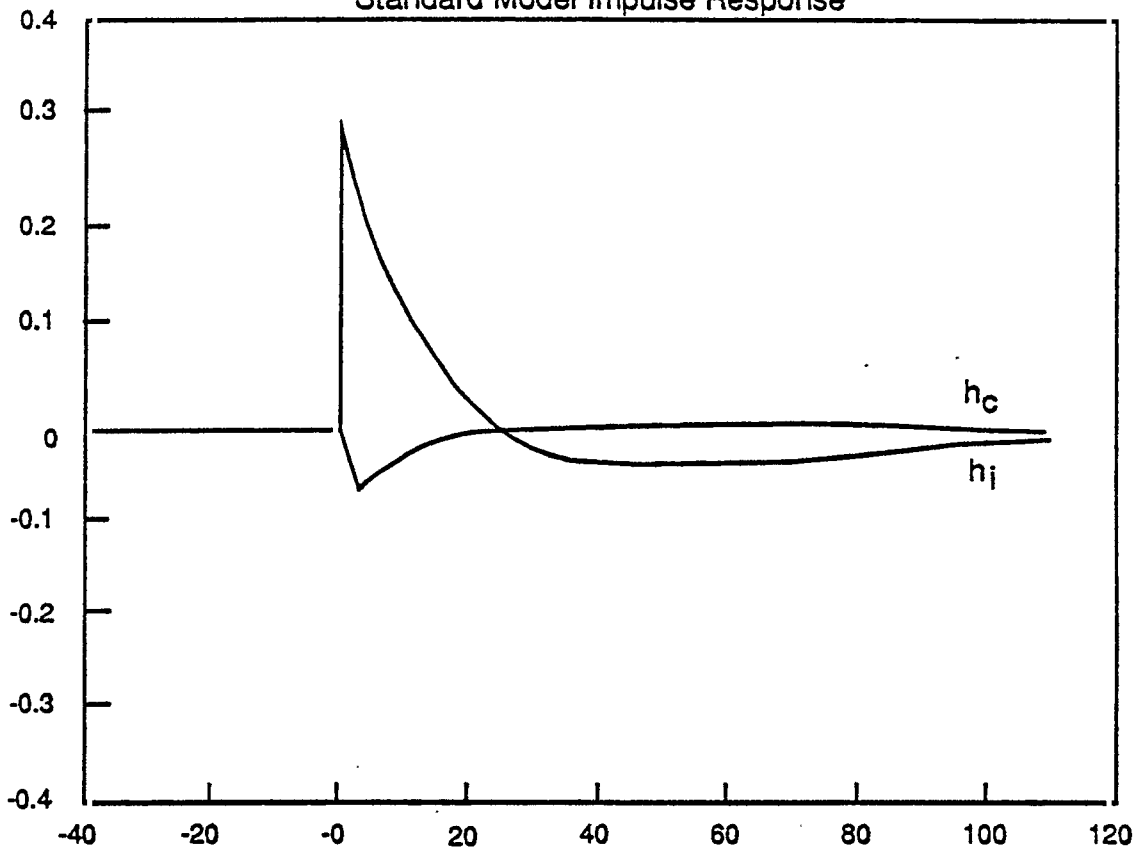
x =	$c_m$	i	w	k	$h_m$	$h_i$	$h_c$
$\frac{\text{std}(x)}{\text{std}(y)}$	.30	3.14	.52	.26	.50	2.66	.25
cor(x,y)	.90	.99	.99	.05	.98	.98	-.98

c) Home Production Model:  $\text{std}(y) = 1.71$

x =	$c_m$	i	w	k	$h_m$	$h_i$	$h_c$	$h_n$
$\frac{\text{std}(x)}{\text{std}(y)}$	.51	2.73	.39	.23	.75	2.40	.59	.70
cor(x,y)	.69	.94	.75	.09	.94	.95	.10	-.76

Figure 1

Standard Model Impulse Response



Home Production Model Impulse Response

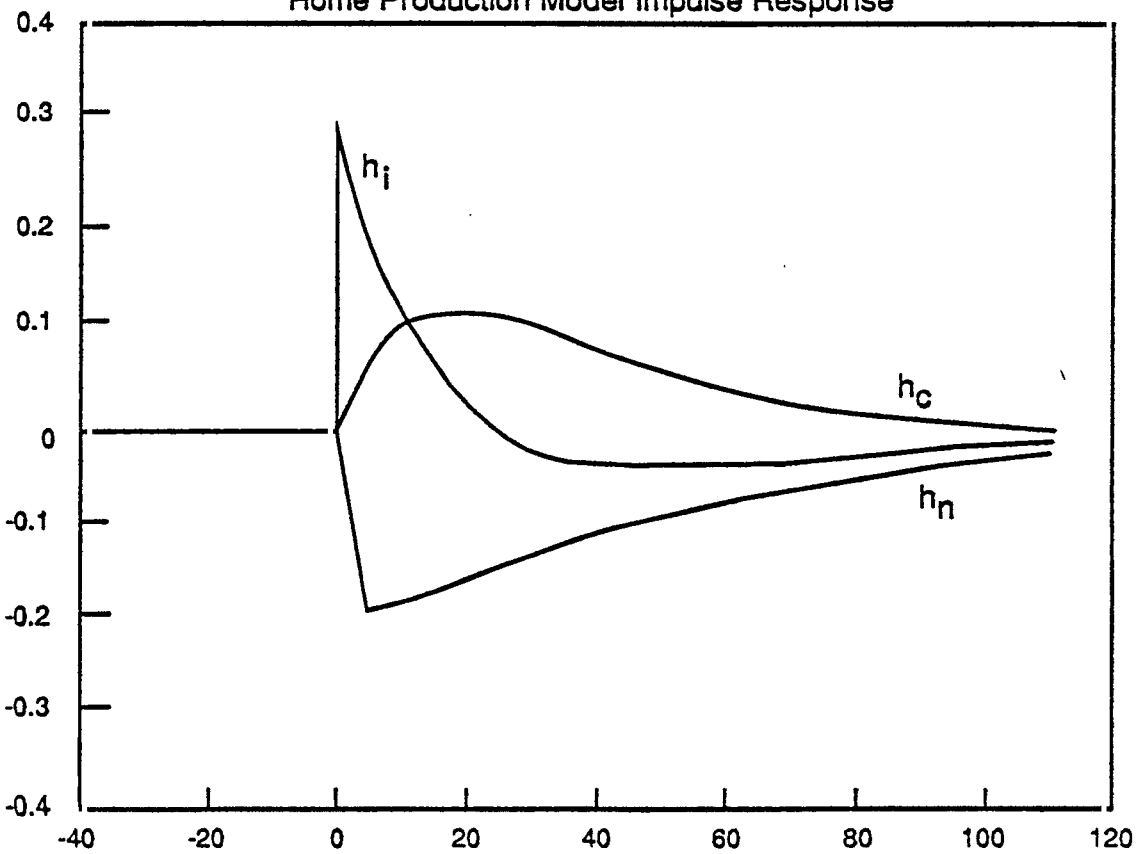


Figure 2:  $h_m$  versus  $w$  (percent deviations)

