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Homogeneous Systolic Pyramid Automata with *n*-Dimensional Layers

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Abstract

Cellular automata were investigated not only in the viewpoint of formal language theory, but also in the viewpoint of pattern recognition. Cellular automata can be classified into some types. A systolic pyramid automata is also one parallel model of various cellular automata. A homogeneous systolic pyramid automaton with *n*-dimensional layers (*n-HSPA*) is a pyramid stack of *n*-dimensional arrays of cells in which the bottom *n*-dimensional layer (level 0) has size an (a≥1), the next lowest (a-1)n, and so forth, the (a-1)st *n*-dimensional layer (level (a-1)) consisting of a single cell, called the root. Each cell means an identical finite-state machine. The input is accepted if and only if the root cell ever enters an accepting state. An *n-HSPA* is said to be a real-time *n-HSPA* if for every *n*-dimensional tape of size a^n (a≥1) it accepts the *n*-dimensional tape in time a-1. Moreover, a 1- way *n*-dimensional cellular automaton (1-*nCA*) can be considered as a natural extension of the 1-way two- dimensional cellular automaton to *n*-dimension. The initial configuration is accepted if the last special cell reaches a final state. A 1-*nCA* is said to be a real- time 1-*nCA* if when started with *n*-dimensional array of cells in nonquiescent state, the special cell reaches a final state. In this paper, we propose a homogeneous systolic automaton with *n*-dimensional layers (*n-HSPA*), and investigate some properties of real-time *n-HSPA*. Specifically, we first investigate a relationship between the accepting powers of real-time *n-HSPA*'s and real-time 1-*nCA*'s. We next show the recognizability of *n*-dimensional connected tapes by real-time *n-HSPA*'s.

Keywords: cellular automaton, diameter, finite automaton, *n*-dimension, parallelism, pattern recognition, real time.

1. Introduction and Preliminaries

The question of whether processing *n*-dimensional digital patterns is much more difficult than (n-1) dimensional ones is of great interest from the theoretical and practical standpoints. Thus, the study of n-dimensional automata as a computational model of n-dimensional pattern processing has been meaningful[4-23]. Cellular automata were investigated not only in the viewpoint of formal language theory, but also in the viewpoint of pattern recognition. Cellular automata can be classified into some types [2]. A systolic pyramid automaton is also one parallel model of various cellular automata. In this paper, we propose a homogeneous systolic automaton with *n*-dimensional layers (*n-HSPA*), and investigate some properties of real-time *n-HSPA*.

Let Σ be a finite set of symbols. An *n-dimensional* tape over Σ is an $(n-1)$-dimensional array of elements of Σ. The set of all n-dimensional tapes over Σ is denoted by $\Sigma^{(n)}$. Given a tape $x \in \Sigma^{(n)}$, for each $j(1 \leq j \leq n)$, we let $l_j(x)$ be the length of x along the jth axis. When $1 \leq i_j \leq l_j(x)$ for each $j(1 \leq j \leq n)$, let $x(i_1, i_2, \ldots, i_n)$ denote the symbol in x with coordinates $(i_1, i_2, \ldots, i_n)$. We concentrate on the input tape x with $l_1(x) = l_2(x) = l_3(x) = \cdots = l_n(x)$. A homogeneous systolic pyramid automaton with n-dimensional layers (*n-HSPA*) is a pyramidal stack of n-dimensional arrays of cells in which the bottom n-dimensional layer (level 0) has size a^n $(a \geq 1)$, the next lowest $(a-1)^n$, and so forth, the $(a-1)$st n-dimensional layer (level $(a-1)$) consisting of a single cell, called the root . Each cell means an identical finite-state machine, $M = (Q, \Sigma, \delta, \#, F)$, where Q is a finite set of states, $\Sigma \subseteq Q$ is a finite set of input states, $\# \in Q - \Sigma$ is the *quiescent state*, $F \subseteq Q$ is the set of *accepting states*, and δ: $Q^{2^n+1} \to Q$ is the *state transition function*, mapping the current states of M and its 2^n son cells in a $2 \times 2 \times \cdots \times 2$ block on the n-dimensional layer below into M's next state. The input is accepted if and only if the root cell ever enters an accepting state. An *n-HSPA* is said to be a real-time *n-HSPA* if for every n-dimensional tape of size a^n $(a \geq 1)$ it accepts the n-dimensional tape in time $a-1$. By $\pounds^R[n\text{-}HSPA]$ we denote the class of the sets of all the n-dimensional tapes accepted by a real-time *n-HSPA*[1]. A 1-*way n-dimensional cellular automaton* (1-*nCA*) can be considered as a natural extension of the 1-way two-dimensional cellular automaton to n dimensions [3]. The initial configuration of the cellular automaton is taken to be an $l_1(x) \times l_2(x) \times \cdots \times l_n(x)$ array of cells in the nonquiescent state. The initial configuration is accepted if the last special cell reaches a final state. A 1-*nCA* is said to be a real-time 1-*nCA* if when started with an $l_1(x) \times l_2(x) \times \cdots \times l_n(x)$ array of cells in the nonquiescent state, the special cell reaches a final state in time $l_1(x) + l_2(x) + \cdots + l_n(x) - 1$. By $\pounds^R[1\text{-}nCA]$ we denote the class of the sets of all the n-dimensional tapes accepted by a real-time 1-*nCA* [3].

2. Main Results

We mainly investigate a relationship between the accepting powers of real-time *n-HSPA*'s and real-time 1-*nCA*'s. The following theorem implies that real-time *n-HSPA*'s are less powerful than real-time 1-*nCA*'s.

Theorem 2.1. $\pounds^R[n\text{-}HSPA] \subsetneq \pounds^R[1\text{-}nCA]$.

Proof : Let $V = \{x\ x \in \{0,1\}^{(n)} \mid l_1(x) = l_2(x) = \cdots = l_n(x) \& [\forall i_1, \forall i_2, \ldots, \forall i_{n-1}\ (1 \leq i_1 \leq l_1(x), 1 \leq i_2 \leq l_2(x), \ldots, 1 \leq i_{n-1} \leq l_{n-1}(x))[x(i_1, i_2, \ldots, i_{n-1}, 1) = x(i_1, i_2, \ldots, i_{n-1}, l_n(x))]]\}$.

It is easily shown that $V_1 \in \pounds^R[1\text{-}nCA]$. Below, we show that $V \notin \pounds^R[n\text{-}HSPA]$. Suppose that there exists a real-time *n-HSPA*$(n = 3)$ accepting V . For each $t \geq 4$, let

$W(n) = \{ x \in \{0,1\}^{(3)} \mid l_1(x) = l_2(x) = \cdots = l_n(x) \& [x(1, 2, 1), (t, t-1, t)] \in \{0\}^{(3)}\}$.

Eight sons of the root cell $A_{(t-1,1,1,1)}$ of M $A_{(t-2,1,1,2)}$, $A_{(t-2,1,2,2)}$, $A_{(t-2,2,1,2)}$, $A_{(t-2,2,2,2)}$, $A_{(t-2,1,1,3)}$, $A_{(t-2,1,2,3)}$, $A_{(t-2,2,1,3)}$, $A_{(t-2,2,2,3)}$ are denoted by C_{UNW}, C_{USW}, C_{USE},C_{UNE}, C_{DNW}, C_{DSW}, C_{DSE}, C_{DNE}, respectively. For each x in $W(n)$, $x(UNW)$, $x(USW)$, $x(USE)$, $x(UNE)$, $x(DNW)$, $x(\text{USW})$, $x(USE)$, $x(UNE)$ are the states of *CUNW, CUSW, CUSE, CUNE, CDNW, CDSW, CDSE, CDNE*, at time t-2, respectively. Let $\sigma(x) = (x(UNW), x(USW), x(DNW), x(DSW))$, $\gamma(x) = (x(USE), x(UNE), x(DSE), x(DNE))$. and $\rho(x) = (x(UNW), x(USW), x(DNW), x(DSW), x(USE), x(UNE), x(DSE), x(DNE))$. Then, the following two propositions must hold:

Proposition 2.1. (i) *For any two tapes x, $y \in W(n)$ whose 1st(1−3) planes are same, $\sigma(x) = \sigma(y)$.* (ii) *For any two tapes x, $y \in W(n)$ whose n-th(1−3) planes are same, $\gamma(x) = \gamma(y)$*

[**Proof :** From the mechanism of each cell, it is easily seen that the states of C_{UNW}, C_{USW}, C_{DNW}, C_{DSW} are not influenced by the information of $x(1-3)_t$'s. From this fact, we have (i). The proof of (ii) is the same as that of (i). ☐]

Propositon 2.2. *For any two tapes x, $y \in W(t)$ whose 1st (1-3) planes are different, $\sigma(x) \neq \sigma(y)$.*

[**Proof :** Suppose to the contrary that $\sigma(x) = \sigma(y)$. We consider two tapes x', $y' \in W(t)$ satisfying the following :
(i) $x(1-3)_1$ and $x(1-3)_t$, are equal to $x(1-3)_1$ of x, respectively
(ii) $y'(1-3)_1$ is equal to $y(1-3)_1$, and $y'(1-3)_t$ is equal to $x(1-3)_1$.

As is easily seen, $x' \in V$ and so x' is accepted by M. On the other hand, from Proposition 2.1(ii), $\gamma(x') = \gamma(y')$. From Proposition 2.1(i), $\sigma(x) = \sigma(x')$, $\sigma(y) = \sigma(y')$. It follows that y' must be also accepted by M. This contradicts the fact that y' is not in V . ☐]

Proof of Theorem 2.1 (*continued*) : Let $p(t)$ be the number of tapes in $W(t)$ whose 1st (1-3) planes are different, and let $Q(t) = \{ \sigma(x) \mid x \in W(t)\}$, where k is the number of states of each cell of M. Then, $p(t) = 2^{t^2}$, and $Q(t) \leq k^4$. It follows that $p(n) > Q(t)$ for large t. Therefore, it follows that for large t, there must be two tapes x,y in $W(t)$ such that their 1st (1-3) planes are different and $\sigma(x) = \sigma(y)$. This contradicts Proposition 2.2, so we can conclude that $V \notin £^R[3\text{-}HSPA]$. In the case of n-dimension, we can show that $V \notin £^R[n\text{-}HSPA]$ by using the same technique. This completes the proof of Theorem 2.1. □

We next show the recognizability of n-dimensional connected tapes by real-time *n-HSPA*'s by using the name technique of Ref.[3]. Let x in $\{0,1\}^{(n)}$. A maximal subset P of $\boldsymbol{N}^n$ satisfying the following conditions is called a 1-*component* of x.

(i)For any $(i_1,i_2, \ldots ,i_n \in P$, we have $1\leq i_1\leq l_1(x)$, $1\leq i_2\leq l_2(x)$,. . .,$1\leq i_n\leq l_n(x)$, and $x(i_1,i_2, \ldots ,i_n) = 1$.

(ii) For any $(i_1,i_2, \ldots ,i_n)$, $(i_1', i_2', \ldots , i_n') \in P$, there exists a sequence $(i_{1,0},i_{2,0}, \ldots ,i_{n,0}),(i_{1,1},i_{2,1}, \ldots ,i_{n,1}), \ldots, (i_{1,n},i_{2,n}, \ldots ,i_{n,n})$ of elements in P such that $(i_{1,0},i_{2,0}, \ldots ,i_{n,0}) = (i_1,i_2, \ldots ,i_n)$, $(i_{1,n},i_2,n, \ldots ,i_{n,n}) = (i'_1,i'_2, \ldots ,i'_n)$, and $|i_{1,j} - i_{1,j-1}| + |i_{2,j} - i_{2,j-1}| + \ldots + |i_{n,j} - i_{n,j-1}| \leq 1 (1 \leq j \leq n)$. A tape $x \in \{0, 1\}^{(n)}$ is called *connected* if there exists exactly one 1-component of x.

Let T_c be the set of all the n-dimensional connected tapes. Then, we have

Theorem 2.2. $T_c \notin £^R[n\text{-}HSPA]$.

3. Conclusion

We investigated a relationship between the accepting powers of homogeneous systolic pyramid automaton with *n*-dimensional layers(*n-HSPA*) and one-way *n*-dimensional cellular automata (1-*nCA*) in real time, and showed that real-time *n-HSPA*'s are less powerful than real time 1-*nCA*'s.

References

1. K.Culik II, J.Gruska and Salomaa, Sytolic trellis automata, Part I, *International Journal of Computer Mathmatics*, 55:pp.99-121, 1991.
2. C. Iwamoto, K.Inoue, and I.Takanami, Some Properties of Homogeneous Pyramid Automata, *The IEICE Transactions on Information and Systems* (*Japanese Edition*), J73-D-I(9) : 778-780, 1990.
3. C. Iwamoto, K. Inoue, and I. Takanami, Some Properties of Homogeneous Pyramid Automata, The IEICE *Transactions on Information and Systems* (*Japanese Edition*), J73-D-I(9) : 778- 780, 1990.
4. K. Krithivasan and M. Mahajan, Systolic pyramid automata, cellular automata and array languages, *International Journal of Pattern Recognition and Artificial Intelligence*, 3(3 4):405-433, 1989.
5. M.Sakamoto, Three-Dimensional Alternating Turing Machines, Ph.D. *Thesis, Yamaguchi University*, 1999.
6. M.Sakamoto, H.Okabe, and K.Inoue, Some properties of four-dimansional finite automata, *in* 2002 *Chugoku − Section Joint Convention Record of Insistes of Electrical and Information Engineerings, Japan*, p.351,2002.
7. M.Sakamoto, S.Nagami, K.Kono, A relation- ship between the accepting powers of three- dimentional layers, *Trans. of SCI*(*Japan*), Vol.17, No.10, pp.451-458,2004.
8. M.Sakamoto, H.Okabe, S.Nagami, S.Taniguchi, T.Maki, Y.Nakama, M.Saito, M.Kono, and K.Inoue, A note on four-dimensional finite au- tomata, *WSEAS Transactions on Computers*, Issue 5,Vol.3, pp.1651-1656,2004.
9. M.Sakamoto, T.Ito, N.Tomozoe, and H.Furutani, Asurvey of three-dimensional automata, *The papers of Technical Meeting on Information Systems, IEE, Japan*, IS-07-12, pp.11-17,2007.
10. M.Sakamoto, T.Ito, H.Furutani, and M.Kono, Some accepting powers of three-dimensional parallel Turning machines, *in 1st European Workshop on Artificial Life and Robotics*, V ienna, Austria, pp.73-79,2007.
11. M.Sakamoto, N.Tomozoe, H.Furutani, M.Kono, T.Ito, Y.Uchida, and H.Okabe, A survey of automata on three-dimensional input tapes, *WSEAST Transactions on Computers,Issue* 10,Vol.7, pp.1638-1647,2008.
12. M.Sakamoto, T.Ito, H.Furutani, and M.Kono, Some accepting powers of three-dimensional parallel Turning machines, *International Journal of Artificial Life and Robotics*,Vol.13,No.1, pp.27-30,2008.
13. M.Sakamoto, M.Fukuda, S.Okatani, T.Ito, H.Furutani, and M.Kono, Path-bouded Three- dimensional finite automata, *International Journal of Artificial life and Robotica, Springer*,Vol.13,No.1,pp.54-57,2008.
14. M.Sakamoto, S.Okatani, M.Fukuda, T.Ito, and H.Furutani, and M.Kono, A relationship between Turning machines and finite automata on four-dimensional input tapes, *International Journal of Artificial life and Robotica, Springer*, Vol.13,No.1, pp.58-60,2008.
15. M.Sakamoto, S.Okatani, K.Kajisa, M.Fukuda, T.Matsuawa, A.Taniue, T.Ito, H.Furutani, and M.Kono, Hierarchies based on the number of cooperating systems of three-dimensional finite automata,International Journal of Artificial Life and Robotics,Vo.4,No.3, pp.425-428.2009.
16. M.Sakamoto, T.Matsukawa, R.Katamune, H.Furutani, M.Kono, S.Ikeda, T.Ito, Y.Uchida, and T.Yoshinaga, Four-dimensional synchronized alternating Turning machines,*Proceedings of the 10th American Conference*

on Applied Mathmatics, Harvard University, Cambridge, USA, pp.195-200,2010(CD-ROM).

17. M.Sakamoto, R.Katamune, T.Matsukawa, H.Furutani, M.Kono, S.Ikeda, T.Ito, Y.Uchida, and T.Yoshinaga, Some result about hierarchy and recognizability of four-dimensional sychronized alternating Turning machines, *Proceedings of the 10th American Conference on Applied Mathmatics, Harvard University, Cambridge, USA*, pp.201-205, 2010(CD-ROM).
18. M.Sakamoto, T.Matsukawa, R.Katamune, H.Furutani, M.Kono, S.Ikeda, T.Ito, Y.Uchida, and T.Yoshinaga, Synchronized Alternating Turning Machines on Four-Dimensional In- put Tapes, *WSES TRANSACTIONS on COMPUTERS*,Issue 4,Vol.9, pp.319-328,2010.
19. M.Sakamoto, R.Katamune, T.Matsukawa, H.Furutani, M.Kono, S.Ikeda, T.Ito, Y.Uchida, and T.Yoshinaga, Hardware Hierarchies and Recognizabilities of Four-Dimensional Synchro- nized Alternating Turning Machines, *WSES TRANSACTIONS on COMPUTERS*,Issue 4. pp.329-338,2010.
20. M.Sakamoto, T.Ito, X.Qingquan, Y.Uchida, T.Yochinaga, M.Yokomichi, S.Ikeda, and H.Furutani, A Note on Three-dimensional Prob- abilistic Finite Automata, *The Seventeenth International Symposium on Artificial Life and Robotics* 2012, Oita, Japan, pp.492-495,2012(CD-ROM).
21. M.Sakamoto, Y.Uchida, T.Hamada, T.Ito, T.Yoshinaga, M.Yokomichi, S.Ikeda, and H.Furutani, A Relationship between the Accept- ing Powers of Nondeterministic Finite Automata and Probabilistic Finite Automata on Three- Dimensional Input Tapes, *in the 2012 IEICE General Conference, Okayama University*, p.4,2012(CD-ROM).
22. T. Toffoli and N. Margolus, *Cellullar automaton machines − new environment for modeling*, MIT Press, 1987.
23. J. Wiedermann, Parallel Turing machines, Technical Report RUU-CS-84-11, *Department of Computer Science, University of Utrecht, the Netherlands*, 1984.