

Homogeneous Turbulence Dynamics

PIERRE SAGAUT

Université Pierre et Marie Curie

CLAUDE CAMBON

Ecole Centrale de Lyon



CAMBRIDGE
UNIVERSITY PRESS

Contents

Abbreviations Used in This Book

page xvi

1 Introduction	1
1.1 Scope of the Book	1
1.2 Structure and Contents of the Book	3
Bibliography	9
2 Statistical Analysis of Homogeneous Turbulent Flows: Reminders . . .	10
2.1 Background Deterministic Equations	10
2.1.1 Mass Conservation	10
2.1.2 The Navier–Stokes Momentum Equations	12
2.1.3 Incompressible Turbulence	13
2.1.4 First Insight into Compressibility Effects	14
2.1.5 Reminder About Circulation and Vorticity	15
2.1.6 Adding Body Forces or Mean Gradients	16
2.2 Briefs About Statistical and Probabilistic Approaches	19
2.2.1 Ensemble Averaging, Statistical Homogeneity	19
2.2.2 Single-Point and Multipoint Moments	19
2.2.3 Statistics for Velocity Increments	20
2.2.4 Application of Reynolds Decomposition to Dynamical Equations	20
2.3 Reynolds Stress Tensor and Related Equations	22
2.3.1 RST Equations	22
2.3.2 The Mean Flow Consistent With Homogeneity	24
2.3.3 Homogeneous RST Equations. Briefs About Closure Methods	26
2.4 Anisotropy in Physical Space. Single-Point and Two-Point Correlations	27
2.5 Spectral Analysis, From Random Fields to Two-Point	
2.5.1 Correlations. Local Frame, Helical Modes	28
2.5.2 Second-Order Statistics	28
2.5.2 Poloidal–Toroidal Decomposition and Craya–Herring Frame of Reference	31
2.5.3 Helical-Mode Decomposition	32
2.5.4 Use of Projection Operators	33
2.5.5 Nonlinear Dynamics	35
2.5.6 Background Nonlinearity in Different Reference Frames	36

2.6	Anisotropy in Fourier Space	38
2.6.1	Second-Order Velocity Statistics	38
2.6.2	Some Comments About Higher-Order Statistics	43
2.7	A Synthetic Scheme of the Closure Problem: Nonlinearity and Nonlocality	43
	Bibliography	47
3	Incompressible Homogeneous Isotropic Turbulence	49
3.1	Observations and Measures in Forced and Freely Decaying Turbulence	49
3.1.1	How to Generate Isotropic Turbulence?	49
3.1.2	Main Observed Statistical Features of Developed Isotropic Turbulence	51
3.1.3	Energy Decay Regimes	57
3.1.4	Coherent Structures in Isotropic Turbulence	58
3.2	Self-Similar Decay Regimes, Symmetries, and Invariants	59
3.2.1	Symmetries of Navier–Stokes Equations and Existence of Self-Similar Solutions	59
3.2.2	Algebraic Decay Exponents Deduced From Symmetry Analysis	62
3.2.3	Time-Variation Exponent and Inviscid Global Invariants	64
3.2.4	Refined Analysis Without PLE Hypothesis	65
3.2.5	Self-Similarity Breakdown	66
3.2.6	Self-Similar Decay in the Final Region	67
3.3	Reynolds Stress Tensor and Analysis of Related Equations	68
3.4	Classical Statistical Analysis: Energy Cascade, Local Isotropy, Usual Characteristic Scales	70
3.4.1	Double Correlations and Typical Scales	70
3.4.2	(Very Brief) Reminder About Kolmogorov Legacy, Structure Functions, “Modern” Scaling Approach	71
3.4.3	Turbulent Kinetic-Energy Cascade in Fourier Space	73
3.5	Advanced Analysis of Energy Transfers in Fourier Space	76
3.5.1	The Background Triadic Interaction	76
3.5.2	Nonlinear Energy Transfers and Triple Correlations	79
3.5.3	Global and Detailed Conservation Properties	80
3.5.4	Advanced Analysis of Triadic Transfers and Waleffe’s Instability Assumption	81
3.5.5	Further Discussions About the Instability Assumption	85
3.5.6	Principle of Quasi-Normal Closures	86
3.5.7	EDQNM for Isotropic Turbulence. Final Equations and Results	89
3.6	Topological Analysis, Coherent Events, and Related Dynamics	97

3.6.1	Topological Analysis of Isotropic Turbulence	98
3.6.2	Vortex Tube: Statistical Properties and Dynamics	102
3.6.3	Bridging with Turbulence Dynamics and Intermittency	107
3.7	Nonlinear Dynamics in the Physical Space	109
3.7.1	On Vortices, Scales, Wavenumbers, and Wave Vectors – What are the Small Scales?	109
3.7.2	Is There an Energy Cascade in the Physical Space?	111
3.7.3	Self-Amplification of Velocity Gradients	112
3.7.4	Non-Gaussianity and Depletion of Nonlinearity	116
3.8	What are the Proper Features of Three-Dimensional Navier–Stokes Turbulence?	117
3.8.1	Influence of the Space Dimension: Introduction to d -Dimensional Turbulence	117
3.8.2	Pure 2D Turbulence and Dual Cascade	118
3.8.3	Role of Pressure: A View of Burgers' Turbulence	120
3.8.4	Sensitivity with Respect to Energy-Pumping Process: Turbulence with Hyperviscosity	122

Bibliography 123

4	Incompressible Homogeneous Anisotropic Turbulence:	
	Pure Rotation	127
4.1	Physical and Numerical Experiments	127
4.1.1	Brief Review of Experiments, More or Less in the Configuration of Homogeneous Turbulence	129
4.2	Governing Equations	131
4.2.1	Generals	131
4.2.2	Important Nondimensional Numbers. Particular Regimes	131
4.3	Advanced Analysis of Energy Transfer by DNS	133
4.4	Balance of RST Equations. A Case Without "Production." New Tensorial Modeling	135
4.5	Inertial Waves. Linear Regime	139
4.5.1	Analysis of Deterministic Solutions	139
4.5.2	Analysis of Statistical Moments. Phase Mixing and Low-Dimensional Manifolds	143
4.6	Nonlinear Theory and Modeling: Wave Turbulence and EDQNM	145
4.6.1	Full Exact Nonlinear Equations. Wave Turbulence	145
4.6.2	Second-Order Statistics: Identification of Relevant Spectral-Transfer Terms	148
4.6.3	Toward a Rational Closure with an EDQNM Model	149
4.6.4	Recovering the Asymptotic Theory of Inertial Wave Turbulence	150
4.7	Fundamental Issues: Solved and Open Questions	153
4.7.1	Eventual Two-Dimensionalization or Not	153

4.7.2	Meaning of the Slow Manifold	155
4.7.3	Are Present DNS and LES Useful for Theoretical Prediction?	156
4.7.4	Is the Pure Linear Theory Relevant?	157
4.7.5	Provisional Conclusions About Scaling Laws and Quantified Values of Key Descriptors	158
4.8	Coherent Structures, Description, and Dynamics	159

Bibliography 164

5 Incompressible Homogeneous Anisotropic Turbulence: Strain 167

5.1	Main Observations	167
5.2	Experiments for Turbulence in the Presence of Mean Strain. Kinematics of the Mean Flow	169
5.2.1	Pure Irrotational Strain, Planar Distortion	170
5.2.2	Axisymmetric (Irrotational) Strain	172
5.2.3	The Most General Case for 3D Irrotational Case	173
5.2.4	More General Distortions. Kinematics of Rotational Mean Flows	173
5.3	First Approach in Physical Space to Irrotational Mean Flows	174
5.3.1	Governing Equations, RST Balance, and Single-Point Modeling	174
5.3.2	General Assessment of RST Single-Point Closures	177
5.3.3	Linear Response of Turbulence to Irrotational Mean Strain	178
5.4	The Fundamentals of Homogeneous RDT	180
5.4.1	Qualitative Trends Induced by the Green's Function	183
5.4.2	Results at Very Short Times. Relevance at Large Elapsed Times	183
5.5	Final RDT Results for Mean Irrotational Strain	184
5.5.1	General RDT Solution	184
5.5.2	Linear Response of Turbulence to Axisymmetric Strain	185
5.6	First Step Toward a Nonlinear Approach	186
5.7	Nonhomogeneous Flow Cases. Coherent Structures in Strained Homogeneous Turbulence	187

Bibliography 189

6 Incompressible Homogeneous Anisotropic Turbulence: Pure Shear 192

6.1	Physical and Numerical Experiments: Kinetic Energy, RST, Length Scales, Anisotropy	192
6.1.1	Experimental and Numerical Realizations	193
6.1.2	Main Observations	193

6.2	Reynolds Stress Tensor and Analysis of Related Equations	197
6.3	Rapid Distortion Theory: Equations, Solutions, Algebraic Growth	199
6.3.1	Some Properties of RDT Solutions	201
6.3.2	Relevance of Homogeneous RDT	204
6.4	Evidence and Uncertainties for Nonlinear Evolution: Kinetic-Energy Exponential Growth Using Spectral Theory	206
6.5	Vortical-Structure Dynamics in Homogeneous Shear Turbulence	207
6.6	Self-Sustaining Turbulent Cycle in Homogeneous Sheared Turbulence	209
6.7	Self-Sustaining Processes in Nonhomogeneous Sheared Turbulence: Exact Coherent States and Traveling-Wave Solutions	210
6.8	Local Isotropy in Homogeneous Shear Flows	214
	Bibliography	217
7	Incompressible Homogeneous Anisotropic Turbulence: Buoyancy and Stable Stratification	219
7.1	Observations, Propagating and Nonpropagating Motion. Collapse of Vertical Motion and Layering	219
7.2	Simplified Equations, Using Navier-Stokes and Boussinesq Approximations, With Uniform Density Gradient	223
7.2.1	Reynolds Stress Equations With Additional Scalar Variance and Flux	224
7.2.2	First Look at Gravity Waves	225
7.3	Eigenmode Decomposition. Physical Interpretation	226
7.4	The Toroidal Cascade as a Strong Nonlinear Mechanism Explaining the Layering	229
7.5	The Viewpoint of Modeling and Theory: RDT, Wave Turbulence, EDQNM	231
7.6	Coherent Structures: Dynamics and Scaling of the Layered Flow, "Pancake" Dynamics, Instabilities	235
7.6.1	Simplified Scaling Laws	235
7.6.2	Pancake Structures, Zig-Zag, and Kelvin-Helmholtz Instabilities	237
	Bibliography	241
8	Coupled Effects: Rotation, Stratification, Strain, and Shear	243
8.1	Rotating Stratified Turbulence	243
8.1.1	Basic Triadic Interaction for Quasi-Geostrophic Cascade	246
8.1.2	About the Case With Small but Nonnegligible f/N Ratio	247

8.1.3	The QG Model Revisited. Discussion	248
8.2	Rotation or Stratification With Mean Shear	250
8.2.1	The Rotating-Shear-Flow Case	253
8.2.2	The Stratified-Shear-Flow Case	255
8.2.3	Analogies and Differences Between the Two Cases	255
8.3	Shear, Rotation, and Stratification. RDT Approach to Baroclinic Instability	256
8.3.1	Physical Context, the Mean Flow	256
8.3.2	RDT Equations	258
8.4	Elliptical Flow Instability From "Homogeneous" RDT	259
8.5	Axisymmetric Strain With Rotation	265
8.6	Relevance of RDT and WKB RDT Variants for Analysis of Classical Instabilities	266

Bibliography 270

9 Compressible Homogeneous Isotropic Turbulence 273

9.1	Introduction to Modal Decomposition of Turbulent Fluctuations	273
9.1.1	Statement of the Problem	273
9.1.2	Kovaszny's Linear Decomposition	274
9.1.3	Weakly Nonlinear Corrected Kovaszny Decomposition	278
9.1.4	Helmholtz Decomposition and Its Extension	279
9.1.5	Bridging Between Kovaszny and Helmholtz Decomposition	281
9.1.6	On the Feasibility of a Fully General Modal Decomposition	281
9.2	Mean-Flow Equations, Reynolds Stress Tensor, and Energy Balance in Compressible Flows	281
9.2.1	Arbitrary Flows	281
9.2.2	Simplifications in the Isotropic Case	285
9.2.3	Quasi-Isentropic Isotropic Turbulence: Physical and Spectral Descriptions	288
9.3	Different Regimes in Compressible Turbulence	291
9.3.1	Quasi-Isentropic Turbulent Regime	292
9.3.2	Weakly Compressible Thermal Regime	309
9.3.3	Nonlinear Subsonic Regime	315
9.3.4	Supersonic Regime	318
9.4	Structures in the Physical Space	319
9.4.1	Turbulent Structures in Compressible Turbulence	320
9.4.2	A Probabilistic Model for Shocklets	321

Bibliography 324

10 Compressible Homogeneous Anisotropic Turbulence	327
10.1 Effects of Compressibility in Free-Shear Flows. Observations	327
10.1.1 RST Equations and Single-Point Modeling	328
10.1.2 Preliminary Linear Approach: Pressure-Released Limit and Irrotational Strain	330
10.2 A General Quasi-Isentropic Approach to Homogeneous Compressible Shear Flows	332
10.2.1 Governing Equations and Admissible Mean Flows	333
10.2.2 Properties of Admissible Mean Flows	335
10.2.3 Linear Response in Fourier Space. Governing Equations	336
10.3 Incompressible Turbulence With Compressible Mean-Flow Effects: Compressed Turbulence	342
10.4 Compressible Turbulence in the Presence of Pure Plane Shear	344
10.4.1 Qualitative Results	344
10.4.2 Discussion of Results	345
10.4.3 Toward a Complete Linear Solution	348
10.5 Perspectives and Open Issues	349
10.5.1 Homogeneous Shear Flows	350
10.5.2 Perspectives Toward Inhomogeneous Shear Flows	350
10.6 Topological Analysis, Coherent Events and Related Dynamics	351
10.6.1 Nonlinear Dynamics in the Subsonic Regime	352
10.6.2 Topological Analysis of the Rate-of-Strain Tensor	354
10.6.3 Vortices, Shocklets, and Dynamics	355
Bibliography	356
11 Isotropic Turbulence-Shock Interaction	358
11.1 Brief Survey of Existing Interaction Regimes	358
11.1.1 Destructive Interactions	358
11.1.2 Nondestructive Interactions	359
11.2 Linear Nondestructive Interaction	360
11.2.1 Shock Modeling and Jump Relations	360
11.2.2 Introduction to the Linear Interaction Approximation Theory	361
11.2.3 Vortical Turbulence-Shock Interaction	363
11.2.4 Acoustic Turbulence-Shock Interaction	370
11.2.5 Mixed Turbulence-Shock Interaction	373
11.2.6 On the Use of RDT for Linear Nondestructive Interaction Modeling	378
11.3 Nonlinear Nondestructive Interactions	379
11.3.1 Turbulent Jump Conditions for the Mean Field	379
11.3.2 Jump Conditions for an Incident Isotropic Turbulence	381
Bibliography	382

12 Linear Interaction Approximation for Shock–Perturbation Interaction	384
12.1 Shock Description and Emitted Fluctuating Field	384
12.2 Calculation of Wave Vectors of Emitted Waves	386
12.2.1 General	386
12.2.2 Incident Entropy and Vorticity Waves	386
12.2.3 Incident Acoustic Waves	389
12.3 Calculation of Amplitude of Emitted Waves	391
12.3.1 General Decompositions of the Perturbation Field	391
12.3.2 Calculation of Amplitudes of Emitted Waves	393
12.4 Reconstruction of the Second-Order Moments	395
12.4.1 Case of a Single Incident Wave	395
12.4.2 Case of an Incident Turbulent Isotropic Field	399
12.5 <i>A posteriori</i> Assessment of LIA	403
Bibliography	405
13 Linear Theories. From Rapid Distortion Theory to WKB Variants	406
13.1 Rapid Distortion Theory for Homogeneous Turbulence	406
13.1.1 Solutions for ODEs in Orthonormal Fixed Frames of Reference	406
13.1.2 Using Solenoidal Modes for a Green's Function with a Minimal Number of Components	408
13.1.3 Prediction of Statistical Quantities	409
13.1.4 RDT for Two-Time Correlations	412
13.2 Zonal RDT and Short-Wave Stability Analysis	412
13.2.1 Irrotational Mean Flows	413
13.2.2 Zonal Stability Analysis With Disturbances Localized Around Base-Flow Trajectories	413
13.2.3 Using Characteristic Rays Related to Waves Instead of Trajectories	415
13.3 Application to Statistical Modeling of Inhomogeneous Turbulence	417
13.3.1 Transport Models Along Mean Trajectories	417
13.3.2 Semiempirical Transport “Shell” Models	418
13.4 Conclusions, Recent Perspectives Including Subgrid-Scale Dynamics Modeling	419
Bibliography	421
14 Anisotropic Nonlinear Triadic Closures	423
14.1 Canonical HIT, Dependence on the Eddy Damping for the Scaling of the Energy Spectrum in the Inertial Range	423

14.2	Solving the Linear Operator to Account for Strong Anisotropy	425
14.2.1	Random and Averaged Nonlinear Green's Functions	425
14.2.2	Homogeneous Anisotropic Turbulence with a Mean Flow	426
14.3	A General EDQN Closure. Different Levels of Markovianization	428
14.3.1	EDQNM2 Version	429
14.3.2	A Simplified Version: EDQNM1	430
14.3.3	The Most Sophisticated Version: EDQNM3	431
14.4	Application of Three Versions to the Rotating Turbulence	433
14.5	Other Cases of Flows With and Without Production	437
14.5.1	Effects of the Distorting Mean Flow	437
14.5.2	Flows Without Production Combining Strong and Weak Turbulence	438
14.5.3	Role of the Nonlinear Decorrelation Time Scale	440
14.6	Connection with Self-Consistent Theories: Single Time or Two Time?	441
14.7	Applications to Weak Anisotropy	443
14.7.1	A Self-Consistent Representation of the Spectral Tensor for Weak Anisotropy	443
14.7.2	Brief Discussion of Concepts, Results, and Open Issues	445
14.8	Open Numerical Problems	446

Bibliography 447

15 Conclusions and Perspectives 449

15.1	Homogenization of Turbulence. Local or Global Homogeneity? Physical Space or Fourier Space?	449
15.2	Linear Theory, "Homogeneous" RDT, WKB Variants, and LIA	451
15.3	Multipoint Closures for Weak and Strong Turbulence	453
15.3.1	The Wave-Turbulence Limit	454
15.3.2	Coexistence of Weak and Strong Turbulence, With Interactions	455
15.3.3	Revisiting Basic Assumptions in MPC	455
15.4	Structure Formation, Structuring Effects, and Individual Coherent Structures	456
15.5	Anisotropy Including Dimensionality, a Main Theme	457
15.6	Deriving Practical Models	459

Bibliography 460

Index 461