Prog. Theor. Phys. Vol. 64, No. 4, October 1980, Progress Letters

Horizon of the Universe and the Broken-Symmetric Theory of Gravity

—The Shortest Time—

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(Received July 7, 1980)

The conjecture that the homogeneity of the universe might be explained by assuming $G \sim T^{-2}$ for $T >$ Planck Energy is analyzed. This conjecture would be viable only if we accept such a ridiculously short time like (Planck time) $\times$ ($\exp - 10^{30}$).

Recently, Zee[1] pointed out that the particle horizon of the universe would become infinitely large if the gravitational constant G behaves like $G \sim T^{-2}$ for $T \gg T_*$ (Planck temperature) $\sim 10^{19}$GeV, T being the temperature of the universe. Such a behavior of G would be expected if we replaced G by $(\epsilon\varphi^2)^{-1}$, where φ is the vacuum expectation value of some scalar field and, at high temperature, this symmetry-breaking is restored.

A similar idea has been given by Stecker.[2]

The expansion equation of the homogeneous, isotropic model is given as

$$\left(\frac{da}{dt}\right)^2 = \frac{8\pi G}{3}\rho_r a^2 , \tag{1}$$

where a is the radius of curvature and $\rho_r a^4$ = constant for the radiation universe. If G behaves like

$$G = \frac{\epsilon^{-1}}{T_*^2 + T^2} \tag{2}$$

as Zee assumed, (1) becomes for $T \gg T_*$

$$\left(\frac{da}{dt}\right)^2 = \frac{8\pi}{3\epsilon}\frac{(\rho_r a^4)}{T_*^2 a_*^2} , \tag{3}$$

where $a_* = a(T_*)$. Then, we get such an expansion law as

$$a = \sqrt{\frac{8\pi}{3\epsilon}}\frac{\sqrt{(\rho_r a^4)}}{T_* a_*}t \tag{4}$$

and, as Zee pointed out, the comoving coordinate of the horizon χ_{H} diverges because

$$\chi_{\mathrm{H}} = \int_0^{t_0}\frac{dt}{a(t)} = \infty . \tag{5}$$

From this result, he conjectured that a large scale homogeneity of the universe might be attainable at the beginning.

Various doubts might be aroused to this argument such as validity of the classical "field" $a(t)$ even for $T \gg T_*$ inspite of the new effect of quantum gravity, and others. However, it is even very attractive idea since we have not known more attractive ideas which explain this puzzle of the standard universe model.

In this short note, we check how small time is necessary for this idea to be viable. From this argument, we will meet with a ridiculously short time which has never appeared in physics.

Instead of (5), we write the horizon after some time t_i as

$$\chi_{\mathrm{H}} \sim \frac{t_0}{a(t_0)} + \int_{t_i}^{t_*}\frac{dt}{a(t)} . \tag{6}$$

From the observation, we have checked that the universe is homogeneous until the horizon at the present at least, which implies that $\chi_{\mathrm{H}} \sim O(\pi)$. Therefore we require that

$$O(\pi) \sim \sqrt{\frac{3\epsilon}{8\pi}}\frac{T_* a_*}{(\rho_r a^4)^{1/2}}\ln t_*/t_i \tag{7}$$

or

$$t_i \sim t_* e^{-A} \tag{8}$$

with

$$A \sim (g/\epsilon)^{1/2} N_\gamma^{1/3}, \tag{9}$$

because

$$A \sim \left(\frac{8\pi}{3\epsilon}\right)^{1/2} \sqrt{g n_\gamma a^3} / (T_* a_*)$$

$$\sim (g/\epsilon)^{1/2} N_\gamma^{1/3},$$

taking the total number of photon in the universe as

$$N_\gamma \sim n_\gamma a^3 \sim (Ta)^3, \tag{10}$$

g being the number of particle species.[3] Here, the definition of the total photon number will be clear for the closed model. But we may use this number even for the open model redefining N_γ as the photon number within the radius of horizon at the present since $t_0 \lesssim a_0$. This number is estimated from the observed values of the Hubble constant and the cosmic black body temperature at the present, and it is given as

$$N_\gamma \sim 10^{90}.$$

Then, t_i becomes extremely small as

$$t_i \sim t_* e^{-10^{30}}, \tag{11}$$

and the temperature at t_i is

$$T_i \sim T_* e^{10^{30}}. \tag{12}$$

If the assumption (2) is true, the expansion law becomes

$$a \sim a_* (t/t_*)^{1/2} \quad \text{for } T < T_*,$$

$$\sim a_* (t/t_*) \quad \text{for } T > T_*,$$

where

$$a_* = N_\gamma^{1/3} t_* \sim 10^{-3} (N_\gamma/10^{90})^{1/3} \text{cm}$$

is a at $T = T_*$. The size of the whole universe at T_* is still a macroscopic size. The time when $a \sim t_*$ is given by $t_* N_\gamma^{-1/3}$ and, if the expansion started from this time, the horizon is very small as

$$\chi_H \sim \frac{1}{N_\gamma^{1/3}} \ln N_\gamma^{1/3}.$$

In previous papers,[3] the author emphasized an importance of the total photon number or the total entropy of the universe, which is nearly the same with N_γ except the g factor. This N_γ characterizes the size of the universe and it is not determined by the physical constants. When we discuss the local interactions among the particles in such problems as He-formation and baryon number generation, N_γ did not appear since the T-t relation does not contain N_γ. However, in our problem of horizon, the size of the universe is crucial. This is the reason why N_γ has appeared in t_i.

We feel something ridiculous about these numbers. We have not met with such large number as $e^{10^{30}}$ except the Poincaré recurrent time of $10^{10^{24}}$. However, Zee's conjecture would work only if we accept such a ridiculous number.

1) A. Zee. Phys. Rev. Letters **44** (1980), 703.
2) F. W. Stecker, Astrophys. J. **235** (1980), L1.
3) The author has written the articles, in which the universe model is described in terms of the Planck units and the total photon number N_γ; H. Sato, Prog. Theor. Phys. **63** (1980), 1971; in *Proceedings of the workshop held in Feb. 1979, KEK-79-18;* preprint RIFP-380 (1979).