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# Horizontal Mergers, Structural Remedies and Consumer Welfare in a Cournot Oligopoly with Assets

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# Horizontal Mergers, Structural Remedies and Consumer Welfare in a Cournot Oligoply with Assets [1]

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**Abstract**

Competition authorities sometimes require that firms divest some of their assets to rivals in order to allow a merger to take place. This paper extends the results of Farrell and Shapiro [1990a] and shows that, in the absence of technological synergies, a merger is highly unlikely to benefit consumers, even if it is subjected to appropriate structural remedies. For instance, a merger may ultimately lead to a lower price only if at least two different firms acquire the divested assets, and if the merging parties had relatively important pre-merger market shares.

**Keywords:** mergers, structural remedies, efficiency gains, Cournot oligopoly.

**Résumé**

Les autorités de concurrence imposent régulièrement aux entreprises de revendre une partie de leur capacité de production à leurs rivaux (existants ou potentiels) pour voir leur fusion autorisée. Nous montrons dans ce papier que de tels remèdes structurels n'ont que très peu de chances de permettre de compenser les effets anti-concurrentiels de la fusion si celle-ci ne crée pas de synergies. De telles fusions devraient ainsi être bloquées par les autorités de concurrence. Ce papier étend donc à l'analyse des remèdes structurels, le résultat de Farrell et Shapiro [1990a].

**Keywords:** fusions, remèdes structurels, gains d'efficacité, concurrence en quantités.

**JEL Codes / Classification JEL:** D43, K21, L13, L41.

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# 1 Introduction

When a proposed merger raises competition concerns, antitrust authorities can either attempt to prohibit the merger or negotiate commitments with the merging parties. In Europe, although detailed investigations have been launched for only a small proportion of proposed mergers (3.4% of the 3004 mergers notified between 1999 and 2008 went into "phase II"), firms are often subjected to commitments before obtaining the authorities' clearance. Out of the 101 mergers that went into "phase II" between 1999 and 2008, 61 were cleared after the European Commission accepted the remedies offered by the parties, whereas only 30 were cleared without commitments (10 were prohibited). Such commitments can be either behavioral (e.g., termination of exclusive agreements, licensing agreements, access to an essential facility) or structural (divestment of assets or brands to competitors). There is however a clear preference for structural remedies, because they are easier to implement and less difficult to monitor than behavioral commitments.[1]

This paper provides a theoretical contribution to the analysis of the impact of structural remedies in horizontal mergers between Cournot competitors in industries where the ownership of some tangible assets is essential. I first develop a framework similar to the model of quantity competition with assets first proposed by Farrell and Shapiro [1990b] and derive some properties of the equilibrium. These preliminary results are then applied to merger control and I show that, when the pre-merger market consists of three firms only, divestitures can never compensate for the negative impact that the merger itself has on consumer surplus.[2] I finally consider larger oligopolies and identify conditions that suffice to ensure that there does not exist a remedy that would be accepted by competition authorities. By allowing authorities to restore some symmetry between the remaining firms, remedies can compensate part of the negative impact of

[1] See the contributions in Lévêque and Shelanski [2003] for an overview of the use of remedies in the U.S. and the E.U., and Duso et al. [2006] for an empirical investigation of the effects of such remedies.

[2] Given that the total quantity of assets is assumed to be fixed, I do not consider post-merger entry. For an analysis of this issue, see Spector [2003] who extends the result of Farrell and Shapiro [1990a] to entry. Werden and Froeb [1998] analyze the same issue in the context of price competition with differentiated products.

the merger. However, I show that this cannot prevent an overall price increase when the divested assets are sold to only one rival, or when the merging parties are initially too small or do not divest a sufficiently high proportion of the acquired assets.

Although there exists a large body of literature focusing on the analysis of horizontal mergers, little attention has been given to the formal analysis of (structural) remedies. Cabral [2003] uses the example of the *Staples / Office Depot* merger to show that forcing the merging firms to sell stores to rivals can be a bad idea, because it might dissuade these rivals from opening new stores, thereby reducing consumer welfare. Medvedev [2004] shows, in a Cournot setting where a firm's marginal cost is inversely proportional to the quantity of assets it owns, that asset divestments increase the set of mergers that can be approved by competition authorities. Remedies may moreover simultaneously increase the merging firms' profits and consumer surplus when competition authorities decide which rival should acquire the divested assets. Using an almost identical framework but assuming that assets are indivisible, Vasconcelos [2007] shows that divestitures can lead to over-fixing, that is, reduce the price below its pre-merger level.

Unfortunately, it is difficult to avoid making ad-hoc assumptions about synergies when considering remedies. For instance, Medvedev [2004] and Vasconcelos [2007] adopt a very specific cost function which implies that synergies only depend on the quantity of assets that firms hold, not on whether these assets are compatible or not. However, synergies are very likely to depend on the identity of merging firms or of the divested assets' buyers. Rather than making specific assumptions about synergies, I focus in this paper on the case where mergers do not create technological synergies. This is admittedly a restrictive assumption and my model is thus not fully general. However, it proves convenient to analyze the role of structural remedies and allows me to extend previous results. In particular, my analysis supports the view that consumers are unlikely to benefit from a merger if firms cannot provide strong evidence that technological synergies can be created by the merger and/or the asset divestments.

# 2 The Model

## 2.1 Demand and Cost Functions

In order to analyze the impact of mergers and asset divestitures, I adopt the model of Cournot oligopoly with assets first developed by Farrell and Shapiro [1990b]. Firms are Cournot oligopolists and sell a homogeneous good in a market where the inverse demand function is given by $P(Q)$, $Q$ being the aggregate output. Throughout this paper, the following standard assumptions are maintained:

$(P_1)$ : There exists $\overline{Q} \in [0, +\infty]$ such that $P(Q) > 0$ for $Q \in \left[0, \overline{Q}\right[$ and $P(Q) = 0$ for any $Q \geq \overline{Q}$.

$(P_2)$ : For any $Q < \overline{Q}$, the inverse demand function, $P(Q)$, is twice-continuously differentiable and decreasing. Moreover, $P'(Q) + QP''(Q) < 0$.

The total number of firms, $N$, is fixed, as is the industry-wide stock of assets (e.g., tangible capital or essential input), reflecting important barriers to entry. Firm $i$ (with $i = 1, ..., N$) initially owns a quantity of assets $k_i > 0$, and the aggregate quantity of assets is denoted $K$. Entry and acquisition of additional assets are therefore impossible, and growth can only be achieved externally (i.e., through mergers). Although these are strong assumptions, I believe that they characterize reasonably well some industries where capacity is fixed (or very difficult to increase) and entry virtually impossible, at least in the short term.[3]

Following Perry and Porter [1985], I assume that asymmetries between firms (pre- and post-merger) only come from the differences in the quantities of assets that they own. I thus suppose that all firms have access to the same technology and that the (common) cost function is $C(q, k)$, where $q$ is the quantity produced and $k$ is the firm's quantity of assets. I make the following assumptions about this cost function:

[3]These conditions may apply, to some extent, to airlines (landing slots at some congested airports are a scarce resource) or to grocery retailers (in France, planning permission restrict the opportunities to open new supermarkets). Motta and Vasconcelos [2005] suggest that they may also apply to the cement or mineral water industries.

$(C_1)$ : The cost function is twice-continuously differentiable and the marginal cost of production is positive ($C_q \geq 0$), strictly increasing ($C_{qq} > 0$) and convex ($C_{qqq} \geq 0$).

$(C_2)$ : The cost function is homogeneous of degree 1 in $(q, k)$.

$(C_3)$ : There is no fixed cost, that is, $C(0, k) = 0$.

Homogeneity of degree 1 (Assumption $(C_2)$) implies that $qC_{qq} + kC_{qk} = 0$[4] and therefore (under Assumption $(C_1)$) $C_{qk} < 0$ : an increase in the stock of capital lowers the marginal cost curve, thereby creating additional incentives to merge. Assumption $(C_2)$ is the central feature of this model, and, in particular, under the condition that $C_{qq} > 0$, it is equivalent to assuming that the cost function satisfies:[5]

$$C(q, k_i + k_j) = \min_{q_i + q_j = q} \left( C(q_i, k_i) + C(q_j, k_j) \right)$$

This property allows me to interpret a merger as the combination of the merging firms' assets and the merged entity is fully characterized by the quantity of assets it owns. Using the terminology of Farrell and Shapiro [1990a], the mergers I consider in this paper do not generate synergies: the best result that a merged entity can achieve, through reallocation of production across facilities, is exactly what the merging parties could have jointly achieved pre-merger by coordinating their production decisions.

Although this critical homogeneity (or "no synergy") assumption means that my framework is not fully general, it is a particularly convenient and reasonable way to analyze structural remedies. As I mentioned in the introduction, it is extremely difficult (if at all possible) to consider a general model with technological synergies, since synergies are likely to depend upon the identity of the acquiring firm (whether for the merger itself or when considering divestments). The objective of this paper is therefore

[4] Euler's theorem implies that $qC_q + kC_k = C$. Differentiating this equation with respect to $q$ yields the equality $qC_{qq} + kC_{qk} = 0$.

[5] Using the first-order condition of the minimization program, it is straightforward to show that Assumption $(C_2)$ implies absence of synergies. To prove the reverse statement, I first show, by induction, that $C(xq, xk) = xC(q, k)$ for any integer $x$. It then easily follows that the property also holds for any rational number $x$, and therefore for any real number $x$ since the cost function is continuous. A complete proof is available from the author upon request.

to propose an analysis of the role of structural remedies in a simplified setting. Moreover, eliminating synergies from the model also serves the purpose of extending Farrell and Shapiro's results to the analysis of structural remedies.

Assumption $(C_3)$ (i.e., no fixed cost) allows me to keep the presentation as simple as possible but is not crucial for the analysis. I briefly discuss the role of fixed costs at the end of the paper. Finally, under Assumption $(C_2)$, the marginal cost function is homogeneous of degree 0, and $C_q(0,k)$ is therefore independent of $k$ (for any $k>0$). I denote by $c_0$ this constant and assume that it is small enough, so that I only consider equilibria for which a positive quantity is produced:

$(P_3):\ P(0) > c_0 \equiv C_q(0,k)\,.$

## 2.2 Concentration and Price

As shown by Gaudet and Salant [1991], the above-mentioned assumptions ensure that, for any allocation of assets, there exists a unique Cournot-Nash equilibrium. Moreover, Assumption $(P_3)$ guarantees that at least one firm is active. This in turn implies that the equilibrium price must be larger than $c_0$, which gives incentives to all other firms to be active:

**Lemma 1** *For any interior allocation of the assets, (i.e., $k_i > 0$ for any $i = 1,...,N$), the unique Cournot-Nash equilibrium is such that all $N$ firms are active (i.e., $q_i > 0$ for all $i = 1,...,N$) and equilibrium quantities are characterized by the first-order conditions of the producers' profit maximization programs:*

$$P(Q^*) + q_i^* P'(Q^*) = C_q(q_i^*, k_i)\,, \qquad \text{for any } i = 1,...,N. \tag{1}$$

Despite being a straightforward result, Lemma 1 is important because it ensures that a transfer of assets from one firm to another never forces any firm out of the market. Moreover, Firm $i$'s equilibrium quantity tends to 0 as $k_i$ tends to 0. Assumption $(C_2)$ indeed implies that the marginal cost function is homogeneous of degree 0 and therefore:

$$C_q(q,k) = C_q\left(\frac{q}{k}, 1\right), \text{ for any } q \text{ and any } k > 0.$$

Since $C_q(q_i, k_i)$ is increasing and convex in $q_i$, and $P + q_i P'$ is decreasing, it must be the case that $q_i$ tends to 0 as $k_i$ tends to 0, because $C_q(q_i, k_i)$ would otherwise tend to infinity. This result allows me to interpret a merger (or any reallocation of assets) as a sum of infinitesimal transfers.

I now show that an increase in concentration is always bad news for consumers. I thus consider the impact of a small transfer of assets $dk$ from Firm 2 to Firm 1, and want to determine the sign of:

$$p^*(k_1 + dk, k_2 - dk, k_3, ..., k_N) - p^*(k_1, k_2, k_3, ..., k_N),$$

where $p^*(k_1, ..., k_N)$ denotes the equilibrium price of the quantity-competition game when the allocation of assets is $\mathbf{k} = (k_1, ..., k_N)$.[6] Following Farrell and Shapiro [1990b], I denote by $\mathbf{dq} = (dq_1, ...dq_N)$ the impact of the transfer of assets on the individual quantities and by $dQ$ the change in total output. Differentiating the first-order conditions (1), I get (with $dk_i = 0$ for any $i = 3, ..., N$) :

$$(P'(Q) + q_i P''(Q))\, dQ + (P'(Q) - C_{qq}(q_i, k_i))\, dq_i = C_{qk}(q_i, k_i)\, dk_i,$$

condition which can also be written as:

$$dq_i = \frac{P'(Q) + q_i P''(Q)}{C_{qq}(q_i, k_i) - P'(Q)} dQ + \frac{-C_{qk}(q_i, k_i)}{C_{qq}(q_i, k_i) - P'(Q)} dk_i. \tag{2}$$

Summing up equations (2) for all $i = 1, ..., N$, yields:

$$\frac{dQ}{dk} = \frac{\frac{-C_{qk}(q_1,k_1)}{C_{qq}(q_1,k_1)-P'(Q)} - \frac{-C_{qk}(q_2,k_2)}{C_{qq}(q_2,k_2)-P'(Q)}}{1 + \sum_{i=1}^{n} \frac{-(P'(Q)+q_i P''(Q))}{C_{qq}(q_i,k_i)-P'(Q)}}. \tag{3}$$

Let me now denote $\sigma(q, k) \equiv \frac{C_{qq}-P'}{-C_{qk}}$ $(> 0)$. The denominator of the right-hand side term of Equation (3) being positive, an (infinitesimal) transfer of assets from Firm 2 to Firm 1 decreases output (that is, leads to a price increase) if and only if $\sigma(q_1, k_1) > \sigma(q_2, k_2)$ (as already shown by Farrell and Shapiro [1990b, Proposition 2]). Farrell and Shapiro [1990b, Proposition 3] also show that increasing the concentration of capital ownership (i.e., transferring assets from a smaller firm to a bigger one) always reduces output if the marginal cost is a strictly convex function of output and capital. However, this

[6] Since firms are identical except for the quantity of assets they own, any permutation keeps the equilibrium price unchanged.

condition is never satisfied under Assumptions $(C_1)-(C_3)$.[7] Nevertheless, the following lemma shows that more asymmetry leads to a higher price in my setting:

**Lemma 2** *Given assumptions* $(P_1)-(P_3)$ *and* $(C_1)-(C_3)$, $\sigma(q_1,k_1) \geq \sigma(q_2,k_2) \Leftrightarrow k_1 \geq k_2$, *so that a transfer of assets from a firm with smaller assets to a firm with larger assets leads to a price increase. As a consequence, for any given number of firms* $(N)$ *and total quantity of asset* $(K)$, *the equilibrium price is minimized when the allocation of the asset is symmetric, that is,* $k_i = \frac{K}{N}$ *for any* $i = 1, ..., N$.

**Proof.** See Appendix A. ■

The production process is more efficient when firms are symmetric because the marginal costs of production are then equalized in equilibrium. Moreover, symmetry is also good for consumers because competition is more intense when firms are symmetric than when a big firm (with a low marginal cost) competes with a much smaller (and therefore less efficient) rival.[8]

The result that the equilibrium price is minimized when assets are symmetrically distributed across firms relies on the fact that a reallocation of assets never induces a firm to exit. Had I considered the same cost function as Medvedev [2004], who assumes that a firm's marginal cost is inversely proportional its capital stock $\left(\text{i.e., } C(q,k) = \frac{cq}{k}\right)$, increasing concentration would still increase the equilibrium price. However, because a firm may now exit if it becomes too small relative to its rivals,[9] a social planner aiming at minimizing price might find it optimal allocate assets asymmetrically.[10]

As a consequence, in this setting without synergies, any merger harms consumers, since it makes the market more asymmetric:

**Proposition 1** *(Farell and Shapiro [1990a, Proposition 2]) Given assumptions* $(P_1)-(P_3)$ *and* $(C_1)-(C_3)$, *in the absence of technological synergies, any merger without structural remedy causes a price increase.*

[7] For instance, it is violated for the quadratic cost function $\left(\frac{\gamma q^2}{2k}\right)$ used by Perry and Porter [1985].

[8] Smaller firms are less efficient in equilibrium because they internalize the impact of their output decision on price levels less than larger firms, so that they choose greater output / capital ratios.

[9] The marginal cost (which does not depend on $q$) becomes infinite as $k$ tends to 0, and therefore $C_q(0,k) > P(0)$ for $k$ small enough.

[10] All active firms would nevertheless be symmetric. Moreover, it might even be socially optimal to have a monopolist.

# 3 Mergers and Structural Remedies

I now analyze the impact of mergers and structural remedies on the equilibrium price. There are two main reasons to focus on consumer surplus rather than on total welfare. First, many authors agree that most competition authorities and courts use a consumer surplus standard.[11] This bias towards consumers is even explicitly mentioned in the merger guidelines in the European Union and in the United States. Second, it proves impossible to provide any general result using a total welfare standard.[12]

In my setting, Proposition 1, which basically replicates the results of Farrell and Shapiro [1990a, Proposition 2], shows that mergers harm consumers and should therefore be blocked by competition authorities whose objective is to protect consumers. Lemma 2 shows, however, that structural remedies (i.e., asset divestitures) that restore symmetry (i.e., reduce the post-merger concentration level) lower the post-merger price. Therefore, conditionally on approving a merger, any remedy that restores symmetry lowers the price below its post-merger level, and competition authorities should therefore favor such remedies. Firms may not be willing to participate (i.e., to offer or accept such remedies) since restoring symmetry may reduce their profits, possibly even below the pre-merger level. As I already mentioned earlier, it proves extremely complicated to derive general results about profits, either at individual or industry level, even a simple model with linear demand and marginal cost functions. Identifying remedies that firms are willing to offer or accept is therefore impossible. However, if firms are willing to offer a remedy that helps restoring symmetry, competition authorities should condition their clearance decision on the actual sale of the corresponding assets to a smaller competitor. In the case where no (small) buyer can be found because the joint-profits of the merging parties and any potential buyer(s) do not increase, then the merger will simply not go-ahead. Otherwise, the remedy will lead to a price that is lower post-remedy than post-merger. This discussion is summarized in the following

[11] According to Lyons [2002], competition authorities *"overwhelmingly focus on consumers (...) to the exclusion of the welfare of merging firms."* Neven and Röller [2000] also find it "*striking that some of the major antitrust agencies actually operate with objectives that differ from welfare maximization."*

[12] In a simple model with linear demand and marginal cost functions, McAfee and Schwartz [1992] could only obtain relatively limited results when considering the impact of mergers on total welfare.

proposition:

**Proposition 2** *Conditionally on accepting the merger, competition authorities should always accept a remedy offered by the merging parties, provided that the corresponding transfer of assets restores symmetry.*

This result is a straightforward consequence of Lemma 2. It is nevertheless interesting because it implies that remedies can be used to reduce the negative impact of a merger. Remedies can therefore be a substitute for the existence of important synergies. For instance, if firms can prove that the merger will create synergies, but cannot convince competition authorities about the magnitude of these cost reductions, they may still be allowed to merge provided that they commit to sell part of their assets to smaller competitors. Firms should of course be able to identify the right assets to divest, since the remedy may otherwise detract the synergy available from the merger.

The objective of the rest of the paper to see whether structural remedies can be enough to compensate for the negative impact of the merger, even in the absence of synergies. I thus try to identify structural remedies that can outweigh the price increase caused by a given merger, in which case competition authorities should identify appropriate asset divestitures before clearing the proposed merger. In a setting with technological synergies, Medvedev [2004] shows that structural remedies indeed extend the range of parameters for which consumers may benefit from a merger. As I show in what follows, this is rarely the case in the absence of technological synergies.

Without loss of generality, consider a merger between Firm $N-1$ and Firm $N$, and assume that $k_{N-1} \geq k_N$ and $k_1 \geq k_2 \geq \ldots \geq k_{N-2}$. The post-merger allocation of assets is thus $(k_1, \ldots, k_{N-2}, k_{N-1} + k_N, 0)$, and a remedy is a transfer of assets from the merging firms *(the insiders)* to a subset of the non-merging firms *(the outsiders)* that leads to the final allocation $\left(k'_1, \ldots, k'_{N-2}, k'_I, 0\right)$. The following definition identifies conditions that I want for the structural remedy and that competition authorities and firms involved in the process (i.e., insiders as well as the non-merging firms that acquire the divested assets) should all be willing to accept:

**Definition 1** ***(Acceptable Remedy)*** *An acceptable remedy is a reallocation of assets that satisfies the following conditions:*

($R_1$) *The new entity is bigger than its biggest constituent and no outsider is smaller than before the merger.*

($R_2$) *The post-remedy price is not higher than the pre-merger price.*

($R_3$) *The joint profits of insiders (i.e., merging firms) and buyers (i.e., outsiders acquiring the divested assets) are higher post-remedy than pre-merger.*

Condition ($R_1$) simply means that competition policy cannot be used to reshape the industry, i.e., it is not industrial policy. Competition authorities can evaluate the proposed merger, they may be able to require that merging firms sell part of the acquired assets to (some of) their rivals, but they cannot propose an alternative merger. Conditions ($R_2$) and ($R_3$) respectively ensure that consumers are not harmed and that involved parties (insiders and buyers) are willing to accept the proposed remedy. If their joint profits increase in the process, it is indeed possible to find prices for the transferred assets.[13]

## 3.1 The Three-firms Case

I first consider the simple case where there are only three firms in the pre-merger market, and show that, although a structural remedy decreases the post-merger price, it cannot prevent the price from rising above its pre-merger level when the merger does not generate technological synergies. To prove this result, it is enough to show that there does not exist any reallocation of assets that simultaneously satisfies ($R_1$) and ($R_2$), which is equivalent to checking that:

$$\min_{k_2 \leq k' \leq k_2 + k_3} p^* \left(K - k', k', 0\right) \geq p^* \left(k_1, k_2, k_3\right).$$

Suppose first the outsider (Firm 1) holds more than half of the assets. Since Firm 1 remains the larger firm after the merger, any structural remedy would only reinforce asymmetry and thus generate an additional price rise. The situation is similar when the biggest insider (Firm 2) initially holds more than half of the assets. The lowest

[13] I implicitly assume that prices can be negative. Imposing non-negative prices for the divested assets makes it less likely to find an acceptable remedy.

(post-remedy) price would then be achieved by replacing the merger between Firm 2 and Firm 3 with a merger between Firm 1 and Firm 3. However, Proposition 1 would then apply, the price thus being greater post-remedy than pre-merger.

The only remaining situation is one where no firm initially owns 50% or more of the capital $\left(0 < k_1, k_2, k_3 < \frac{K}{2}\right)$. The competition authorities' preferred remedy (i.e., the remedy satisfying Condition $(R_1)$ that generates the lowest post-remedy price) is then to restore perfect symmetry. However, Lemma 2 shows that increasing concentration causes a price rise. Keeping $k_3$ constant, reallocating the remaining assets to increase the quantity of assets held by either Firm 1 or Firm 2 to its upper bound $\frac{K}{2}$ can only increase price. Hence:

$$\max_{0\leq k_1,k_2,k_3\leq \frac{K}{2}} p^*\left(k_1,k_2,k_3\right) = \max_{0\leq k_3\leq \frac{K}{2}} p^*\left(\frac{K}{2},\frac{K}{2}-k_3,k_3\right).$$

Repeating this argument thus yields:

$$\max_{0\leq k_1,k_2,k_3\leq \frac{K}{2}} p^*\left(k_1,k_2,k_3\right) = p^*\left(\frac{K}{2},\frac{K}{2},0\right) = \min_{k_2\leq k'\leq k_2+k_3} p^*\left(K-k',k',0\right),$$

where the last equation holds since $k_2 \leq \frac{K}{2} \leq k_2 + k_3$. The best post-remedy situation is therefore the worst pre-merger scenario, implying that structural remedies are never enough to prevent the price from increasing above its pre-merger level. This result is summarized in the following proposition:

**Proposition 3** *When $N = 3$, any merger causes the price to rise, even when competition authorities can impose structural remedies.*

For a merger to have a positive impact on consumer surplus once it has been subjected to the appropriate remedy, the pre-merger situation must be such that no firm initially owns more than half of the available assets. The merger creates a new largest firm and the best remedy satisfying condition $(R_1)$ is thus to create two symmetric firms. However, because asymmetry is bad for consumers, the worst pre-merger scenario is one where two of the three firms are extremely asymmetric, that is, when one firm is infinitely small, situation which corresponds to the best post-remedy outcome. Therefore, no remedy can totally compensate for the negative impact of the merger.

This result holds even if competition authorities are allowed to require that the merging firms divest assets to a new entrant. A transfer of assets causes the price to

decrease only if the newly formed entity transfers more than the acquired assets (i.e., more than $k_3$) to the new entrant in order to restore symmetry. But this is industrial policy rather competition policy, since Firm 2 acquires Firm 3 only to divest some of its own assets in addition to the acquired assets.

## 3.2 Four or More Firms

Mergers rarely occur in markets with only three players, which seems to limit the applicability of Proposition 3. However, the analysis carried out in the three-firms case can also be applied to derive more general conclusions. A first implication of Proposition 3 is that a remedy involving only one outsider is never acceptable: to prove this result, it simply suffices to replicate the previous analysis keeping the allocation of assets between the remaining $N-3$ firms constant.

Moreover, there must initially be enough asymmetry between firms for an acceptable remedy to exist. In particular, if the biggest firm is not big enough, acceptable remedies can never be found. Consider an initial (interior) allocation such that $\max_i k_i \leq \frac{K}{N-1}$. In this case, the competition authorities' preferred remedy (i.e., the remedy satisfying $(R_1)$ that yields the lowest price) is to restore symmetry, in which case the post-remedy price is $p^*\left(\frac{K}{N-1}, \frac{K}{N-1}, \ldots, \frac{K}{N-1}, 0\right)$. I now show that this is always larger than the highest pre-merger equilibrium price, that is:

$$\max_{0\leq k_1,k_2,\ldots,k_N\leq\frac{K}{N-1}} p^*(k_1,\ldots,k_N) \leq p^*\left(\frac{K}{N-1},\ldots,\frac{K}{N-1},0\right).$$

Because price rises when the market becomes more asymmetric, keeping $k_3,\ldots,k_N$ constant, the price is maximized when either $k_1=\frac{K}{N-1}$ or $k_2=\frac{K}{N-1}$. This implies that:

$$\max_{0\leq k_1,k_2,\ldots,k_N\leq\frac{K}{N-1}} p^*(k_1,\ldots,k_N) = \max_{0\leq k_2,k_3,\ldots,k_N\leq\frac{K}{N-1}} p^*\left(\frac{K}{N-1},k_2,\ldots,k_N\right)$$

Repeating this argument $N-2$ times then yields:

$$\max_{0\leq k_1,k_2,\ldots,k_N\leq\frac{K}{N-1}} p^*(k_1,\ldots,k_N) = p^*\left(\frac{K}{N-1},\ldots,\frac{K}{N-1},0\right).$$

Just like in the three-firms case, the analysis shows that the authorities' preferred remedy yields the same outcome as the worst pre-merger scenario. Therefore, no remedy

can prevent a price increase. The above analysis is summarized in the following proposition:

**Proposition 4** *There never exists any acceptable remedy if only one outsider is involved or if the largest firm initially owns less than one* $(N-1)^{th}$ *of available assets (i.e.,* $(N-1)\max_i k_i \leq K$*).*

Although few mergers involve markets were there are only three competitors, it is relatively common for remedies to involve one outsider only, especially when the parties offer to divest some assets at an early stage of the investigation.[14] The analysis suggests that, unless important synergies can be identified, the merger can only cause a price increase. Competition authorities should therefore block the merger or identify alternative remedies involving more outsiders. Since restoring symmetry is important to maximize the effect of divestments on price, the remedy should involve as many outsiders as possible (as long as firms are willing to participate, i.e., that Condition $(R_3)$ holds). This may be difficult to execute in practice, especially if some assets are indivisible, which only reinforces the feeling that (Cournot) mergers that do not exhibit strong synergies should not be cleared by competition authorities.

Proposition 4 also shows that acceptable remedies do not exist unless the biggest firm is large enough. It is unfortunately rather difficult to derive more general results and I thus turn to a simpler situation limiting my attention to the four-firms case. Although this seems restrictive, it allows me to analyze situations where the remedy involves two outsiders only, and is therefore more general than it may seem at first glance. Indeed, once the identity of the four firms − the two merging firms and the two buyers − is known, necessary conditions for acceptable remedies to exist can be identified (focusing on Conditions $(R_1)$ and $(R_2)$), holding the quantities of assets owned by the remaining outsiders constant. In what follows, I first identify general necessary

[14]It may even be the case that a remedy takes the form of a second merger notified by the parties at the same time as the first merger. In June 2000, in a case involving newsprint and magazine paper producers, *UPM-Kymmene* notified to the European Commission its proposed acquisition of *Haindl*. Simultaneously, *Norske Skog* notified the acquisition of two of *Haindl*'s mills, conditioning this acquisition on the first concentration being cleared by competition authorities (see cases COMP/M.2498 and COMP/M.2499).

(but not always sufficient) conditions for acceptable remedies to exist in the four-firm case, before providing an illustration in a more restrictive setting with linear demand and marginal cost functions. As I then show, acceptable remedies may exist but only in rather peculiar situations.

Suppose from now on that $N = 4$, $k_1 \geq k_2$ and $k_3 \geq k_4$, and consider a merger between Firm 3 and Firm 4. If Firm 3 does not hold more than a third of the available assets (i.e., $k_3 \leq \frac{K}{3}$), either the largest outsider (Firm 1) holds at most one third of the assets, or the competition authorities' preferred remedy does not involve Firm 1 (since the impact of divestitures on price is larger when it involves a smaller buyer). In both cases, Proposition 4 applies; therefore, there does not exist any acceptable remedy.

For acceptable remedies to exist, it must thus be the case that $k_3 > \frac{K}{3}$. This in turn implies that $k_1 < \frac{K}{3}$, since the competition authorities' preferred remedy would otherwise involve Firm 2 only. The pre-merger allocation of assets must therefore be such that:

$$k_2 \leq k_1 < \frac{K}{3} < k_3 \tag{4}$$

Since the smallest insider and the outsiders jointly own less than two thirds of the assets, the competition authorities' preferred remedy is to force Firm 3 to divest all of Firm 4's assets. This remedy involves both outsiders only if the following condition holds:

$$k_2 + k_4 > k_1 \;\Leftrightarrow\; k_1 < \frac{K - k_3}{2}. \tag{5}$$

The post-remedy allocation of assets is then $\left(\frac{K-k3}{2}, \frac{K-k3}{2}, k_3, 0\right)$. The same allocation would have been reached had Firm 1 proposed to acquire Firm 4 and to divest $k_1 + k_4 - \frac{K-k_3}{2}$ to Firm 2. But, this remedy involves a single outsider and Proposition 4 thus implies that the equilibrium price is greater post-remedy than pre-merger. Therefore, acceptable remedies may exist only if the following condition holds:

$$k_1 + k_4 - \frac{K - k_3}{2} > \min\left(k_1, k_4\right) \;\Leftrightarrow\; \max\left(k_1, k_4\right) > \frac{K - k_3}{2}. \tag{6}$$

Conditions (4), (5) and (6) are necessary (but not always sufficient) for acceptable remedies to exist in the four-firms case, and combining them leads to:

**Proposition 5** *When $N = 4$, there does not exist any acceptable remedies when either the merger does not involve the two largest firms, or the largest insider holds at most a third of the assets $\left(k_3 \leq \frac{K}{3}\right)$, or the outsiders jointly own more assets than the smallest insider $(k_1 + k_2 \geq k_4)$.*

A merger is a transfer of assets that creates more asymmetry and as such has a negative impact on consumer surplus, and the larger the *target* (i.e., the firm acquired during the merger process), the larger this impact. Divestments can restore some symmetry and thus limit the negative impact of the merger. If the remedy involves only one firm, it cannot fully compensate for the negative effect of the merger since the effect (in absolute value) of a transfer is larger the larger the difference in size between the buyer and the seller. However, if divested assets are sold to two or more outsiders, it is possible to generate an important positive effect on consumer surplus, especially if the buyers are initially very small. But if the buyers are small, the target must be a relatively big firm.

For instance, remedies that satisfy Conditions $(R_1)$ and $(R_2)$ exist when each insider initially holds almost half of the available assets (i.e., $k_3 = k_4 = \frac{K}{2} - \varepsilon$, with $\varepsilon > 0$ small enough). This however implies that the two outsiders are initially extremely small $(k_1 = k_2 = \varepsilon)$. Indeed, for $\varepsilon$ close to 0, it must be the case that:

$$p^*\left(\varepsilon, \varepsilon, \frac{K}{2} - \varepsilon, \frac{K}{2} - \varepsilon\right) > p^*\left(\frac{K}{4}, \frac{K}{4}, \frac{K}{2}, 0\right).$$

Pre-merger, the situation is very similar to a symmetric duopoly: the two outsiders being very small, they face very step marginal cost curves and thus do not exert any real competitive pressure on the two large firms. The merger between the two large firms thus looks very bad for consumers, since it leads to a near monopoly. However, because the target is a large firm, divestments can be substantial. Eventually, the proposed merger (and the associated remedy) has almost the same effect as reversing a merger between two medium-size firms, each holding a quantity of assets equal to $\frac{K}{4}$, which thus leads to a price decrease. Although the initial merger has a large negative effect on consumer surplus, the remedy is more beneficial because it restores effective competition between the merged entity and the two outsiders.

It is however not obvious that firms are willing to accept the proposed remedy (i.e.,

the remedy might violate Condition $(R_3)$). It is unfortunately extremely difficult to consider the impact of remedies on profits in the general case. I therefore restrict my attention to a specific linear - quadratic example à la Perry and Porter [1985]. I normalize the total quantity of assets to $K = 1$, and assume a linear demand function, $P(Q) = 1 - Q$, and a quadratic cost function, $C(q, k) = \frac{\gamma q^2}{2k}$, where the parameter $\gamma$ measures the importance of assets in this Cournot setting. To keep the analysis tractable, I focus on situations where insiders, as well as outsiders, are initially symmetric, that is: $k_1 = k_2 = \frac{1-k}{2}$ and $k_3 = k_4 = k$.[15]

I then compute, for any value of $\gamma$, conditions on $k$ for which acceptable remedies exist. The results are illustrated in Figure 1 which shows that the range of possible values for the insiders' pre-merger asset endowments is extremely limited: insiders need to be rather large, and, as soon as assets play a significant role in the model (i.e., for values of $\gamma$ that are not too small), the range of possible initial allocations of assets is extremely restricted (each insider must own between 40% and 44% of the assets).

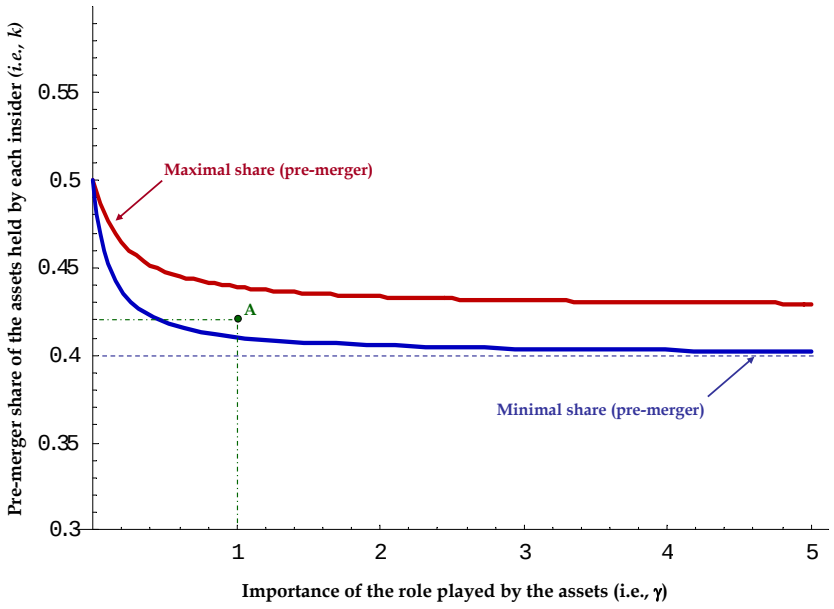


Figure 1: Insiders' initial allocation for which acceptable remedies exist

[15] In an earlier version of this paper, I also looked at asymmetric situations. The analysis is extremely cumbersome but yields very similar results: acceptable remedies exist, but only for a restricted set of initial allocations.

I also compute the minimal remedy, that is, the share of the acquired assets that have to be divested to ensure that the equilibrium price is not greater post-remedy than pre-merger and that industry profits do not decrease.[16] Figure 2 illustrates the results of this computation and shows that a very large proportion of the acquired assets have to be divested for the merger to be acceptable.

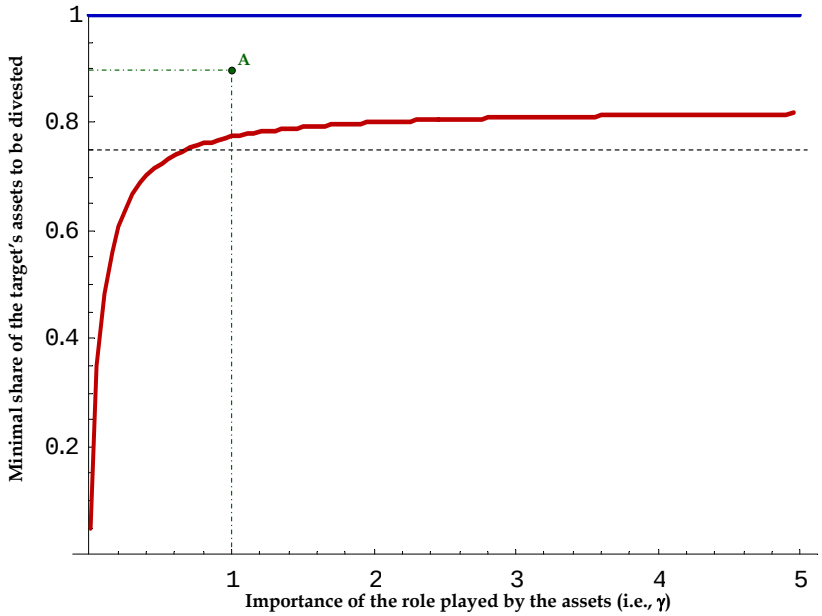


Figure 2: Minimal share of the acquired assets to be divested

Figure 2 shows in particular that all of the acquired assets would need to be transferred to the outsiders when the insider's pre-merger share is too small (i.e., on the lower curve of Figure 1). Moreover, the share of the acquired assets that need to be divested remains always above 75% (at least when $\gamma$ is not too close to 0).

Finally, point A on both figures provides an exemple of a merger for which an acceptable remedy exists when $\gamma = 1$. The proposed situation is one where each insider initially own 42% of the assets and 90% of the acquired assets are equally divided between the two outsiders. Post-remedy, the new entity thus owns only 46.2% of the assets.[17]

[16] Since all firms are involved in the process, their joint profits are equal to the industry profits.

[17] The corresponding pre-merger market shares are 40% for each insider and 10% for each outsider, whereas the post-remedy market shares are 42.6% for the new entity and 28.7% for each outsider.

As the above analysis and the example show, few mergers meet the requirements for acceptable remedies to exist. Moreover, these situations do not appear to be extremely realistic given the magnitude of the necessary divestments. In this Cournot setting with assets, a rule that bans all mergers unless the merging parties can convincingly argue that important synergies can be achieved through the merger (and only through this merger) is therefore unlikely to generate many type I errors.

# 4 Conclusions

This paper shows that, when mergers do not involve technological efficiencies, structural remedies are useful in only a very limited set of cases. If there are only three firms, or divested assets are transferred to a single outsider, or the largest firm is not big enough, the merger has a negative impact on consumer surplus overall and should therefore be blocked. When the pre-merger market is very asymmetric, remedies can be acceptable if the merger involves big firms and a large proportion of the acquired assets are divested to small rivals. However, such extreme situations seem to be relatively rare. I should also mention that this result does not apply to all mergers but only to industries where the total quantity of tangible assets is fixed (or cannot be increased in the short term). This feature implies that the results remain valid even if the divested assets are transferred to new entrants.

There is nevertheless a more positive lesson that can be drawn for my analysis. Although remedies might not be enough to compensate for the merger's negative impact on consumer surplus, they can be a useful instrument to limit this impact. Therefore, if competition authorities are willing to accept mergers as long as the price does not increase too much, structural remedies that reduce post-merger concentration may allow firms to obtain clearance for a merger that would otherwise have been blocked. However, this requires a relatively complex evaluation of the merger's and remedy's effects on price.

In this paper, I abstracted from considering fixed costs. However, taking them into account would not dramatically affect the analysis. With fixed costs, the issue of exit following a merger may arise. However, if a merger or a proposed remedy induces a firm

to exit, this can only have an additional negative impact on consumer surplus since it reduces competition between the remaining firms. Therefore, as long as I assume that all firms are active in the pre-merger equilibrium (i.e., there is no firm that is inactive pre-merger and may be willing to "re-enter" post-remedy), introducing fixed costs should only reinforce my results.

Finally, consumer surplus may not always be the standard adopted by competition authorities. Unfortunately, taking profits into consideration in this setting is extremely complex even when restricting attention to linear demand and marginal cost functions. Even in a three-firms setting, it may well be the case that some mergers - one their own or subjected to the appropriate remedies - enhance total welfare. Since, total industry profits would then necessarily be increasing, firms would always be willing to accept such remedy. However, I carried out some simulations that seem to suggest that mergers are either welfare-improving, in which case they should be cleared by competition authorities without remedy, or that acceptable remedies do not exist.

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# A Proof of Lemma 2

The cost function $C(q,k)$ being homogeneous of degree 1, we have:

$$qC_{qq} + kC_{qk} = 0, \tag{7}$$

and $\sigma(q,k)$ can thus be written as:

$$\sigma(q,k) = \frac{k}{q}\frac{C_{qq} - P'}{C_{qq}}.$$

We now apply the *cross-sectional differentiation* technique presented by Farrell and Shapiro [1990b, p. 291]. The objective is to compare the values of $\sigma(q,k)$ for different firms *"looking across firms in a given equilibrium,"* meaning that we compare different values of $k$ (and $q$) keeping the total quantity $Q$ constant. This analysis yields:

$$\frac{d^{cs}\sigma}{dk} = \frac{1}{q}\left(1 - \frac{k}{q}\frac{d^{cs}q}{dk}\right)\frac{C_{qq} - P'}{C_{qq}} + \frac{k}{q}\frac{P'\left(C_{qqq}\frac{d^{cs}q}{dk} + C_{qqk}\right)}{(C_{qq})^2}.$$

From the proof of Proposition 4 of Farrell and Shapiro [1990b, p. 291], we get that:

$$\frac{d^{cs}q}{dk} = \frac{-C_{qk}}{C_{qq} - P'} = \frac{q}{k}\frac{C_{qq}}{C_{qq} - P'},$$

implying:

$$\begin{aligned}
\frac{d^{cs}\sigma}{dk} &= \frac{1}{q}\left(1 - \frac{C_{qq}}{C_{qq} - P'}\right)\frac{C_{qq} - P'}{C_{qq}} + \frac{k}{q}\frac{P'}{(C_{qq})^2}\left(\frac{q}{k}\frac{C_{qq}}{C_{qq} - P'}C_{qqq} + C_{qqk}\right) \\
&= \frac{-P'}{qC_{qq}} + \frac{P'}{q(C_{qq})^2(C_{qq} - P')}\left(qC_{qq}C_{qqq} + k(C_{qq} - P')C_{qqk}\right) \\
&= \frac{-P'}{q(C_{qq})^2(C_{qq} - P')}\left((C_{qq} - P')(C_{qq} - qC_{qqq} - kC_{qqk}) - qC_{qqq}P'\right)
\end{aligned}$$

Differentiating equation (7) with respect to $q$ yields $C_{qq} + qC_{qqq} + kC_{qqk} = 0$, and therefore:

$$\frac{d^{cs}\sigma}{dk} = \frac{-P'}{q(C_{qq})^2(C_{qq} - P')}\left(2C_{qq}(C_{qq} - P') + qC_{qqq}(-P')\right) > 0.$$

This implies that the expression $\sigma(q,k)$ is greater for larger firms than for smaller firms (in a given equilibrium).[18]

[18] Remark that the result still holds if $C_{qqq}$ is negative but small.