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Horizontal Symmetry and Masses of Neutrinos

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Assuming a horizontal symmetry $SU_F(n)$, we demonstrate a possibility that the left-handed neutrinos receive masses of order 1 eV, which is consistent with recent experiments on neutrino oscillations. The smallness of the neutrino masses is an indication of a large breaking of the horizontal symmetry.

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A striking and puzzling phenomenon is that neutrinos appear nearly massless in contrast to all other leptons and quarks. The left-handed neutrinos may combine with their chiral partners (if they exist) to form Dirac fermions with masses comparable to the usual lepton and quark masses. It has been pointed out[1] by the present author and independently by Gell-Mann, Ramond and Slansky that this can be avoided if the right-handed neutrinos receive superheavy Majorana masses. This situation can happen[1)~3)] in some models with continuous horizontal symmetries[4),5)] or grand unified $SO(10)$ models.[6)]

In the $SO(10)$ models, the large Majorana masses M of right-handed neutrinos can be generated by a 126-plet of Higgs scalar. The value of M would be naturally of order of grand unification mass, 10^{15} GeV,[*)] which leads to undetectably small masses of left-handed neutrinos, $\sim 10^{-6}$eV. The smallness of neutrino masses is a consequence of the strong breaking of $SO(10)$.

On the other hand, in the models with horizontal symmetries, the large Majorana masses are related[2)] to a breaking of the horizontal symmetries. The main point of this paper is to show a possibility that the left-handed neutrinos have masses of order 1 eV, which may be consistent with recent experiments[8)] on neutrino oscillations.

In order to illustrate this point we take an $SU_F(n)\times SU(2)\times U(1)$ model, where the subgroup $SU_F(n)$ represents a horizontal symmetry and n, ≥ 3, is the number of generations. The weak $SU(2)$-doublet and -singlet fermions are assigned to n-plets of $SU_F(n)$. It should be noted that the right-handed neutrinos are required[2)] in order to remove triangle anomalies in the lepton sector.

Now, we introduce two kinds of Higgs scalars $\chi=(n(n+1)/2, 1)$ and $\phi=(n^2-1, 2)$, where the values in each parenthesis are the representation dimensions of $SU_F(n)\times SU(2)$. The scalar χ responsible for a breaking of the horizontal $SU_F(n)$ has a Yukawa coupling with the right-handed neutrinos and their charge-conjugated fields. The Higgs ϕ breaks the electroweak $SU(2)\times U(1)$ down to the electromagnetic $U(1)$ and also give Dirac masses to the fermions.

Then, mass term of neutrinos is obtained as

$$\mathcal{L}_{\text{mass}}=-\bar{\nu}_R m \nu_L-\frac{1}{2}\overline{\nu_R{}^c}M\nu_R+\text{h.c.}\,, \quad (1)$$

*) If we introduce a 16-plet of Higgs scalar instead of a Higgs 126, the right-handed neutrinos can remain massless at the tree level. As pointed out by Witten,[7)] two loop diagrams produce Majorana masses of the right-handed neutrinos, which may be much less than 10^{15} GeV.

where $m=G\lambda^a\langle\phi^a\rangle$ and $M=G'\langle\chi\rangle$ are $n\times n$ matrices and $\nu_{L,R}=(\nu_e,\nu_\mu,\cdots\nu_n)_{L,R}$. As pointed out in the previous paper,[2)] we have the following relations on the Dirac mass matrices:

$$m=\kappa m_l^\dagger=\kappa' m_u=\kappa'' m_d^\dagger\,, \tag{2}$$

$$\mathrm{Tr}(m)=0\,. \tag{3}$$

Here κ, κ' and κ'' are numerical parameters and m_l, m_u and m_d are mass matrices of leptons, u-type and d-type quarks, respectively. Equations (2) and (3) lead to a strong constraint on weak-mixing angles, which rejects[9)] the six-quark model. Thus, we consider the case of four generations in this paper.

It is convenient to use two component Weyl spinors, ξ and ζ, defined by

$$\nu_R=\begin{pmatrix}\xi\\0\end{pmatrix}\quad\text{and}\quad\nu_L=\begin{pmatrix}0\\\zeta\end{pmatrix}. \tag{4}$$

Then, the mass matrices of Eq. (1) is rewritten as

$$\mathcal{L}_{\text{mass}}=-\frac{1}{2}(\tilde{\xi}^\dagger\zeta^\dagger)\begin{pmatrix}M & m^*\\ m^\dagger & 0\end{pmatrix}\begin{pmatrix}\xi\\-\tilde{\zeta}\end{pmatrix}+\text{h.c.}\,, \tag{5}$$

where $\tilde{\xi}$ or $\tilde{\zeta}=-i\sigma_2(\xi^*$ or $\zeta^*)$. For $M\gg m$, the mass eigenmatrices in ξ-$\tilde{\zeta}$ space are given by approximately M and $m^\dagger M^{-1}m^*$. The nearly left-handed neutrino, $\zeta'=\zeta+O(m/M)\tilde{\xi}$, therefore, receives a small Majorana mass matrix as

$$\mathcal{L}^{\zeta'}_{\text{mass}}\approx-\frac{1}{2}\zeta'^\dagger m^\dagger M^{-1}m^*\tilde{\zeta}'+\text{h.c.} \tag{6}$$

We take the mass matrix $M=G'\langle\chi\rangle$ diagonalized by the $SU_F(n)$-gauge transformations. For an estimation of the neutrino masses, we simply assume that each component of $\langle\chi\rangle$ is of the same order of magnitude. The values of $\langle\chi\rangle$ can be determined[4)] as $\langle\chi\rangle\approx O(10^7\text{GeV})$ by using the data on CP violation in $K_L{}^0\to2\pi$ decay.

We, then, obtain the following relation on neutrino masses from Eqs. (6) and (2):

$$\begin{aligned}\det m_{\zeta'}&\approx[\det m]^2\det M^{-1}\\&=(\kappa^2/G')^4[\det m_l]^2\det\langle\chi\rangle^{-1}\\&=(\kappa^2/G')^4 O[(10\,\text{eV})^4].\end{aligned} \tag{7}$$

Here we have assumed the mass of fourth lepton, τ', to be 10^2GeV. Unless (κ^2/G') is too large, we can take all neutrino masses less than the cosmological upper bound.[10)] The result of $(\kappa^2/G')\approx0.1$ may be compatible with recent data[8)] on neutrino oscillations. In our model, the smallness of neutrino masses $m_{\zeta'}$ is attributed to mainly the large breaking of the horizontal $SU_F(n)$.

If the transition elements in m_l between the lighter leptons (e,μ) and heavier ones (τ,τ') are fortunately small, we can have a prediction on m_{ν_e} and m_{ν_μ} independently of the heavy lepton masses as,

$$\begin{aligned}m_{\nu_e}m_{\nu_\mu}&\approx\left(\frac{\kappa^2}{G'}\frac{m_e m_\mu}{\langle\chi\rangle}\right)^2\\&\approx O(1\,\text{eV}^2)\end{aligned} \tag{8}$$

for $(\kappa^2/G')\approx10^2$. Here ν_i denotes ζ_i' $(i=e,\mu,\cdots)$. Correspondingly to the case $m_{\nu_e}m_{\nu_\mu}\approx O(1\,\text{eV}^2)$, we also find $m_{\nu_\tau}m_{\nu_{\tau'}}\approx O(1\,\text{MeV}^2)$ for $m_{\tau'}\approx10^2$GeV, which is roughly consistent with the cosmological bounds.[10)] This possibility is more attractive if there are several heavy leptons with masses of order 10^2GeV.

It is possible to extend our discussions to the ground unified models including horizontal symmetries. In some schemes of such unifications, the heavy Majorana masses of right-handed neutrinos are correlated essentially with a breaking of the horizontal symmetries. For example, in an $SO_F(6)\times SO(10)$ model with fermions $\psi=(4,16)$ transforming as a spinor representation, a Higgs scalar $\chi=(1,126)$ has no Yukawa coupling to the right-handed neutrinos. The Majorana masses of the right-handed neutrinos are generated by

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a Higgs $\chi=(20, 126)$ which also breaks the horizontal $SO_F(6)$. The large Majorana masses are, therefore, related to a breaking of the horizontal group.

In conclusion, we have demonstrated a possibility that the masses of some or all left-handed neutrinos are of order 1 eV. However, our numerical estimates are very crude admittedly because of great uncertainties concerning the mass matrices m and M. We expect high-precision experiments on neutrino oscillations to shed light on the subjects in this paper.

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