NBER WORKING PAPER SERIES

HOW BIG IS THE TAX ADVANTAGE TO DEBT?

Alex Kane

Alan J. Marcus

Robert L. McDonald

Working Paper No. 1286

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
March 1984

The authors are grateful to Betty Yobaccio for expert research assistance, and to Wayne Mikkelson and Daniel Siegel for comments on an earlier draft. The research reported here is part of the NBER's research program in Financial Markets and Monetary Economics. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Working Paper #1286
March 1984

How Big is the Tax Advantage to Debt?

ABSTRACT

This paper uses an option valuation model of the firm to answer the question, "What magnitude tax advantage to debt is consistent with the range of observed corporate debt ratios?" We incorporate into the model differential personal tax rates on capital gains and ordinary income. We conclude that variations in the magnitude of bankruptcy costs across firms can not by itself account for the simultaneous existence of levered and unlevered firms. When it is possible for the value of the underlying assets to jump discretely to zero, differences across firms in the probability of this jump can account for the simultaneous existence of levered and unlevered firms. Moreover, if the tax advantage to debt is small, the annual rate of return advantage offered by optimal leverage may be so small as to make the firm indifferent about debt policy over a wide range of debt-to-firm value ratios.

Alex Kane, Alan J. Marcus
and Robert L. McDonald
School of Management
Boston University
704 Commonwealth Avenue
Boston, MA 02215

HOW BIG IS THE TAX ADVANTAGE TO DEBT?

The observed range of debt-to-firm-value ratios in the U.S. economy is, roughly speaking, from zero to 60 percent. Traditional explanatory models of capital structure have focused on the tradeoff between the tax shield and bankruptcy costs arising from the use of debt finance [2,9,10,18,19]. Other models have focused on information issues, such as agency costs [7,16] and signalling [17], though it is fair to say that bankruptcy costs are still thought to be an important determinant of debt structure.

One important recent criticism of the bankruptcy cost models was that of Miller [15], who argued that such costs were too small relative to the tax benefits of debt to explain the existence of unlevered firms. Instead, Miller argued that taking personal as well as corporate taxation into account eliminated any net tax advantage of debt finance, so that individual firms would be indifferent about financial policy. DeAngelo and Masulis [4], however, argued that if there were bankruptcy costs (which Miller ignored), then the net tax advantage to debt would in equilibrium be low but positive, exactly offsetting marginal bankruptcy costs. Every firm could have a determinate debt structure based on the tax advantage-bankruptcy cost tradeoff, no matter how small bankruptcy costs were.

Surprisingly, there is no evidence as to whether bankruptcy-cost models which take account of personal as well as corporate taxes can explain the observed range of debt ratios in the U.S. economy. It is known that a bankruptcy cost model will predict high debt ratios--even with large bankruptcy costs--when the corporate tax rate is set at 50 percent and personal taxes are ignored (Kim, [9]). It is not known whether such a model has reasonable explanatory power when the net tax advantage to debt is much lower, as it would be in the DeAngelo-Masulis equilibrium.

In this paper we develop a valuation model for a levered firm with bankruptcy costs, incorporating personal taxes, and attempt to see whether such a model can potentially account for the observed range of debt-equity ratios in the U.S. We find that differences across firms in bankruptcy costs alone cannot account for the simultaneous existence of levered and unlevered firms. Next, we use simulation analysis to determine a reasonable cross-sectional range for optimal debt ratios, given the tax advantage to debt. The simulation results indicate that if the tax advantage to debt is small, then the cost of substantially deviating from the optimal debt ratio is small (in a sense we make precise). In this sense, the model is consistent with a wide range of observed debt ratios. These results leave open the possibility that other factors, such as moral hazard considerations, may be more important determinants of debt policy than traditional tax and bankruptcy-cost considerations.

Several authors have developed similar models to examine the tradeoff between the tax advantage and potential bankruptcy cost attributable to debt finance. In particular, Turnbull [19] and Brennan and Schwartz [2] use option-valuation models to value a levered firm and derive optimal debt positions. The advantage of their contingent claims framework is that the valuation formula requires only easily interpreted and estimated parameters.

Our model uses the contingent claims methodology, but differs significantly from that used by Turnbull and Brennan and Schwartz. We incorporate personal taxes and allow for the possibility that the underlying asset -- unlevered capital -- follows a mixed jump-diffusion process rather than a simple diffusion process. In addition, if there is a net tax advantage to debt, and if there are no arbitrage opportunities in the market for physical assets, then it will be suboptimal for investors to hold unlevered capital. Consequently, unlike the earlier papers, we assume that unlevered assets are not held by investors. Incorporating this into the derivation provides us with a measure of the advantage to leverage which is a flow measure (the extra rate of return earned by a levered firm) rather than the standard stock measure (the extra price that would be paid for a levered firm). The flow measure provides an easily interpreted notion of what constitutes a "large" advantage to debt finance.[1] Also, the solution implicitly accounts for the firm's rebalancing its capital structure at periodic intervals. Finally, the previous papers assume that the tax deduction attributable to debt is based on an exogenously given coupon rate. In this paper, the tax deduction is endogenous A weakness of the model is that we do not allow the firm to go bankrupt except at maturity (c.f. [2]).

Section 1 of the paper sets out the model and analytic solutions for debt and equity values. Section 2 proves that marginal bankruptcy costs are generally zero at a zero debt level, which implies that an all-equity capital structure is suboptimal regardless of the magnitude of bankruptcy costs as long as there is some tax advantage to debt. Section 3 presents simulations showing equilibrium debt to value ratios for a variety of personal tax rates. The results suggest that the marginal personal tax rate on interest income must be quite close to the corporate tax rate to account for the range of debt ratios commonly observed in the U.S. and that at those tax rates the equilibrium advantage to leverage is small. Section 4 concludes.

1. Valuation of the Firm

We will value the levered firm relative to the value of its unlevered assets, the market value of which is assumed to evolve according to the stochastic process:[2]

$$dA = (\alpha+\lambda)Adt + \sigma Adz + Adq \qquad (1)$$

where dz is a Wiener process, α is the instantaneous expected rate of return on A and σ is the instantaneous standard deviation of the rate of return, and dq is a Poisson process which takes on value -1 with probability λdt and value 0 otherwise. The jump process is a generalization of the usual diffusion process used in earlier contingent-claims analysis of debt structure. We will assume that dq is uncorrelated with the market in order to obtain closed-form solutions.

For simplicity, we model the levered firm as issuing pure discount bonds with maturity T, which can be rolled over at maturity. The debt affects the value of the firm via two channels. First, it creates the possibility of bankruptcy with associated costs denoted by B; these potential costs reduce the current value of the firm. However, offsetting the bankruptcy cost is a tax shield generated by the tax deductibility of interest payments.

To calculate the tax shield, let D denote the face value of the bonds and P_0 the market value of the debt at the issue date, 0. Then the yield on the bond is defined by $\rho = \frac{1}{T}\ln(D/P_0)$ and the interest attributed for tax purposes at time t is $\rho P_t = \rho P_0 e^{\rho t}$, where P_t is the value ascribed to the bond by the tax authorities at time t. (This value need not equal the actual market value of the bond at t except at t=0.) The implicit interest generates a cash flow of $\phi\rho P_t$, where ϕ denotes the corporate tax rate.

We assume that the firm is prevented by a bond covenant from paying out dividends before the debt matures. The tax deduction is invested in a special account at rate r_{TS}.[3] The funds thus invested grow at an after tax rate of $r(1-\phi)$, so that the tax shield, TS, accumulates by T to a value of

$$TS = \int_0^T \phi\rho P_t e^{r_{TS}(1-\phi)(T-t)} dt = \frac{\rho\phi}{\rho - r_{TS}(1-\phi)} [D - P_0 e^{r_{TS}(1-\phi)T}] \qquad (2)$$

We assume a full loss-offset provision, so that the tax shield accumulates in all states of nature, except those in which the firm is bankrupt at maturity.

We model bankruptcy costs, B (conditional on bankruptcy occurring), as increasing with the face value of debt, D:

$$B = b_0 + b_1 D \qquad 0 \leq b_0,\ 0 \leq b_1 < 1$$

The model we develop below is partial equilibrium, but is consistent with a general equilibrium in which all firms and investors face the same statuory tax rates respectively, with bankruptcy costs differing across firms. It is obvious that if the tax advantage differs across firms, debt ratios will differ, so we do not consider this possibility.

A. Boundary Conditions

Let P(A,D,T), S(A,D,T) and V(A,D,T) denote the market values of debt, equity and firm value at T. The payoffs to claimants at time T depend upon whether or not the tax shield is lost in bankruptcy.[4] We treat both cases:

Case 1: Tax shield lost in event of bankruptcy.

Event	P(A,D,T)	S(A,D,T)	V(A,D,T)	
A+TS>D	D	A–D+TS	A+TS	
D>A+TS>B+TS	A–B	0	A–B	(3a)
D>A+TS and B>A	0	0	0	

In the event that B+TS>D, the second event does not occur.

Case 2: Tax shield retained in bankruptcy.

Event	P(A,D,T)	S(A,D,T)	V(A,D,T)	
A+TS>D	D	A–D+TS	A+TS	
D>A+TS>B	A+TS–B	0	A+TS–B	(3b)
B>A+TS	0	0	0	

In the event that B>D, the second event again does not occur. In Case 2, it is easiest to think of the tax shield proceeds as literally being invested in a special account or financial asset. Thus, even if firm value jumps to zero, the tax shield remains.

The foregoing are terminal boundary conditions. It is also necessary to characterize the relationship between V_0 and A_0. Brennan and Schwartz and Turnbull do so by assuming that both V and A are the prices of assets which are willingly held by investors. This assumption implies the ongoing existence of unlevered assets even in a world with a tax advantage to debt. In this world, $V_0 > A_0$, and it would be possible for investors to perform arbitrage by purchasing unlevered assets and levering them. We assume, to the contrary, that equilibrium in the market for unlevered assets precludes the existence of such arbitrage opportunities, so that $V_0 = A_0$. Our assumption implies that no investor would buy and hold an unlevered asset without levering it. Thus A is not the price of an asset which would be willingly be held by investors.

B. Valuation Equations for Debt and Equity

We now derive the partial differential equations (p.d.e.s) which must be satisfied by the values of debt and equity. We will follow Constantinides [3] in using the CAPM[5] to derive the p.d.e. for each security. The CAPM approach is appropriate in this case because the underlying asset, A, would never be held as an unlevered asset in and of itself, since to hold A would be to forego valuable tax shields. The usual formation of the Black-Scholes [1] hedge portfolio is therefore inappropriate. In addition, the possibility of a jump in A precludes the formation of a riskless hedge portfolio. The equilibrium (versus arbitrage) approach avoids these problems.

Denote by u and g the tax rates on debt and equity income. We assume that all returns on debt and equity are taxed on accrual, and that all investors are in the same tax bracket. The after tax expected rates of return on the market and on instantaneously riskless bonds therefore can be written as $(1-g)r_M$ (assuming all returns to equity are taxed at the capital gains rate) and $(1-u)r$. Written in terms of after-tax rates of return, the intertemporal CAPM of Merton [14] then implies that

$$(1-g)\,\frac{1}{dt}\,E\left(\frac{dS}{S}\right) = (1-u)r + \beta_S[(1-g)r_M - (1-u)r] \qquad (4)$$

where E(.) is the expectations operator, S denotes the value of equity, S(A, D, t), and β_S is the beta of the after-tax return on levered equity with the after tax market return. There are no dividend payments.

To derive an expression for β_S, note that the diffusion components of S and A are instantaneously perfectly correlated, with $\sigma_s = \sigma A S_A / S$ (Galai and Masulis, [6]). Therefore, since the jump is non-systematic,

$$\beta_S = \frac{AS_A}{S} \beta_A$$

where β_A is the beta of the after-tax rate of return on the underlying asset, and where $S_A = \partial S / \partial A$. Therefore, (4) may be rewritten

$$(1-g) \frac{1}{dt} E(\frac{dS}{S}) = (1-u)r + \frac{AS_A}{S}[(1-g)\alpha^* - (1-u)r] \tag{5}$$

where $\alpha^*(1-g) = r(1-u) + \beta_A[r_M(1-g) - r(1-u)]$, i.e., α^*, is the before-tax equilibrium required rate of return on an asset with a beta of β_A. α^* differs from α if and only if there is value to leverage. (See below.)

Finally using Ito's lemma[6] to derive E(dS/S) and substituting that expression into equation (5) yields:

$$\frac{1}{2} S_{AA} A^2 \sigma^2 + S_t - (\lambda + \frac{1-u}{1-g} r) S + S_A A [r(\frac{1-u}{1-g}) - \delta + \lambda] = 0 \tag{6}$$

where δ is defined as $\alpha^* - \alpha$, subscripts denote partial derivatives, and the fact that S(0,D,T) = 0 is used to eliminate one of the jump terms. δ will be positive precisely when the underlying asset is priced to reflect tax shields which the asset in and of itself does not pay. δ measures the rate of return deficiency of A, which arises because A is priced to reflect the debt (and concomitant tax shields) that it can support, while the capital gains rate on A by itself does not include those tax shields. The rate of appreciation on A alone (as opposed to the total rate of return on the levered firm) will therefore not be sufficient to compensate the investor for holding the asset.[7] δ may thus be interpreted as the penalty rate per period for maintaining zero leverage, or equivalently, as the net tax advantage of debt in terms of rate of return. If there were only corporate taxes (g=u=0), δ would approximately equal the amortized tax shield less amortized expected bankruptcy costs.

Equation (6) is a p.d.e. that must be satisfied by the equity valuation formula together with the boundary conditions of Section A. Using an analogous derivation but recalling that interest is taxed at u, the p.d.e. for debt can also be derived:

$$\frac{1}{2}P_{AA}A^2\sigma^2 + P_t - r_1P + P_AA[r_1-\delta^*] = -\lambda\frac{1-g}{1-u}P(0) \tag{7}$$

where:

$$r_1 = r + \lambda\frac{1-g}{1-u}$$

$$\delta^* = \delta - (\lambda+\alpha^*)\frac{g-u}{1-u}$$

P(0) = the nonstochastic value of debt in the event of a jump to zero.

Notice that the p.d.e.'s for equity and debt are not identical, and also that the required rate of return on the unlevered assets, α^*, appears in the p.d.e. for debt. Both of these facts are attributable to the different tax treatment of debt and equity. For u = g, the p.d.e.'s are identical in form.

C. Solution

The solution to equation (6) subject to the terminal boundary conditions (3a) or (3b) is:

$$S(A,D,0) = Ae^{-\delta T}N(d_1) + (TS-D)e^{-r_2T}N(d_2) \tag{8}$$

where

$$r_2 = r_1(\frac{1-u}{1-g})$$

$$d_1 = \frac{\ln(A/(D-TS)) + (r_2-\delta+\sigma^2/2)T}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

The debt solution, however, depends on whether (3a) or (3b) holds:[8]

Case 1: Tax Shield lost in bankruptcy

Because the tax shield is lost in bankruptcy, we have $P(0,D,T)=0$. Nevertheless, we must distinguish two cases. If $D-TS>B$, the solution to (7) is

$$P(A, D, 0) = Ae^{-\delta^*T}N(d_3) - e^{-r_1T}BN(d_4) + e^{-r_1T}(B+D)N(d_6) - Ae^{-\delta^*T}N(d_5) \quad (9)$$

If $D-TS<B$, on the other hand, the solution is

$$P(A, D, 0) = De^{-r_1T}N(d_6) \quad (10)$$

where

$$d_3 = \frac{\ln(A/B) + (r_1 - \delta^* + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_4 = d_3 - \sigma\sqrt{T}$$

$$d_5 = \frac{\ln(A/(D-TS)) + (r_1 - \delta^* + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_6 = d_5 - \sigma\sqrt{T}$$

Case 2: Tax Shield retained in bankruptcy.

If $TS<B$, then $P(0,D,T)=0$ as before, and the solution to (7) is

$$P(A,D,0) = Ae^{-\delta^*T}N(d_7) + (TS-B)e^{-r_1T}N(d_8) + e^{-r_1T}(B-TS+D)N(d_6)$$

$$-Ae^{-\delta^*T}N(d_5) \quad (11)$$

On the other hand, if $TS>B$, then $P(0,D,t)=(TS-B)e^{-r(T-t)}$, and the solution to (7) is

$$P(A,D,0) = Ae^{-\delta^{*}T} + (TS-B)e^{-rT} + e^{-r_1 T}(B-TS+D)N(d_6) - Ae^{-\delta^{*}T}N(d_5) \quad (12)$$

where

$$d_7 = \frac{\ln(A/(B-TS)) + (r_1-\delta^{*}+\sigma^2/2)T}{\sigma\sqrt{T}}, \qquad d_8 = d_7 - \sigma\sqrt{T}$$

The value of the firm in all cases is given by

$$V(A, D, 0) = P(A, D, 0) + S(A, D, 0). \quad (13)$$

These equations can only be solved implicitly, since the tax shield enters the cumulative normal density, and it is a function of P. Furthermore, δ is also determined endogenously. As noted above, we will also impose the requirement that $V_0 = A_0$, which amounts to ruling out arbitrage resulting from levering up unlevered assets. The market values of assets are assumed to fully reflect the value of debt they can support.

The solutions presented above are fully consistent with a multiperiod interpretation in which the firm reoptimizes its debt position at the end of every period, although it appears to be cast in a one-period (of duration T) context. Because the firm can freely rebalance capital structure at the maturity date, it can make a leverage decision looking ahead only one period. The rebalancing feature is embodied in the boundary condition: Firm value at T reverts to A. A equals the value of the optimally levered firm, because equilibrium in the market for assets requires that the underlying asset be bid to a level which reflects the potential of leverage. Thus, even if debt policy is temporarily suboptimal, the market value of the underlying asset at maturity will reflect future gains from leverage.

2. Is Zero Debt Ever Optimal?

We now ask whether bankruptcy costs can help explain the simultaneous existence of levered and unlevered firms. The general conclusion of this section is that when the tax advantage to debt is positive, however small, it is never optimal for firms to have strictly zero debt, however large their bankruptcy costs. If the tax advantage is literally zero, then in one case (when the tax shield is not lost in bankruptcy and TS>B) it is optimal for all firms to have positive leverage. In all other cases they are indifferent about the debt-equity ratio in the vicinity of zero debt if the Miller condition holds. Thus, unless the Miller condition holds, bankruptcy cost models are inconsistent with the simultaneous existence of both levered and unlevered firms.

A. Case 1 and Case 2 (TS<B).

In these cases, the equations for P are given by equations (9), (10), and (11). In each case, as D approaches zero, the market value of debt, P, approaches $D\exp(-r_1 T)$ and the yield to maturity on the debt, ρ, approaches r_1. If we also assume that $r_{TS} = r_1$, then as D approaches zero the value of the tax shield approaches

$$TS = D[1 - e^{-r_1 T + r_{TS}(1-\phi)T}] \tag{14}$$

The assumption $r_{TS}=r_1$ implies that the firm earns an actuarially fair rate of return on the tax shield. Since in each case the firm loses the tax shield in bankruptcy (in Case 2 bankruptcy costs exhaust the tax shield), the proceeds from the tax shield must be invested not at r, but at $r+\lambda$ (ignoring taxes) to account for bankruptcy risk. Using (14), it is possible to show that

$$\left.\frac{\partial V}{\partial D}\right|_{D=0} = e^{-r_2 T}\,[1 - e^{r_1[(1-\phi) - \frac{(1-u)}{(1-g)}]T}] \tag{15}$$

If the Miller [15] condition holds [i.e., $1-\phi=(1-u)/(1-g)$], the marginal gain from issuing debt is zero at zero debt. Therefore, if $r_{TS} = r_1$, $\partial V/\partial D = 0$ at zero debt regardless of the magnitude of bankruptcy costs. This result has the interpretation that marginal bankruptcy costs are always zero at zero debt. The marginal contribution from the tax shield is positive if

$1-\phi<(1-u)/(1-g)$. If there is a positive tax advantage to debt, all firms should issue positive debt, since (16) is independent of bankruptcy costs. If the tax shield proceeds are invested at a rate less than r_1, then it will be optimal for all firms to issue zero debt when the Miller condition holds. Indifference requires that the Miller condition hold and that $r_{TS}=r_1$.

B. Case II (TS>B).

In the cases just considered, the tax shield is always lost in the event the firm bankrupts, so that the firm receives the tax deduction for interest payments only in those states of the world where full interest is actually paid on the debt. If the tax shield is not lost in bankruptcy and TS>B, then issuing debt generates at least a partial tax deduction for interest payments even in states where the firm does not actually make interest payments. This is clearly a gain for the firm's security holders at the expense of the government in these states, since corporate tax deductions are taken on the interest without personal taxes being paid. Thus, even if the Miller condition holds, the firm will issue a positive amount of debt in order to take advantage of this gain in loss states.

The discussion in this section has assumed that $r_{TS} = r_1$. If $\lambda > 0$, it is possible that $r_{TS} < r_1$. In this case, differences in λ across firms can account for different debt policies. In particular, if there were a small positive tax advantage to debt, firms with $\lambda=0$ would issue positive debt (with the amount depending on firm-specific parameters such as the level of bankruptcy costs), and firms with sufficiently great λ would issue no debt.

3. Simulation Results

To derive the optimal debt ratio for any set of parameters, we maximize V with respect to D. As we have argued, however, if the value A is bid up in equilibrium to reflect the optimal debt position assets can support, then it must be true that V(A, D, 0) = A at the optimal level of D. To satisfy these conditions we must find the value of δ which solves the problem

$$\max_{D} V(D;\delta) \text{ subject to } V(D^*;\ \delta) = A.$$

The value of δ is endogenously determined as part of the solution, and is precisely the equilibrium net tax advantage of debt expressed as a rate of return. The problem of optimal debt structure is equivalent to that of choosing D to maximize δ, subject to the constraint that $A_0 = V_0$. The level of debt which maximizes firm value is also the level which maximizes the tax advantage net of bankruptcy penalties.

We report simulation results corresponding to a real interest rate of 2 percent, bankruptcy costs of 15 percent of debt ($b_0 = 0$, $b_1 = .15$), a rebalancing interval of 1 year, and an annual standard deviation of 25 percent. Note that this value of σ must be interpreted as the standard deviation of the rate of return of the unlevered assets. We also allow λ, the probability that the value of the assets jump to zero, to equal both 0 and .01. It is assumed that the tax shield is lost in bankruptcy and that $r_{TS}=r$, independent of λ. Both assumptions are designed to minimize the importance of debt. Nevertheless, optimal debt ratios are quite high.

Figure 1 displays the optimal debt ratios as a function of the personal tax rate on debt (g is set equal to zero). The corporate tax rate is set equal to .46 and we then calculate debt ratios and δ for different personal tax rates. The tax rate on the horizontal axis in the figures should be interpreted as $1 - (1-u)/(1-g)$, which equals the actual rate of tax on interest income for g=0. In the Miller equilibrium $(1-u)/(1-g)$ equals 1 minus the corporate tax rate, at which point there is no tax advantage to debt. In the graph, it can be seen that the debt ratio falls to zero at this point, reflecting the fact that with possible bankruptcy and no tax advantage, optimal debt must be zero. The graphs for different λ's are almost indistinguishable, although debt ratios are higher for the firm with $\lambda=.01$.

The most striking characteristic of the graph is the extremely sharp increase in debt ratios as the personal tax rate falls just below the corporate rate and the fairly flat increase in the ratio thereafter. We have found that this general shape occurs for virtually any set of parameters and that the optimal debt ratio is relatively insensitive to a change in bankruptcy costs. The debt-to-value ratio reaches approximately 40 percent as soon as there is a small difference in the corporate and personal tax rates. These results suggest that the tax advantage to debt would need to be approximately zero for the range of observed debt ratios to be consistent with the model.

Figure 2 displays the annual rate of return advantage to debt, δ, for $\lambda=0$ and $\lambda=.01$. When λ rises, both debt ratios and δ rise. With greater λ the risk premium on debt, and hence the tax shield, are greater. This induces more debt. Bankruptcy is also more probable, but this does not discourage debt; a jump to zero will induce bankruptcy, however little debt the firm has issued, so that more debt does not increase the chance of a bankruptcy due to a jump. Our estimates of the rate of return advantage to debt are extremely small. The net advantage falls below one-half of one percent per year for personal tax rates above 35 percent. At personal tax rates of 40 percent, the net advantage is generally below two-tenths of a percent. The penalty of suboptimal leverage would thus be small.

Other comparative static properties of the model are straightforward.[9] The optimal debt level and the rate of return advantage to debt are larger for smaller bankruptcy costs, for smaller standard deviations and for shorter rebalancing intervals. The last result is attributable to the fact that shorter rebalancing intervals reduce the probability of bankruptcy for any level of debt. A reduction in T thus allows the firm to maintain a higher debt ratio with a greater tax shield. With no debt issue costs, the optimal rebalancing interval would be zero and firms would always be fully levered.

Figure 3 (in which $\lambda=0$) shows the rate of return penalty (the amount by which δ is less than the maximum possible) for firms with positive, but below-optimal debt, and for firms with above-optimal debt.[10] The figure shows clearly that it is more costly to exceed the optimal debt ratio by a given amount than to fall below that ratio. The penalty for being unlevered when the personal tax rate is 30 percent is less than 20 basis points, while with a personal tax rate of 45.5 percent, the penalty is under one basis point. The asymmetry suggests that we would be likelier to observe firms with too little debt, as opposed to firms with too much debt.

These simulation results suggest that, on the whole, the bankruptcy cost/tax advantage model provides little insight into the determination of capital structure. When calculated at values consistent with observed debt ratios of below 50 percent, the equilibrium rate of return advantage of optimal leverage is so small as to be nearly unnoticeable.

4. Conclusion

We have presented a model of optimal debt policy which incorporates personal taxes and bankruptcy costs. The solution of the model suggests that the advantage of debt finance is best measured as a rate of return per period. Simulation results indicate that the advantage of debt measured in this way is quite small for reasonable parameters. The personal tax rate must be extremely close to the corporate rate in order to explain the existence of unlevered firms, and, at those rates, the annual rate of return advantage to debt is small. We conclude that the tax advantage/bankruptcy cost tradeoff is unlikely to play a major role in explaining observed leverage patterns.

Footnotes

1. Consider the Modigliani-Miller formula for the value of a levered firm: $V_L = V_U + \theta D$, where θ is the corporate tax rate and D is the value of debt issued by the firm. The formula can be misleading: an unlevered firm would sell for V_U only if it was certain that the firm would never become levered in the future. Thus, the cost to a firm of failing to become levered today is not θD. Rather it is the reduction in value associated with the length of time for which the firm will be unlevered.
2. This process fully accounts for the effect of corporate taxation on value, except for the incremental value from the tax shield on debt. In assuming a full loss offset, we ignore the possibility that inability to use tax shields may affect debt policy (c.f. [4]).
3. At an opposite extreme, it also would have been possible to model the tax shield as paid out to equity holders in an initial dividend. In this case, the tax shield would be certain, and we would have inferred an even lower range for the net tax advantage to debt than that reported below.
4. Does the firm in fact lose the tax shield in bankruptcy? To the extent that the firm reinvests previous deductions for interest expense, it is reasonable to treat this part of the tax shield like any other asset that is not lost in bankruptcy. If the tax shield is literally kept in a special separate account, then it would not be lost even if firm value jumped to zero. However, if the firm has unused interest deductions and it goes bankrupt, these would likely be lost, since under the Internal Revenue Code the unused deductions would be offset against the gain from repaying the debt at a discount. (This gain would otherwise not be taxed.) Thus given our assumption that the tax shield is contained in a special account, we conclude that if the debt is long maturity most of the

tax shield would not be lost in bankruptcy, while if the debt was short maturity, it would be.

5. Any mean-variance of security pricing would generate the same result.
6. Merton [12] discusses Ito's Lemma for mixed jump-diffusion processes.
7. The importance of the rate-of-return shortfall in option pricing models is elaborated in McDonald and Siegel [11].
8. A stochastic bankruptcy cost can be incorporated into the valuation formula. Assume that bankruptcy costs follow

   $$dB = \mu B dt + sB dw$$

   This specification allows b_1 to be random in the proportional cost case and it would allow b_0 to be random in the fixed bankruptcy cost case. In general, the solution will depend (because B is a non-traded asset) upon the correlation of B with the market. If we consider the special case where $\mu = 0$, and where B is uncorrelated with both A and with the market, then the solution (9) is unchanged, except that in d_3 and d_4, σ^2 is replaced by $\sigma^2 + s^2$. It is straightforward to find the solution in cases of non-zero drift, and with general correlations between B, A, and the market, by applying the methods in Fischer [5].
9. Kane, Marcus and McDonald [8] study the properties of a similar model in greater detail.
10. The rate of return penalty for a nonoptimal debt ratio is calculated as the percentage decline in firm value for maintaining that debt ratio for one period, and then optimally rebalancing. This penalty is calculated assuming that the prices of assets in the economy are bid up to reflect the gains from optimal leverage policy. The value of δ used in the calculations is therefore the rate of return premium to optimal leverage, as in the other simulations. As with the other figures, we have found that the shape of Figure 3 is not sensitive to parameter choice.

References

1. Black, Fischer and Myron Scholes. "The Pricing of Options and Corporate Liabilities, Journal of Political Economy 81 (May/June 1973), 637-654.

2. Brennan, Michael J. and Eduardo S. Schwartz. "Corporate Income Taxes, Valuation, and the Problem of Optimal Capital Structure," Journal of Business 51 (January 1978), 103-114.

3. Constantinides, George M. "Market Risk Adjustment in Project Valuation," Journal of Finance 33 (May 1978), 603-616.

4. DeAngelo, Harry and Ronald Masulis. "Optimal Capital Structure Under Corporate and Personal Taxation," Journal of Financial Economics, 8 (1980), 3-29.

5. Fischer, Stanley. "Call Option Pricing When the Exercise Price is Uncertain, and the Valuation of Index Bonds," Journal of Finance 33 (March 1978), 169-176.

6. Galai, Dan and R.W. Masulis. "The Option Pricing Model and the Risk Factor of Stock," Journal of Financial Economics 34 (1976) 53-81.

7. Jensen, Michael C. and William H. Meckling. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure," Journal of Financial Economics 3 (1976), 305-360.

8. Kane, Alex, Alan J. Marcus and Robert L. McDonald. "Debt Policy and the Rate of Return Premium to Leverage," Boston University mimeo, 1983.

9. Kim, E. Han. "A Mean-Variance Theory of Optimal Capital Structure and Corporate Debt Capacity," Journal of Finance 33 (March 1978), 5-63.

10. Kraus, A. and R. Litzenberger. "A State Preference Theory of Optimal Financial Leverage," Journal of Finance 28 (September 1973), 911-922.

11. McDonald, Robert and Daniel Siegel. "Option Pricing When the Underlying

Asset Earns a Below-Equilibrium Rate of Return: A Note," Journal of Finance, forthcoming (1984).

12. Merton, Robert C. "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," Journal of Ecomomic Theory 3 (1971), 373-413.

13. ________. "Option Pricing when Underlying Stock Returns are Discontinuous," Journal of Financial Economics 3 (1976),125-144.

14. ________. "An Intertemporal Capital Asset Pricing Model," Econometrica 41 (September 1973), 867-886.

15. Miller, Merton. "Debt and Taxes," Journal of Finance 32 (May 1977), 261-275.

16. Myers, Stewart. "Determinants of Corporate Borrowing," Journal of Financial Economics 5 (1977), 147-175.

17. Ross, Stephen A. "The Determination of Financial Structure: The Incentive-Signaling Approach," Bell Journal of Economics 8 (Spring 1977), 23-40.

18. Scott, James H. Jr. "A Theory of Optimal Capital Structure", Bell Journal of Economics, Vol. 7, No. 1 (Spring, 1976), pp. 33-54.

19. Turnbull, Stuart M. "Debt Capacity," Journal of Finance 34 (September 1979), 931-940.

FIGURE 1

Debt Ratios as a Function of the Personal Tax Rate

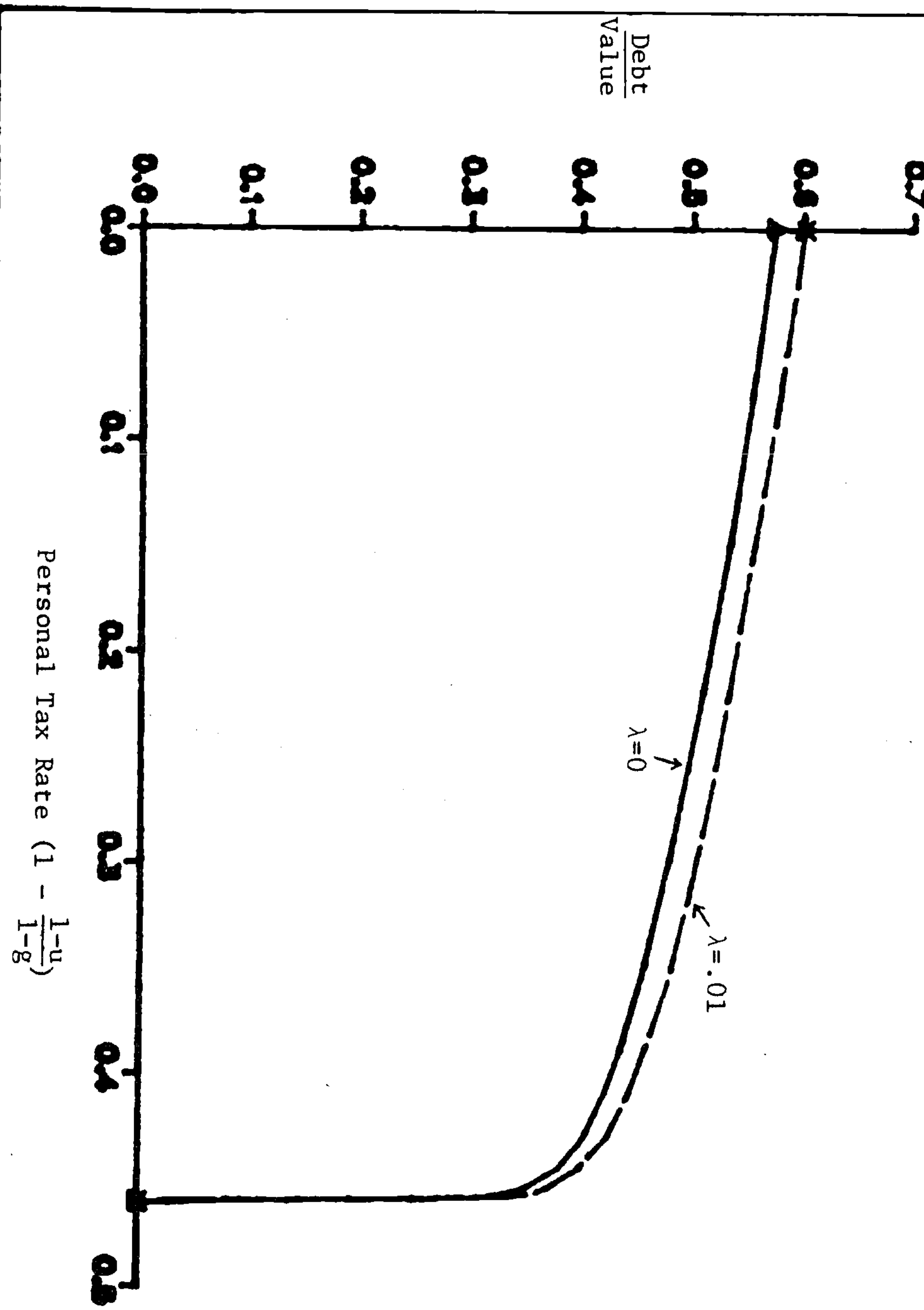
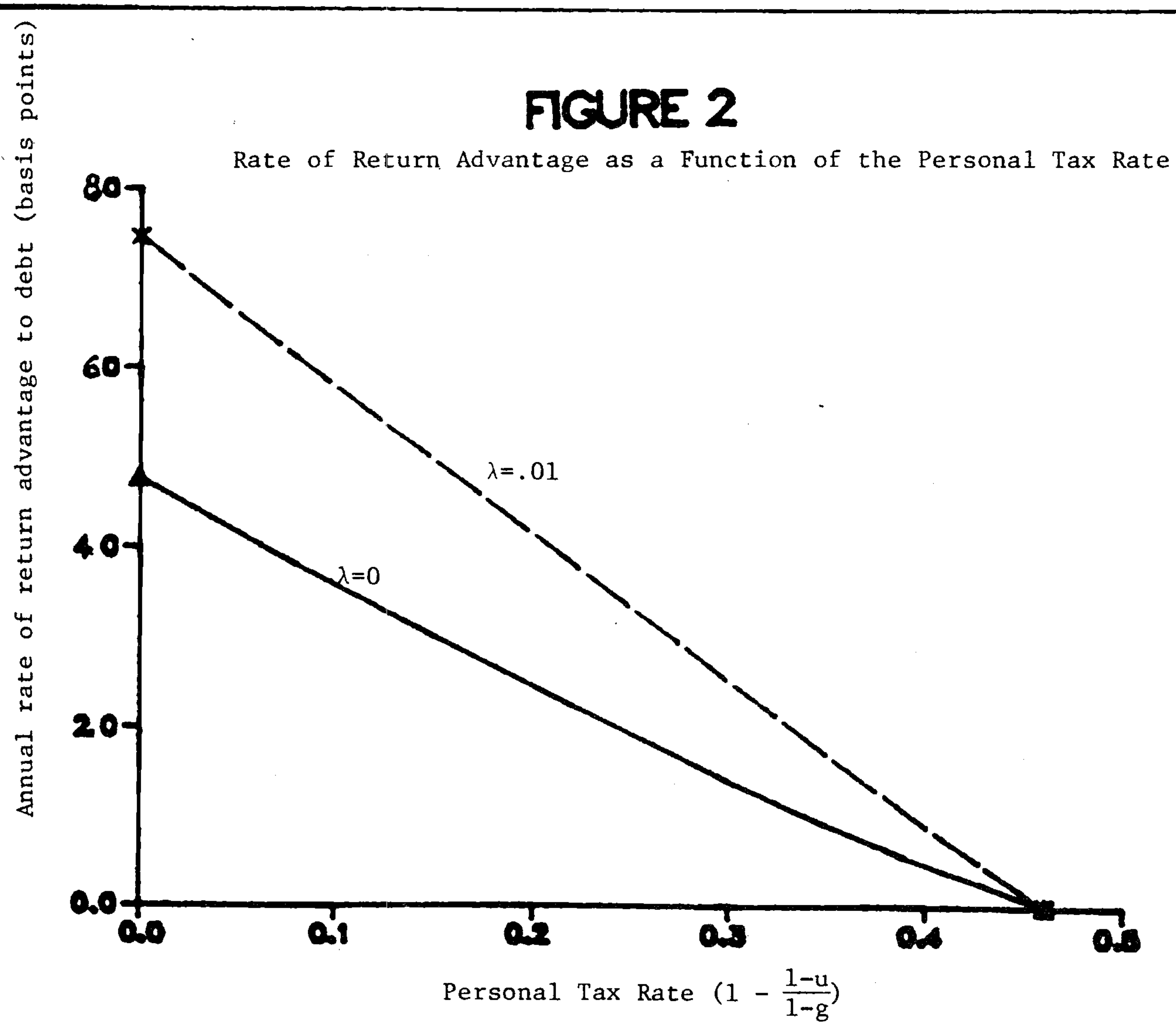


FIGURE 2

Rate of Return Advantage as a Function of the Personal Tax Rate



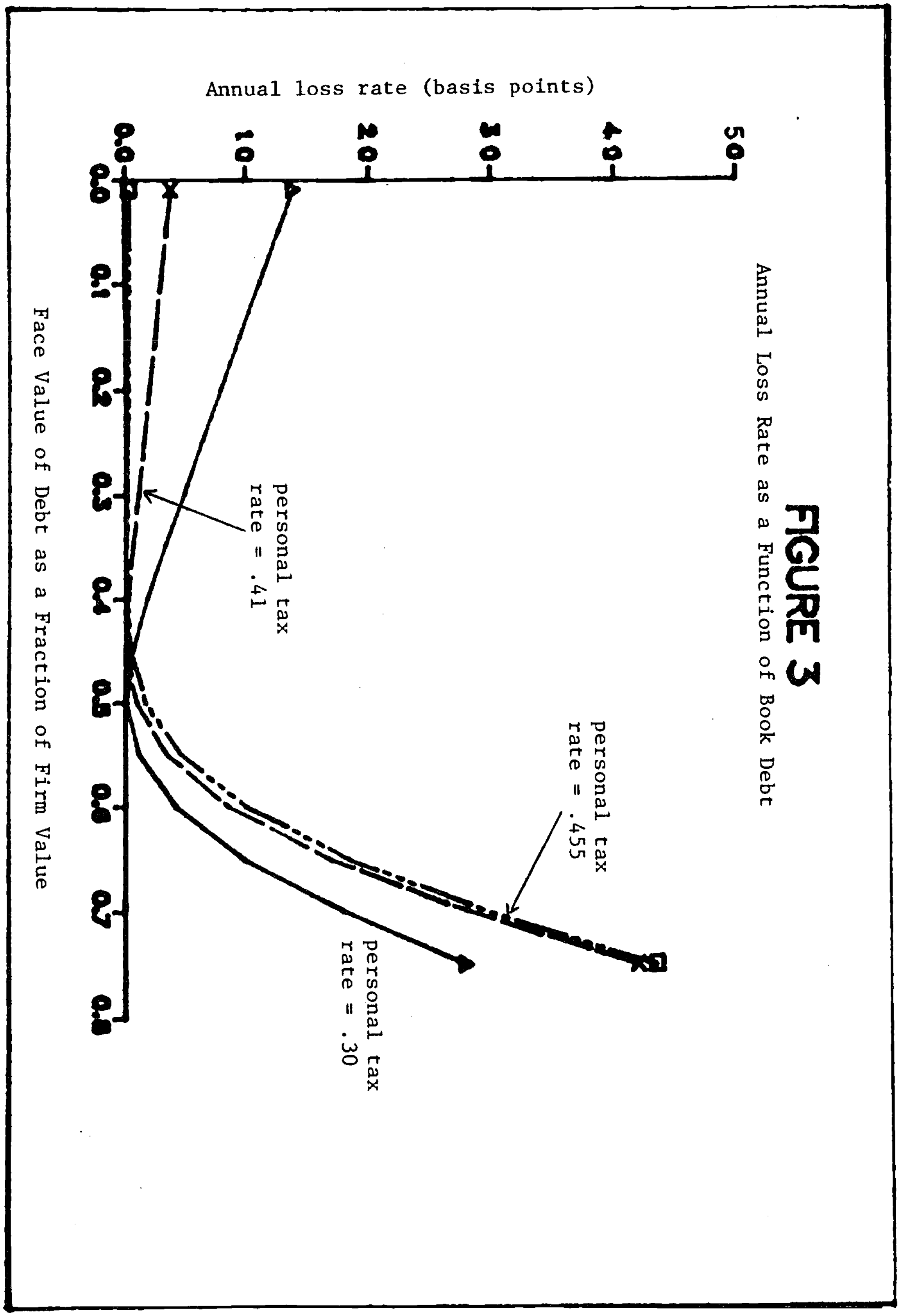


FIGURE 3
Annual Loss Rate as a Function of Book Debt