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HOW BIRTH CONTROL AFFECTS BIRTHS

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The Institute was established in November 1969. This series of Working Papers, begun in September 1970, is designed to facilitate early circulation and discussion of research materials originating from the Institute.

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No one knows better than students of population that scientific achievements create problems. Biological research over the past half century has enabled individuals to live into and through reproduction who in earlier regimes would have died in childhood. The consequence is a large increase of births and hence of people. The acceleration of biology and medicine has been followed at a surprisingly short interval by an acceleration of population, and this in turn threatens us with biological as well as other kinds of catastrophe.

It is natural that medicine should interest itself in rectifying this consequence of its success, and doctors have been active in the development and diffusion of the means of birth control. That the fitting of loops and the inducing of abortions have become widespread medical activities shows how the professional, legal, and moral framework can adapt itself to changed objective requirements--in this instance runaway population.

Parallel to the medical research and diffusion now going into contraception are important developments in demographic theory. These center on the individual woman as she proceeds through successive conceptions, abortions, and births, but their main interest is not the individual. Demographers are interested in populations, and the mathematics applied shows

that results for populations can be very different from those for individuals. A new field is growing up, created in response to the needs of the times by Ansley Coale, Robert Potter, Norman Ryder, Mindel Sheps, Christopher Tietze, and others. Though their work is highly technical, some of it at least is translatable into words and simple arithmetic.

Reduction of births may come by way of abortion, an intervention between conception and birth, or contraception that prevents the new life before it starts. Some societies have made abortion their principal method of birth control, and most use it in one degree or another when birth control fails. Let us start this exposition by seeing how abortion affects births.

#### Births Averted by Induced Abortion

An induced abortion of a pregnancy that is proceeding to term prevents a birth; do 1000 induced abortions performed in a population prevent 1000 births? The answer is no, not even approximately. Here as elsewhere the logic of individuals becomes grossly misleading when applied to populations.

#### An arithmetical example

Consider a couple that has just conceived and then decides to have an abortion; they might have the abortion in the second month, and be sterile for one further month, a total time from conception of three months. To calculate the

effect of the abortion we need to consider what happens after. Suppose that the couple is young and fertile again, and resumes intercourse without contraceptive protection. Then the expected number of months before the wife is pregnant again is about five. The three infertile months before and after the abortion, plus the five months to another pregnancy-- eight months in all--have brought the couple back to the same condition of just having conceived that it was in when we started our imaginary observation. The eight months represent the time out from childbearing due to one abortion. Only if this length of time was sufficient to have a child would one abortion prevent one birth in the population.

In fact, having a child takes much longer. It requires the number of months until conception without protection, say five once again, plus the mean time to term, nine months, plus the postpartum sterile period, which may be as much as another eight months, or 22 months in all. Only at the end of this 22 months is the woman back in the same condition as at the start of the cycle and ready to begin intercourse again.

On this model, childbearing is a matter of time. The eight months accounted for by the abortion are seen as having kept the woman out of exposure for a little more than one third of the 22 months that it would have taken her to have a child. Put the other way, the 22 months would have produced one child without the abortion; to prevent childbearing during that

period by means of abortions alone would have required  
 $22/8 = 2.75$  abortions.

Hence the answer to the question of how many births  
1000 abortions in a population prevent is, in this example,  
 $1000 \times 8/22 = 364$ . Similar arithmetic shows that if the  
abortion were delayed until the fetus was four or five months  
old it would keep the woman out of childbearing longer, and  
even abortions taking place just before natural birth are not  
equivalent to births prevented insofar as the succeeding  
infertile period is less than after a birth.

If on the other hand our model had admitted possible  
pregnancy terminated by miscarriage, the average time between  
successive normal births would have been even longer, and the  
effect of the induced abortion would have been accordingly  
further reduced. Moreover, the abortion of a conception that  
was destined to end up as a stillbirth would not have pre-  
vented a birth at all, but could even have brought the suc-  
ceeding pregnancy closer. Far from reducing births, the abor-  
tion in this case would on the average have added a fraction  
of a birth.

The issues here are of general enough application  
that they have been the subject of a more precise formal model.

#### The theoretical analysis

The model is a renewal process, one that repeatedly  
reverts to the same situation. Think of a just-married

couple; they start intercourse, say without contraception, then they have a pregnancy, childbirth, postpartum anovulatory period, and at the end of the cycle they are in the same condition as when they began; they are ready to start over. This is the meaning of "renewal" in this context. Our analysis will be confined to expected values, though a similar argument could tell us about variances or other moments as well. In effect we will be comparing the renewal process of intercourse, pregnancy, childbearing, intercourse, ..., with the renewal process of intercourse, pregnancy, abortion, intercourse, .... We will disregard miscarriage and stillbirth, and take account only of conceptions leading to live births.

First, to find the mean time to conception: Suppose that month by month the chance of conceiving is  $p$  for a woman in the fertile condition. Then among a number of couples the fraction  $p$  will be expected to conceive in the first month, and  $1 - p$  will proceed into the second month. Of these  $p$  will then become pregnant, or  $(1 - p)p$  of the original women will become pregnant in the second month. Similarly the fraction  $(1 - p)^2 p$  will become pregnant in the third month, etc. The expected number of months of exposure until pregnancy occurs will be found by multiplying  $p$  by 1,  $(1 - p)p$  by 2,  $(1 - p)^2 p$  by three, ..., and adding:

$$p + 2(1 - p)p + 3(1 - p)^2 p + \dots$$



To sum this expression we differentiate both sides of the identity  $(1 - x)^{-1} = 1 + x + x^2 + \dots$ , true for values of  $x$  between 0 and 1, and obtain

$$(1 - x)^{-2} = 1 + 2x + 3x^2 + \dots$$

Then we put  $1 - p$  for  $x$  and multiply by  $p$ . The total turns out to be  $\left[1 - (1-p)\right]^{-2} p = 1/p$ , and this is the mean number of months for exposure if the probability of conception in any month is  $p$ .

The feature of a renewal process that makes our task simple is its additivity. Suppose that the number of months of sterility following the onset of pregnancy is also a random variable, with mean value  $s$ . Then the expected length of time from the start of intercourse to the end of pregnancy is the sum of these two mean values,  $1/p + s$ . This is not as obvious as it looks, but we need not stop to prove it here. It follows from the general proposition that if we have to wait for one event, and when that occurs we have to wait for another, and each event follows its own random distribution, then the mean time to the second event is the sum of the mean times for the two events separately.

The same proposition applies to abortion, which involves first the random time to pregnancy and then the random time to performance of the abortion. We will find it convenient to incorporate with the time to abortion the length of the post-abortion sterile period. Suppose these two add up to a total infecund period of random length whose expected

value is  $a$ . Then the total length of the cycle is on the average  $(1/p) + a$ .

Now let us see how many births are prevented by abortions taking place at such time after the onset of pregnancy that the woman is infecund for an expected  $a$  months. Since the total length of the cycle involving one abortion averages  $1/p + a$ , and the total length of the cycle involving one birth averages  $1/p + s$ , the number of the former that would fit into the latter is

$$\frac{\frac{1}{p} + s}{\frac{1}{p} + a}$$

This is the number of abortion cycles required to fill the time that would be taken by one birth cycle. It is therefore the number of abortions that would prevent one birth, and its reciprocal is the fraction of a birth prevented by one abortion. If  $p = 0.2$ ,  $s = 17$ , and  $a = 3$ , we have  $8/22$  births prevented by one abortion, as in the numerical calculation that preceded this algebra.

The same result may be obtained by thinking of two women with the same constant probability  $p$  of conception leading to live birth in any month. The first of the women carries all the conceptions to term, and according to the above argument will have a baby on the average every  $1/p + s$  months. The other woman wants no children and uses abortion as her sole method of contraception. She would have an abortion every

$1/p + a$  months. Over any long period of time  $T$  the first woman would expect to have  $T/(1/p + s)$  births, the second woman would expect to have  $T/(1/p + a)$  abortions. The second ratio divided by the first, i.e.,  $(1/p + s)/(1/p + a)$ , again tells us how many abortions are equivalent to one birth.

#### Abortion as a backup to contraception

The constants in the above calculation are about right for human populations that do not use contraception. With contraception the effect of abortion is very much greater, a fact that may not be intuitively obvious. We will see how it follows from the above discussion.

To apply the argument to our new problem we suppose that the efficiency of contraception is  $e$ . If  $e$  is 0.95, then instead of the probability of conceiving in a particular month being 0.2, as for unprotected fertile couples, it is reduced to  $0.2(1 - 0.95)$  or about 0.01. More generally, in place of  $p$  we write  $p(1 - e)$  as the probability of conceiving in a particular month. So much for the definition of efficiency of contraception.

Now we go through the whole of the preceding argument again, this time with  $p(1 - e)$  instead of  $p$ . Nothing else is changed: the mean length of time to pregnancy for fertile couples becomes  $1/p(1 - e)$ , and the number of abortions that would prevent one birth is now

$$\frac{\frac{1}{p(1-e)} + s}{\frac{1}{p(1-e)} + a}$$

Entering  $p = 0.2$ ,  $e = 0.95$ ,  $s = 17$  months and  $a = 3$  months gives

$$\frac{\frac{1}{0.01} + 17}{\frac{1}{0.01} + 3} = 1.14$$

abortions to prevent one birth.

This is a very different outcome from the no-contraception case. With unprotected intercourse it takes nearly three abortions to prevent one birth. With 95 percent efficient contraception it takes only about one and one seventh abortions to prevent one birth. If the efficiency of contraception was higher than 0.95 an abortion would be even more effective.

#### Limitations of the deterministic model

The argument by which we have found how many abortions in a population are required to avert one birth is deterministic: we compared two women going through repeated cycles, one involving births and the other involving abortions, but without allowing for variation in the length of cycle. We also disregarded variation in fecundability among women. If  $p$  is not constant from woman to woman the resulting expression for births averted has additional terms. The complexities offered by the real world can be met by making the mathematics more elaborate; the art of this work is to stop the elaboration once the

model is realistic enough to draw practical conclusions.

Some kinds of complexity can be defined out of the model. Stillbirths, for example, have been eliminated in what precedes by the device of considering only those conceptions that lead to live births.

#### Births Averted by Contraception

A similar logic serves to establish the effect of contraception. How many births are averted by the insertion of a loop, or by giving a woman a supply of pills? Again let us confine ourselves to conceptions leading to live births, of which the probability in any month is taken as  $p$ ; again we require the proposition proved above that the expected time to conception is  $1/p$  months.

#### Contraception by one method continuing indefinitely

The couple who use a contraceptive of efficiency  $e$  have a probability of conceiving in any month of  $p^* = p(1 - e)$ . With the same sterile period after conception of  $s$  months they will expect a child once in  $\frac{1}{p^*} + s = \frac{1}{p(1 - e)} + s$  months, while an unprotected couple would expect a child each  $\frac{1}{p} + s$  months.

This tells us that inefficient contraception cannot reduce the birth rate very much. One might think that contraception of 50 percent efficiency would lower the birth rate by half, but its effect is much less. On the present model, with

the constants used before, a birth would take place every  $\frac{1}{0.2} + 17 = 22$  months without contraception. With 50 percent efficient contraception a birth would take place every  $\frac{1}{(0.2)(1 - 0.5)} + 17 = 27$  months. Contraception of 50 percent efficiency has reduced the birth rate from  $1/22$  to  $1/27$ , a reduction of only 19 percent.

#### Frequency of intercourse

The same model also tells us that childbearing is by no means proportional to frequency of sexual intercourse. To see this, suppose that a woman is fertile one day out of 30, and that she has intercourse ten times per month on occasions that are random in relation to susceptibility. Then the probability of conception is  $10/30$  in any particular month and the expected time to conception 3 months. If the infertile period totals 17 months, then she would have a child every 20 months; her monthly birth rate would be  $1/20$ .

Suppose now that she reduces her frequency of intercourse to five times per month. If the occasions are still random then the probability in any month is reduced to  $1/6$ , and the expected exposure time is increased to six months. She will have a baby on the average every  $6 + 17 = 23$  months, that is, a monthly birth rate of  $1/23$ . By reducing intercourse 50 percent, from 10 to 5 times per month, people reduce their birth rate from  $1/20$  to  $1/23$ , or only 13 percent on our assumption. Restraint is not an effective way of holding down births. We revert to contraception.

To assess the consequence of one fitting of a loop or one provision of a stock of pills to a woman who has patronized a birth control clinic, it would be extreme optimism to assume that the loop will stay in place indefinitely, or the pills be used forever. Plentiful data now exist on discontinuance rates, and we need to elaborate our model to take these into account.

#### Discontinuance of contraception

Let the probability of discontinuing the use of the contraceptive for a reason other than pregnancy be  $d$  in any month if pregnancy is not considered, and suppose again that among users during a fecundable month  $p^* = p(1 - e)$  accidentally conceive. Then an expected  $d^* = d(1 - p^*)$  drop the contraceptive at the end of the month while still susceptible, and  $1 - p^* - d^*$  continue its use into the following month.

Now the probability of pregnancy by an accident during the use of the contraceptive is

$$p^* + (1 - p^* - d^*)p^* + (1 - p^* - d^*)^2 p^* + \dots = \frac{p^*}{p^* + d^*} .$$

The probability of dropping the contraceptive is exactly the same series, but with  $d^*$  rather than  $p^*$  at the end of each term, so we have

$$d^* + (1 - p^* - d^*)d^* + (1 - p^* - d^*)^2 d^* + \dots = \frac{d^*}{p^* + d^*} .$$

If we suppose the process to continue until pregnancy occurs

before or after dropping the contraceptive, then the probability that it occurs after is also  $d^*/(p^* + d^*)$ . The ratio of the probability of conceiving while wearing the loop, say, is to the probability of conceiving after discontinuance as  $p^*$  is to  $d^*$ .

To obtain the probabilities month by month for those who are destined to become pregnant while wearing the contraceptive, we divide the unconditional probability of becoming pregnant in each month by  $p^*/(p^* + d^*)$ . Among these women  $p^*/(p^*/(p^* + d^*))$  would become pregnant in the first month,  $(1 - p^* - d^*)p^*/(p^*/(p^* + d^*))$  in the second month, etc. Cancelling in these ratios we find  $p^* + d^*$  for the first month,  $(1 - p^* - d^*)(p^* + d^*)$  for the second month, etc. Hence for these the mean length of exposure is

$$p^* + d^* + 2(1 - p^* - d^*)(p^* + d^*) + \dots,$$

which works out to

$$\frac{1}{p^* + d^*},$$

using the same argument that summed  $p + 2(1 - p)p + \dots$  to give us  $1/p$ .

Similarly the distribution to discontinuance by that fraction  $d^*/(d^* + p^*)$  who are destined to discontinue before they become pregnant is the same, and the mean exposure to discontinuance is again  $1/(p^* + d^*)$ .

This agreement of the two distributions is by no means obvious. It is at first contra-intuitive, and we should try to reorganize our intuition to conform to the algebra. Think again of a group of non-pregnant women



subject to a chance  $p^*$  of pregnancy in any month, as well as to a chance  $d^*$  of dropping the contraceptive. Now in any month, out of  $W$  women who are still neither pregnant nor discontinued, the number who are expected to discontinue at some future time before becoming pregnant is  $Wd^*/(p^* + d^*)$ , and the number who are expected to become pregnant while wearing the contraceptive is  $Wp^*/(p^* + d^*)$ . The number who are expected to discontinue in this month is  $Wd^*$ . As a fraction of all the women they are  $d^*$  of course, but as a fraction of the  $Wd^*/(p^* + d^*)$  women who are expected to discontinue they are

$$\frac{Wd^*}{Wd^*/(p^* + d^*)} .$$

Cancelling in this fraction gives  $p^* + d^*$  as the probability of discontinuing in any particular month for those destined to discontinue at some time before they become pregnant. The same argument shows that the probability of becoming pregnant in a particular month for any women destined to become pregnant while using the contraceptive is the same  $p^* + d^*$ .

This proves that the two conditional distributions are the same, and therefore the mean number of months must be the same for those discontinuing as for those becoming pregnant while wearing the contraceptive.

#### Effect of contraception as against natural fertility

Suppose the  $d^*/(p^* + d^*)$  individuals who drop out through discontinuance of the contraceptive use no contraceptive afterwards. They will have a further period of exposure while

they are unprotected, and this will average  $1/p$  months. Hence the time to conception is the time with the contraceptive for everybody,  $1/(p^* + d^*)$ , and for the fraction  $d^*/(p^* + d^*)$  who discontinue is an additional  $1/p$ . The mean exposure time to conception averaged over all the women in our hypothetical population is

$$t^* = \frac{1}{p^* + d^*} + \frac{d^*}{p^* + d^*} \left( \frac{1}{p} \right) = \frac{1 + (d^*/p)}{p(1 - e) + d^*} .$$

This simple and fundamental result due to Potter enables us to find the effect in births averted of a segment of contraception, for example fitting a loop to a woman. If we know the three quantities,  $p$ ,  $e$ , and  $d^*$  of the expression for mean exposure, then we can calculate the mean months of intercourse until conception occurs. This is to be contrasted with  $1/p$ , the mean months of intercourse without contraception, and the difference is the effect of the contraception.

For the interval between successive births we add to the expressions for exposure to intercourse the mean sterile period associated with pregnancy, say  $s$  as before, including the time both before and after the birth. The expected inter-birth period associated with contraception is then  $t^* + s$ . Thus one birth takes place under contraception (including an allowance for dropping the contraceptive) every  $t^* + s$  months. If there were no contraception one birth would take place every  $1/p + s$  months, and the expected number of births without contraception during the  $t^* + s$  months would be  $(t^* + s)/(1/p + s)$ . Hence the births averted by the segment of

contraception must number this quantity less the one that did take place:

$$\frac{t^* + s}{\frac{1}{p} + s} - 1 = \frac{t^* - \frac{1}{p}}{\frac{1}{p} + s} = \frac{p - p^*}{(1 + ps)(p^* + d^*)} .$$

The general method for finding births averted by a segment of contraception is thus to calculate the expected birth rate with and without the segment and take the ratio less unity. We can only find the effect of some action by comparing what happens if the action is taken with what would happen if the action was not taken. To say what would happen with or without some proposed action requires a model, and indeed no causal imputation can be made without a model. The more realistic the model the more precise and certain is the imputation.

To continue with our simple example, suppose that  $p = 0.2$ ,  $s = 17$ , and (as an approximation for the IUD)  $p^* = (0.05)(0.2) = 0.01$ ,  $d^* = 0.03$ . Then in the absence of contraception a birth would take place every  $1/p + s = 5 + 17 = 22$  months. With the IUD,  $t^*$  would be 28.75, and a birth would take place every  $t^* + s = 45.75$  months. Hence the births averted by the segment of the IUD are

$$\frac{45.75}{22} - 1 = 1.08,$$

supposing it to be replaced by no contraception after it falls out.

When modern contraception takes the place of pre-modern

One direction of realism in the model is recognition that many of those who are fitted with a modern means like the loop have already been using contraception of lower efficiency. The number of births averted that can properly be credited to the loop is only the difference resulting from the superiority of the loop over the method used previously; it would be quite improper to credit the loop with all the births averted through its use, as calculated above, where no other method was taken into account.

By a simple alteration of the meaning of the preceding symbols we can deal with the case where the modern contraceptive does not replace natural fertility, but supplants some other less efficient method already in use. Suppose that when the modern method is discontinued--the loop falls out, for instance--the couple goes back to the earlier method. All that is needed to make the above argument apply is to redefine  $p$ , making it refer now not to natural fertility, but to the probability of conception in any given month under the earlier method of contraception.

If the alternative to which a woman falls back on discontinuance is 90 percent efficient, the  $p$  would become  $(0.2)(1 - 0.9) = 0.02$ , and  $1/p + s$  is  $50 + 17 = 67$ . Then  $t^*$  is now  $(1 + d^*/p)/(d^* + p^*) = (1 + 0.03/0.02)/(0.04) = 62.5$ , and  $t^* + s$  is 79.5. Births averted by the segment of contraception in this situation are  $(79.5/67) - 1 = 0.19$ .

The calculation of births averted by contraception is important enough to be worth summarizing in a table for our two cases:

	Loop superimposed on	
	Natural fertility	90% efficient contraception
p	0.2	0.02
$p^* = p(1 - e)$	0.01	0.01
d*	0.03	0.03
s	17	17

and it provides

$\frac{p - p^*}{(1 + ps)(p^* + d^*)}$	1.08	0.19
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as before.

The contribution of a fitting of a loop to a birth control program is less than one fifth as great if the alternative is 90 percent efficient contraception as if it is natural fertility.

#### Delay in exposure

Often a loop is fitted just after a child is born. It is an administrative convenience that the woman is already present in the hospital, and after having a baby she is likely

to be receptive to birth control. For the months of post-partum sterility the loop cannot affect the chance of childbearing, and on the other hand it is subject to the risk of accidental or deliberate removal. If this risk is  $d$  per month in the absence of pregnancy, and the woman has  $A$  months to go before she becomes fertile again, then the chance that the contraceptive will begin to work as assumed in the above model is  $(1 - d)^A$ .

The argument needs to be broken down for (1) the conditional probability if the loop stays in place  $A$  months, and (2) the conditional probability if it is discontinued. If the loop stays in place  $A$  months, then the subsequent expected exposure time is  $t^*$ , and if it does not the subsequent expected exposure time is  $1/p$ . Hence the unconditional expected exposure time is

$$t_A^* = (1 - d)^A t^* + (1 - (1 - d)^A)(1/p),$$

where we start counting at the beginning of the fertile period. Now births averted would be a smaller quantity than before:

$$\text{Births averted given } A = \frac{t_A^* + s}{\frac{1}{p} + s} - 1 = \frac{t_A^* - \frac{1}{p}}{\frac{1}{p} + s}.$$

This is a smaller quantity because  $t_A^*$  is a decreasing function of  $A$ , so that for positive  $A$  the births averted will also be a decreasing function of  $A$ . Intuition must be in accord: loops inserted in temporarily sterile women

cannot make as much difference to the birth rate as loops in fertile women.

These considerations pose an operations research type of problem for the family planning administrator. He can more easily arrange for loops to be fitted while women are in the hospital, but some of the loops will be lost before they can come into effective use. What part of his effort should go into this immediate postpartum fitting, as against the part that goes into other birth control activities? If the woman leaves the hospital unfitted, when should she be reminded to return for fitting? The lactation period and the temporary sterility associated with it is a random variable, and one would aim to come as close to the end as possible. The optimum solution would compromise between the waste of loops inserted too early and the risk of an unprotected fertile gap.

Again, what amount of effort should be put into providing contraception to women in their twenties, and what to those in the thirties? We know that the dropout rate is higher for younger women, so the expected time they will wear the loop after its insertion is shorter. Yet they are more fertile, and so while the loop is in place it prevents more births. Aside from this, a birth prevented to a younger woman helps to lengthen the generation, which, as we shall see later, in itself lowers the annual rate of increase of the population.

The large amount of data on costs and effectiveness now being gathered ought to provide a basis for rational decision of these and other points.

### A contraception balance sheet

The theory sketched here, along with data now becoming available to provide the constants, suggests an overall account of the childbearing activities of a country. Starting with the number of married couples of different ages and of different degrees of susceptibility, we can say how many children would be produced if there were no contraception. From the actual number born before the advent of modern loops and pills we can estimate the efficiency of older style contraceptive methods. With estimates of the number of loops fitted and pills provided we can say what additional births are being averted, using the above outlined theory. The balance sheet starts in effect with the amount of intercourse, and accounts for all of it, including the part that results in the bearing of children.

With a target number of births set as acceptable at the end of 20 years, say, we can calculate what amount of further modern birth control activity will be necessary, recognizing alternative routes to the target.

### Other Effects of Birth Control

#### Child-spacing and the efficiency of contraception

Those advocating contraception have argued that it would advance the welfare of mothers by allowing them to space out their children, and indeed "child-spacing" was a euphemism for contraception in the days when mention of the direct term was injudicious. Today we observe in groups using contraception



a tendency to concentrate children within a few years of the mother's reproductive life. The interchild interval seems to go down as the efficiency of contraception goes up, so that if we needed a euphemism for contraception today, child-concentration would be more appropriate than child-spacing. The reason is a further aspect of the mathematics of birth control.

To present a simplified case, suppose that a couple want no more than three children, that they command a technology of birth control that is 95 percent efficient, and they have a reproductive life of 25 years. They would have a probability of conceiving in any month of  $(0.2)(1 - 0.95) = 0.01$  with their 95 percent efficient contraceptive, and over the 25 years they would have an expected three pregnancies. (We consider once again only pregnancies leading to live births.) The couple could not afford to let up for any part of their fecund life; using contraception for every one of the 300 months they would have three children on the average, and these would appear at random intervals spread through the 25 years.

At the other extreme, a couple that command a sure method of contraception can go ahead and have their three children early in their married lives, or at any juncture they find convenient, with complete confidence that they could then stop. Not needing to allow for accidental pregnancies, they can enjoy the advantages of

having their children in a clump. These advantages include especially the economies in the mother's time--she has to stay home whether she is looking after one child or three-- and the companionship that the children provide for one another. Instead of the mother being tied up for 25 or 30 years, until her youngest is aged 10, she can raise the same three children in 12 years if they are closely concentrated. After that she can resume the career that she has interrupted to bear the children.

A very high efficiency of contraception is necessary for this; even 99 percent efficiency is not good enough. For with 99 percent efficiency the chance of conceiving in a particular month with  $p = 0.2$  is  $(0.2)(1 - 0.99) = 0.002$ . The couple who have all three children in a clump and then have twenty years of fertile life remaining must realize that their chance of avoiding a fourth pregnancy over 240 months is  $0.998^{240} = 0.618$ . The chance is almost 40 percent that they will bear an unwanted child with a contraceptive of 99 percent efficiency.

When contraception of 0.999 efficiency is attained then the chance of the unwanted child in the same circumstances is reduced to  $1 - 0.9998^{240} = 0.047$ , that is, five percent or one in twenty. At about this point in the technology of contraception couples can fully accept the advantage of concentrating their children and disregard the risk of unwanted pregnancies. Before this point they may in some

degree concentrate their children, but prudence requires deferring one wanted child as an allowance for accidents.

#### Preference for boys

With the advent of the perfect contraceptive most of the problems dealt with above will disappear. The perfect contraceptive would be 100 percent efficient, so no one would have to allow for accidental pregnancy; not only would it always work, but it would be so simple and automatic that no one could forget to apply it; there would be no need for abortion as a back-up. It would be entirely harmless and inoffensive, so no one could feel a disinclination to use it. Such a perfect instrument is still some years of research distant.

Certain problems of the present day will, however, persist into the era of perfect contraception. An enduring contradiction seems to exist between the desire of parents for children and the capacity of the earth's crust to maintain population.

One aspect of the contradiction arises out of parental preferences for children of both sexes. If parents are satisfied to have two children that would grow to maturity, but insist that the two include one boy and one girl, it turns out that they would need to average more than three children in all. Let us see by what probability mechanism the two children, which on the average would keep the population at a desirable constancy, become three children and, ultimately,

an intolerable burden on the ecology, merely because parents continue to reproduce until they have a child of each sex.

If the probability of a boy on a particular birth is  $b$ , then the chance that the couple will have first a girl and then a boy is  $(1 - b)b$ . Let us continue with those instances where the couple end up with a boy. The probability of having two girls in succession and then a boy is  $(1 - b)^2b$ ; of having three girls and then a boy is  $(1 - b)^3b$ ; ... The expected number of children in this series is

$$2(1 - b)b + 3(1 - b)^2b + \dots$$

Comparing this with our analysis of the mean months of exposure to conception, in which we required the sum  $p + 2(1 - p)p + 3(1 - p)^2p + \dots$ , we see that the two are identical, except that now our sum lacks the initial term  $b$ . The sum of the  $p$  series was  $1/p$ , and hence we now have the sum  $1/b$ , but need to subtract  $b$ , to obtain  $(1/b) - b$ .

The argument for the contingency where the couple achieves one child of either sex but ends up with a girl is exactly the same, but we have to replace  $b$  by  $1 - b$ , to obtain  $(1/(1 - b)) - (1 - b)$ .

The expected number in total is the sum of the expected number ending with a boy and that ending with a girl, or

$$\frac{1}{b} - b + \frac{1}{1 - b} - (1 - b) = \frac{1}{b} = \frac{1}{1 - b} - 1.$$

If  $b$  equals  $1/2$ , the mean number of children on this last works out to 3; if  $b$  departs from  $1/2$  the mean is greater than 3. With a departure of  $(b - \frac{1}{2})$  the mean number of children can be shown to be approximately  $3 + 16(b - \frac{1}{2})^2$ , which gives an average number of children of 3.05 for the usual sex ratio at birth of 105 boys per 100 girls.

What in fact are the sex preferences of parents? The first stage of a study in Hull, England, shows that 45.7 percent of recently married couples want one boy and one girl; 15.4 percent want two boys and one girl; 12.6 percent want two boys and two girls. Considering the three quarters of parents that fall into these three categories, we find that on the average they want 2.55 children. But because of their preferences for certain boy-girl combinations they will on the average actually attain about 3.7 children.

All this would change drastically with the advent of control of sex in the offspring. Parents wanting one boy and one girl would average two children rather than three or more. The mean for the surveyed group above would go down from 3.7 to 2.55, a drastic reduction.

Moreover, a secondary effect would appear with the ability to determine sex. Preferences for boys being stronger on the average, the next generation would have a predominance of men--anything up to 60 percent is conceivable. The proportion of the population that could get married would decline. Abstracting from any change in the total population, a drop to

40 percent women from the usual 48 percent would in itself cause a fall in the birth rate of one sixth, supposing the birth rates of those women who do marry to be the same as before.

A further effect would follow in what one might call the dynamics of the sex ratio: with abundance of boys the tastes of parents would alter. Before even the first generation of children under the regime of free choice of sex grew to marrying age parents would have an increased appreciation of girls. They might even respond with an excess of girls. Some swaying back and forth between boys and girls would subsequently occur.

A model for the transitional behavior of parents following on the invention of sex control is not yet in existence, but we can say with some confidence that the immediate effect of sex control would be an excess of boys, and the ultimate effect something close to the present sex ratio. This would mean a considerable temporary reduction in the birth rate over the transitional period, and a permanent more substantial reduction through the mechanism earlier described by which parents no longer need to average three children in order to be assured of one boy and one girl.

#### Family size and the birth rate

The new demography, written with the accent on birth rather than death, and in terms of decisions by individual couples on having and not having children, still requires a

link to the old, which stressed populations and their annual rates of increase. We want a way of going from average family size to the rate of increase of the population.

Suppose a community in which married couples average  $c$  children, and where the age at which they have these does not change over time. How fast will that community grow from year to year? The answer to this question depends on the proportion of individuals who marry, the average age at which they have children, the distribution of their children by sex, and the probability of babies surviving to have children themselves. Let us consider the female side of the process only, though an analogous argument would apply for males.

If married couples have  $c$  children, and a fraction  $m$  of women marry, then the average number of children per (married or single) woman is the product of these,  $cm$ . If the proportion of births that are girl babies is  $f$ , a number close to 0.48 for most populations, then the average number of female progeny to woman is  $cmf$ . If the proportion of these that live to reproductive age is on the average  $\ell$ , the surviving number per woman number  $cmf\ell$ . Finally, if the age at childbearing is  $T$ , then the ratio per generation  $cmf\ell$  is converted to a ratio of increase per year by taking the  $T$ th root:  $\sqrt[T]{cmf\ell}$ .

In order to allow for variation in ages at childbearing we have to take as the generation a number slightly greater than the observed arithmetic mean age of childbearing, but the excess

is usually a year or less. The mean chance of survival has similarly to be an average of survivorships weighted by the ages of childbearing, but with low mortality that is close to the probability of surviving to the mean age of childbearing. Moreover, the annual increase is usually reckoned on a compounding basis, with the period of compounding infinitely small, while we have calculated a simple ratio of increase per year. These refinements alter the results very slightly, and we need not be detained by such details here.

The result, that the ratio of population in one year to that in the preceding year is  $\frac{T}{\sqrt{cmfl}}$ , tells us that when mortality was high variation in proportion surviving could make a good deal of difference. The fact that the proportion of babies surviving to bear children themselves has gone up from about 0.5 just three or four decades back in many less developed countries to 0.9 now is what underlies the contemporary population problem; further increase cannot make much difference.

The proportion  $f$  of births that are girls is up to now a biological fact and does not change much. The mean age of childbearing  $T$  has gone down with today's earlier marriage; in Sweden of the 19th century  $T$  was as high as 32 years, and now for some countries is as low as 26. If everything else remains the same, the younger average age means a shorter generation and correspondingly more rapid turnover, which has exactly the same effect on growth as an increased number of children per family. In at least one recent year Canada showed



a higher ratio of births to women than the United States, but a lower rate of increase because the mothers were older.

A further development of this logic shows that children born to a woman of age 40 have only about half the impact on the long-term rate of increase that children born to women of age 20 have. Birth control programs tend to attract older women first, and only later do they attract young women. Insofar as the new confidence in birth control encourages couples to have their children in a clump while they are young, rather than having the same number spread over 25 years, the birth control program could actually increase the rate of growth of the population. Fortunately, most couples use birth control to have fewer children as well as to have them younger.

Recent statistics of births in Hong Kong, Taiwan, and other places where massive programs of modern contraception have been introduced show a sharp reduction in childbearing at the oldest reproductive ages of women, while younger women are slower to respond. Aside from the shortening of the generation, any forward projection has to take account of the fact that the age-specific rates for older women cannot fall below zero. It needs no mathematics to see that a levelling of the birth rate will occur unless the birth control movement spreads to younger women. There are signs that it is doing so in certain countries around the rim of South and East Asia.

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## ABSTRACT

Advances have been made in the technique of contraception, and these are being adopted in various parts of the world. The effect that the adoption of the loop or the pill has in averting births that would occur without protection, or with older forms of contraception, is a main interest of contemporary demography. To estimate this effect one must regard conception and birth as a renewal process for the individual woman. Though the models of the succeeding pages are very simple, they show the lines along which correct calculations can be made. Further advances in theory are producing more realistic models, and large amounts of data collection now going forward provide better estimates of such parameters of the process as probability of conceiving in a given month without protection and with various types of contraceptives.