# How Many Universes Do There Need To Be?

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## Abstract

In the simplest cosmological models consistent with General Relativity, the total volume of the Universe is either finite or infinite, depending on whether or not the spatial curvature is positive. Current data suggest that the curvature is very close to flat, implying that one can place a lower limit on the total volume. In a Universe of finite age, the 'particle horizon' defines the patch of the Universe which is observable to us. Based on today's best-fit cosmological parameters it is possible to constrain the number of observable Universe sized patches, $N_{\rm U}$. Specifically, using the new *WMAP* data, we can say that there are at least 21 patches out there the same volume as ours, at 95% confidence. Moreover, even if the precision of our cosmological measurements continues to increase, density perturbations at the particle horizon size limit us to never knowing that there are more than about $10^5$ patches out there.

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Understanding how big all of space is and our place within its immensity, has long been one of the great mysteries for human beings to ponder. Within simple cosmological models, consistent with General Relativity, the total volume depends on the curvature of the spatial sections, while the volume of the Universe which is accessible to observation is determined by the expansion history of the Universe. Since we now have good empirical knowledge of both global curvature and cosmological dynamics, we can constrain the fraction of the Universe comprised by our observable 'patch'.

Although introductory cosmology courses teach the principles of non-Euclidean spaces, we now know that our Universe is very close to having flat spatial geometry. The newest results from the Wilkinson Microwave Anisotropy Probe (*WMAP*, [1]) measurements of the cosmic microwave background (CMB) combined with other cosmological data show that the average density of the Universe is within a few percent of the value required for flatness.

Sometimes it is assumed that the Universe *is* spatially flat. However, this is an empirical question, and right now we do not know whether the Universe is slightly closed or slightly open. A closed universe, appealing on theoretical grounds (see [2] and references therein), would have finite volume, while an open or exactly flat space would have infinite volume. Because of this, we do not know whether our own observable part is a negligible or significant fraction of the whole volume, although as we discuss below, it is now possible to place a rather robust limit on this fraction.

Tegmark has used the claim that the Universe is known to have infinite volume to argue [3] that there are as many 'observable universe' patches as there are 'multiverses' in the quantum mechanical 'many-worlds' picture, and hence we are already forced to imagine $\gtrsim 10^{100}$ copies of 'the Universe', perhaps more than the number of different possible particle configurations. Hence the suggestion is that modern cosmological observations lead one to imagine that there are many near-copies of our patch out there in the vastness of the entire Universe. However, as we will see below, the empirically-motivated limit is more like 10 than $10^{100}$. Hence there is no reason, purely on the grounds of cosmological observations, to be forced to believe in other versions of oneself, differing by only a few bits of information.

Of course, we have no possibility for observationally determining the character of the Universe on scales larger than the (apparent) particle horizon – that is, we cannot see farther away (using photons at least) than the last scattering surface, where the CMB radiation was released. Nevertheless, we *can* determine that the observable volume is very close to homogeneous and isotropic, and, by applying the cosmological principle, conclude that the Universe continues to be well approximated by a homogeneous and isotropic Robertson-Walker (RW) metric on scales much larger still. As we stated above, such a spacetime can be spatially open and infinite, in which case the arguments of Tegmark may be relevant, or it can be spatially closed and finite. In this latter case, our observable volume is a finite fraction of the total volume, and those arguments are invalid.

If the universe is closed, current cosmological observations put an upper limit on the spatial curvature, and hence can be used to place a lower limit on the radius of curvature of such a model. By using an observational probability distribution for curvature (together with other relevant cosmological parameters), and treating the Universe globally as an RW spacetime, it becomes a straightforward problem to determine the minimum ratio of the total volume to the observable volume consistent with observations, i.e. the minimum number of observable-universe-sized 'patches' that must exist.

Based on growing circumstantial evidence, together with strong theoretical motivation, there is good reason to believe that a period of inflation in the early Universe drove the

spatial curvature very close to zero. However, we take the approach here of determining what can be said about the curvature based on *empirical* evidence alone, coupled with the minimal extra ingredient of the assumption of global homogeneity and isotropy. Indeed we will see that inflation itself *predicts* that we will never be able to determine the spatial curvature precisely. In addition, non-trivial topologies can render a spatially flat universe finite in volume, but that possibility only strengthens our contention that we cannot conclude that the Universe is spatially infinite. Current limits on the topology size are in fact a little larger than the particle horizon distance [4].

The metric for a spatially closed RW cosmology can be written

$$ds^2 = a^2(\eta)\left(-d\eta^2 + d\chi^2 + \sin^2\chi\, d\Omega^2\right), \tag{1}$$

where $\eta$ is the conformal time, $d\Omega^2$ is the solid angle element, $\chi$ is the comoving radial coordinate which runs from zero at our location to $\pi$ at the opposite 'pole' of a homogeneous closed spatial slice, and we have set $c = 1$. In this particular form of the metric, the scale factor $a$ is not arbitrary, but in fact equals the physical radius of the spatial slices. This conformal form makes it trivial to calculate the comoving distance to the particle horizon, i.e. the greatest value $\chi_{\rm ph}$ within our past light cone. The result is

$$\chi_{\rm ph} = \int_0^{\eta_0} d\eta = \int_0^{a_0} \frac{da}{\dot{a}a} = (-\Omega_{\rm K})^{1/2} D(\Omega_{\rm M}, \Omega_\gamma, \Omega_\Lambda), \tag{2}$$

where the subscript $_0$ indicates a current value. Here $\Omega_{\rm M}$, $\Omega_\gamma$, and $\Omega_\Lambda$ are the present-day energy densities in matter, radiation, and cosmological constant, respectively, as fractions of the critical density today, $\rho_{\rm crit} \equiv 3H_0^2/8\pi G$. The parameter $\Omega_{\rm K}$ measures the contribution of curvature to the dynamics ($\Omega_{\rm K} = 1 - \Omega_{\rm M} - \Omega_\gamma - \Omega_\Lambda$ by the energy constraint equation). The integral $D$ is given by

$$D(\Omega_{\rm M}, \Omega_\gamma, \Omega_\Lambda) = \int_0^1 \left(\Omega_\gamma + \Omega_{\rm M}a + \Omega_{\rm K}a^2 + \Omega_\Lambda a^4\right)^{-1/2} da. \tag{3}$$

The volume of the current spatial slice out to distance $\chi_{\rm ph}$ is

$$V_{\rm ph} = \int \sqrt{h}\, d^3x = \pi a_0^3\left[2\chi_{\rm ph} - \sin(2\chi_{\rm ph})\right], \tag{4}$$

where $h$ is the determinant of the spatial part of the metric (1). Thus in particular the volume of the entire slice is $V_{\rm tot} \equiv 2\pi^2 a_0^3$, and so the number of particle horizon volumes that can fit in the entire slice is

$$N_{\rm U} \equiv \frac{V_{\rm tot}}{V_{\rm ph}} = \frac{2\pi}{2\chi_{\rm ph} - \sin(2\chi_{\rm ph})}. \tag{5}$$

The parameters $\Omega_{\rm M}$, $\Omega_\gamma$, and $\Omega_\Lambda$ are now well-measured [1] and they fix $\Omega_{\rm K}$ through the energy constraint equation, assuming Einstein gravity. Choosing the best-fit WMAP 3 year data set parameters $\Omega_{\rm M} = 0.24$, $\Omega_\gamma = 5.5 \times 10^{-5}$, and $\Omega_\Lambda = 0.76$, we find, performing the integral (3), that $D \simeq 3.5$. This tells us that the apparent particle horizon exceeds the current Hubble radius by the factor 3.5. Note that this calculation, through Eq. (3), depends on the assumption of radiation domination into the arbitrary past. Of course standard inflationary models violate this assumption, and the particle horizon can actually diverge in these models. But we are interested here in the *apparent* particle horizon, i.e. the

3

comoving distance to the last scattering surface, as this determines the size of the *observable* Universe.

We can estimate the limit on the number of observable Universe patches by looking at the observed probability distributions for the cosmological parameters, and through Eqs. (2) and (5), convert these to a distribution for $N_{\rm U}$. Since, of the relevant parameters, $\Omega_{\rm K}$ has the greatest relative uncertainty, the uncertainty in $N_{\rm U}$ will be dominated by that of $\Omega_{\rm K}$ through Eq. (2). Using the Markov Chain Monte Carlos provided by the *WMAP* team for non-flat cosmologies (together with additional information on the Hubble constant from the Hubble Space Telescope (*HST*) Key Project [5]) we calculated the likelihood function for $N_{\rm U}$; the result is presented in Fig. 1. Based on this distribution we find a 95% confidence lower limit of $N_{\rm U} > 21$. That is, 95% of models consistent with the *WMAP* and *HST* data are closed models with more than 21 observable-universe-sized patches, or are open models. Different choices of cosmological data, model spaces, and Bayesian priors yield somewhat different distributions for $N_{\rm U}$, but all reasonable choices yield a lower limit of $N_{\rm U} \gtrsim 10$.

Ultimately we are limited by cosmic variance in our observable patch. Due to the spectrum of perturbations produced by standard inflationary models, the curvature perturbation on the scale of our patch today is of order $10^{-5}$ (see also for example [6]), and hence we are unlikely to ever know with confidence that $|\Omega_{\rm K}| < 10^{-4}$ (since we could live a few $\sigma$ into the tail of the distribution). In that case Eq. (5) becomes

$$N_{\rm U} \simeq \frac{3\pi}{2\chi_{\rm ph}^3} \simeq 0.1(-\Omega_{\rm K})^{-3/2}. \tag{6}$$

Hence the *best* lower limit that we will ever be able to place is $N_{\rm U} \gtrsim 10^5$, unless of course improved measurements find that the Universe is actually open. But assuming that we
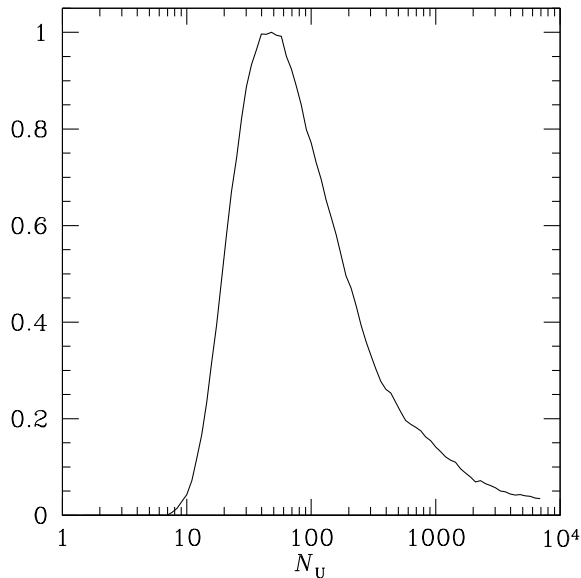


FIG. 1: Probability distribution (in arbitrary units) for the number of universe patches $N_{\rm U}$ based on *WMAP + HST* data. The 95% confidence lower limit is $N_{\rm U} > 21$. Although by eye this distribution appears to favour values near $N_{\rm U} \sim 100$, this is actually not the case due to the logarithmic scale and because of a large peak at infinite $N_{\rm U}$ corresponding to open models.

4

continue to measure that the curvature is close to flat, then cosmology alone will only enable us to infer that there are *at least* 100,000 other observable patches out there. Determining that the Universe is genuinely infinite will remain beyond the reach of purely empirical studies.

**Acknowledgments**

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