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How Should Benefits and Costs Be Discounted in an Intergenerational Context?

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Abstract

In September 2011, the US Environmental Protection Agency asked 12 economists how the benefits and costs of regulations should be discounted for projects that affect future generations. This paper summarizes the views of the panel on three topics: the use of the Ramsey formula as an organizing principle for determining discount rates over long horizons, whether the discount rate should decline over time, and how intra- and intergenerational discounting practices can be made compatible. The panel members agree that the Ramsey formula provides a useful framework for thinking about intergenerational discounting. We also agree that theory provides compelling arguments for a declining certainty-equivalent discount rate. In the Ramsey formula, uncertainty about the future rate of growth in per capita consumption can lead to a declining consumption rate of discount, assuming that shocks to consumption are positively correlated. This uncertainty in future consumption growth rates may be estimated econometrically based on historic observations, or it can be derived from subjective uncertainty about the mean rate of growth in mean consumption or its volatility. Determining the remaining parameters of the Ramsey formula is, however, challenging.

Key Words: discount rate, uncertainty, declining discount rate, benefit-cost analysis

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*At a workshop held at Resources for the Future in September of 2011 twelve of the authors were asked by the USEPA to give advice on principles to be used in discounting the benefits and costs of projects that affect future generations. Maureen L. Cropper chaired the workshop.

1. Introduction

In project analysis, the rate at which future benefits and costs are discounted often determines whether a project passes the benefit-cost test. This is especially true of projects with long horizons, such as projects to reduce greenhouse gas (GHG) emissions. The benefits of reduced GHG emissions last for centuries, but mitigation costs are borne today. Whether such projects pass the benefit-cost test is especially sensitive to the rate at which future benefits are discounted. In evaluating public projects, France and the United Kingdom use discount rate schedules in which the discount rate applied to benefits and costs in future years declines over time: the rate used to discount benefits from year 200 to year 100 is lower than the rate used to discount benefits in year 100 to the present (see Figure 1).[1] In the United States, the Office of Management and Budget (OMB) recommends that project costs and benefits be discounted at a constant exponential rate, although this rate may be lower for projects that affect future generations. This raises a familiar, but difficult, question: how should governments discount the costs and benefits of public projects, especially projects that affect future generations?

In this paper we ask what principles should be used to determine the rates at which to discount the costs and benefits of regulatory programs when costs and benefits extend over very long horizons. We address this issue by considering three sets of questions.[2] The first set deals with the use of the Ramsey formula as an organizing principle for determining discount rates over long horizons. The second deals with the literature on declining discount rates. This literature suggests that if there is a persistent element to the uncertainty in the rate of return to capital or in the growth rate of the economy, then it will result in an effective discount rate that declines over time. The final set of questions concerns how intra- and intergenerational discounting practices can be made compatible.

We begin by elaborating on the questions addressed and briefly summarize our answers. The remainder of the paper goes into more detail on the points we make and summarizes the relevant literature on each topic. The final section of the paper presents our main conclusions.

[1] The discount rates pictured in Figure 1 are instantaneous certainty-equivalent discount rates.
[2] These are the questions posed to the panel by the USEPA at a workshop held at Resources for the Future in September of 2011.

Question 1: Use of the Ramsey Formula.

We revisit the foundations of benefit-cost analysis, which dictate that the sure benefits and costs of a project should be converted to consumption units and discounted to the present at the consumption rate of interest—the rate at which society would trade consumption in year t for consumption in the present.[3] Under certain assumptions—and ignoring uncertainty—this approach leads to the Ramsey discounting formula, in which the discount rate applied to net benefits at time t, ρ_t, equals the sum of the utility rate of discount (δ) and the rate of growth in consumption between t and the present (g_t), weighted by (minus) the elasticity of marginal utility of consumption (η): $\rho_t = \delta + \eta \cdot g_t$..

Should the Ramsey formula for the consumption rate of discount be used to determine discount rates over long horizons? If so, how should it be parameterized? What are the implications of uncertainty about future growth in per capita consumption for the consumption rate of discount?

All of us agree that the Ramsey formula provides a useful conceptual framework for examining intergenerational discounting issues. The key question is whether it can be put to practical use to generate a path of discount rates for use in benefit-cost analysis, and if so how. One approach is to view the parameters δ and η as representing policy choices (the "prescriptive" approach to discounting); another is to base estimates of δ and η on market rates of return (the "descriptive" approach) (Arrow et al. 1996). Those who favor the prescriptive approach argue that the parameters of the Ramsey formula could be based on ethical principles. Alternatively, the parameters could be based on public policy decisions (η could, for example, be inferred from the progressivity of the income tax structure, as has been done in the UK) or on attempts to elicit social preferences using stated preference methods. Others of us who favor the descriptive approach suggest that η (or ρ itself) could be inferred from decisions in financial markets, although behavior in financial markets, even for longer-term assets such as 30-year bonds, is likely to reflect intragenerational rather than intergenerational preferences.

Although we realize that the consumption rate of discount need not equal the rate of return to risk-free investment due to a variety of potential market imperfections (see, e.g.,

[3] Throughout the paper we ignore uncertainty in the steam of benefits and costs associated with a project, effectively assuming that these have been converted to certainty-equivalents.

Kocherlakota 1996), if these imperfections are judged to be not too severe then practical considerations may argue for using the rate of return to investment to measure the discount rate. This is, indeed, the approach currently taken by OMB (2003), which refers to the real rate of return on long-term government debt as "the social rate of time preference."[4] Although the rate of return to risk-free investment may not equal the consumption rate of discount, market rates of return are observable (at least for some maturities), and some of us believe that they provide a reality check on results obtained by estimating δ and η by other means.

Although we have not reached a consensus on how, empirically, to parameterize the Ramsey formula, we agree, in principle, on the impact of uncertainty in the rate of growth in consumption (g_t) on the rate of discount. As is well known, the standard Ramsey formula for the consumption rate of discount can be extended to handle uncertainty about the rate of growth in consumption by subtracting a third "precautionary" term from the formula (Mankiw 1981; Gollier 2002). If growth is subject to independently and identically distributed shocks, this is unlikely to significantly alter the consumption rate of discount. If, however, shocks to growth are positively correlated over time, the precautionary term in the Ramsey formula may become sizeable in absolute value for long horizons, leading to a declining term structure of discount rates (Gollier 2008). The Ramsey formula can also be extended to evaluate policies that would reduce catastrophic risks to the economy (Pindyck and Wang 2012). In this case, the impact of possible catastrophes could also have a significant impact on the discount rate.

Question 2: Discounting Using a Declining Discount Rate Schedule

Two branches of the literature about declining discount rates have emerged over the last decade. The first branch is based on the extended Ramsey rule. If, contrary to the standard assumption, shocks to the growth rate of consumption per capita are positively correlated over time, the precautionary term in the Ramsey rule becomes sizeable in absolute value for long horizons, leading to a decreasing term structure (see Gollier 2012 for an extended survey).

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[4] OMB (2003) further defines the social rate of time preference as "the rate at which 'society' discounts future consumption flows to their present value."

The other branch of this literature, the Expected Net Present Value (ENPV) approach, was initially developed by Weitzman (1998, 2001, 2007b). He showed that the uncertainty about future discount rates justifies using a decreasing term structure of discount rates today. Computing ENPVs with an uncertain but constant discount rate is equivalent to computing NPVs with a certain but decreasing "certainty-equivalent" discount rate. More specifically, a probability distribution over the discount rate under constant exponential discounting in the future should induce us to use a declining term structure of discount rates today. Other literature has used a reduced-form approach to estimating certainty-equivalent discount rates based on historical time series of interest rates (for example, Newell and Pizer 2003; Groom et al. 2007; Hepburn et al. 2009). Gollier and Weitzman (2010) have attempted to reconcile these two branches of the literature (the ENPV approach and the extended Ramsey rule).

How should the results of the declining discount rate (DDR) literature be reflected in benefit-cost analyses? Should a schedule of discount rates be derived from theoretical principles and/or simulation models? Should discount rates be estimated empirically? Or, should both approaches be used? Will the use of a DDR lead to time inconsistent policy decisions?

We note that, if there is uncertainty about the discount rates that will be applied in the future and if probabilities can be assigned to these discount rates, this will result in a declining schedule of certainty-equivalent discount rates. The question is how such probabilities should be assigned. The consensus view of the panel is that if disagreement among experts reflects disagreement about preferences (e.g., as in Weitzman 2001)—rather than underlying uncertainty about the economy—it is not appropriate to use this disagreement to set probabilities.

If uncertainty about the discount rate reflects uncertainty about the state of the economy and if there are persistent shocks to growth, this will lead (ceteris paribus) to a declining discount rate schedule. Such declining discount rates do not lead to time inconsistencies, as any desired policy revision would depend on new information and is not certain.

There are two approaches that could be used to estimate discount rates in this case:

1. An approach based on the extended Ramsey formula, which would entail choosing δ and η and modeling, either numerically or analytically, the process that generates g_{t}; or
2. An approach that focuses on estimating reduced-form models of market interest rates.

The empirical literature, which we summarize later in the paper, has focused on the latter approach, using time series data to estimate the stochastic process generating market interest rates. Another approach suggested by some panelists is to obtain forecasts of future market interest rates from experts knowledgeable about the future state of the economy. Should the USEPA wish to follow either of these approaches, the Agency should have its Science Advisory Board approve criteria set forth by the Agency for model selection and for combining results from the literature.

Question 3: Assessing Intra- and Inter-Generational Benefits and Costs within a Rulemaking.

The final set of questions that we examine pertains to the consistent treatment of inter- and intra-generational benefits in benefit-cost analysis. In a recent regulatory impact analysis for Corporate Average Fuel Economy (CAFE) standards (USEPA 2010), the benefits of reduced carbon emissions were discounted at constant exponential rates of 2.5, 3, and 5 percent, following the rates adopted by the US Inter-Agency Task Force on the Social Cost of Carbon.[5] The benefits of reduced fuel consumption were discounted at rates of 3 and 7 percent. The use of a 2.5 percent discount rate for intergenerational benefits and a rate of 3 percent for intra-generational benefits implies that different sources of benefits occurring in the same year could be discounted to the present at different rates, which is inconsistent.

Are the approaches to discounting over long horizons suggested in Questions 1 and 2 consistent with current approaches to intra-generational discounting? Is it appropriate to add the present value of benefits and costs, calculated according to one set of discount rates to the present value of benefits calculated using an alternate discount rate?

[5] The final 2010 interagency report on Social Cost of Carbon is available at: http://go.usa.gov/3fH.

Our answer to Question 3 is simple: It is clearly inappropriate to discount benefits and/or costs occurring in the same year to the present using different discount rates. One solution to this problem is to apply a declining discount rate schedule to all regulations. This would result in consistency between intra- and intergenerational discounting practices. Another approach is to apply the same constant exponential discount rate to all categories of benefits and costs.

2. The Ramsey Formula as a Basis for Intergenerational Discounting

In the context of intergenerational discounting, the consumption rate of discount is usually approached from the perspective of a social planner who wishes to maximize the social welfare of society (Dasgupta 2008; Goulder and Williams 2012). The utility of persons alive at t, u_t, is given by an increasing, strictly concave function of consumption (which can be broadly defined to include both market and non-market goods and services), c_t, i.e., $u_t = u(c_t)$, and it is assumed that the planner maximizes the discounted sum of the utilities of current and future generations. In evaluating investment projects, a social planner would be indifferent between \$1 received at time t and \$ε today if the marginal utility of \$ε today equaled the marginal utility of \$1 at time t.[6]

$$u'(c_0)\varepsilon = e^{-\delta t} u'(c_t) \tag{1}$$

Equation (1) assumes that the utility received from a given level consumption is constant over time, but that future utility is discounted at the rate δ. Solving equation (1) for ε, the present value of \$1 in year t, yields

$$\varepsilon = \frac{e^{-\delta \cdot t} u'(c_t)}{u'(c_0)} = e^{-\rho_t \cdot t} \tag{2}$$

[6] In this paper c_t represents the average consumption of people alive at time t. In an intergenerational context t is often interpreted as indexing different generations; however, it need not be. It can simply represent average consumption in different time periods, some of whom may be the same people. A discussion of models that distinguish individuals within and across generations is beyond the scope of our paper.

where ρ_t denotes the annual consumption rate of discount between periods 0 and *t*. If we assume that *u(c)* exhibits constant relative risk aversion (CRRA) [$u(c) = c^{(1-\eta)}/(1-\eta)$] then ρ_t can be written using the familiar Ramsey formula

$$\rho_t = \delta + \eta \cdot g_t$$
(3)

where η is both the coefficient of relative risk aversion and (minus) the elasticity of marginal utility with respect to consumption, and g_t is the annualized growth rate of consumption between time 0 and time t.[7]

In equation (3) δ is the rate at which society (i.e., the social planner) discounts the utility of future generations. A value of δ =0 says that we judge the utility of future generations to contribute as much to social welfare as our utility. η describes (for any generation) how fast the marginal utility of consumption declines as consumption increases. Higher values of η imply that the marginal utility of consumption declines more rapidly as consumption increases. The standard interpretation of (3) is that the social planner will discount the utility of consumption of future generations at a higher rate because future generations are wealthier (i.e., the higher is the rate of growth in consumption, g_t). To illustrate, if g_t =1.3% annually, per capita consumption in 200 years will be 11 times higher than today. So, it makes sense to discount the utility of an extra dollar of consumption received 200 years from now. And, the planner will discount it at a higher rate the faster the marginal utility of consumption decreases as consumption rises.

How to Parameterize the Ramsey Formula?

To parameterize the Ramsey formula requires estimates of δ and η as well as information about the process governing the growth of per capita consumption. Below we discuss both prescriptive and descriptive approaches to quantifying δ and η.

δ and η as Policy Parameters

Many of us regard the Ramsey approach to discounting, which underlies the theory of cost-benefit analysis, as a normative approach. This implies that its parameters should

[7] In the Ramsey formula η captures the inter-temporal elasticity of substitution between consumption today and consumption in the future, the coefficient of relative risk aversion and inequality aversion. More sophisticated treatments (Epstein and Zin 1991; Gollier 2002) separate attitudes towards time and risk.

reflect how society values consumption by individuals at different points in time; i.e., that δ and η should reflect social values. The question is how these values should be measured.

Many (but not all) of the panelists agree with Frank Ramsey that it is ethically indefensible to discount the utility of future generations, except possibly to take account of the fact that these generations may not exist. This implies that $\delta = 0$, or a number that reflects the probability that future generations will not be alive. Stern (2006), for example, assumes that the hazard rate of extinction of the human race is 0.1% per year.

The parameter η, which determines how fast the marginal utility of consumption declines as consumption increases, can be viewed as a measure of intertemporal inequality aversion (Dasgupta 2008; Gollier et al. 2008): it reflects the maximum sacrifice one generation should make to transfer income to another generation. To make this more concrete, Table 1 describes the maximum sacrifice that society believes a higher income group (A) should make to transfer 1 dollar to the poorer income group (B), as a function of η. When group A is twice as rich as group B and $\eta = 1$ the maximum sacrifice is \$2; when $\eta = 2$, the maximum sacrifice is \$4.[8]

Should δ and η Reflect Observed Behavior in Public Policy?

How, empirically, should η be determined? One approach is to examine the value of η implied by decisions that society makes to redistribute income, such as through progressive income taxes. In the UK, socially revealed inequality aversion, based on income tax schedules, has fluctuated considerably since the Second World War, with a mean of 1.6 (Groom and Maddison 2012). Applying this value to climate policy would assume (1) that the UK government has made the "right" choice in income redistribution and (2) that income redistribution within a country and period is the same as income redistribution between countries and over time. Tol (2010) estimates the consumption rate of international inequity aversion, as revealed by decisions on the level and allocation of development aid, at 0.7.

It is also possible to elicit values of η and δ using stated preference methods. The issue here is whose preferences are to be examined and how. As Dasgupta (2008) has

[8] As noted in footnote 5, η also measures the coefficient of relative risk aversion and the inter-temporal elasticity of substitution of consumption. Atkinson et al. (2009) note that these parameters need not coincide when individuals are asked questions to elicit their social preferences.

pointed out, it is important to examine the implications of the choice of η and δ for the fraction of output that a social planner would choose to save. Ceteris paribus, a lower value of η implies that society would choose to save a larger proportion of its output to increase the welfare of future generations. The implications of the choice of δ and η would need to be made clear to the subjects queried.

Some of us were, however, skeptical of the validity of using stated preference methods, especially as applied to lay people who may not appreciate theoretical constructs such as "pure time preference," "risk-free investment" and "benevolent social planner." We suggest that a check on the reasonableness of results obtained from direct questioning methods would be to compare the resulting values of ρ_t with the return on risk-free investment. This may not represent the consumption rate of discount; however, as noted above, it is currently viewed as a surrogate for the consumption rate of discount by OMB (2003), and is more readily observable.

Should δ and η Reflect Observed Behavior in Financial Markets?

In the simple Ramsey formula the parameter η also represents the coefficient of relative risk aversion, suggesting that η could be estimated from observed behavior in financial markets.[9] Although some panel members favored this approach, other members objected to the use of these estimates on two grounds: they reflect the preferences of people proportionate to their activity in financial markets, and they do not reflect intergenerational consumption tradeoffs, making them inappropriate as estimates of η in a social welfare function.

The use of financial market data to estimate η raises the broader issue of whether the consumption rate of discount should reflect observed behavior and/or the opportunity cost of capital. The descriptive approach to social discounting (Arrow et al., 1996), epitomized by Nordhaus (1994, 2007), suggests that δ and η should be chosen so that ρ_t approximates market interest rates. In base runs of the Nordhaus DICE 2007 model, $\delta = 1.5$ and $\eta = 2$. DICE is an optimal growth model in which g_t and ρ_t are determined endogenously. In DICE 2007 ρ_t ranges from 6.5% in 2015 to 4.5% in 2095 as consumption growth slows over time (Nordhaus 2007).

[9] The macroeconomic literature on the coefficient of relative risk aversion is summarized by Meyer and Meyer (2005).

This raises the question: should we expect the consumption rate of discount in equation (3) to equal the rate of return to capital in financial markets, and, if not, what should we do about this? In an optimal growth model (e.g., the Ramsey model), the consumption rate of discount in (3) will equal the marginal product of capital along an optimal consumption path. If, for example, the social planner chooses the path of society's consumption in a one-sector growth model, ρ_t will equal the marginal product of capital along an optimal path. What if society is not on an optimal consumption path? Then theory tells us that we need to calculate the social opportunity cost of capital—we need to evaluate the present discounted value of consumption that a unit of investment displaces—and use it to value the capital used in a project when we conduct a cost-benefit analysis (Dasgupta, Marglin and Sen 1972). But, once this is done—once all quantities have been converted to consumption equivalents—the appropriate discount rate to judge whether a project increases social welfare is the consumption rate of discount (ρ_t).

A potential problem, as some panel members pointed out, is that converting all costs and benefits to consumption units can, in practice, be difficult. This argues for using the rate of return to capital to measure the discount rate, when a project displaces private investment. This is, in effect, what OMB recommends when it suggests using a 7% real discount rate. A discount rate of 7% is "an estimate of the average pretax rate of return on private capital in the U.S. economy" (OMB 2003) and is meant to capture the opportunity cost of capital when "the main effect of the regulation is to displace or alter the use of capital in the private sector."

The Ramsey Formula When the Growth Rate of Consumption is Uncertain

The rate of growth in consumption is likely to be uncertain, especially over long horizons. Allowing for uncertainty in the rate of growth in per capita consumption alters the Ramsey formula. We begin with the case in which shocks to consumption are independently and identically distributed, which yields the extended Ramsey formula. Formally, suppose that $ln(c_t/c_0) = \sum_{i=1,t} ln(c_i/c_{i-1})$, where $ln(c_i/c_{i-1})$, the proportionate change in consumption at time i, is independently and identically normally distributed with mean μ_g and variance σ_g^2. This adds a third term to the Ramsey formula (Gollier 2002):[10]

$$\rho = \delta + \eta\mu_g - 0.5\, \eta^2\sigma_g^2. \tag{4}$$

[10] This result goes back at least as far as Mankiw (1981).

The last term in (4) is a precautionary effect: uncertainty about the rate of growth in consumption reduces the discount rate, causing the social planner to save more in the present.[11] The magnitude of the precautionary effect is, however, likely to be small, at least for the United States. Suppose that $\delta = 0$, and $\eta=2$, as suggested by Gollier (2008) and Dasgupta (2008). Using annual data from 1889 to 1978 for the United States, Kocherlakota (1996) estimated μ_g to be 1.8% and σ_g to equal 3.6%. This implies that the precautionary effect is 0.26% and that ρ = 3.34% (rather than 3.6%, as implied by equation (3)).[12]

Shocks to consumption may have a larger impact on the discount rate if they represent catastrophic risks. Pindyck and Wang (2012) examine the discounting implications of the risk of global events that could cause a substantial decline in the capital stock, or in the productivity of the capital stock. Examples of catastrophes include an economic collapse on the order of the Great Depression, nuclear or bio-terrorism, a highly contagious "mega-virus" that kills large numbers of people, or an environmental catastrophe, such as the melting of the West Antarctic Ice Sheet.

Suppose that catastrophic risk is modeled as a Poisson process with mean arrival rate λ, and that, if a catastrophe occurs, consumption falls by a random percentage ξ.[13] This subtracts $\eta\lambda E(\xi)$ from the right-hand side of (4), thus reducing the discount rate. How important is this last term? Recent estimates of λ and $E(\xi)$ based on panel data by Barro (2006, 2009) and others put $\lambda \approx .02$ and $E(\xi) \approx 0.3$ to 0.4. Thus if $\eta = 2$, the adjustment would be about -1.2% to -1.6%.

Uncertain Consumption and the DDR

As equation (4) illustrates, independently and identically normally distributed shocks to consumption with known mean and variance result in a constant consumption rate of discount. As discussed below, the consumption rate of discount may decline if

[11] A necessary condition for this to hold is that the planner be prudent (i.e., that the third derivative of $u(c)$ be positive), which is satisfied by the CRRA utility function.
[12] Gollier (2011) finds that the size of the precautionary effects is much larger for other countries, especially developing countries.
[13] Equation (4) assumes that, in continuous time, the log of consumption evolves according to arithmetic Brownian motion: $d\ lnc_t = \mu_g dt + \sigma_g dz$. Catastrophic risk could be modeled by adding a term $-\lambda dq$ to this expression where dq represents a Poisson (jump) process with mean arrival rate λ.

shocks to consumption are correlated over time, or if the rate of change in consumption is independently and identically distributed with unknown mean or variance.

Gollier (2008) proves that if shocks to consumption are positively correlated and $u(c)$ exhibits CRRA, ρ_t will decline.[14] The intuition behind this is that positive shocks to consumption make future consumption riskier, increasing the strength of the precautionary effect in equation (4) as t increases. To illustrate, a possible form that shocks to consumption could take is for $ln(c_t/c_{t-1}) \equiv x_t$, the percentage growth in consumption at t, to follow an AR(1) process

$$x_t = \varphi x_{t-1} + (1-\varphi)\mu + u_t \qquad (5)$$

where u_t is independently and identically normally distributed with constant variance. Mathematically, equation (5) will generate a declining discount rate, provided $0 < \varphi < 1$. To be precise, the precautionary effect is multiplied by the factor $(1-\varphi)^{-2}$ as t goes to infinity (Gollier 2008).

Various models of per capita consumption growth have been estimated for the US (e.g., Cochrane 1988; Cecchetti , Mark and Lam 2000) and these could be used to empirically estimate a DDR using the extended Ramsey formula. The persistence in the rate of change in per capita consumption in the US, based on historic data, is likely to result in a discount rate that declines slowly over time. In the case of equation (5), Gollier (2008) reports an estimate of $\varphi=0.3$, based on the literature, which implies a very gradual decline in the discount rate. The same is true of the certainty-equivalent discount rate based on the regime-switching model of Cecchetti, Mark and Lam (2000). The certainty-equivalent rate in the positive growth regime declines from 4.3% today to 3.4% after 100 years.

The approach to parameterizing the extended Ramsey formula described in the previous paragraphs is based on the assumption that the nature of the stochastic consumption-growth process can be adequately characterized by econometric models estimated using historical data. The consumption-based asset pricing literature suggests

[14] Formally Gollier shows that if $ln(c_i/c_{i-1})$ exhibits positive first-order stochastic dependence and u‴(c) > 0, ρ_t will decline as t increases.

that this is not the case.[15] To quote Weitzman (2007b), "People are acting in the aggregate like there is much more . . . . subjective variability about future growth rates than past observations seem to support." This argues for treating μ_g and σ_g as uncertain. Subjective uncertainty about the trend and volatility in consumption growth, as modeled in Weitzman (2007, 2004) and Gollier (2008), will lead to a declining discount rate.

Weitzman (2004) considers the case in which x_t is independently and identically normally distributed with mean μ_g and variance σ_g^2. The planner is uncertain about μ_g and updates his diffuse prior using n observations on x_t. This leads to the following equation for the certainty-equivalent discount rate,

$$\rho_t = \delta + \eta\, \mu_g - 0.5\, \eta^2 \sigma_g^2 - 0.5\, \eta^2 \sigma_g^2 (t/n). \qquad (6)$$

Bayesian updating adds a fourth term to the extended Ramsey rule—a "statistical forecasting effect"—which causes ρ_t to decline with t, conditional on n and σ_g^2. Intuitively, Bayesian learning generates positive correlation in the perceived growth of consumption.[16]

The form of the planner's subjective uncertainty about the mean rate of growth in consumption clearly influences the path of the certainty-equivalent discount rate. The assumptions in Weitzman (2004) cause the certainty-equivalent discount rate to decline linearly, eventually becoming negative (see equation (6)). Gollier (2008) presents examples that yield non-negative paths for the certainty-equivalent discount rate.

Gollier (2008) proves that, when the rate of growth in log consumption follows a random walk and the mean rate of growth depends on θ [$\mu_g = \mu_g(\theta)$], the certainty-equivalent discount rate, R_t, is given by

$$R_t = \delta + \eta M_t$$

(7)

where M_t is defined by

[15] The extended Ramsey formula does a poor job of explaining the equity premium puzzle: the large gap between the mean return on equities and risk-free assets.
[16] Weitzman (2007b) also considers the case where σ_g^2 is unknown and is assumed to have an inverted Gamma distribution. In this case, Bayesian updating transforms the distribution of x_t from a normal into a Student-t distribution, which has fatter tails. The certainty-equivalent discount rate also declines to $-\infty$ in this case.

$$exp(-\eta t\, M_t) = E_\theta\, exp\, [-\eta t(\mu_g(\theta) - 0.5\eta\sigma_g^{\,2})].$$

(8)

Due to Jensen's inequality, M_t (and R_t) will decline over time. Figure 2 demonstrates the path of R_t for case of $\delta = 0$, $\eta = 2$ and $\sigma_g = 3.6\%$. The mean rate of growth in consumption is assumed to equal 1% and 3% with equal probability. This yields a certainty-equivalent discount rate that declines from 3.8% today to 2% after 300 years—a path that closely resembles the French discounting schedule in Figure 1. The choice of other distributions for θ will, of course, lead to other DDR paths.

3. Directly Estimating Discount Rates Over Long Time Horizons

The Ramsey formula provides a theoretical basis for intergenerational discounting and also suggests that the discount rate schedule is likely to decline over time due to uncertainty about the rate of growth in per capita consumption. The extended Ramsey formula does not, however, readily yield an empirical schedule of certainty-equivalent discount rates.

An alternate approach to modeling discount rate uncertainty that is more empirically tractable is the expected net present value (ENPV) approach. Suppose that an analyst discounts net benefits at time t, Z(t), to the present at a constant exponential rate r, so that the present value of net benefits at time t equals Z(t)exp(-rt).[17] If the discount rate r is fixed over time but uncertain then the expected value of net benefits is given by

$$A(t)Z(t) = E(exp(-rt))Z(t)$$

(9)

where expectation is taken with respect to r. $A(t)$ is the expected value of the discount factor and $R_t \equiv -(dA_t/dt)/A_t$ is the instantaneous certainty-equivalent discount rate. If the probability distribution over r is stationary, then, because the discount factor is a convex function of r, the certainty-equivalent discount rate, R_t, will decline over time (Weitzman 1998, 2001).[18]

[17] We assume that Z(t) represents certain benefits. If benefits are uncertain we assume that they are uncorrelated with r and that Z(t) represents certainty-equivalent benefits.

[18] Gollier and Weitzman (2010) discuss the theoretical underpinnings for the expected net present value approach. The approach is consistent with utility maximization in the case of a logarithmic utility function.

This is illustrated in Table 2, which contrasts the present value of $1,000 received at various dates using a constant discount rate of 4 percent versus a constant discount rate that equals 1% and 7% with equal probability. Jensen's inequality guarantees that the present value computed using the mean discount rate of 4% is always smaller than the expected value of the discount factor, i.e.,

$$E(exp(-rt)) > exp(-E(r)t)). \quad (10)$$

This effect is magnified as t increases, implying that R_t declines over time.

This was first pointed out in the context of intergenerational discounting by Weitzman (1998, 2001). In "Gamma Discounting," Weitzman showed that, if uncertainty about r is described by a gamma distribution with mean μ and variance σ^2, the certainty-equivalent discount rate is given by

$$R_t = \mu/[1 + t\sigma^2/\mu] \ . \quad (11)$$

The gamma distribution provides a good fit to the responses Weitzman obtained when he asked over 2,000 Ph.D. economists what rate should be used to discount the costs and benefits associated with programs to mitigate climate change. The associated mean (4 percent) and standard deviation (3 percent) of responses lead to the schedule of certainty-equivalent discount rates in Table 3.

It is, however, important to consider the underlying source of uncertainty that generates a declining discount rate schedule. On the one hand there are differences in opinion concerning how the future will turn out with regard to the returns to investment, growth and hence the discount rate. Over such time horizons there is genuine uncertainty about these quantities which will be resolved in the future. Weitzman (2001), however, argued that this source of disagreement among experts represents the "tip of the iceberg" compared to differences in normative opinions on the issue of intergenerational justice. Rather than reflecting uncertainty about the future interest rate, which falls naturally into the positive/descriptive school, variation in normative opinions reflects irreducible differences on matters of ethics. Here variation reflects heterogeneity rather than uncertainty. In the unanimous view of the panel, disagreement among experts that reflects differing preferences, rather than underlying uncertainty about the economy, should not form the basis for establishing a declining discount rate schedule.

In contrast, if expert responses represent forecasts, Freeman and Groom (2012) argue that they should be combined to reduce forecasting error, as is typical in the literature on combining forecasts (e.g., Bates and Granger, 1969). Where experts are unbiased and independent the appropriate measure of variation is the standard error, which is smaller by a factor of the inverse of the sample size. Freeman and Groom (2012) show that, in this case, the most appropriate methods of combining forecasts lead to a much slower decline in the discount rate than the original Gamma Discounting approach. Generally their methods suggest that the decline will be more rapid the greater the dependence between expert forecasts.

The declining certainty-equivalent discount rate in Gamma Discounting follows directly from Jensen's inequality and the fact that the distribution over the discount rate is constant over time. In the more general case in which the discount rate varies over time, so that

$$A(t) = E[exp(-\sum_{\tau=1\ldots t} r_\tau)], \qquad (12)$$

the shape of the R_t path depends on the distribution of the $\{r_\tau\}$. If $\{r_\tau\}$ are independently and identically distributed, the certainty-equivalent discount rate is constant. In equation (12), there must be persistence in uncertainty about the discount rate for the certainty-equivalent rate to decline. If, for example, shocks to the discount rate are correlated over time, as in equation (13),

$$r_t = \pi + e_t \quad and \quad e_t = ae_{t-1} + u_t, \quad |a| \leq 1, \qquad (13)$$

the certainty-equivalent discount rate will decline over time (Newell and Pizer 2003).[19]

Empirical Estimates of the DDR Schedule for the US

[19] In equation (13) the interest rate follows an AR(1) process. In estimating (13) it is typically assumed that $\pi \sim N(\mu_\pi, \sigma_\pi^2)$ and $\{u_t\} \sim$ i.i.d. $N(0, \sigma_u^2)$.

The approach followed in the empirical DDR literature is to view r_t as representing the risk-free return to investment (e.g., the return on Treasury bonds), and to develop models to forecast r_t. The empirical DDR literature includes models of interest rate determination for the US (Newell and Pizer 2003; Groom et al. 2007), Australia, Canada, Germany and UK (Hepburn et al. 2009; Gollier et al. 2008) and France, India, Japan and South Africa (Gollier et al. 2008). We focus on the empirical DDR literature as applied to the US.

Newell and Pizer (2003) estimate reduced-form models of bond yields for the United States, using two centuries of data on long-term, high-quality, government bonds (primarily US Treasury bonds), and use them to estimate certainty-equivalent discount rates over the next 400 years. The authors assume that interest rates follow an autoregressive process. This is given by equation (13) in the case of AR(1).[20] Equation (13) implies that the mean interest rate is uncertain, and that deviations from the mean interest rate will be more persistent the higher is a.

The authors demonstrate that the instantaneous certainty-equivalent interest rate corresponding to (13) is given by

$$R_t = \mu_\pi - t\sigma_\pi^2 - \sigma_u^2 f(a,t) , \quad (14)$$

where $f(a,t)$ is increasing in a and t. How fast the certainty-equivalent interest rate declines depends on the variance in the mean interest rate as well as on how persistent are shocks to the mean interest rate (i.e., on a). When $a = 1$ interest rates follow a random walk. To illustrate the implications of persistence, if $a = 1$, $\mu_\pi = 4\%$, $\sigma_\pi^2 = 0.52\%$ and $\sigma_u^2 = 0.23\%$, the certainty-equivalent discount rate declines from 4% today to 1% 100 years from now. In contrast, a value of $a < 1$ (a mean-reverting model) implies that interest rates will revert to μ_π in the long run. When $a = 0.96$, $\mu_\pi = 4\%$, $\sigma_\pi^2 = 0.52\%$ and $\sigma_u^2 = 0.23\%$, the certainty-equivalent discount rate is 4.0% today and 3.6% 100 years from now (Newell and Pizer 2003).

[20] The authors estimate autoregressive models in the logarithms of the variables ($\ln r_t = \ln\pi + e_t$) to avoid negative interest rates. Their preferred models are AR(3) models in which $e_t = a_1e_{t-1} + a_2e_{t-2} + a_3e_{t-3} + u_t$.

Newell and Pizer use results from their preferred specifications of random walk and mean reverting models to simulate the path of certainty-equivalent discount rates.[21] In the random walk model (see Figure 3) the certainty-equivalent discount rate falls from 4% today to 2% in 100 years; in the mean reverting model a certainty-equivalent discount rate of 2% is reached only in 300 years. The authors cannot reject the random walk hypothesis, but investigate the implications of both models for calculating the marginal social cost of carbon.[22]

The subsequent literature, following the literature in Finance, has estimated more flexible reduced-form models of interest rate determination. Groom et al. (2007) estimate five models for the US using the same data as Newell and Pizer (2003). The first two are random walk (RW) and mean-reverting (MR) models identical to those in Newell and Pizer (2003); the third is an autoregressive IGARCH model that allows the conditional variance of the interest rate (held fixed in equation (13)) to vary over time; the fourth is a regime-switching model that allows the interest rate to shift randomly between two regimes that differ in their mean and variance. The final model, which out-performs the others in within- and out-of-sample predictions, is a state space model. This is an autoregressive model that allows both the degree of mean reversion and the variance of the process to change over time.[23]

Groom et al. (2007) use their estimation results to simulate certainty-equivalent discount rates. Figure 4 displays the paths of certainty-equivalent discount rates based on all five models, starting from a discount rate of 4%. The certainty-equivalent rates from the state space model decline more rapidly than rates produced by the random walk model (see Figure 4) for the first 100 years, leveling off at about 2%. The random walk model yields a certainty-equivalent discount rate of 2% at 100 years and 1% in year 200, declining to about 0.5% when t=400.

The DDR schedules estimated by Newell and Pizer (2003) and Groom et al. (2007) make a considerable difference to estimates of the social cost of carbon, compared to using

[21] The preferred models are AR(3) models, estimated using the logarithms of the variables (see footnote 18). The random walk model imposes the restriction that $a_1 + a_2 + a_3 = 1$.

[22] The point estimate of $a_1 + a_2 + a_3 = 0.976$ with a standard error of 0.11. The authors also note that the mean-reverting model, when estimated using data from 1798 through 1899, over-predicts interest rates in the first half of the 20th century.

[23] In the state-space model $r_t = \pi + a_t r_{t-1} + e_t$, where $a_t = \sum \lambda_i a_{t-1} + u_t$. e_t and u_t are serially independent, zero-mean, normally distributed random variables, whose distributions are uncorrelated. The authors compare the models using the root mean squared error of within- and out-of-sample predictions.

a constant exponential discount rate. Using damage estimates from Nordhaus (1994), both sets of authors compute the marginal social cost of carbon as the present discounted value of global damages from emitting a ton of carbon in 2000, discounted at a constant exponential rate of 4% and using certainty-equivalent rates from their models. Using a constant exponential rate of 4% the social cost of carbon is $5.29 (1989 USD). It is $14.44 per ton of carbon using the state space model v. $10.32 per ton using the random walk model (1989 USD).[24]

[24] Freeman et al. (2012) show that these results are robust using real interest rate series pre-1955 to replace the nominal rates used by Newell and Pizer (2003) and Groom et al., (2007). They find a value of the social cost of carbon of $12/ton (1989 USD).

How Should an Empirical DDR Schedule Be Selected?

An important question is how, practically, a DDR schedule should be determined if a DDR is to be used for project analysis. Before proceeding, we note that OMB states that Agencies should provide estimates of net benefits using a 3 percent discount rate when a regulation primarily affects consumption, and is to be measured by the real rate of return on long-term government debt. One possibility would be to replace a constant exponential rate of 3 percent with a DDR estimated using econometric models similar to those described in the previous section. Results from the empirical DDR literature are, however, sensitive to the model estimated, the data series used to estimate the model, and how the data are smoothed and corrected for inflation. If the EPA wishes to pursue this approach in constructing a DDR for use in regulatory impact analysis it should ask its Science Advisory Board to approve criteria for data and model selection.

An alternate approach to establishing a DDR using the extended Ramsey formula would require establishing values for δ and η as well as the other parameters in the formula. The UK discounting schedule pictured in Figure 1 assumes that $\delta = 1.5$ and $\eta = 1$ (HM Treasury, Annex 6 2003).[25] Should EPA wish to follow this approach it will need to determine a method for establishing δ and η and the process generating g_t. We advise the Agency to consult the Science Advisory Board on the appropriate procedures for estimating δ and η and the process generating per capita consumption growth.

The DDR and Time Consistency

An issue that frequently arises in the context of the DDR is whether the use of a declining discount rate will lead to time inconsistent decisions. It is well known (Gollier et al. 2008) that a policy chosen by a decision maker who maximizes a time separable expected utility function will be time consistent if expected utility is discounted at a constant exponential rate.[26]

The problem of time consistency can, however, arise in an ENPV framework if the DDR schedule does not change over time. If an analyst were to evaluate future net benefits

[25] The initial value of the discount rate assumes $g = 2\%$, implying $\rho = 3.5\%$. The DDR path is a step function that approximates the rate of decline in the discount rate in Newell and Pizer's random walk model (OXERA 2002).

[26] Constant exponential discount is a sufficient condition for time consistency but not a necessary one. See Heal (2005) for other conditions that will yield time consistent decisions. It is a necessary condition for an optimal policy to be both time consistent and stationary.

using the discounting schedule in "Gamma Discounting" (Weitzman 2001) in 2012 and, if the schedule did not change over time, a program that passed the benefit cost test in 2012 would not necessarily pass it in 2022, depending on the time pattern of net benefits.

Of course, if new information becomes available that alters the DDR schedule, the analyst will want to re-evaluate the ENPV of the program. Because new information is available, a reversal of the outcome of the BCA would not constitute time inconsistency. Newell and Pizer (2003) argue that an analyst, when using historical data to estimate a DDR, will naturally update estimates of the DDR as more information becomes available. This obviates the problem of time inconsistency. In a regulatory setting, however, such updating may occur only infrequently.[27] The question of updating is an issue to which more thought should be given.

4. Consistency in the Evaluation of Intra-Generational and Inter-Generational Benefits

OMB currently requires projects involving intra-generational benefits and costs to be discounted at rates of 3% and 7%. The former is meant to approximate the consumption rate of discount and the latter the real, pretax return on private investment. For regulations with important intergenerational benefits or costs, OMB advises that the analyst "might consider a further sensitivity analysis using a lower but positive discount rate in addition to calculating net benefits using discount rates of 3 and 7 percent" (OMB 2003).

The panel notes that applying this guidance consistently requires that benefits and costs occurring in the same year be discounted to the present using the same discount rate. This was not done in a recent regulatory impact analysis of Corporate Average Fuel Economy standards: benefits associated with reduced GHG emissions were discounted to the present at a lower rate than the fuel savings associated with the proposed standards. This is clearly inappropriate. As OMB's guidance dictates, the same constant exponential rate must be applied to all benefits, whether intra- or inter-generational. An alternative approach would be to use a declining discount rate schedule, as discussed in the previous section.

[27] The UK discount rate schedule in Figure 1 has been in place since 2003 (HM Treasury 2003).

5. Concluding Remarks

In this paper we have considered three sets of questions. The first set asked whether the Ramsey formula could serve as a basis for discounting over long horizons and, if so, how the parameters of the formula should be determined. We all agree that the Ramsey formula provides a useful framework for thinking about intergenerational discounting. We do not, however, agree as to how the parameters of the Ramsey formula might be determined empirically. In many ways our discussion about how to parameterize the formula revisits a long-standing debate about the "descriptive" v. "prescriptive" approach to discounting—the former approach arguing that discount rates should reflect observed behavior in markets, and the latter that ethical considerations should be used to set the utility rate of discount and the elasticity of marginal utility of consumption. This debate was described many years ago by Arrow et al. (1996) and is more recently reflected in the literature critiquing the *Stern Review* (Nordhaus 2007; Weitzman 2007a).

The third set of questions asked how approaches to intra-generational and inter-generational discounting could be made consistent. This is an easy question: If benefits and costs are to be discounted at a constant exponential rate, the same rate must be used to discount costs, intra-generational and inter-generational benefits. An alternate approach would be to use a declining discount rate schedule, applied to both costs and benefits, as discussed below.

It is the second set of questions—about the impact of uncertainty on the discount rate and whether a declining discount rate schedule should be used in project evaluation—that are the most interesting. The answers to these questions have important implications for how regulations are evaluated in the United States. Currently, OMB requires that benefits and costs be discounted at a constant exponential rate, although a lower discount rate than the required 3% and 7% may be used as a sensitivity analysis when a project yields benefits to future generations. In contrast, France and the UK use declining discount rate schedules in which all costs and benefits occurring in the same year are discounted at a rate that declines over time. Who is correct?

Theory provides compelling arguments for a declining certainty-equivalent discount rate. In the Ramsey formula, uncertainty about the future rate of growth in per capita consumption can lead to a declining consumption rate of discount, assuming that shocks to consumption are positively correlated. This uncertainty in future consumption growth rates may be estimated econometrically based on historic observations, or it can be derived from subjective uncertainty about the mean rate of growth in mean consumption or its volatility.

The path from theory to a numerical schedule of the certainty-equivalent consumption rate of discount is, however, difficult. It requires estimates of δ, η and the process generating g_t. These are all difficult to estimate and to defend to regulators. This suggests that a second-best approach might be used. The ENPV approach is less theoretically elegant, and does not measure the consumption rate of discount as given by the Ramsey formula. It is, however, empirically tractable and corresponds to the approach currently recommended by OMB for discounting net benefits, when expressed in consumption units (OMB 2003).

The empirical ENPV literature has focused on models of the rate of return on long-term, high quality, government debt. And, in the US, the literature suggests that the certainty-equivalent rate is declining over time. Results from the empirical DDR literature are, however, sensitive to the model estimated, the data series used to estimate the model, and how the data are smoothed and corrected for inflation. Should the USEPA wish to follow this approach, it should consult its Science Advisory Board to approve criteria for model selection and for combining results from the literature.

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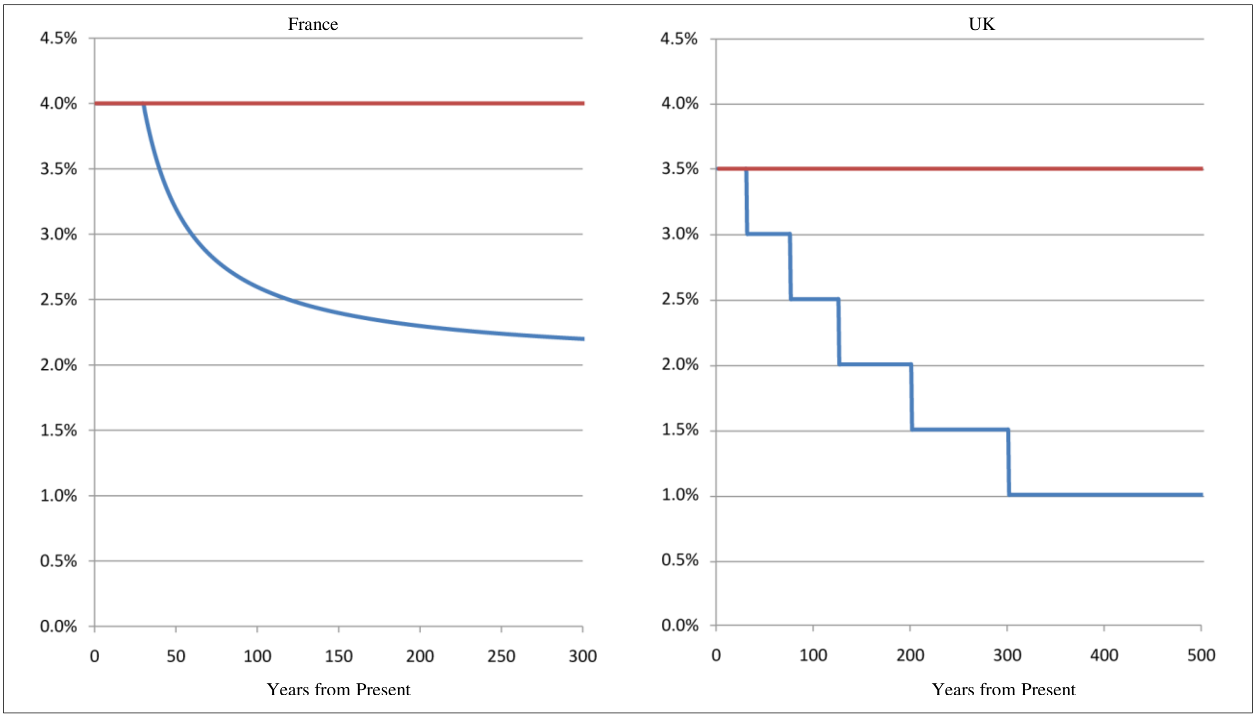
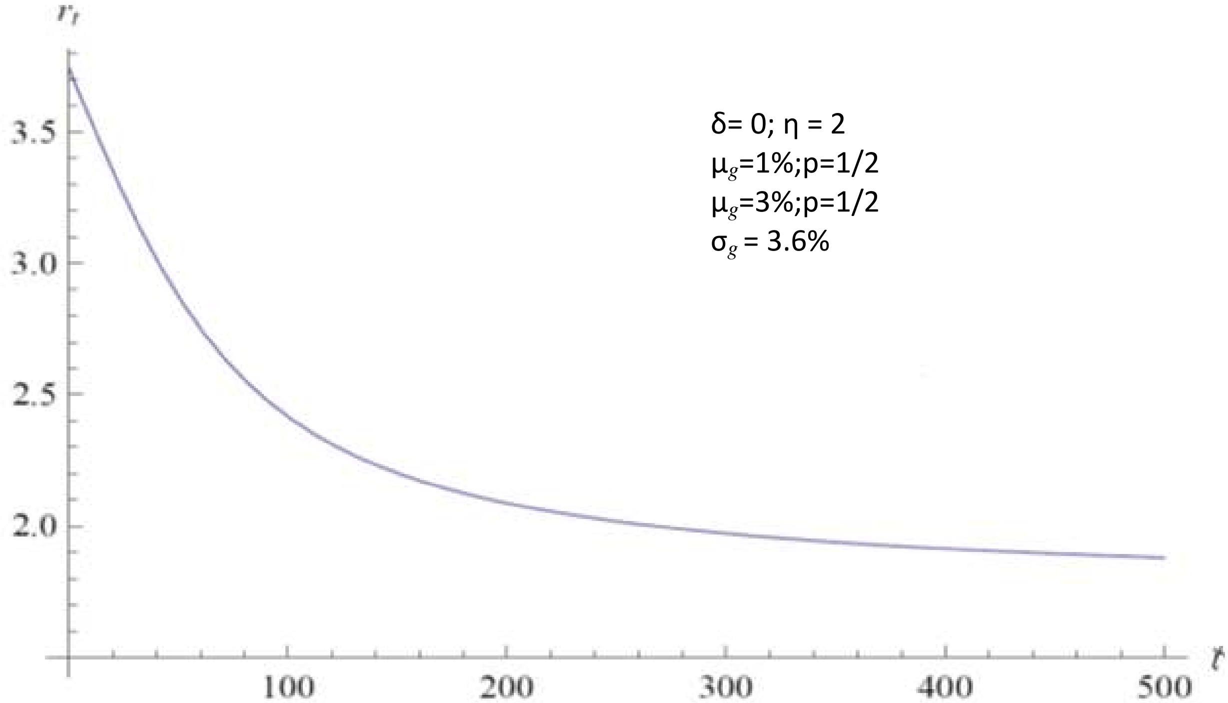
Figure 1: Declining Discount Rates in France and the UK



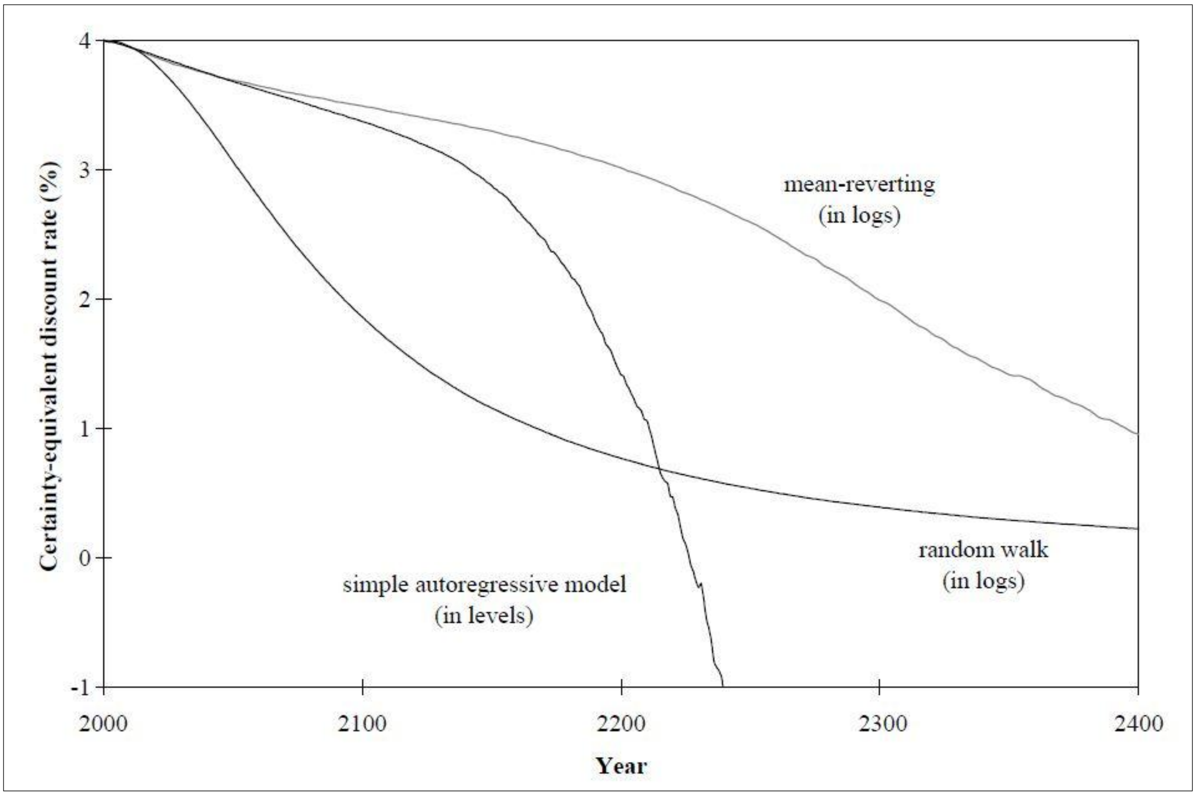
Source: Sterner, Damon, and Mohlin (2012)

Figure 2: Certainty-Equivalent Discount Rate Assuming Per Capita Consumption Follows a Random Walk With Uncertain Mean ($\mu = \mu(\theta)$)



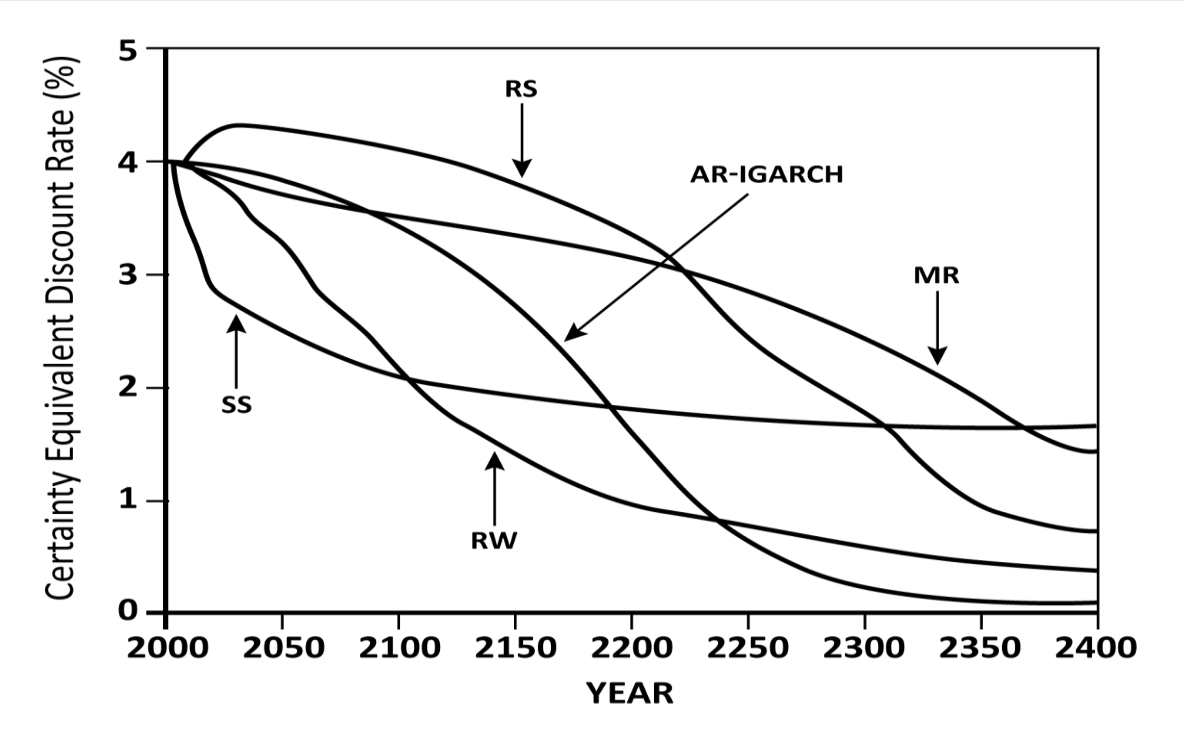
Source: Gollier (2008)

Figure 3: Forecasts of Certainty-Equivalent Discount Rates from Newell and Pizer (2003)



Source: Newell and Pizer (2003)

Figure 4: Forecasts of Certainty-Equivalent Discount Rates from Groom et al. (2007)



Note: RS = Regime Switching, MR = Mean-Reverting, SS=State Space, RW=Random Walk, AR-IGARCH= Autoregressive Integrated Generalized Autoregressive Conditional Heteroskedasticity
Source: Groom et al. (2007)

Table 1: Maximum Acceptable Sacrifice from Group A to Increase Income of Group B by $1

H	Group A Income = 2*Group B Income	Group A Income = 10*Group B Income
0	1.00	1.00
0.5	1.41	3.16
1	2.00	10.00
1.5	2.83	31.62
2	4.00	100.00
4	16.00	10000.00

Gollier et al. (2008)

Table 2: Present Value of a Cash Flow of $1000 Received After t Years

T	Scenario A: 4%	Scenario B: 1% or 7%	Certainty-Equivalent Discount Rate (R_t)
1	960.7894	961.2218	0.0394
10	670.3200	700.7114	0.0313
50	135.3353	318.3640	0.0128
100	18.3156	184.3957	0.0102
150	2.4788	111.5788	0.0101
200	0.3355	67.6681	0.0101
300	0.0061	24.8935	0.0101
400	0.0001	9.1578	0.0101

Source: Gollier et al. (2008)

Table 3: Discount Rate Schedule from Weitzman (2001)

Time period	Name	Marginal discount rate (Percent)
Within years 1 to 5 hence	*Immediate* Future	4
Within years 6 to 25 hence	*Near* Future	3
Within years 26 to 75 hence	*Medium* Future	2
Within years 76 to 300 hence	*Distant* Future	1
Within years more than 300 hence	*Far-Distant* Future	0

Source: Weitzman (2001)