How should private investors diversify? - An empirical evaluation of alternative asset allocation policies to construct a "world market portfolio"

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Abstract

This study evaluates the out-of-sample performance of numerous asset allocation strategies from the perspective of a Euro zone investor. Besides an increased sample period from January 1973 to December 2008, our contribution to the literature is twofold. First, we compare the performance of a broad spectrum of heuristic portfolio policies with a large set of well-established model extensions of the Markowitz (1952) mean-variance framework. Second, we explicitly differentiate between two prominent ways of diversification that are usually analyzed separately: international diversification in the stock market and diversification over different asset classes. Our analysis allows us to compare and discuss different diversification strategies to construct a "world market portfolio" that is as ex-ante efficient as possible. For international equity diversification, we find that none of the Markowitz-based portfolio models is able to significantly outperform simple heuristics. Among those, the GDP weighting dominates the traditional cap-weighted approach. In the asset allocation case, Markowitz models are again not able to beat a broad spectrum of fixed-weight alternatives out-of-sample. Analyzing more than 5000 heuristics, we find that in fact almost any form of well-balanced allocation over asset classes offers similar diversification gains as even very sophisticated and recently developed portfolio optimization approaches. Based on our findings, we suggest a simple and cost-efficient allocation approach for private investors.

Keywords: portfolio theory, asset allocation, investment management, international diversification, heuristics, fundamental indexing

JEL Classification Code: G11

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1 Introduction

Since the path-breaking work of Markowitz (1952), diversification has been considered the key to mean-variance portfolio optimization. The concept of diversification as "the only free lunch in investment" has not only become part of the accepted wisdom among practitioners, but also motivated extensive research. Nevertheless, regarding the solution to a major implementation problem, there is still little consensus: From the perspective of a private investor in real-life situations, which assets should be combined according to which allocation policy to maximize diversification benefits?[1] While the traditional academic approach has been to rely on scientific portfolio choice models, some practical guides such as Siegel (2008) and Svensen (2005) propose simplistic alternatives for private investors. So how to diversify optimally in reality? In the following, we analyze this key question from the perspective of a Euro zone individual investor. To do so, we run a horse race between a broad range of competing asset allocation policies. In particular, we compare the performance of eleven well-established model extensions of the Markowitz (1952) mean-variance framework with plausible heuristics derived from the academic as well as practical literature. Moreover, we explicitly combine two prominent ways of diversification that are usually analyzed separately: International diversification in the stock market and diversification over different asset classes. Combining multiple asset classes and numerous allocation policies, our analysis allows us to compare and discuss different diversification strategies to construct an investable "world market portfolio" which is as ex-ante efficient as possible.

So far, potential diversification benefits have primarily been analyzed for internationally diversified stock portfolios. The focus here has been on the special viewpoint of US investors (e.g. Bekaert and Urias (1996), De Roon et al. (2001), De Santis and Gerard (1997), Harvey (1995)). An international perspective for the period from 1985 to 2002 is taken by Driessen and Laeven (2007).

However, recent findings on the correlation structure of international stocks markets imply that even worldwide equity market diversification can offer only limited benefits for at

[1]The importance of this question is highlighted by Campbell and Viceira (2002), page 25: "One of the most interesting challenges of the 21st century will be the development of systems to help investors carry out the task of strategic asset allocation."

least two reasons. First, increasing return correlations within the stock universe over the last decades (Goetzmann et al. (2005)) lead to decreasing diversification gains (Driessen and Laeven (2007)). Second, correlations tend to be particularly high in periods of poor performance (Erb et al. (1994), Longin and Solnik (2001)). Thus, benefits from global diversification in the stock market tend to be smallest when they are most needed. By exclusively focussing on stocks, most studies on portfolio optimization thus neglect the additional diversification potential offered by other asset classes. As asset allocation has been shown to be the main determinant of portfolio performance (see e.g., Brinson et al. (1986) or Ibbotson and Kaplan (2000)), this limitation seems harmful.

Besides the question which assets to incorporate in portfolio optimization, the question which allocation policy to use is a second issue. In assessing the benefits of global diversification, and thus in deriving recommendations for allocation policies, the vast majority of studies exclusively rely on the Markowitz (1952) model. In this context, both in-sample[2] and out-of-sample analyses can be found in the literature. The in-sample procedure, however, neglects that Markowitz approaches, while being optimal in theory, suffer from estimation error in expected returns, variances and covariances when implemented in practice. There is a large literature explicitly dealing with how to improve the out-of-sample performance of Markowitz approaches - with partly disillusioning results. In a recent study focussing on stock portfolios, DeMiguel et al. (2009b) conclude that the estimation error is so severe that it offsets the gains from using models of optimal asset allocation as opposed to a naive equal-weighting of all portfolio components. In the light of potentially poor out-of-sample results and our focus on private investors, it seems insufficient to limit the analysis to the Markowitz-based methods of portfolio optimization.

Instead, we also incorporate simple asset allocation heuristics in the analysis. Their performance is particularly relevant for private investors. First, most of them will not have the knowledge and resources to implement sophisticated extensions of the Markowitz model. Second, a simple holistic asset allocation strategy might be considered as a possible rem-

[2]In-sample analyses assume that investors already know the realizations of the necessary input parameters at the point of portfolio optimization. Thus, a backward optimization is performed. Results are hardly feasible in reality. In contrast, out-of-sample analyses test the performance of optimization methods under realistic conditions. In the simplest case, the sample period is divided into two disjunct subperiods. Based on the data of the first period, the input parameters are estimated. The performance of the resulting portfolio is then (and exclusively) analyzed in the second period.

edy against widespread costly biases such as a lack of diversification (e.g., French and Poterba (1991) and Grinblatt and Keloharju (2001)) or excessive trading (e.g., Barber and Odean (2000) and Odean (1999)). Third, for Euro zone private investors, hardly any of the products available on the market satisfactorily meet our requirements of a transparent, cost-efficient and broadly diversified portfolio.[3]

To sum up, the question how to construct and implement a "world market portfolio" consisting of multiple asset classes is not sufficiently answered in the literature. To the best of our knowledge, there is no study evaluating competing asset allocation policies for such a scenario. We aim to take a step in this direction by analyzing the performance of well-established model extensions of the Markowitz (1952) approach as opposed to a broad range of plausible heuristics. We focus on Euro zone private investors within a yearly rebalanced buy-and-hold approach. To achieve comparability with the previous literature, a two-step procedure is employed: First, we concentrate on global diversification in the stock market. Specifically, we compare the performance of equally-, market value- and GDP-weighted portfolios with eleven extensions of the Markowitz (1952) model favored in the literature. Such an analysis might be considered an important complement of the results of DeMiguel et al. (2009b), in particular since we also provide an out-of-sample test of the norm-constrained portfolio models which have been proposed recently in DeMiguel et al. (2009a). Second, we extend our analysis to the multi-asset class case incorporating bonds and commodities. In the baseline scenario, we derive simple fixed-weight policies from the academic as well as practical literature and compare them to the optimization models. A number of sensitivity checks assures the robustness of our results. Specifically, we subsequently analyze the performance of more than 5000 alternative fixed-weight strategies covering every possible proportion of stocks, bonds and commodities in 1% steps.

As we focus on private investors, we pay particular attention to the practicability of our analysis. We concentrate on the economic profitability (in contrast to expected utility considerations), which we mainly measure by the monthly out-of-sample Sharpe ratio

[3]On the one hand, there are passive products like index funds or exchange-traded funds which are based on pure stock, bond or commodity indices. Even within the respective asset class, they are often not comprehensively diversified. On the other hand, there are actively managed multi-asset class funds. However, actively managed funds on average underperform passive benchmarks after costs (e.g., Carhart (1997) and Comer et al. (2009)).

after transaction costs. Moreover, our analysis is based on renowned indices, which are investable for private investors at low costs via exchange-traded funds.

The central results of our study can be summarized as follows: For international equity diversification, we find that none of the Markowitz-based portfolio models is able to significantly outperform simple heuristics. Among those, the popular cap-weighted approach, which is at the heart of most stock indices, is dominated by alternative stock weighting schemes. In the asset allocation case, almost any well-balanced fixed-weight proportion of stocks, bonds and commodities is able to realize substantial diversification gains. Again, Markowitz models are not able to beat these simplistic alternatives out-of-sample. Based on our findings, we suggest a simple and cost-efficient asset allocation approach for private investors.

The remainder of this paper is organized as follows. Section 2 describes our data and illustrates the risk-return characteristics of the asset classes. Section 3 discusses popular extensions of the Markowitz approach, which leads to the selection of promising optimization models for the construction of a "world market portfolio". Subsequently, we derive alternative heuristic asset allocation policies. Section 4 contains the empirical analysis of the competing strategies and provides a number of robustness checks. A summary of the results is given in section 5.

2 Data and Descriptive Statistics

2.1 Asset Classes and Data

We include stocks, bonds and commodities in our analysis. All asset classes are represented by indices whose selection is based on the criteria transparency, representativeness, investability, liquidity and data availability.[4]

[4]We require the index composition and index rules to be disclosed by the index provider (transparency). The index should already cover most of the market within an asset category to reduce complexity (representativeness). In doing so, the "world market portfolio" can be constructed with only few highly diversified indices. Moreover, low-cost exchange-traded funds tracking these indices should exist to enable private investors to actually implement our suggestions (investability and liquidity). Finally, we require a long return data history to conduct powerful statistical tests (data availability).

Based on these requirements, we rely on the renowned Morgan Stanley Capital International (MSCI) index family, which has been widely used in previous studies (e.g., Driessen and Laeven (2007), De Roon et al. (2001)), to cover the global stock universe. In the baseline analysis, stocks in the "world market portfolio" are represented by the four regional indices MSCI Europe, MSCI North America, MSCI Pacific as well as MSCI Emerging Markets. Taken together, they currently cover 45 countries and track the performance of several thousand stocks. The MSCI indices are designed to cover 85% of the free float-adjusted market capitalization of the respective investable equity universe.

Bonds are incorporated because of their low correlation with stocks (see section 2.2). In the baseline analysis, they are represented by the iBoxx Euro Overall index, which consists of Euro zone bonds of different maturities and credit ratings.[5] The index currently tracks the performance of more than 2,200 bonds. In robustness checks, we also make use of the iBoxx Euro Sovereign Index, which only consists of government bonds, the JPM Global Bond Index as well as the ML European Monetary Union Index.

Partly due to a lack of investability, commodities have long been neglected by private investors. However, many studies provide evidence of the high diversification potential of broad-based commodity futures indices.[6] Moreover, diversification benefits tend to be especially pronounced in times of unexpected inflation and declining stock markets. In the baseline analysis, commodities are represented by the S&P GSCI Commodity Total Return Index.[7] This world-production weighted index currently includes 24 commodity nearby futures contracts, tracking the performance of energy products, industrial and

[5] As we aim to derive suggestions for private investors, we do not consider currency hedging. For internationally diversified bond portfolios, Black and Litterman (1992) and Eun and Resnick (1994) find that currency risk needs to be controlled for. We thus restrict our analysis to Euro-denominated bonds. As the iBoxx index universe is only available from 1999 on, we replace the return of the iBoxx Euro Overall Index with the return of the REXP for the time period before 1999. Our approach is justified by a monthly return correlation of 0.965 between these two indices after 1999.

[6] Historically, these indices delivered equity-like returns and volatilities. At the same time, they provided low and partly even negative correlations with stocks and bonds (e.g., Erb and Harvey (2006) and section 2.2). Other commodity exposure such as physical trading, individual commodity futures or stocks of companies owning and producing commodities does not offer the specific risk, return, and correlation features of broad-based commodity futures indices (e.g., Erb and Harvey (2006) and Gorton and Rouwenhorst (2006)). Thus, they are less suitable for our analysis.

[7] Goldman Sachs calculates the index back till January 1969, which makes it the commodity future index with the longest available data history. In unreported results, we find a high correlation to alternative indices. For an overview of differences and commonalities of commodity futures indices we refer to Gordon (2006).

precious metals, agricultural products as well as livestock.

We do not incorporate real estate in our analysis as we want to derive suggestions for the asset allocation of individual investors. These are often already heavily exposed to real estate risk (e.g., Calvet et al. (2007), Campbell (2006)) so that the additional inclusion of real estate in the overall portfolio might lead to a lack of diversification. Moreover, we do not incorporate alternative asset classes such as hedge funds and private equity for two reasons. First, their diversification potential in the multi asset case is often found to be limited (e.g., Amin and Kat (2003), Ennis and Sebastian (2005), Patton (2009) and Phalippou and Gottschalg (2009)). Second, we could not identify indices satisfactorily meeting our selection criteria.

Our sample period covers the period from February 1973 through December 2008 and thus extends previous studies on international diversification in the stock market (e.g., Driessen and Laeven (2007), De Roon et al. (2001) or De Santis and Gerard (1997)). For all indices, we use Euro-denominated total return indices extracted from Thomson Reuters Datastream. Hence, our findings refer to an investment without currency hedging, which is a realistic assumption for private investors.[8]

To implement our heuristic portfolio strategies in the stock universe, we require the gross domestic product (in current U.S. dollars) as well as the stock market capitalization of the MSCI index regions. We obtain these data from the World Bank, the International Monetary Fund and Thomson Reuters Datastream, respectively. We use the three month FIBOR as published by the OECD as a proxy for the risk-free asset. Historical stock market capitalization data are available from 1973 on, which marks the lower bound of our sample period.

2.2 Descriptive Statistics

Table 1 gives an overview of the monthly return parameters of the asset classes. Shown are characteristics for the iBoxx Euro Overall index, the S&P GSCI Commodity Total

[8]To convert index levels in Euro we refer to the time series of synthetical Euro/USD exchange rates as calculated by Thomson Reuters Datastream. In robustness checks, we redo the analysis using the historical DEM/USD exchange rate as published by Deutsche Bundesbank. The qualitative nature of our results does not change.

Return index and a number of stock indices. The latter comprise the four regional MSCI indices and, for comparison purposes, the country-specific MSCI indices for the G7 states as well as a global capitalization-weighted stock index. The global index is constructed from the four regional stock indices. The MSCI Emerging Markets are only incorporated from 1988 on, as this is the starting point of the index calculation.[9]

Please insert table 1 here

Table 1 shows only small differences in the average monthly Sharpe ratio of the country-specific (0.079) as well as regional stock indices (0.084) as compared to the global stock index (0.087). Interestingly, over the last 20 years, the Sharpe ratio of the capitalization weighted global stock index was even lower than the average Sharpe ratios of the regional and country stock indices. These findings indicate that the standard approach of a cap-weighted stock index might not add much value. Moreover, the benefits of geographic diversification in the stock universe seem to decline. This motivates, first, the analysis of alternative allocation mechanisms for the stock market and, second, the incorporation of additional asset classes.

In this context, table 1 verifies that both bonds and commodities yield attractive Sharpe ratios. To asses their diversification potential in a "world market portfolio", we briefly analyze the correlation between the regional stock indices, bonds and commodities. To this end, the sample period is divided in the two subperiods before and after 1988. Table 2 shows the results.

Please insert table 2 here

In the second subperiod, pairwise correlations between the MSCI regional indices tend to increase substantially. The analysis of the commonality across asset classes, however, shows a different picture. Returns from bonds and commodities are low correlated both with each other and with returns from a global cap-weighted stock index. Moreover, we do not find a disadvantageous increase in correlations over time.

[9] Driessen and Laeven (2007) emphasize that investment restrictions were imposed on many emerging markets till the mid 80-ties and that reliable index calculations are only available since then. Thus, the return of our global stock index can be considered a proxy for the performance of worldwide investable equity.

Figure 1 and figure 2 illustrate the time-series behavior of correlations within the stock markets and across asset classes, respectively. Correlation coefficients are computed using a rolling window approach based on the previous 60 months.

Please insert figures 1 and 2 here

Figure 1 reveals an almost steadily increase in the comovement of international stock markets since the 1980ies. However, as figure 2 illustrates, there is no increase in correlations across asset classes. Nevertheless, correlations vary considerably through time, which points to potential estimation errors in Markowitz-based optimization methods (see section 3.1). We discuss promising optimization approaches in the next section.

3 Asset Allocation Models

The models considered for portfolio selection in the case of both global stock market diversification and diversification over asset classes are briefly summarized in table 3. The last column of this table gives the abbreviation that we use to refer to the model in the results section.

3.1 Markowitz-based Optimization Models

We use a variety of different Markowitz (1952)-based optimization models that have been suggested in the existing literature to deal with the well-known problem of estimation error which is ignored in the tradional mean-variance model of Markowitz (1952).[10] These models either impose additional constraints in the optimization process, shrink the estimated input parameters in order to mitigate the effect of estimation error, or they do both. Shortsale constraints prevent the optimization model from taking too extreme long and short positions to exploit even small differences in the return structure of assets. Shrinkage models correct the estimated parameters toward a common value. In doing so, they

[10] Consistent with previous empirical evidence, we find that the traditional mean-variance optimization without constraints leads to extreme long and short positions and exorbitant high turnover. Therefore, we refrain from reporting results for this model.

try to reduce the error-maximizing property of the mean-variance model when historical data is used for parameter estimation (e.g., Michaud (1989)). As shown by Jagannathan and Ma (2003) both approaches work similarly by increasing the number of assets with non-negative portfolio weights which enforces a certain extent of diversification.

The first model we implement is the mean-variance framework with non-negativity condition (*maxsr*). The objective of this model is to maximize the Sharpe ratio of the portfolio, which allows us to refrain from individual risk preferences in the optimization process. Mathematically, the following problem is solved:

$$\max_{\mathbf{x}^\top} \frac{\mathbf{x}^\top(\mu - r)}{\sqrt{\mathbf{x}^\top \Omega \mathbf{x}}}, \tag{1}$$

$$\text{s.t. } \mathbf{x}^\top 1 = 1,$$
$$\text{s.t. } x_i \geq 0,$$

where r ist the risk-free rate, $\mathbf{x}^\top$ denotes the transposed N-vector of portfolio weights, μ reflects the N-vector of expected returns, and Ω symbolizes the $N \otimes N$ variance-covariance matrix. μ and Ω are estimated from historical return data (see subsection 4.1).

In addition, we employ three extensions of this model that either shrink the sample means ($js - maxsr$), the sample variance-covariance matrix ($ccm - maxsr$), or both ($js - ccm$). The shrinkage estimation of expected returns is based on the work of James and Stein (1961). In our study, we use the following estimator which has been proposed by Michaud (1998):

$$E(\mu_i) = \overline{\mu} + \beta_i(\overline{\mu}_i - \overline{\mu}), \tag{2}$$

$$\beta_i = \max\{0, 1 - (k-3)\sigma_i^2/(\Sigma(\overline{\mu}_i - \overline{\mu})^2)\}, \tag{3}$$

where $\overline{\mu}_i$ and σ_i^2 are the sample mean and sample variance of asset i, $\overline{\mu}$ is the sample average across all assets and k reflects the number of different assets. Equation 3 shows that the shrinkage parameter β_i is lower for assets with a high sample variance which increases the shrinkage of the estimated mean according to equation 2. We shrink the elements of the variance-covariance matrix employing the constant correlation model developed in Ledoit and Wolf (2004).[11]

Besides models which try to maximize the Sharpe ratio, we employ several models which aim at constructing minimum variance portfolios. Among those are the traditional minimum variance approach with and without short-sale constraints ($minvar$, $minvar - nb$), the minimum variance approach with shrinkage estimation of the variance-covariance matrix using the constant correlation model and short-sale restriction ($ccm - minvar$), and a set of extensions to the general minimum variance framework ($nc1v, nc1r, nc2v, nc2r$) which have recently been developed by DeMiguel et al. (2009a). DeMiguel et al. (2009a) impose the additional constraint that the sum of the absolute values of the portfolio weights (known as 1-norm) or the sum of the squared values of the portfolio weights (known as 2-norm) must be smaller than a given parameter threshold δ. Effectively, this constraint allows portfolios to have some short positions, but restricts the total amount of short-selling. In order to calibrate the value of the threshold parameter δ, DeMiguel et al. (2009a) use two different methods. First, they choose the parameter δ which minimizes the portfolio variance if the sample is cross-validated. Second, they set δ to maximize the portfolio return in the last period in order to exploit positive autocorrelation in portfolio returns.[12] In their empirical analysis, DeMiguel et al. (2009a) are able to show that their models often outperform the traditional minimum variance approach as well as other existing portfolio-strategies at a significant margin.

The superior performance of minimum variance optimization, in particular compared to models that do not ignore information about sample mean returns, has been demonstrated in various studies (see, e.g., Haugen and Baker (1991), Chopra et al. (1993), and Jagannathan and Ma (2003)). Moreover, with the inclusion of the extensions developed by

[11] The authors provide the code on their web-site (http://www.ledoit.net/shrinkCorr.m). We assume a constant correlation equal to the historical correlation average for the stock market indices and a correlation of 0 between different asset classes. Our results are unchanged if we simply use the historical correlation average over all indices irrespective of the asset class underlying the index.

[12] For further information about the derivation of the portfolio models as well as the motivation of DeMiguel et al. (2009a), we refer the reader to their study. We do not evaluate other portfolio models considered in their paper, because the design of these models is very similar to the ones tested in our study and all models achieve very similar results in terms of out-of-sample portfolio variance, Sharpe ratio and turnover.

DeMiguel et al. (2009a), we provide additional evidence on the performance of new models which seem to be able to achieve a higher Sharpe ratio than a naive (1/N)-strategy, which is a severe hurdle rate for most other Markowitz (1952)-based applications according to DeMiguel et al. (2009b). Therefore, we believe to use a promising set of scientific portfolio choice models against which we test the heuristic construction rules, which are illustrated in the next subsection.

Insert table 3 here

3.2 Heuristic Models

3.2.1 International Stock-Market Diversification

We consider three different weighting schemes for a global stock portfolio: Equal-weighting (1/N heuristic), market-value weighting and GDP-weighting.

An equally-weighted portfolio might be considered a natural benchmark for more sophisticated methods of portfolio optimization. First, it is very easy to implement. Second, in particular private investors have been shown to apply this allocation rule (e.g., Benartzi and Thaler (2007)).

Another strategy is to base portfolio weights on the relative market capitalization of the constituents. This concept is at the heart of most major stock market indices. Liquidity and investment capacity arguments are important benefits of market-value weighted indices; however, these considerations are only of minor relevance for our objective. An undisputed advantage of this approach is its very low turnover as portfolio weights automatically rebalance when security prices fluctuate.

Nevertheless, concerns against this weighting scheme have recently been raised. Figure 3 gives the intuition behind these arguments. It shows the time series of portfolio weights of a market-value weighted stock index constructed from the MSCI indices for North America, Europe, the Pacific region and the Emerging Markets. Figure 3 illustrates that the resulting global stock index tends to be dominated by single regions. Between 1998 and 2007, for example, the weight of North America was on average about 45%. As the MSCI indices themselves are cap-weighted, US large caps substantially drove the performance of the global stock universe during that period. In contrast, the portfolio weights in the previous decade were heavily influenced by the bull and subsequent bear market of the Japanese stock market. The fraction of the Japan-dominated Pacific region was more than 52% in 1989 and heavily dropped to about 15% in 1998. These examples illustrate the pro-cyclical nature of market-value weighted indices.

Please insert figure 3 here

Motivated by many studies arguing that price fluctuations sometimes do not fully reflect changes in company fundamentals (e.g., Shiller (1981)), a growing literature questions the efficiency of cap-weighted indices (e.g., Treynor (2005), Siegel (2006)). Recently, alternative index concepts aimed at better approximating true firm values have been proposed. These indices rely on smoothed cap weights (Chen et al. (2007)) or are weighted by fundamental measures such as earnings, dividends or book values (Arnott et al. (2005)). The intuition here is that a weighting scheme based on fundamentals might be less volatile and less driven by sentiment. Consistent with this rationale, back-testing shows that fundamentally-weighted country-specific indices have outperformed cap-weighted indices in the past (e.g., Arnott et al. (2005), Hsu (2006)).

These findings justify the inclusion of a fundamentally constructed global stock market index in our analysis. To transfer the idea from the firm to the regional level, we weight the four MSCI indices based on the relative gross domestic product (GDP) of their covered countries. As can be seen from figure 4, this procedure indeed results in a less volatile, more balanced allocation.

Please insert figure 4 here

As the MSCI indices themselves are market-value weighted, this policy might be considered a compromise between a cap-weighted and a fundamentally weighted approach. We rely on this concept for several reasons. First, it ensures the implementability of our findings for private investors. Second, changing portfolio weights for hundreds of firms in each regional index is costly and laborious. Third, one might argue that a GDP-weighting scheme relies partly on value and size premiums (Jun and Malkiel (2008)). The literature controversially discusses the nature of these premiums. If they are a rational compensation for systematic risk (e.g., Fama and French (1992, 1993)), then there will be no outperformance of strategies tilting towards these factors. If they at least partly represent market inefficiencies (e.g., Lakonishok et al. (1994)), then their future persistence is doubtful. Both rationales thus caution not to blindly rely on purely fundamental strategies.

3.2.2 Diversification over asset classes

After adopting one of these stock-weighting schemes, one has to decide on the asset allocation policy. The easiest strategy for private investors would arguably be to assign time-invariant weights to stocks, bonds and commodities. The high number of potential fixed-weight asset allocation strategies requires the definition of a benchmark against which Markowitz-based models can be tested. As selecting any specific fixed-weight strategy is clearly a somewhat arbitrary choice, we employ a two step procedure. First, we screen the literature to derive a promising baseline allocation policy which we exemplarily use in the baseline empirical tests in subsection 4.2.2. Second, we analyze the performance of more than 5000 alternative portfolios with any possible fixed-weights (in 1% steps) in section 4.3 to assess the robustness of time-invariant allocation policies.

Regarding the ratio of stocks and bonds, we try to determine a best practice solution as a benchmark. Specifically, we study the security market advice of major investment bankers and brokerage firms as reported in e.g. Annaert et al. (2005) and Arshanapalli et al. (2001) as well as institutional holdings as reported in e.g. Blake et al. (1999), Brinson et al. (1986) and Ibbotson and Kaplan (2000)). Most of these studies analyze the allocation over cash, bonds and stocks and do not consider other asset classes. We focus on the time-series average of the cross-sectional mean of the asset allocations described in these studies, as Annaert et al. (2005) and Arshanapalli et al. (2001) document the efficiency of such a strategy. Based on the overall picture, we derive a consensus recommendation of roughly 60% stocks and 40% bonds.

Next, we analyze the literature that explicitly deals with commodities in an asset allocation context. Based on the studies of e.g. Erb and Harvey (2006) and Anson (1999), we estimate a consensus weight of roughly 15% for commodities.

Constructing an ex-ante baseline portfolio from these results leaves us with some degrees of freedom. Specifically, commodities could be incorporated at the expense of less stocks, less bonds or less stocks and bonds. Given this arbitrary choice, we use stocks, bonds and commodities in a fixed proportion of 60%, 25% and 15%. Note again that our objective is just to derive a plausible ex-ante strategy as a starting point for the empirical analysis, not an ex-post optimal portfolio.[13]

4 Empirical Analysis

4.1 Performance Evaluation Methodology

The performance of the portfolio strategies is assessed using their out-of-sample Sharpe ratio over the sample period from February 1973 till December 2008. Our implementation of the Markowitz-based models relies on a "rolling-window" approach, i.e. we distinguish between estimation and evaluation period. Specifically, at the begin of each February, we use return data of the previous 60 months to calculate the input parameters needed to determine the portfolio weights of each index. Using these weights, we then calculate the portfolio returns over the next 12 months without rebalancing. Next February, portfolio weights are readjusted using the updates of the parameter estimates. This process yields a time series of out-of-sample returns which is used to measure the Sharpe ratio of each strategy. The ratio is defined as the average monthly excess return over the risk free rate, divided by the standard deviation of monthly excess returns in the whole sample period. To assess the significance in differences of Sharpe ratios between the strategies, we follow the approach used in e.g. DeMiguel et al. (2009b).[14]

[13] In fact, we find that our baseline heuristic performs slightly worse than the other two alternatives. Hence, from an ex-post perspective, the benchmark against which we test scientific asset allocation models might be regarded conservative.

[14] Consider two portfolios i and j with input parameters $\widehat{\mu}_i$, $\widehat{\mu}_j$, $\widehat{\sigma}_i$, $\widehat{\sigma}_j$ and $\widehat{\sigma}_{i,j}$, which are estimated over a sample period of t. The hypothesis $H_0 : \widehat{\mu}_i/\widehat{\sigma}_i - \widehat{\mu}_j/\widehat{\sigma}_j = 0$ can then be tested using the test statistic $\widehat{Z}_{i,j}$, which is asymptotically distributed as a standard normal:

$$\widehat{Z}_{i,j} = \frac{\widehat{\sigma}_j\widehat{\mu}_i - \widehat{\sigma}_i\widehat{\mu}_i}{\sqrt{\widehat{\upsilon}}}, with : \widehat{\upsilon} = \frac{1}{t}[2\widehat{\sigma}_i^2\widehat{\sigma}_j^2 - 2\widehat{\sigma}_i\widehat{\sigma}_j\widehat{\sigma}_{i,j} + \frac{1}{2}\widehat{\mu}_i^2\widehat{\sigma}_j^2 + \frac{1}{2}\widehat{\mu}_j^2\widehat{\sigma}_i^2 - \frac{\widehat{\mu}_i\widehat{\mu}_j\widehat{\sigma}_{i,j}^2}{\widehat{\sigma}_i\widehat{\sigma}_i}$$

.

For the market-value weighting scheme, we calculate the portfolio weights at the rebalancing date using market values as of January, 1st. The one month lag has the aim of ensuring real-time data availability. The GDP-weighting is based on GDP-data from the previous year. We also compute the portfolio turnover of each strategy which results from the annual adjustment of the portfolio weights. This allows us to estimate the amount of transaction costs associated with each strategy and to calculate the out-of-sample Sharpe ratio after costs. In order to do so, we set a proportional bid-ask-spread equal to 40 basis points per transaction.[15] Then, the costs c_t due to portfolio rebalancing in month t can be estimated as follows:

$$c_t = s \cdot \sum_{i=1}^{N} |w_{i,t} - w_{i,t-}|, \tag{4}$$

where $w_{i,t}$ is the intended portfolio weight, $w_{i,t-}$ is the portfolio weight before rebalancing and the expression $\sum_{i=1}^{N} |w_{i,t} - w_{i,t-}|$ defines total portfolio turnover.

Since differences in the Sharpe ratio are hard to interpret from an economic point of view, we also rely on the return gap as additional performance measure. To compute the return gap, the portfolios are combined with the risk free asset until their volatility equals the volatility of the GDP-weighted stock portfolio or the 60-25-15 asset allocation portfolio, our baseline heuristics. This allows us to directly compare the risk-adjusted return of the portfolio to the return of our benchmark heuristics. More specifically, the return gap, $Return\,Gap_t$, in month t is obtained from the following equation:

$$Return\,Gap_t = r_{bm,t} - [\frac{\sigma_{bm}}{\sigma} r_t + (1 - \frac{\sigma_{bm}}{\sigma}) r_{f,t}], \tag{5}$$

where $r_{f,t}$ is the risk-free rate in t, $r_{bm,t}$ stands for the return of the benchmark (i.e. the GDP-weighted stock portfolio or the 60-25-15 asset allocation portfolio) and σ and σ_{bm} denote the monthly standard deviation of the portfolio and benchmark return over the sample period.

[15]The spread is assumed to be the same for each index. It is based on the average bid-ask spread in 2007 for selected exchange-traded funds tracking the indices used in our analysis. Other trading costs as well as a potential price impact are neglected. These costs should be marginal for broad-based indices, though.

4.2 Baseline Results

4.2.1 International Stock-Market Diversification

We start the empirical analysis with a comparison of the performance of the eleven Markowitz-based models and the various heuristic models for an internationally diversified stock portfolio. Results are reported in table 4.

Please insert table 4 here

Several results are noteworthy. The mean annual turnover of all Markowitz-based models is substantially larger than the turnover of all heuristics. While the economic impact is not very strong due to the specification of our baseline analysis, assuming higher transaction costs and more frequent rebalancing generally works in favor of the heuristic models. After-costs mean returns and standard deviations tend to be quite similar for most models. With the exception of the market-weighted stock portfolio (0.87%) and the maximum Sharpe ratio approach with short sale constraints (1.07%), all monthly mean returns are in the range of 0.90% to 0.99%. The minimum variance approach and its various extensions exhibit, as expected, the lowest fluctuation in returns. However, in economic terms, the reduction in risk as compared to the standard deviation of the three heuristics, seems small. Consequently, after-costs Sharpe ratios between February 1973 and December 2008 tend to be similar for most approaches. Using the GDP-weighted stock portfolio as benchmark, the only significant result is the underperformance of the popular market-weighted strategy. Analyzing all pairwise differences in Sharpe ratios in unreported results, we find that the market-weighted approach is also significantly inferior to the equal-weighted strategy as well to the minimum variance approach with short sale constraints. While the market capitalization based approach can thus be identified as a less efficient diversification strategy, there is no model which clearly arises as the most powerful approach. For example, as can be seen from table 4, there is no consistency in ranking across subperiods. Nevertheless, for most models, the performance is better than in the undiversified case as given in table 2.2.

The main results so far can thus be summarized as follows. First, most approaches are able to realize diversification gains. Second, none of the Markowitz-based optimization models dominates simple heuristics. Third, among those, the GDP-weighting and the equal-weighting significantly outperform the traditional cap-weighted approach. Fourth, in the overall picture, there is no dominating approach.

4.2.2 Diversification over asset classes

In the following, we include bonds and commodities in the baseline analysis. Again, we compare the performance of eleven scientific portfolio choice models with three heuristics. The latter only differ in their stock weighting scheme (value-weighted, equal-weighted, GDP-weighted). The proportion invested in bonds (25%) and commodities (15%) is the same across heuristics and motivated by the literature survey in section 3.2.2. In section 4.3, we extensively vary these portfolio weights to assess the sensitivity of our findings.

Please insert table 5 here

Table 5 shows the main results for the baseline analysis. Compared to the international diversification in the stock market, there is less homogeneity in mean returns and standard deviations across models. This also translates into larger differences in Sharpe ratios. Not all approaches are able to realize the diversification potential of additional asset classes. While the minimum variance approaches as well as the fixed-weight heuristics yield substantially higher Sharpe ratios than before, the other Markowitz-based strategies largely fail to achieve better risk-adjusted excess returns.[16] However, scientific portfolio choice models again are not able to outperform a passive benchmark. Within the heuristics, the stock weighting scheme still matters: The value-weighted approach underperform the GDP-weighted strategy at a significant margin. Nevertheless, in the overall picture, there is no approach which clearly arises as the most powerful one.

To illustrate the economic significance of our findings, we compute the return gap of various indices and compare them to both the GDP-weighted stock portfolio and the 60-25-15 asset allocation portfolio. Using the GDP-weighted strategy as a benchmark allows us to assess the benefit of heuristic diversification in the stock universe. Relying on

[16]In unreported results, we study the portfolio weights induced by Markowitz-based models. As expected, minimum variance approaches in general heavily invest in bonds. Moreover, they are characterized by a rather stable allocation, resulting in low turnover and thus low transaction costs.

the 60-25-15 strategy as a benchmark is intended to exemplarily quantify the additional benefits obtained from a naive fixed-weight allocation over different asset classes. Table 6 verifies that heuristic diversification, both in the stock market and in the asset allocation case, adds value. With the exception of the MSCI Emerging Markets, the GDP-weighted strategy outperforms every stock index as well as bonds and commodities in terms of risk-adjusted return. Including additional asset classes, as implemented in the 60-25-15 portfolio, strengthens these results. The outperformance ranges here from 7.8 to 24.2 basis points per month (or roughly 95 to 290 basis points per year) and thus is economically meaningful. Table 6 might be interpreted as exemplified evidence that relying on simple rules of thumb in diversifying substantially improves the risk-return profile of the overall portfolio. We more thoroughly address this issue in the next section.

Please insert table 6 here

4.3 Variations in the fixed weight asset allocation strategy

We derive the 60-25-15 asset allocation strategy from the existing literature and use it as a benchmark for the different Markowitz models. One potential concern to this approach may be that the good performance of our baseline heuristic results from backward optimization. To examine whether other possible heuristic strategies perform much worse than our baseline, we therefore calculate the Sharpe ratio after costs for a variety of different fixed-weight asset allocation schemes as well. In constructing the portfolios, we increase the portfolio weight of each asset class in steps of 1% from 0% to 100%, reduce the weight of the second class by the same amount and hold the weight of the third portfolio constituent constant. Imposing a non-negativity constraint for portfolio weights, this approach yields 5150 different portfolios. The stock component of the portfolios is still based on the GDP-weighting approach. Figure 5 displays our results. In order to interpret the figure, note that the portfolio weight of the commodity component indirectly follows from the weights of the two other asset classes. For instance, the portfolio with 0% in stocks and 0% in bonds is completely invested in the commodity index.

Please insert figure 5 here

Figure 5 shows a substantial increase in Sharpe ratios when moving away from portfolios with an extreme portfolio allocation (e.g., 100% of only one asset class). Moreover, the slope in the Sharpe ratio more and more becomes flat, if we move to the middle of the graphical presentation. This pattern suggests that a wide range of well-balanced allocation approaches over asset classes are able to offer substantial diversification gains. In fact, of the 5150 tested portfolios, approximately 42% perform better or equal than our baseline heuristic and 58% perform worse. Those that perform worse are very often heavily tilted towards only one asset class. If we subdivide the sample period into the subperiods from 1973-1988 and 1988-2008, the resulting figures look very similar. It follows that the 60-25-15 asset allocation policy is only one out of many different fixed-weight asset allocation schemes which achieve a good performance and which are not dominated by sophisticated academic portfolio models. These are good news for private investors: Although it is not possible to identify the best performing portfolio ex ante, almost any form of well-balanced allocation of asset classes already offers Sharpe ratios similar to the best performing strategy.

4.4 Robustness checks

To ensure the robustness of our results, we conduct numerous robustness checks. These tests differ with respect to the data set, the rebalancing frequency, the input parameter estimation method for the Markowitz models, the implementation of the GDP-weighting heuristic and the performance measure used. Moreover, we also investigate the impact of the recent financial crisis on our findings.

Variation in the data set

We extensively vary the data set to examine whether our findings are robust with respect to the indices used to represent the asset classes. First, we exclude the MSCI Emerging Markets index which is not available prior to 1988 from the calculations. Second, we rely on the country-specific MSCI indices for the G7 states instead of the regional MSCI indices. Third, we redo our analysis in the asset allocation context using only the MSCI world as the stock market component. Fourth, we also use alternative indices for bonds and commodities.[17] Overall, we find that the variation in the data set does not alter any conclusions drawn in this paper.

Rebalancing frequency

Monthly instead of annual rebalancing does not lead to significantly better results before costs for both the scientific portfolio models as well as the heuristics. After transactions costs, performance tends to deteriorate for most approaches. In general, the performance drop is more severe for the Markowitz models, which have a higher turnover. For the heuristics, the rather minor importance of the rebalancing frequency can also be inferred from figure 5, which shows that shifts in the portfolio weights are not harmful as long as the portfolio is not too much tilted towards only one asset. In this regard, the major benefit of portfolio rebalancing is to avoid extreme portfolios consisting of mainly only one asset.

Parametrization

In the baseline analysis, we use a time window of 60 months to estimate the input parameters for the Markowitz-based models. To examine whether the performance of these models improves when a longer time-series of historical returns is used for parametrization, we base the estimation method also on a rolling-window approach with 1) 120 months and with 2) all historical data available in a particular month. We do not observe a consistent improvement in the results of the Markowitz models in the additional tests. Moreover, the out-of-sample Sharpe ratios are still not significantly different from those of the heuristic models.

Implementation of the GDP-weighting heuristic

We change the methodology of the GDP-weighting scheme in two ways. First, we base portfolio weights on the relative GDP of the next year to proxy rational expectations. Second, we use GDP weights derived from purchasing power parity (PPP) valuations as provided by the World Bank and the International Monetary Fund. The performance of

[17] Specifically, we replace the iBoxx Euro Overall Index with the iBoxx Euro Sovereign Index, the JPM Global Bond Index and the ML European Monetary Union Index, respectively. Commodities are also represented by the Reuters/Jefferies Total Return Index and the DB Commodity Euro Index, respectively. In most cases this leads to a reduction in the sample size, since most index alternatives have a shorter return data history.

the GDP-weighting scheme is virtually unchanged in the first check and slightly improves in the second check.

Other performance measures

The recent literature has proposed a number of alternative performance ratios. We thus repeat our analysis utilizing asymmetrical performance measures which have been shown to be particularly suited for non-normal return distributions (e.g., Biglova et al. (2004), Farinelli et al. (2008), Farinelli et al. (2009)). Specifically, we employ the Sortino ratio, the Rachev ratio and the Generalized Rachev ratio.[18] The Sortino ratio is computed as the average excess return over the risk free rate divided by the downside volatility of the excess return. The Rachev ratio relies on the conditional value at risk of the excess return. Portfolios with the highest Rachev ratios are the ones which best manage to simultaneously deliver high returns and get insurance for high losses. The General Rachev ratio additionaly takes investors' degree of risk aversion into account. Utilizing these alternative measures does not change the qualitative nature of our results. A broad spectrum of heuristic portfolio allocation mechanisms still yields similar results as scientific portfolio choice models. Moreover, there is no consistency in ranking across performance ratios, which again indicates that there is no overall dominating approach.

The effect of the financial crisis

Inspection of figure 5 indicates that higher bond weights tend to be associated with a higher Sharpe ratio, as long as the portfolio exhibits some degree of diversification. To examine whether this result is driven by the recent financial crisis, we redo the complete analysis excluding the year 2008 which was accompanied by sharpe declines in stock and commodity prices. While the pattern of the resulting figure looks very similar, i.e. we also observe a flat optimum, portfolios with lower weights in bonds tend to have a higher ex post Sharpe ratio now. This demonstrates once again that a number of heuristic portfolio strategies make sense ex ante, but that it is virtually impossible to identify the best performing strategy. Moreover, the 60-25-15 asset allocation portfolio is still not dominated by any of the scientific portfolio choice models tested in this paper.

[18]For a detailed description of these ratios, we refer the reader to Biglova et al. (2004) and Rachev et al. (2007). To implement the ratios, we apply the parametrization described in Biglova et al. (2004) and Farinelli et al. (2008).

5 Conclusion

In this study, we analyze how to construct an efficient "world market portfolio" from the viewpoint of a Euro zone private investor. To this end, we compare eleven Markowitz-based optimization methods favored in the literature with a broad range of heuristic allocation strategies, both for international stock market diversification and in the asset allocation case.

Our main results can be summarized as follows. First, for global equity diversification, the GPD-weighting scheme significantly outperforms the popular value-weighed approach. Second, prominent Markowitz extensions do not add significant value above this under realistic conditions. Third, the inclusion of additional asset classes is, in general, highly beneficial. Diversification gains are here mainly driven by a well-balanced allocation over different asset classes. As long as the portfolio is not heavily tilted towards one asset class, almost any form of naive fixed-weight allocation strategy realizes diversification potential. Fourth, Markowitz-based optimization methods again do not add substantial value.

Our findings are good news for private investors: Relying on simple time-invariant asset allocation policies significantly improves upon the performance of any single asset class portfolio. Moreover, following these easily implementable rules of thumb does not lead to lower risk-adjusted returns as compared to even very sophisticated and recently proposed portfolio choice models.

Our study suggests several directions for further research. First, provided the availability of reliable data, the analysis could be extended to other asset classes. Eun et al. (2008) and Petrella (2005), for example, argue that investors can gain additional diversification benefits from small and mid caps. Second, alternatives to the estimation of input parameters from historical data could be analyzed. Third, our findings suggest that combining minimum variance concepts with heuristic allocation schemes might be a fruitful direction. Within a bottom-up approach, for example, minimum variance models could be implemented on an individual asset level (see e.g., Jagannathan and Ma (2003)), while plausible heuristics might be used on an index or asset class level.

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Table 1: Descriptive Statistics for the Different Indices

This table reports the return distribution of the various indices which we consider for portfolio construction. Returns are calculated using Datastream's total return index (code: RI) and denominated in Euro. Global Stock Index is a market-weighted stock index comprising the four different regional stock indices MSCI Europe, MSCI North America, MSCI Pacific, and MSCI Emerging Markets.

Asset Class/ Region	Sample Period	Mean Return	Std. Dev. Return	VaR 95%	Sharpe Ratio
Stocks: Country Indices					
Germany	73-08	1.03%	5.90%	-8.42%	0.097
France	73-08	1.06%	6.17%	-9.35%	0.098
Italy	73-08	0.89%	7.36%	-10.37%	0.059
United Kingdom	73-08	1.02%	6.51%	-9.02%	0.087
United States	73-08	0.91%	5.42%	-8.30%	0.083
Canada	73-08	0.92%	6.28%	-8.65%	0.074
Japan	73-08	0.81%	6.31%	-9.04%	0.056
Average	**73-08**	**0.95%**	**6.28%**	**-9.02%**	**0.079**
Average	**88-08**	**0.70%**	**5.86%**	**-9.31%**	**0.054**
Stocks: Regional Indices					
Emerging Markets	88-08	1.12%	7.41%	-12.61%	0.098
Europe	73-08	0.97%	4.79%	-7.75%	0.106
North America	73-08	0.92%	5.37%	-8.11%	0.086
Pacific	73-08	0.81%	5.93%	-8.61%	0.059
Average	**73-08**	**0.90%**	**5.36%**	**-8.16%**	**0.084**
Average	**88-08**	**0.72%**	**5.84%**	**-9.69%**	**0.056**
Global Stock Index	**73-08**	**0.87%**	**4.75%**	**-8.44%**	**0.087**
Global Stock Index	**88-08**	**0.59%**	**4.84%**	**-8.73%**	**0.040**
Other Asset Classes					
Bonds	73-08	0.57%	1.12%	-1.29%	0.099
Commodities	73-08	0.92%	6.30%	-9.91%	0.074

Table 2: Return Correlations between Different Indices

This table reports correlation coefficients between the stock -, bond - and commodity indices which we consider for portfolio construction for the total sample period (Panel A) and two sub-sample periods (Panel B and C). Returns are calculated using Datastream's total return index (code: RI) and denominated in Euro. Global Stock Index is a market-weighted stock index comprising the four different regional stock indices MSCI Europe, MSCI North America, MSCI Pacific, and MSCI Emerging Markets.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: 01.01.1973 - 31.12.2008							
(1) Europe	1.00						
(2) North America	0.72	1.00					
(3) Pacific	0.57	0.48	1.00				
(4) Emerging Markets	.	.	.	.			
(5) Global Stock Index	0.84	0.90	0.77	.	1.00		
(6) Bonds	0.10	0.00	0.02	.	0.02	1.00	
(7) Commodities	0.13	0.24	0.16	.	0.23	-0.10	1.00
Panel B: 01.01.1973 - 31.01.1988							
(1) Europe	1.00						
(2) North America	0.61	1.00					
(3) Pacific	0.50	0.41	1.00				
(4) Emerging Markets	.	.	.	.			
(5) Global Stock Index	0.76	0.91	0.71	.	1.00		
(6) Bonds	0.17	-0.04	0.01	.	0.02	1.00	
(7) Commodities	0.07	0.24	0.05	.	0.19	-0.12	1.00
Panel C: 01.02.1988 - 31.12.2008							
(1) Europe	1.00						
(2) North America	0.80	1.00					
(3) Pacific	0.61	0.54	1.00				
(4) Emerging Markets	0.70	0.71	0.60	1.00			
(5) Global Stock Index	0.90	0.89	0.81	0.80	1.00		
(6) Bonds	0.03	0.02	0.00	-0.06	0.01	1.00	
(7) Commodities	0.17	0.24	0.22	0.28	0.24	-0.09	1.00

Table 3: List of Portfolio Models

This table lists the various Markowitz-based optimization models from the existing literature (Panel A) and heuristic models (Panel B) which we consider for portfolio construction. δ is the threshold parameter developed in DeMiguel et al. (2009a) to limit the norm of the portfolio weight vector. The last column gives the abbreviation that we use to refer to the model.

No.	Portfolio Model	Abbreviation
	Panel A: Markowitz-based portfolio optimization models from the existing literature	
1	Maximum Sharpe ratio approach with shortsale constraints	maxsr
2	Minimum variance approach without shortsale constraints	minvar-nb
3	Minimum variance approach with shortsale constraints	minvar
4	James/Stein estimator of expected returns with shortsale constraints	js
5	James/Stein estimator of expected returns plus Ledoit/Wolf constant correlation model with shortsale constraints	js-ccm
6	Maximum Sharpe ratio approach plus Ledoit/Wolf constant correlation model with shortsale constraints	ccm-maxsr
7	Minimum variance approach plus Ledoit/Wolf constant correlation model with shortsale constraints	ccm-minvar
8	1-norm constrained minimum variance portfolio with δ calibrated using cross-validation over portfolio variance	nc1v
9	1-norm constrained minimum variance portfolio with δ calibrated by maximizing portfolio return in previous period	nc1r
10	2-norm constrained minimum variance portfolio with δ calibrated using cross-validation over portfolio variance	nc2v
11	2-norm constrained minimum variance portfolio with δ calibrated by maximizing portfolio return in previous period	nc2r
	Panel B: Heuristic portfolio models considered in this paper	
12	GDP-weighted stock portfolio	gdp
13	Market-weighted stock portfolio	macap
14	Equally-weighted stock portfolio	naiv
15	Asset Allocation Model with the following weights: 60% stocks, 25% bonds and 15% commodities; stock portfolio is GDP-weighted	60-25-15; gdp
16	Asset Allocation Model with the following weights: 60% stocks, 25% bonds and 15% commodities; stock portfolio is market-weighted	60-25-15; macap
17	Asset Allocation Model with the following weights: 60% stocks, 25% bonds and 15% commodities; stock portfolio is equally-weighted	60-25-15; naiv

Table 4: Markowitz vs. Heuristics: International Stock Market Diversification Results

This table reports means, standard deviations and Sharpe ratios of monthly out-of-sample returns after costs as well as average turnover for the international equity portfolios which are constructed using the various Markowitz-based optimization models and heuristic models. Sharpe ratios are reported for the total sample period (1973-2008) and two sub-sample periods (February 1973-January 1988 and February 1988-December 2008). We assume a bid-ask spread of 40 basis points to calculate after-cost returns. The last columns gives the p-values that the Sharpe ratio for each of these models is different from that for the GDP-weighted stock portfolio, our baseline heuristic. We compute the p-values of the difference using the approach employed by DeMiguel et al. (2009b) and a two-tailed level of significance. See section 3 and table 3 for a description of the models.

Portfolio Model	Mean Return	Std. Dev. Return	Mean Annual Turnover	Sharpe Ratio 1973-2008	Sharpe Ratio 1973-1988	Sharpe Ratio 1988-2008	p-value $H_0 : SR = SR_{gdp}$
Panel A: Markowitz-based optimization models							
maxsr	1.07%	5.79%	59.34%	0.106	0.121	0.099	0.97
minvar-nb	0.93%	4.54%	30.70%	0.104	0.166	0.061	0.90
minvar	0.97%	4.59%	22.68%	0.112	0.166	0.075	0.72
js	0.91%	5.02%	71.88%	0.089	0.121	0.067	0.47
js-ccm	0.90%	5.02%	66.95%	0.087	0.125	0.061	0.41
ccm-maxsr	0.98%	5.55%	60.10%	0.093	0.125	0.076	0.60
ccm-minvar	0.94%	4.58%	18.39%	0.106	0.158	0.070	0.94
nc1v	0.94%	4.54%	31.09%	0.106	0.166	0.065	0.97
nc1r	0.97%	4.57%	26.71%	0.113	0.166	0.076	0.74
nc2v	0.93%	4.56%	27.66%	0.103	0.166	0.059	0.80
nc2r	0.96%	4.57%	25.28%	0.109	0.166	0.070	0.88
Panel B: Heuristic models							
gdp	0.97%	4.78%	11.27%	0.107	0.155	0.076	.
macap	0.87%	4.75%	2.51%	0.087	0.157	0.040	0.06
naiv	0.99%	4.78%	12.92%	0.110	0.176	0.069	0.62

Table 5: Markowitz vs. Heuristics: Asset Allocation Results

This table reports means, standard deviations and Sharpe ratios of monthly out-of-sample returns after costs as well as average turnover for the asset allocation portfolios which are constructed using the various Markowitz-based optimization models and heuristic models. Sharpe ratios are reported for the total sample period (1973-2008) and two sub-sample periods (February 1973-January 1988 and February 1988-December 2008). We assume a bid-ask spread of 40 basis points to calculate after-cost returns. The last columns gives the p-values that the Sharpe ratio for each of these models is different from that for the 60-25-15 asset allocation portfolio with gdp-weighting in the stock market, our baseline heuristic. We compute the p-values of the difference using the approach employed by DeMiguel et al. (2009b) and a two-tailed level of significance. See section 3 and table 3 for a description of the models.

Portfolio Model	Mean Return	Std. Dev. Return	Mean Annual Turnover	Sharpe Ratio 1973-2008	Sharpe Ratio 1973-1988	Sharpe Ratio 1988-2008	p-value $H_0 : SR = SR_{gdp}$
Panel A: Markowitz-based optimization models							
maxsr	0.83%	3.77%	49.71%	0.099	0.179	0.029	0.49
minvar-nb	0.59%	1.14%	13.30%	0.121	0.167	0.081	0.88
minvar	0.62%	1.12%	7.35%	0.150	0.185	0.119	0.70
js	0.74%	2.68%	41.10%	0.107	0.179	0.030	0.63
js-ccm	0.72%	2.72%	40.48%	0.097	0.186	0.008	0.44
ccm-maxsr	0.69%	3.70%	51.01%	0.063	0.180	-0.034	0.12
ccm-minvar	0.63%	1.14%	6.17%	0.151	0.182	0.126	0.66
nc1v	0.60%	1.13%	13.50%	0.123	0.168	0.084	0.91
nc1r	0.61%	1.12%	10.22%	0.140	0.163	0.121	0.84
nc2v	0.61%	1.13%	9.64%	0.132	0.185	0.086	0.96
nc2r	0.62%	1.12%	7.35%	0.150	0.185	0.119	0.70
Panel B: Heuristic models							
60-25-15; gdp	0.88%	3.24%	12.72%	0.129	0.184	0.094	.
60-25-15; macap	0.82%	3.21%	9.93%	0.113	0.185	0.063	0.08
60-25-15; naiv	0.89%	3.24%	13.39%	0.133	0.205	0.088	0.53

Table 6: Return Gaps of Various Indices compared to the GDP-weighted stock portfolio and the 60-25-15 asset allocation portfolio

This table reports the Sharpe ratio and Value-at-Risk at the 95% confidence level of monthly returns for various indices as well as the GDP-weighted stock portfolio and the 60-25-15 asset allocation portfolio with gdp-weighting in the stock market, which are our baseline heuristic models for portfolio construction. Moreover, the table presents the Return Gap of these indices in basispoints (bp) per month compared to our baseline heuristics. Portfolio weights are readjusted every February each year. See section 3 and table 3 for a description of the models and subsection 4.1 for a description of the computation of the Return Gap.

Asset Class/ Region	Sample Period	Sharpe Ratio	VaR 95%	Return Gap (bp per month) GDP-stock portfolio	Return Gap (bp per month) 60-25-15 portfolio
Panel A: Stock indices					
MSCI Germany	73-08	0.097	-8.42%	4.2	10.3
MSCI France	73-08	0.098	-9.35%	3.6	9.8
MSCI Italy	73-08	0.059	-10.37%	24.7	24.2
MSCI United Kingdom	73-08	0.087	-9.02%	11.1	15.0
MSCI United States	73-08	0.083	-8.30%	12.6	16.0
MSCI Canada	73-08	0.074	-8.65%	16.3	18.5
MSCI Japan	73-08	0.056	-9.04%	24.7	24.2
MSCI Emerging Markets	88-08	0.098	-12.61%	-10.5	-0.8
MSCI Europe	73-08	0.106	-7.75%	0.5	7.8
MSCI North America	73-08	0.086	-8.11%	10.8	14.8
MSCI Pacific	73-08	0.059	-8.61%	22.6	22.8
Panel B: Asset classes					
GDP-stock portfolio	73-08	0.107	-7.91%	.	7.5
Bonds	73-08	0.099	-1.29%	3.9	10.1
Commodities	73-08	0.074	-9.91%	15.5	18.0

Figure 1: Time-Series Behavior of Correlations within the Stock Market

This figure depicts the movement in the average correlation over the sample period for the regional stock indices MSCI Europe, MSCI North America, MSCI Pacific and MSCI Emerging Markets with respect to all other stock indices. Correlation coefficients are computed using a rolling window approach based on the previous 60 months.

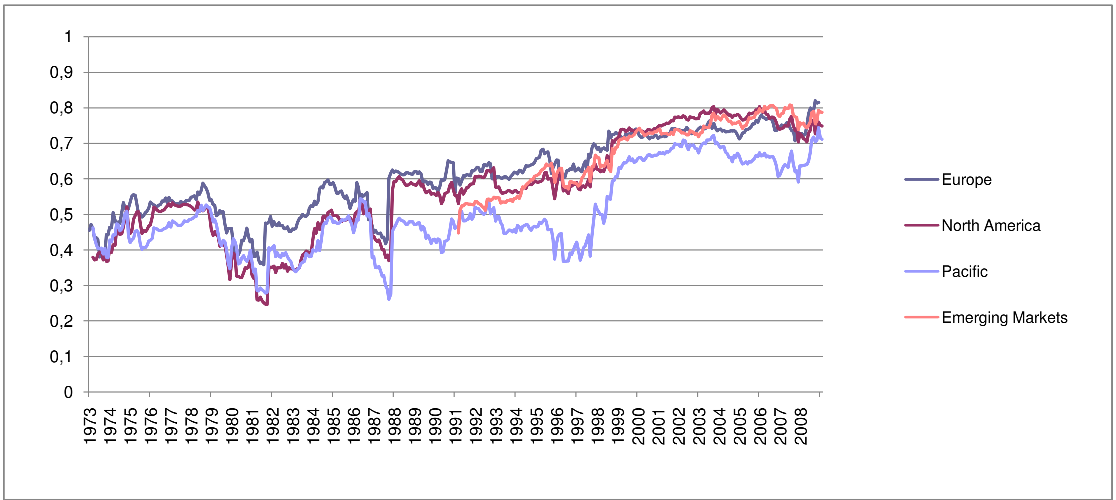

Figure 2: Time-Series Behavior of Correlations between Asset Classes

This figure depicts the movement in the average correlation over the sample period for the iBoxx Euro Overall Index and the S&P GSCI Commodity Total Return Index with respect to the regional MSCI stock indices. Correlation coefficients are computed using a rolling window approach based on the previous 60 months.

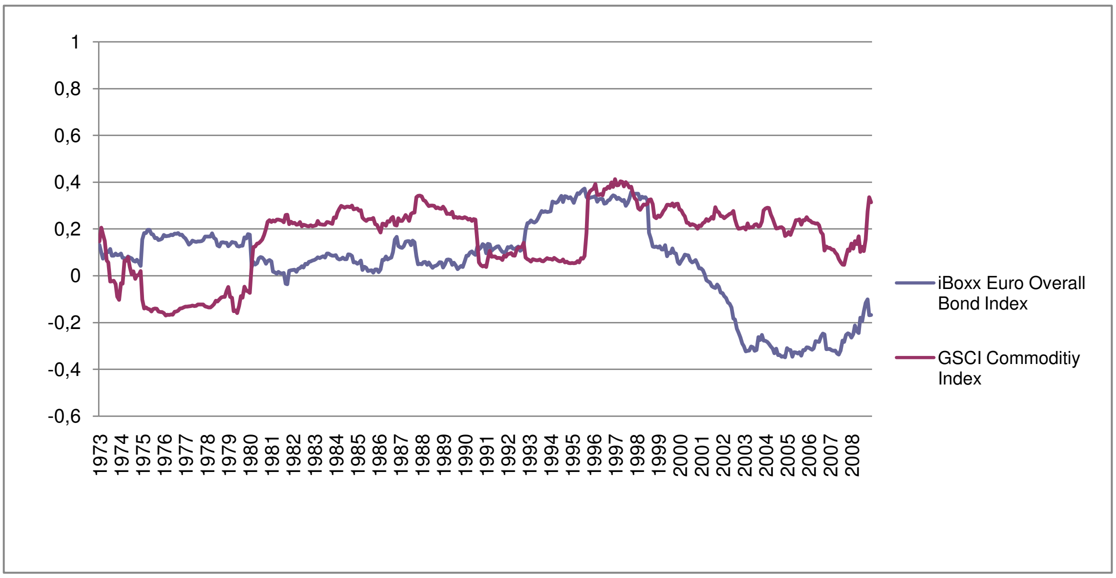

Figure 3: Time-Series Evolution of Portfolio Weights of a Cap-weighted Stock Index

This figure depicts the portfolio weights of a market-value weighted stock index constructed from the MSCI indices for North America, Europe, the Pacific region and the Emerging Markets over the sample period. The data source is Thomson Reuters Datastream.

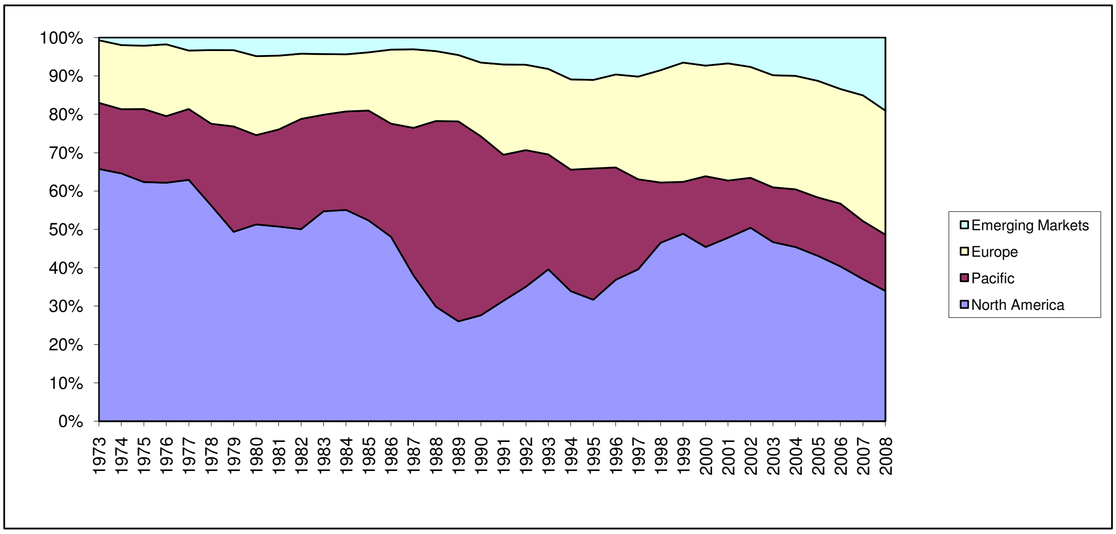

Figure 4: Time-Series Evolution of Portfolio Weights of a GDP-weighted Stock Index

This figure depicts the portfolio weights of a GDP-weighted stock index constructed from the MSCI indices for North America, Europe, the Pacific region and the Emerging Markets over the sample period. Data sources are the World Bank for the period 1973-2005 and the International Monetary Fund for the period 2006-2008.

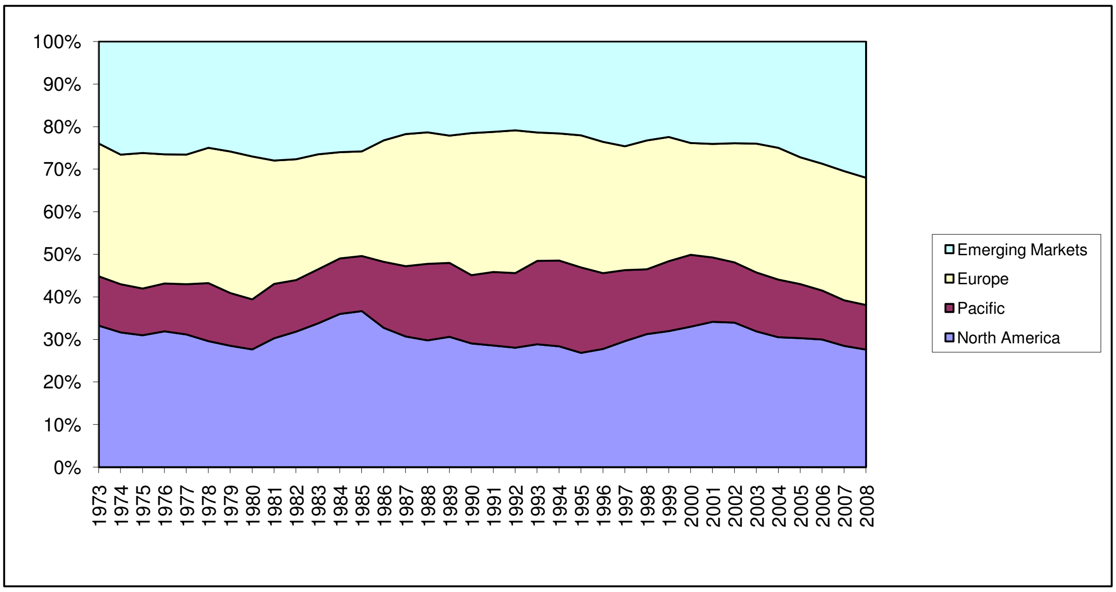

Figure 5: Graphical Presentation of the Performance of Alternative Fixed-Weight Asset Allocation Strategies

This figure depicts the Sharpe ratios of alternative heuristic portfolio strategies in the asset allocation context. In constructing the portfolios, we increase the portfolio weight of each asset class at the rebalancing date in steps of 1% from 0% to 100% and adjust the portfolio weights of the other 2 classes appropriately. This approach yields 5150 different portfolios. The stock component of the portfolios comprises the four regional MSCI indices and is GDP-weighted.