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How Students Learn Statistics

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Summary

Research in the areas of psychology, statistical education, and mathematics education is reviewed and the results applied to the teaching of college-level statistics courses. The argument is made that statistics educators need to determine what it is they really want students to learn, to modify their teaching according to suggestions from the research literature, and to use assessment to determine if their teaching is effective and if students are developing statistical understanding and competence.

Key words: Statistical education; Misconceptions; Teaching and learning

1 Introduction

Many statisticians are involved in teaching statistics either formally in a college classroom or informally in an industrial setting. Regardless of the setting, a major concern of those who teach statistics is how to ensure that the students understand statistical ideas and are able to apply what they learn to real-world situations. Although teachers of statistics often express frustration about difficulties students have learning and applying course material, many may be unaware of the growing body of research related to teaching and learning statistics. In this paper I attempt to summarize this literature and apply it specifically to improving learning outcomes in college-level statistics courses.

2 Theories of Learning

Before looking at research related to learning statistics, it is important to think about how students learn in general. Learning in a course is more complex than merely remembering what students have read or been told, and many of us have found that students do not necessarily learn by having us explain to them how to solve a problem. In fact, it is frustrating to work out a problem elegantly, explaining all the steps clearly, and then find out hardly any of the students understand it.

Many of us have informal learning theories that guide our teaching approaches. Some theories of learning are well defined and have recognizable names such as behaviorism, or cognitivism. In describing how students learn or think, theories of learning serve as a basis for theories of instruction that draw conclusions about how instruction should be carried out (Romberg & Carpenter, 1986). What happens in a particular course can be viewed as an interaction between the teacher's goals for what students should learn, views of students' characteristics and abilities, theory of how students learn, and assumptions about how students should be taught.

A recent theory of learning which has been widely accepted in education communities stems from earlier work by Jean Piaget, and has been labelled 'constructivism.' This theory describes learning as actively constructing one's own knowledge (Von Glasersfeld, 1987). Today, this is the guiding theory for much research and reform in mathematics and science education. Constructivists view students as bringing to the classroom their own ideas, material. Rather than 'receiving' material in class as it is given, students restructure the new information to fit into their own cognitive frameworks. In this manner, they actively and individually construct their own knowledge, rather than copying knowledge 'transmitted', 'delivered' or 'conveyed' to them. A related theory of teaching focuses on developing students' understanding, rather than on rote skill development, and views teaching as a way to provide opportunities for students to actively construct knowledge rather than having knowledge 'given' to them.

Theories of learning and instruction interact with teachers' particular goals for what students should learn in their courses. What are the skills and ideas teachers would really like their students to take away from their statistics courses? These goals do not necessarily correspond to what students are asked on quizzes or exams. If teachers were asked what they would really like students to know six months or one year after completing an introductory statistics course, most would probably not respond that students should know how to compute a standard deviation by hand, know how to convert normal variables to standard normal variables and look up their probabilities on the z table, or compute expected values. Many would indicate that they would like students to understand some basic statistical concepts and ideas, to become statistical thinkers, and to be able to evaluate quantitative information. A poignant way to think about this question is to ask 'what would you feel MOST bad about your former students not knowing about after completing a statistics course?'

3 Goals for Students

I believe that we really want students to gain an understanding of ideas such as the following:

(a) The idea of variability of data and summary statistics.
(b) Normal distributions are useful models though they are seldom perfect fits.
(c) The usefulness of sample characteristics (and inference made using these measures) depends critically on how sampling is conducted.
(d) A correlation between two variables does not imply cause and effect.
(e) Statistics can prove very little conclusively although they may suggest things, and therefore statistical conclusions should not be blindly accepted.

Statisticians are already discussing these general notions as central goals for student learning. A list of prioritized topics is given by Hogg (1990) based on a discussion at a workshop of statisticians regarding what the goals for an introductory statistics course should be. Moore (1991) has also specified, core elements of statistical thinking in terms of what students should be learning in statistics classes.

In addition to concepts, skills, and types of thinking, most statisticians would probably agree that we also have attitude goals for how we would like students to view statistics as a result of our courses. Such attitude goals are:

(a) It is important to learn some fundamentals of statistics in order to better understand and evaluate information in the world.
(b) Anyone can learn important ideas of statistics by working hard at it, using good study habits, and working together with others.
(c) Learning statistics means learning to communicate using the statistical language, solving statistical problems, drawing conclusions, and supporting conclusions by explaining the reasoning behind them.
(d) There are often different ways to solve a statistical problem.
(e) People may come to different conclusions based on the same data if they have different assumptions and use different methods of analysis.

Once we have articulated our goals for students in statistics classes, we need to address the issue of how we enable students to learn these ideas and to change their already established beliefs about statistics. Many college statistics classes consist of listening to lectures and doing assignments in textbooks or in computer labs. Do these activities help achieve the goals for our students? Are students being adequately prepared to use statistical thinking and reasoning, to collect and analyze data, to write up and communicate the results of solving real statistical problems?

Much research has been done that indicates that students are not learning what we want them to. Reviews by Garfield & Ahlgren (1988), by Scholz (1991), and by Shaughnessy (1992), summarize research related to learning and understanding probability and statistics. The studies reviewed tend to fall in two categories: psychological research and statistics education research. In addition, some studies in mathematics education offer additional insights into the teaching and learning of quantitative information. Relevant findings from these three areas of research are summarized briefly below.

4 Psychological Research

Most of the published research consists of studies of how adults understand or misunderstand particular statistical ideas. A seminal series of studies by Kahneman, Slovic & Tversky (1982) revealed some prevalent ways of thinking about statistics that are inconsistent with a correct technical understanding. Some salient examples of these faulty 'heuristics' are summarized below.

Representativeness

People estimate the likelihood of a sample based on how closely it resembles the population. (If you are randomly sampling sequences of 6 births in a hospital, where B represents a male birth and G a female birth; BGGBGG is believed to be a more likely outcome than BBBBBG.) Use of this heuristic also leads people to judge small samples to be as likely as large ones to represent the same population. (70% Heads is believed to be just as likely an outcome for 1000 tosses as for 10 tosses of a fair coin.)

Gamblers fallacy

Use of the representative heuristic leads to the view that chance is a self-correcting process. After observing a long run of heads, most people believe that now a tail is 'due' because the occurrence of a tail will result in a more representative sequence than the occurrence of another head.

Base-rate fallacy

People ignore the relative sizes of population subgroups when judging the likelihood of contingent events involving the subgroups. For example, when asked the probability of a hypothetical student taking history (or economics), when the overall proportion of students in these courses is 0.70 and 0.30 respectively, people ignore these base rates and instead rely on information provided about the student's personality to determine which course is more likely to be chosen by that student.

Availability

Strength of association is used as a basis for judging how likely an event will occur. (E.g., estimating the divorce rate in your community by recalling the divorces of people you know, or estimating the risk of a heart attack among middle-aged people by counting the number of middle-aged acquaintances who have had heart attacks.) As a result, people's probability estimates for an event are based on how easily examples of that event are recalled.

Conjunction fallacy

The conjunction of two correlated events is judged to be more likely than either of the events themselves. For example, a description is given of a 31-year old woman named Linda who is single, outspoken, and very bright. She is described as a former philosophy major who is deeply concerned with issues of discrimination and social justice. When asked which of two statements are more likely, fewer pick *A: Linda is a bank teller,* than *B: Linda is a bank teller active in the feminist movement,* even though A is more likely than B.

Additional research has identified misconceptions regarding correlation and causality (Kahneman, Slovic & Tversky; 1982), conditional probability (e.g., Falk, 1988; Pollatsek, Well, Konold & Hardiman; 1987), independence, (e.g., Konold, 1989b) randomness (e.g., Falk, 1981; Konold, 1991), the Law of Large Numbers (e.g., Well, Pollatsek & Boyce; 1990), and weighted averages (e.g., Mevarech, 1983; Pollatsek, Lima & Well, 1981).

A related theory of recent interest is the idea of the outcome orientation (Konold, 1989a). According to this theory, people use a model of probability that leads them to make yes or no decisions about single events rather than looking at the series of events. For example: A weather forecaster predicts the chance of rain to be 70% for 10 days. On 7 of those 10 days it actually rained. How good were his forecasts? Many students will say that the forecaster did not do such a good job, because if should have rained on all days on which he gave a 70% chance of rain. They appear to focus on outcomes of single events rather than being able to look at series of events—70% chance of rain means that it **should** rain. Similarly, a forecast of 30% rain would mean it will not rain. 50% chance of rain is interpreted as meaning that you cannot tell either way. The power of this notion is evident in the college student who, on the verge of giving it up, made this otherwise perplexing statement: 'I don't believe in probability; because even if there is a 20% chance of rain, it could still happen' (Falk & Konold, 1992, p.155).

The conclusion of this body of research by psychologists seems to be that inappropriate reasoning about statistical ideas is widespread and persistent, similar at all age levels (even among some experienced researchers), and quite difficult to change (Garfield & Ahlgren, 1988).

5 Statistical Education Research

A second area of research conducted primarily by statistics educators, is focused less on general patterns of thinking, and more on how statistics is learned. Some of these studies have contradicted implications of the psychological studies described earlier (e.g., Borovcnik, 1991; Konold et al., 1991; Garfield & delMas, 1991). For example, some of these studies indicate that students' use of heuristics (such as representativeness and availability) seems to vary with problem context.

Garfield & DelMas (1991) examined performance of students in an introductory course on a variety of parallel problems, designed to elicit use of the representative heuristic. Their results suggest that students do not rely exclusively on the representativeness heuristic to answer all problems of a similar type. Konold et al. (1991) hypothesized that inconsistencies in student responses are caused by a variety of perspectives with which students reason. Students appear to understand and reconstruct a problem in different ways, leading them to apply different strategies to solve them. Borovcnik & Bentz (1991) discuss other reasons for inconsistencies in student responses, such as the constraints imposed by artificial experiments and ambiguity of questions used.

Additional research on learning probability and statistics suggests ways to help students learn, as well as problems that need to be considered.

What helps students learn

- Activity-based courses and use of small groups appear to help students overcome some misconceptions of probability (Shaughnessy, 1977) and enhance student learning of statistics concepts (Jones, 1991).
- When students are tested and provided feedback on their misconceptions, followed by corrective activities (where students are encouraged to explain solutions, guess answers before computing them, and look back at their answers to determine if they make sense), this 'corrective-feedback' strategy appears to help students overcome their misconceptions (e.g., believing that means have the same properties as simple numbers) (Mevarech, 1983).
- Students' ideas about the likelihood of samples (related to the representativeness heuristic) are improved by having them make predictions before gathering data to solve probability problems, then comparing the experimental results to their original predictions (Shaughnessy, 1977; delMas & Bart, 1987; and Garfield & delMas, 1989).
- Use of computer simulations appears to lead students to give more correct answers to a variety of probability problems (Garfield & delMas, 1991; Simon, Aktinson & Shevokas, 1976; Weissglass & Cummings, 1991).
- Using software that allows students to visualize and interact with data appears to improve students' understanding of random phenomena (Weissglass & Cummings, 1991) and their learning of data analysis (Rubin, Rosebery & Bruce, 1988).

Problems to be considered

- Training involving application of the Law of Large Numbers may improve students' reasoning about samples of data (Nisbett et al., 1987). Other studies contradicted these results and showed that students' responses to a narrow type of probability problem improved, but their thinking did not (Shaughnessy, 1992).
- Students may answer items correctly on a test because they know what the expected answer is, but still have incorrect ideas. In a study involving students in various courses, students were able to say that different sequences of coin tosses were all equally likely when asked which was most likely to occur. However, when asked which was *least likely* to occur, they unperturbedly selected one or another particular sequence (Konold, 1989b).
- Students' misconceptions are resilient and difficult to change. Instructors cannot expect students to ignore their strong intuitions merely because they are given contradictory information in class (Konold, 1989b; Well et al., 1990; delMas & Garfield, 1991).

6 Mathematics Education Research

In addition to the research on learning and understanding statistical ideas, several studies on methods of improving students' general mathematical competence have relevance for teaching statistics. Many of these studies appear in reviews by Romberg & Carpenter (1986) and Silver (1990) and help reinforce and extend the research on statistical learning. The relevant findings are summarized below:

- More time spent on developing understanding (e.g., discussing why an algorithm works, how skills are interrelated, and how one concept is distinguished from other) leads to increased student performance on problem solving tests.
- Use of small groups leads to better group productivity, improved attitudes, and sometimes, increased achievement.

- Having students read through worked-out examples may be more effective than having them work through many of the conventional exercises assigned as homework.
- Students learn more from working on open-ended problems than from goal-specific problems where there is one right answer.
- 'Writing to learn' mathematics activities appear to be helping students understand mathematics better.
- Research on particularly innovative programs emphasizing problem solving and higher order thinking indicates that students do better on these activities than do students in traditional programs, without suffering any loss on traditional tests.

All of these results may be relevant to learning specifically statistical ideas.

7 Principles of Learning Statistics

Based on the relevant research in the context of constructivist principles, I have formulated some general principles of learning statistics:

Students learn by constructing knowledge

Many research studies both in education and in psychology support the theory that students learn by constructing their own knowledge, not by passive absorption of information (Resnick, 1987, von Glasersfeld, 1987). Regardless of how clearly a teacher or book tells them something, students will understand the material only after they have constructed their own meaning for what they are learning. Moreover, ignoring, dismissing, or merely 'disproving' the students' current ideas will leave them intact—and they will outlast the thin veneer of course content.

Students do not come to class as 'blank slates' or 'empty vessels' waiting to be filled, but instead approach learning activities with significant prior knowledge. In learning something new, they interpret the new information in terms of the knowledge they already have, constructing their own meanings by connecting the new information to what they already believe. Students tend to accept new ideas only when their old ideas do not work, or are shown to be inefficient for purposes they think are important.

Students learn by active involvement in learning activities

Research shows that students learn better if they are engaged in, and motivated to struggle with, their own learning. For this reason, if no other, students appear to learn better if they work cooperatively in small groups to solve problems and learn to argue convincingly for their approach among conflicting ideas and methods (National Research Council, 1989). Small-group activities may involve groups of three or four students working in class to solve a problem, discuss a procedure, or analyze a set of data. Groups may also be used to work on an in-depth project outside of class. Group activities provide opportunities for students to express their ideas both orally and in writing, helping them become more involved in their own learning. For suggestions on how to develop and use cooperative learning activities see Johnson, Johnson & Smith (1991) or Goodsell et al. (1992).

Students learn to do well only what they practice doing

Practice may mean hands-on activities, activities using cooperative small groups, or work on the computer. Students also learn better if they have experience applying ideas in new situations. If they practice only calculating answers to familiar, well-defined problems, then that is all they are likely to learn. Students cannot learn to think critically, analyze information, communicate ideas, make arguments, tackle novel situations, unless they are permitted and encouraged to do those things over and over in many contexts. Merely repeating and reviewing tasks is unlikely to lead to improved skills or deeper understanding (American Association for the Advancement of Science, 1989).

Teachers should not underestimate the difficulty students have in understanding basic concepts of probability and statistics

Many research studies have shown that ideas of probability and statistics are very difficult for students to learn and often conflict with many of their own beliefs and intuitions about data and chance (Shaughnessy, 1992; Garfield & Ahlgren, 1988).

Teachers often overestimate how well their students understand basic concepts

A few studies have shown that although students may be able to answer some test items correctly or perform calculations correctly, they may still misunderstand basic ideas and concepts. For example, Garfield & delMas (1991) found that when students were asked whether a sample of 10 tosses or 100 tosses of a fair coin was more likely to have exactly 70% heads, students tended to correctly choose the small sample, which seemed to indicate that they understood that small samples are more likely to deviate from the population than are large samples. When asked the same questions about whether a large, urban hospital or a small, rural hospital is more likely to have 70% boys born on a particular day, students responded that both hospitals were equally likely to have 70% boys born on that day, indicating that students could not transfer their understanding to a more real-world context.

Learning is enhanced by having students become aware of and confront their misconceptions

Students learn better when activities are structured to help students evaluate the difference between their own beliefs about chance events and actual empirical results (delMas & Bart, 1989; Shaughnessy, 1977). If students are first asked to make guesses or predictions about data and random events, they are more likely to care about the actual results. When experimental evidence explicitly contradicts their predictions, they should be helped to evaluate this difference. In fact, unless students are forced to record and then compare their predictions with actual results, they tend to see in their data confirming evidence for their misconceptions of probability. Research in physics instruction also points to this method of testing beliefs against empirical evidence (e.g., Clement, 1987).

Calculators and computers should be used to help students visualize and explore data, not just to follow algorithms to predetermined ends

Computer-based instruction appears to help students learn basic statistics concepts by providing different ways to represent the same data set (e.g., going from tables of data to histograms to boxplots) or by allowing students to manipulate different aspects of a particular representation in exploring a data set (e.g., changing the shape of a histogram to see what happens to the relative positions of the mean and median) (Rubin, Rosebery & Bruce, 1988). Instructional software may be used to help students understand abstract ideas. For example, students may develop an understanding of the Central Limit Theorem by constructing various populations and observing the distributions of statistics computed from samples drawn from these populations. The computer can also be used to improve students' understanding of probability by allowing them to explore and represent models, change assumptions and parameters for these models, and analyze data generated by applying these models (Biehler, 1991).

Students learn better if they receive consistent and helpful feedback on their performance

Learning is enhanced if students have opportunities to express ideas and get feedback on their ideas. Feedback should be analytical, and come at a time when students are interested in it. There must be time for students to reflect on the feedback they receive, make adjustments, and try again (AAAS, 1989). For example, evaluation of student projects may be used as a way to give feedback to students while they work on a problem during a course, not just as a final judgement when they are finished with the course (Garfield, 1993). Since statistical expertise typically involves more than mastering facts and calculations, assessment should capture students' ability to reason, communicate, and apply, their statistical knowledge. A variety of assessment methods should be used to capture the full range of students' learning (e.g., written and oral reports on projects, minute papers reflecting students' understanding of material from one class session, or essay questions included on exams). Teachers should become proficient in developing and choosing appropriate methods that are aligned with instruction, and should be skilled in communicating assessment results to students (Webb & Romberg, 1992). For a variety of classroom assessment techniques designed to help instructors better understand and improve their students' learning, see Angelo & Cross (1993).

Students learn to value what they know will be assessed.

Another reason to expand assessment beyond the exclusive use of traditional tests, is that students will only apply themselves to those skills and activities on which they know they will be evaluated. If students know they will be evaluated on their ability to critique and communicate statistical information, or to work collaboratively on a group project, they will be more willing to invest themselves in improving skills required by these activities.

Use of the suggested methods of teaching will not ensure that all students will learn the material.

No method is perfect and will work with all students. Several research studies in statistics as well as in other disciplines show that students' misconceptions are often strong and resilient—they are slow to change, even when students are confronted with evidence that their beliefs are incorrect. And this is only part of the problem. Another is whether students are engaged enough to struggle with learning new ideas.

8 Summary: Implications for Teaching

Statistics teaching can be more effective if teachers determine what it is they really want students to know and do as a result of their course—and then provide activities designed to develop the performance they desire. Appropriate assessment needs to be incorporated into the learning process so that teachers and students can determine whether the learning goals are being achieved—in time to do something about shortcomings before the course is over. Teachers need to consider the implications of research findings and determine how they relate to particular courses, students, and available resources. There is not just one blueprint for change.

Statistics educators should think about and continually assess their personal theories of learning and teaching in light of the evidence classroom experience provides. Teachers should experiment with different teaching approaches and activities and monitor the results, not only by using conventional tests but by carefully listening to students and evaluating information reflecting different aspects of their learning. In this way, teachers may continually analyze and refine their theories of how students learn statistics.

Finally, students should be encouraged to assess their own learning as well as their notions of how they learn, by giving them opportunities to reflect on the teaching/learning process.

9 Further Research

Despite the abundance of research studies cited above, most of them have only general implications. Much is still to be learned about particular problems. Important questions that still need to be asked include:

(a) How does the use of computers improve student learning of particular concepts and help overcome particular misconceptions? E.g., what kinds of computer labs work best in developing the idea of particular concepts, such as averages or sampling variability?
(b) What techniques are most effective in confronting and overcoming particular misconceptions?
(c) What specific small-group activities work best in helping students learn particular concepts and develop particular reasoning skills?
(d) What types of assessment procedures and materials best inform teachers about students' understanding?

Results of research studies based on these questions, along with the base of knowledge already summarized, will help us to rethink what in statistics is most important to learn, how it should be taught, and what evidence of success we should seek.

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Résumé

La recherche dans les domaines de la psychologie et de l'éducation en statistique et en mathématique est revue et les résultats sont appliqués à l'enseignement des cours de la statistique au niveau collégial. On énonce l'argument que les éducateurs en statistique doivent déterminer ce qu'ils veulent vraiment enseigner aux étudiants. Ainsi, ils seront en mesure de modifier leur méthode d'enseignement selon les suggestions provenant des documents de la recherche, et ils seront également en mesure d'utiliser les évaluations qui visent à déterminer si leur méthode d'enseignement est efficace et si les étudiants élaborent une compréhension et des compétences en matière de statistiques.

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