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# HOW THE WEALTH WAS WON: FACTORS SHARES AS MARKET FUNDAMENTALS

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**ABSTRACT**

Why do stocks rise and fall? From 1989 to 2017, the real per-capita value of corporate equity increased at a 7.5% annual rate. We estimate that 44% of this increase was attributable to a reallocation of rewards to shareholders in a decelerating economy, primarily at the expense of labor compensation. Economic growth accounted for just 25%, of the increase followed by a lower risk price (18%), and lower interest rates (14%). The period 1952 to 1988 experienced less than one third of the growth in market equity, but economic growth accounted for more than 100% of it.

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# 1 Introduction

Why do stocks rise and fall? Surprisingly little academic research has focused directly on this question.[1] While much of the literature has concentrated on explaining expected quarterly or annual returns, this paper takes a longer view and considers the economic forces that have driven the total value of the market over the post-war era. According to textbook economic theories, the stock market and the broader economy should share a common trend, implying that the same factors that boost economic growth are also the key to rising equity values over longer periods of time.[2] In this paper, we directly test this paradigm.

Some basic empirical facts serve to motivate the investigation. While the US equity market has done exceptionally well in the post-war period, this performance has been highly uneven over time, even at long horizons. For example, real market equity of the US corporate sector grew at an average rate of 7.5% per annum over the last 29 years of our sample (1989 to 2017), compared to an average of merely 1.6% over the previous 23 years (1966 to 1988).[3] At the same time, growth in the value of what was actually produced by the corporate sector has displayed a strikingly different temporal pattern. While real corporate net value added grew at a robust average rate of 3.9% per annum from 1966 to 1988 amid anemic stock returns, it averaged much lower growth of only 2.6% from 1989 to 2017 even as the stock market was booming. This multi-decade disconnect between growth in market equity and output presents a difficult challenge to theories in which economic growth is the key long-run determinant of market returns.

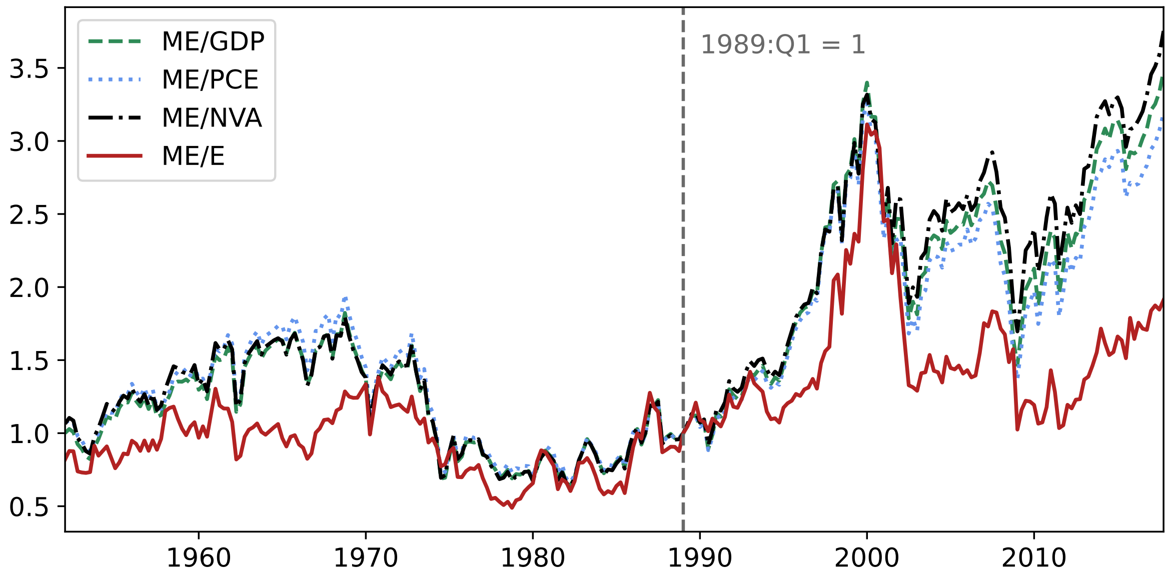
One potential resolution of this puzzle is to posit that economic fundamentals such as cash flows may be relatively unimportant for the value of market equity, with discount rates driving the bulk of growth even at long horizons. In this paper we entertain an alternative hypothesis motivated by an additional set of empirical facts. Within the total pool of net value added produced by the corporate sector, only a relatively small share — averaging 12.3% in our sample — accrues to the shareholder in the form of after-tax profits. Importantly, however, this share varies widely and persistently over time, fluctuating from less than 8% to nearly 20% over our sample. This suggests that swings in the profit share are strong enough to cause large and long-lasting deviations between cash flows and output. If

[1] We refer here to the question of what determines the level of equity values, as opposed to studying determinants of the price-dividend ratio or expected returns.

[2] This tenet goes back to at least Klein and Kosobud (1961), followed by a vast literature in macroeconomic theory that presumes balanced growth among economic aggregates over long periods of time. For a more recent variant, see Farhi and Gourio (2018).

[3] These subperiods are chosen visually to approximate structural breaks in the growth of market valuations and earnings shares, allowing for a clear demonstration of the paper's main mechanism. However, a strength of our ultimate estimation approach is that we are able to decompose the evolution of the value of the stock market over any subperiod, making these particular choices relatively unimportant.

**Figure 1: Stock Market Ratios**




*Notes:* To make the units comparable, each series has been normalized to unity in 1989:Q1. The sample spans the period 1952:Q1-2017:Q4. **ME:** Corporate Sector Stock Value. **E:** Corporate Sector After-Tax Profits. **GDP & C:** Current Dollars GDP and personal consumption expenditures. **NVA:** Gross Value Added of Corporate Sector - Consumption of Fixed Capital.

so, growth in market equity could diverge from economic growth for an extended period of time, even when valuations are largely driven by fundamental cash flows. Indeed, while the 1989-2017 period lagged the 1966-1988 period in economic growth, it exhibited growth in after-tax corporate *profits* of 5.1% per annum that far outpaced the average 1.8% earnings growth of the previous period. Behind these trends are movements in the after-tax profit share of output, which fell from 15.3% in 1966 to 8.9% in 1988, before rising again to 17.4% by the end of 2017. These shifts are in turn made possible by a reverse pattern in labor's share of corporate output, which rises from 67.0% in 1966:Q1 to 72.4% in 1988:Q4, before reverting to 67.7% by 2017:Q4.

The upshot of these trends is a widening chasm between the stock market and the broader economy. This phenomenon is displayed in Figure 1, which plots the ratio of market equity for the corporate sector to three different measures of aggregate economic activity: gross domestic product, personal consumption expenditures, and net value added of the corporate sector. Despite substantial volatility in these ratios, each is at or near a post-war high by the end of 2017. Notably, however, the ratio of market equity to after-tax profits (earnings) for the corporate sector is far below its post-war high.

What role, if any, might these trends have played in the evolution of the post-war stock market? To translate these empirical facts into a quantitative decomposition of the post-war growth in market equity, we construct and estimate a model of the US equity market. Although the specification of a model necessarily imposes some structure, our approach is intended to let the data speak as much as possible. We do this by estimating a flexible parametric model of how equities are priced that allows for influence from a number of mutually uncorrelated latent factors, including not only factors driving productivity and profit shares, but also independent factors driving risk premia and risk-free interest rates.

Equity in our model is priced, not by a representative household, but by a representative shareholder, akin in the data to a wealthy household or large institutional investor. The remaining agents supply labor, but play no role in asset pricing. Shareholder preferences are subject to shocks that alter their patience and appetite for risk, driving variation in both the equity risk premium and in risk-free interest rates. Our representative shareholder consumes cash flows from firms, the variation of which is driven by shocks to the total rewards generated by productive activity, but also by shocks to how those rewards are divided between shareholders and other claimants. Our model naturally generates operating leverage effects due to capital investment, implying that the cash flow share of output moves more than one-for-one with the earnings share (the *leverage effect*), and that cash flow growth is more volatile when the earnings share is low (the *leverage risk effect*).

We estimate the full dynamic model using state space methods, allowing us to precisely decompose the market's observed growth into these distinct component sources. The model is flexible enough to explain the entirety of the change in equity values over our sample and at each point in time. To capture the influence of our primitive shocks at different horizons, we model each as a mixture of multiple stochastic processes driven by low and high frequency variation. To discipline our estimation of these series, we confront the model with a wide range of data, including options-based measures of risk premia and long-term forecasts of interest rates. Because our log-linear model is computationally tractable, we are able to account for uncertainty in both latent states and parameters using millions of Markov Chain Monte Carlo draws. We apply and estimate our model using data on the US corporate sector over the period 1952:Q1-2017:Q4.

Our objective in this paper is to develop a decomposition capable of informing and disciplining subsequent structural modeling. To do so, we measure the variation in the value of market equity that would have occurred varying a single component at a time, while holding all other components fixed. By design, this approach measures the contribution of each component source without requiring one to take a stand on the structural model that generated the stochastic innovations driving our data. This is important if the objective is to obtain a set of empirical facts against which a range of structural models can be evaluated, as opposed to obtaining a set of empirical outcomes predicated on a particular model.

Our main results may be summarized as follows. First, we find that neither economic growth, risk premia, nor risk-free interest rates has been the foremost driving force behind the market's sharp gains over the last several decades. Instead, the single most important contributor has been a string of *factor share* shocks that reallocated the rewards of production without affecting the size of those rewards. Our estimates imply that the realizations of these shocks persistently reallocated rewards to shareholders, to such an extent that they account for 44% of the market increase since 1989. Decomposing the components of corporate earnings reveals that the vast majority of this increase in the profit share came at the expense of labor compensation.

Second, while equity values were also boosted since 1989 by persistent declines in the market price of risk, and in the real risk-free rate, these factors played smaller roles, contributing 18% and 14% respectively to the increase in the stock market over this period.

Third, growth in the real value of corporate sector output contributed just 25% to the increase in equity values since 1989 and 54% over the full sample. By contrast, while economic growth accounted for more than 100% of the rise in equity values from 1952 to 1988, this 37 year period created less than a third of the growth in equity wealth generated over the 29 years from 1989 to the end of 2017.

Fourth, the considerable gains to holding equity over the post-war period can be in large part attributed to an unpredictable sequence of shocks, largely factor share shocks that reallocated rewards to shareholders. We estimate that roughly 2.1pp of the post-war average annual log return on equity is attributable to this string of favorable shocks, rather than to genuine ex-ante compensation for bearing risk. These results imply that the common practice of averaging return data over the post-war sample to estimate an equity risk premium would overstate the true risk premium by 43%.

We extensively check robustness of our main results on the role of profit shares and redistribution in driving the market. First, we test our key model parameters — the persistences of our factor share processes — demonstrating that the autocorrelation structure of our estimated profit share closely matches the data without overstating its persistence. Second, we relax our microfounded and tightly parameterized link between the earnings share and risk premia, and estimate an alternative specification where this link has an arbitrary and freely estimated strength. The results are remarkably similar to our baseline model. Last, we use a modification of the typical Campbell-Shiller decomposition to obtain alternative estimates of the role of factor shares. This decomposition is model-free and only measures direct cash flow effects, ignoring the indirect contributions through risk premia implied by our structural model. The resulting contributions are large at all plausible levels of persistence, and imply that our model estimates if anything understate these direct cash flow contributions.

All told, our results point strongly to a leading role for factor shares in driving market valuations at long horizons.

**Related Literature.** The empirical asset pricing literature has traditionally focused on explaining stock market expected returns, typically measured over monthly, quarterly or annual horizons.[4] But as noted in Summers (1985), and still true today, surprisingly little attention has been given to understanding what drives the real level of the stock market over time. Previous studies have noted an apparent disconnect between economic growth and the rate of return on stocks over long periods of time, both domestically and internationally.[5] But these works have not provided a model and evidence on the economic foundations of this disconnect or on the alternative forces that have driven the market in post-war US data, a gap our study is intended to fill.[6]

In this regard, the two papers closest to this one are Lettau and Ludvigson (2013) and our previous work entitled "Origins of Stock Market Fluctuations," (Greenwald, Lettau and Ludvigson (2014), hereafter GLL), which this paper supplants. Lettau and Ludvigson (2013) was a purely empirical exercise that showed under a natural rotation scheme, shocks from a VAR that push labor income and asset prices in opposite directions explain much of the long-term trend in stock wealth. GLL expanded on this analysis by demonstrating that a calibrated model could reproduce many of these VAR results. At the same time, neither paper undertook a complete structural estimation of an equity pricing model, and thus they could not directly decompose movements in market valuations into fundamental structural forces. Compared to GLL, the model in this paper is both richer and more flexible in terms of its state variables and its cash flow process, is directly estimated on the time series rather than calibrated, and produces a period-by-period accounting of the drivers of market equity.[7]

Like GLL and Lettau, Ludvigson and Ma (2018), the model of this paper adopts a heterogeneous agent perspective characterized by "shareholders," who hold the economy's financial wealth and consume capital income, and "workers" who finance consumption out of wages and salaries. This choice is motivated by empirical observation: the top 5% of

[4]A body of research has addressed the question of whether expected returns or expected dividend growth drive valuation ratios, e.g., the price-dividend ratio, but this analysis is silent on the the primitive economic shocks that drive expected returns or dividend growth. For reviews of empirical asset pricing literature, see Campbell, Lo and MacKinlay (1997), Cochrane (2005), and Ludvigson (2012).

[5]See e.g., Estrada (2012); Ritter (2012); Siegel (2014)).

[6]One exception is Lansing (2021), a paper subsequent to the initial draft of our work, who also estimates a model to exactly match and decompose macroeconomic and financial time series data, and emphasizes the role of sentiment.

[7]The older GLL paper solves a fully nonlinear model in place of an approximate log-linear model, demonstrating that the results in this paper are robust to allowing for these nonlinearities.

the stock wealth distribution owns 76% of the stock market value (and earns a relatively small fraction of income from labor compensation), while around half of households have no direct or indirect ownership of stocks at all.[8] In this sense our model relates to a classic older literature emphasizing the importance for stock pricing of limited stock market participation and heterogeneity.[9] We add to this literature by demonstrating the relevance of frameworks in which investors are concerned about shocks that have opposite effects on labor and capital.

Beyond this work, a growing body of research considers the role of redistributive or factor-augmenting shocks in asset pricing or macro models.[10] In addition to our main distinguishing contribution that we pursue a quantitative decomposition of the drivers of equity values over time using an estimated structural model, we differ from this literature in our treatment of equity risk and pricing. In this literature, labor compensation is a charge to claimants on the firm and therefore a source of cash-flow variation in stock and bond markets, but these theories typically imply that a variant of the consumption CAPM using aggregate consumption still prices equity returns, implying that these frameworks cannot not account for the evidence in Lettau et al. (2018) that the capital (i.e., nonlabor) share of aggregate income is a strongly priced risk factor. By contrast, our framework allows these redistributive shocks to influence not only cash flows but also the quantity of risk faced by investors.[11]

Our work is also closely related to papers studying the sources of macroeconomic and financial transitions over time. Farhi and Gourio (2018) extend a representative agent neoclassical growth model to allow for time varying risk premia, and find a large role for rising market power in driving the high returns to equity over the last 30 years, supporting our

[8]Source: 2016 Survey of Consumer Finances (SCF). In the 2016 SCF, 52% of households report owning stock either directly or indirectly. Stockowners in the top 5% of the net worth distribution had a median wage-to-capital income ratio of 27%, where capital income is defined as the sum of income from dividends, capital gains, pensions, net rents, trusts, royalties, and/or sole proprietorship or farm. Even this low number likely overstates traditional worker income for this group, since the SCF and the IRS count income paid in the form of restricted stock and stock options as "wages and salaries." Executives who receive substantial sums of this form would be better categorized as "shareholders" in the model below, rather than as "workers" who own no (or very few) assets.

[9]See e.g., Mankiw (1986), Mankiw and Zeldes (1991), Constantinides and Duffie (1996), Vissing-Jorgensen (2002), Ait-Sahalia, Parker and Yogo (2004), Guvenen (2009), and Malloy, Moskowitz and Vissing-Jorgensen (2009).

[10]See e.g., Caballero, Farhi and Gourinchas (2017), Danthine and Donaldson (2002), Donangelo, Gourio, Kehrig and Palacios (2019), Eisfeldt and Papanikolaou (2013), Eisfeldt and Papanikolaou (2014), Eggertsson, Robbins and Wold (2021), Farhi and Gourio (2018), Favilukis and Lin (2016, 2013, 2015), Gomez (2016), Hall (2000), Marfe (2016).

[11]The factor share element of our paper is also related to a separate macroeconomic literature that examines the long-run variation in the labor share (e.g., Atkeson (2020), Barkai (2020), Karabarbounis and Neiman (2013), Koh, Santaeulàlia-Llopis and Zheng (2020), and the theoretical study of Lansing (2014)). The factor share findings in this paper also echo those from previous studies that use very different methodologies but find that returns to human capital are negatively correlated with those to stock market wealth (Lustig and Van Nieuwerburgh (2008); Lettau and Ludvigson (2009); Chen, Favilukis and Ludvigson (2014))). Our results have also been supported in subsequent work by Kuvshinov and Zimmermann (2021)

findings on the importance of the factor share shock. Corhay, Kung and Schmid (2018) find a similar result that they likewise attribute to market power using a rich model of the firm investment margin. An appealing feature of these approaches is that they specify a structural model of production that takes a firm stand on the sources of variation in the earnings share. In contrast, our modeling and estimation approach is designed to quantify the role the earnings share has played in stock market fluctuations, without requiring us to take a stand on the structural model that may have produced its underlying variation. As a result, we are able to explain the full transition dynamics of the data period-by-period, while Farhi and Gourio (2018) and Corhay et al. (2018) compare their richer production models only across different steady states. We view this work as complementary, but discuss the important implications of these differing methodological approaches further below.

Our work also relates to the literature estimating log-affine SDFs in reduced form.[12] These studies describe the evolution of the state variables and the SDF in purely statistical terms, for example using a freely estimated vector autoregression (VAR) for state dynamics. While less statistically flexible, our work features more economic structure, using separate and mutually uncorrelated fundamental components, as well as parametric restrictions on the SDF exposures obtained from theory, such as the leverage risk effect. This structure allows a much clearer interpretation of the drivers of asset prices. For example, unlike VAR-based models, which face the difficult task of transforming reduced-form residuals into identified structural shocks, our model allows us to directly read off the contribution of each latent state. We thus complement this literature by providing economic insight on the economic sources of market fluctuations, particularly the role of factor shares.

Our study further connects with a large body on work contrasting the role of expected dividend growth vs.discount rates in driving valuation ratios.[13] Our emphasis on determinants of cash flows (i.e., the earnings share) as a key driver of valuations differs from these papers, which emphasize the role of discount rates, largely because we ask a different question. While this literature finds that cash flows have little impact on the value of equity relative to *dividends*, we focus on the value of equity relative to *output*. Because shifts in the earnings share work precisely by driving a wedge between the earnings/payouts of the corporate sector and its output, we do not view any contradiction between our results.

Last, we link to the broad literature that endogenizes equity risk premia based on the consumption processes of shareholders, by providing a new mechanism through the leverage risk effect.[14] This mechanism shares a deep similarity to the habit specification of Campbell

[12]See e.g., Ang and Piazzesi (2003), Bekaert, Engstrom and Xing (2009), Dai and Singleton (2002), Duffie and Kan (1996), Lustig, Van Nieuwerburgh and Verdelhan (2013).

[13]See e.g., Campbell and Shiller (1989), Cochrane (2011).

[14]See e.g., Bansal and Yaron (2004), Barro (2009), Campbell and Cochrane (1999), Campbell, Pflueger

and Cochrane (1999), but is driven by variation in earnings relative to an external target (reinvestment), rather than by variation in aggregate consumption relative to an external target (habit). This approach thus offers novel quantitative and empirical implications for variation in risk premia over time that differ from consumption-based mechanisms.

**Overview.** The rest of this paper is organized as follows. Section 2 describes the theoretical model. Section 3 presents the data. Section 4 describes our estimation procedure. Section 5 presents our findings. Section 6 considers robustness and extensions. Section 7 concludes.

# 2 The Model

This section presents our structural model of the equity market. Throughout this exposition, lowercase letters denote variables in logs, while bolded symbols represent vectors or matrices.

**Demographics** The economy is populated by a representative firm that produces aggregate output, and two types of households. The first type are "shareholders" who typify owners of most equity wealth in the US (i.e., wealthy households or institutional investors). They may borrow and lend among themselves in the risk-free bond market. The second type are hand-to-mouth "workers" who finance consumption out of wages and salaries.[15]

**Productive Technology** Output is produced under a constant returns to scale process:

$$Y_t = A_t N_t^{\alpha} K_t^{1-\alpha}, \tag{1}$$

where $A_t$ is a mean zero factor neutral total factor productivity (TFP) shock, $N_t$ is the aggregate labor endowment (hours times a productivity factor) and $K_t$ is input of capital, respectively. Workers inelastically supply labor to produce output. We assume that both capital and labor productivity grow deterministically at a gross rate $G = \exp(g)$. Hours of

and Viceira (2014), Constantinides and Duffie (1996), Wachter (2013).

[15]This stylized assumption is motivated in the US data by the high concentration of top wealth shares, the evidence that the wealthiest earn the overwhelming majority of their income from ownership of assets or firms, and the finding that households outside of the top 5% of the stock wealth distribution own far less financial wealth of any kind. In the 2016 SCF, the median household in the top 5% of the stock wealth distribution had $2.97 million in nonstock financial wealth. By comparison, households with no equity holdings had median nonstock financial wealth of $1,800, while all households (including equity owners) in the bottom 95% of the stock wealth distribution had median nonstock financial wealth of $17,480. Additional evidence is presented in Lettau, Ludvigson and Ma (2019).

labor supplied are fixed and normalized to unity, so $N_t = G^t$. These assumptions imply

$$Y_t = A_t(G^t K_0)^\alpha (G^t)^{1-\alpha} = A_t G^t K_0^\alpha \tag{2}$$

where $K_0$ is the fixed initial value of the capital stock.

**Factor Shares** Once output is produced, it is divided among the various factors of production and other entities. We define earnings (after-tax profits) as $E_t = S_t Y_t$, where the *earnings share* $S_t$ represents the fraction of total output that accrues to shareholders in the form of earnings, arising from both foreign and domestic operations. The remaining fraction $1 - S_t$ of output accrues to workers in the form of labor compensation, to the government in the form of tax payments, and to debtholders in the form of interest payments. In our estimation, we assume an exogenous process for $S_t$ that does not directly distinguish between shifts in these components, but return to analyze their separate roles in Section 5.6. For now, we note that most variation in $S_t$ is driven by the labor share of domestic value added. While we do not directly microfound variation in $S_t$, we note that it would be isomorphic to stochastic variation in $\alpha$ in a model where workers are paid their marginal product.

**Investment and Payout Technology.** We assume that attaining balanced growth in capital requires the firm to invest a fixed fraction $\omega$ of its output beyond replacing depreciated capital. We view this as a parsimonious approximation to a richer model with time-varying investment, which allows us to solve the model in closed form without tracking the capital stock or solving the optimal investment problem.[16] While this abstracts from the volatility and cyclicality of the investment-output ratio at high frequencies, this type of transitory variation should have only modest effects on driving the value of forward-looking equity. Instead, we show in Section 5.3 that our parsimonious assumption closely reproduces the observed variation in cash flows at the low frequencies essential to our core results.

Shareholders receive the portion of earnings not reinvested as cash flows:

$$C_t = E_t - \omega Y_t = (S_t - \omega) Y_t. \tag{3}$$

The variable $C_t$ represents net payout, defined as net dividend payments minus net equity issuance. It encompasses any cash distribution to shareholders including share repurchases, which have become a dominant means of returning cash to shareholders in the US For brevity,

[16]Jermann (1998) demonstrates that generating realistic asset pricing moments in production economies requires very large investment adjustment costs. Our investment process can be seen as a limiting case in which any deviation from a constant investment-output ratio is infinitely costly.

we refer to these payments simply as "cash flows."

Importantly, (3) implies that the volatility of cash flow growth is amplified relative to earnings share growth — a form of operating leverage. For a numerical example, if $\omega = 6\%$, then an increase in the earnings share $S_t$ from 12% to 18% increases the cash flow share from 6% to 12%. As a result, proportional growth in the cash flow share (100%) is twice as large as proportional growth in the earnings share (50%), a phenomenon that we call the *leverage effect*. We note that this leverage effect should hold on average even if the reinvestment share is not exactly constant, so long as investment at long horizons is proportional to output rather than earnings.

**Preferences.** Let $C^s_{it}$ denote the consumption of an individual stockholder indexed by $i$ at time $t$. Identical shareholders maximize the function

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \prod_{k=0}^{t} \beta_k u\left(C^s_{it}\right), \qquad u\left(C^s_{it}\right) = \frac{\left(C^s_{it}\right)^{1-x_{t-1}}}{1-x_{t-1}}. \tag{4}$$

This specification effectively corresponds to time separable power utility preferences with a time-varying price of risk $x_t$, and a time-varying time discount factor $\beta_t$. Since shareholders perfectly insure idiosyncratic risk, shareholder consumption $C_{it}$ is identically equal to aggregate cash flows $C_t$.[17] At the same time, because firm cash flows are only a subset of total economy-wide consumption, redistributive shocks to $S_t$ that shift the share of income between labor and capital shift shareholder consumption are a source of systematic risk for asset owners. This implication has been explored by Lettau et al. (2019) who study risk pricing in a large number of cross-sections of return premia.

Aggregating over shareholders, equities are priced by the stochastic discount factor of a representative shareholder, taking the form

$$M_{t+1} = \beta_t \left(\frac{C_{t+1}}{C_t}\right)^{-x_t} \tag{5}$$

This specification is a generalization of the SDFs considered in previous work, (e.g., Campbell and Cochrane (1999) and Lettau and Wachter (2007)). As in these models, the preference shifters $(x_t, \beta_t)$ are taken as exogenous processes that are the same for each shareholder.

We specify the risk price $x_t$ as an independent stochastic process. Since an SDF always

[17]This need not imply that individual shareholders are hand-to-mouth households. They may trade an arbitrary set of assets with each other, including a complete set of state contingent contracts. Because they perfectly share any identical idiosyncratic risk with other shareholders they each consume per capita aggregate shareholder cash flows $C_t$ at equilibrium. See the Appendix for a stylized model.

reflects both preferences and beliefs, an increase in $x_t$ may be thought of as either an increase in effective risk aversion or an increase in pessimism about shareholder consumption. In practice, it can also be viewed as a residual that captures any variation in equity values not directly accounted for by our framework, such as changes in stock market participation. Thus, $x_t$ may occasionally go negative, reflecting the possibility that investors sometimes behave in a confident or risk tolerant manner.[18]

Shareholder preferences are also subject to exogenous shifts in the subjective discount factor $\beta_t$. As is well known, applying a realistic level of variation in the risk price while holding the time discount factor fixed would generate counterfactually high volatility in the risk-free rate. Instead, following Ang and Piazzesi (2003), we specify $\beta_t$ as

$$\beta_t = \frac{\exp(-\delta_t)}{\mathbb{E}_t \exp\left(-x_t \Delta c_{t+1}\right)}.$$

where $\Delta c_{t+1}$ represents log cash flow growth. This specification implies $\mathbb{E}_t M_{t+1} = \exp(-\delta_t)$, ensuring that the log risk-free rate is equal to the exogenous stochastic process $\delta_t$ at all times, regardless of the values for the other state variables of the economy.

## 2.1 Model Solution and Parameterization

**Exogenous Processes.** Our model has four sets of exogenous processes that drive $a_t$, $s_t$, $x_t$, and $\delta_t$, respectively. We specify TFP as a random walk in logs

$$\Delta a_{t+1} = \varepsilon_{a,t+1}, \quad \varepsilon_{a,t+1} \overset{\text{iid}}{\sim} N(0, \sigma_a^2). \tag{6}$$

which, together with (2), implies $\Delta y_{t+1} = g + \varepsilon_{a,t+1}$.

In principle, we could allow expected growth to contain a slow-moving stochastic process, as in Bansal and Yaron (2004) and the subsequent long run risk literature. In Appendix A.5, we show that the effect of this process on the price-payout ratio would be identical to its effect in Bansal and Yaron (2004) in the limiting case as the EIS goes to infinity. Howeveras is well known in this literature, the parameters of the long run risk process are difficult to pin down in a sample of this size, potentially creating large parameter uncertainty in our estimation. To avoid this, we use the simpler random walk process (6) for our estimation, but note that actual variation in a long run risk component would instead be picked up by our risk price process, potentially changing our interpretation of the role of the risk price.

[18]This does not imply a negative unconditional equity risk premium. Investors in the model occasionally behave in a risk tolerant manner while still being averse to risk on average. Indeed, our estimates reported below imply a substantial positive mean equity premium.

For the remaining latent states, we specify each as the sum of two components, each of which are in turn specified as an independent AR(1) process:

$$
\begin{aligned}
s_t &= \bar{s} + \mathbf{1}'\tilde{\mathbf{s}}_t, & \tilde{\mathbf{s}}_{t+1} &= \boldsymbol{\Phi}_s \tilde{\mathbf{s}}_t + \boldsymbol{\varepsilon}_{s,t+1}, & \boldsymbol{\varepsilon}_{s,t+1} &\overset{\text{iid}}{\sim} N(0, \boldsymbol{\Sigma}_s), \\
x_t &= \bar{x} + \mathbf{1}'\tilde{\mathbf{x}}_t, & \tilde{\mathbf{x}}_{t+1} &= \boldsymbol{\Phi}_x \tilde{\mathbf{x}}_t + \boldsymbol{\varepsilon}_{x,t+1}, & \boldsymbol{\varepsilon}_{x,t+1} &\overset{\text{iid}}{\sim} N(0, \boldsymbol{\Sigma}_x), \\
\delta_t &= \bar{\delta} + \mathbf{1}'\tilde{\boldsymbol{\delta}}_t, & \tilde{\boldsymbol{\delta}}_{t+1} &= \boldsymbol{\Phi}_\delta \tilde{\boldsymbol{\delta}}_t + \boldsymbol{\varepsilon}_{\delta,t+1}, & \boldsymbol{\varepsilon}_{\delta,t+1} &\overset{\text{iid}}{\sim} N(0, \boldsymbol{\Sigma}_\delta)
\end{aligned}
$$

where $\tilde{\mathbf{s}}_t, \tilde{\mathbf{x}}_t,$ and $\tilde{\boldsymbol{\delta}}_t$ are $2 \times 1$ vectors, $\boldsymbol{\Phi}_s, \boldsymbol{\Phi}_x,$ and $\boldsymbol{\Phi}_\delta$ are $2 \times 2$ diagonal matrices, and tildes indicate that the latent state vectors are demeaned. While these AR(1) processes are unbounded, in principle allowing for implausible or unusual values, we verify in Section 5.1 that impossible earnings shares in excess of unity effectively never occur, while negative levels of the risk price are uncommon and are not estimated to have occurred over our sample.

We choose a two-component mixture for each process to allow the model to flexibly capture both high and low frequency variation in the latent states. Since equity gives its owners access to profits for the lifetime of the firm, it is a heavily forward-looking asset that is much more influenced by persistent rather than transitory fluctuations. Our mixture specification allows the model to accurately capture both low frequency movements that have greater impact on equity prices, as well as higher frequency movements that have a smaller impact on equity prices but may nonetheless drive much of the variation in the observable series. Correspondingly, we refer to the components of each latent state vector as the high or low frequency component, so that e.g., $\text{diag}(\boldsymbol{\Phi}_{\mathbf{s}}) = (\phi_{s,LF}, \phi_{s,HF})$ with $\phi_{s,LF} > \phi_{s,HF}$.

Stacking this system yields a transition equation for the economy's state vector $\mathbf{z}_t$:

$$
\mathbf{z}_{t+1} = \boldsymbol{\Phi}\mathbf{z}_t + \boldsymbol{\varepsilon}_{t+1}, \qquad \boldsymbol{\varepsilon} \overset{\text{iid}}{\sim} N(0, \boldsymbol{\Sigma}) \tag{7}
$$

where[19]

$$
\mathbf{z}_t = \begin{bmatrix} \tilde{\mathbf{s}}_t \\ \tilde{\mathbf{x}}_t \\ \tilde{\boldsymbol{\delta}}_t \\ \Delta a_t \end{bmatrix}, \quad \boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_s & 0 & 0 & 0 \\ 0 & \boldsymbol{\Phi}_x & 0 & 0 \\ 0 & 0 & \boldsymbol{\Phi}_\delta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{\varepsilon}_t = \begin{bmatrix} \boldsymbol{\varepsilon}_{s,t} \\ \boldsymbol{\varepsilon}_{x,t} \\ \boldsymbol{\varepsilon}_{\delta,t} \\ \varepsilon_{a,t} \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_s & 0 & 0 & 0 \\ 0 & \boldsymbol{\Sigma}_x & 0 & 0 \\ 0 & 0 & \boldsymbol{\Sigma}_\delta & 0 \\ 0 & 0 & 0 & \sigma_a^2 \end{bmatrix}. \tag{8}
$$

**Log-Linearization.** We seek a specification that allows an analytical, log-linear solution for the price-dividend ratio. This solution requires three approximations: (i) a log-linear

[19]The i.i.d. shock $\varepsilon_{a,t}$ is included the state vector $\mathbf{z}_t$ even though it is exactly pinned down by the observable series $\Delta y_t$ so that we can estimate its mean and variance, since these parameters influence our asset pricing equations.

approximation of the equity return, (ii) a log-linear approximation of cash flow growth, and (iii) a second-order perturbation of the log SDF that allows for linear terms in the states and shocks, as well as interactions between the states and shocks. We summarize these approximations below, with full detail relegated to the appendix.

First, we approximate the return on equity

$$R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t}.$$

where $P_t$ denotes total market equity, i.e., price per share times shares outstanding. Following Campbell and Shiller (1989), we approximate the log return as

$$r_{t+1} = \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}, \tag{9}$$

where $pc_t = \ln(P_t/C_t)$, $\kappa_1 = \exp(\overline{pc}) / (1 + \exp(\overline{pc}))$, $\kappa_0 = \ln(\exp(\overline{pc}) + 1) - \kappa_1 \overline{pc}$, and $\overline{pc}$ is the average value of $pc_t$.

Second, we log-linearize the log cash flow to output ratio $c_t - y_t = \log(S_t - \omega)$ to obtain

$$c_t - y_t \simeq \overline{cy} + \xi(s_t - \bar{s}), \qquad \xi \equiv \frac{\bar{S}}{\bar{S} - \omega}$$

where $\overline{cy} = \log(\bar{S} - \omega)$, and $\bar{S}$ is the average value of $S_t$. Differencing this relation and rearranging yields

$$\Delta c_t = \xi \Delta s_t + \Delta y_t. \tag{10}$$

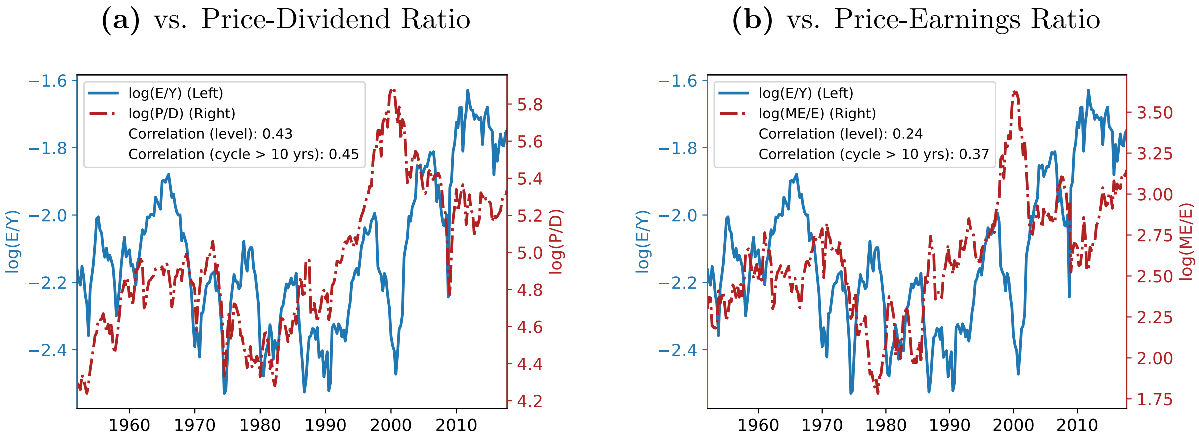
Importantly, for $\omega > 0$ we have $\xi > 1$, so that changes in profit share map more than one-for-one into cash flows, preserving the leverage effect discussed above.

Last, we approximate our nonlinear SDF (5) using a perturbation around the steady state that includes terms linear in $\mathbf{z}_t$ and $\boldsymbol{\varepsilon}_{t+1}$, as well as interactions between $\mathbf{z}_t$ and $\boldsymbol{\varepsilon}_{t+1}$. While the complete solution and derivation can be found in the appendix, we present here the more intuitive form

$$\log M_{t+1} \simeq -\delta_t - \mu_t - \underbrace{x_t \left(\xi \Delta s_{t+1} + \Delta y_{t+1}\right)}_{\text{baseline cash flow risk}} + \underbrace{\bar{x}\xi(\xi - 1)\left(\mathbb{E}_t[s_{t+1}] - \bar{s}\right)\Delta s_{t+1}}_{\text{leverage risk effect}} \tag{11}$$

where $\mu_t$ is implicitly set to ensure a log risk-free rate of $\delta_t$. The "baseline cash flow risk" term represents the price of risk $x_t$ times the change in cash flows under the approximation (10). The final *leverage risk effect* term is a second-order interaction, representing the fact

**Figure 2: Earnings Share and Valuations**




*Notes:* log(E/Y) denotes the logarithm of the after-tax profit (earnings) share of output for the corporate sector. log(ME/E) is the log of the market equity-to-earnings ratio. log(P/D) is the log of the CRSP price-dividend. Each plot present the correlation between the series in levels, as well as the correlation of the components of each series with cycles of at least 10 years, obtained using the random walk band-pass filter of Christiano and Fitzgerald (1999). The sample spans the period 1952:Q1-2017:Q4.

that the same shock to the earnings share $\varepsilon_{s,t+1}$ has a larger proportional impact on cash flows when the current (and thus expected) earnings share is low. For example, if we again assume $\omega = 6\%$, then increasing the earnings share from 8% to 10% would increase the cash flow share of output by 100% (from 2% to 4%), while the same proportional increase in the earnings share from 16% to 20% would only increase the cash flow share of output by 40% (from 10% to 14%). The leverage risk effect captures this phenomenon, causing the SDF to load more negatively on changes in profit shares when the expected profit share is low.

The specification (11) implies that changes in the profit share influence market valuations by affecting both cash flows and risk premia. We view this risk premium channel as strongly supported by the data. Figure 2 displays the time-series variation in the corporate sector log earnings share of output, $e_t - y_t$, alongside either the Center for Research in Securities Prices (CRSP) log price-dividend ratio $p_t - d_t$, or the corporate sector log price-earnings ratio $p_t - e_t$, showing that these variables are positively correlated, particularly at lower frequencies. For example, the correlation between $e_t - y_t$ and $p_t - d_t$ is 47% for band-pass filtered components of the series that retain fluctuations with cycles greater than 10 years.

A model with no correlation between the the earnings share and the price or quantity of risk would instead unambiguously predict that $p_t - c_t$ and $p_t - e_t$ should be negatively correlated with the profit share. Because shocks to the profit share revert over time, they influence prices (discounted forward-looking cash flows) less than current cash flows, resulting in a *negative* correlation between the earnings share and valuation ratios — an intuition we formalize in our equilibrium conditions below. The positive correlation observed in the data instead implies that persistently high earnings shares must coincide with a decline in expected future returns, so that valuation ratios still rise even as the earnings share is rationally expected to decline in the future.[20]

**Equilibrium Stock Market Values.** The first-order-condition for optimal shareholder consumption implies the following Euler equation:

$$\frac{P_t}{C_t} = \mathbb{E}_t \exp\left[m_{t+1} + \Delta c_{t+1} + \ln\left(\frac{P_{t+1}}{C_{t+1}} + 1\right)\right]. \tag{12}$$

We conjecture a solution to (12) taking the form

$$pc_t = A_0 + \mathbf{A}_s'\widetilde{\mathbf{s}}_t + \mathbf{A}_\delta'\widetilde{\boldsymbol{\delta}}_t + \mathbf{A}_x'\widetilde{\mathbf{x}}_t. \tag{13}$$

where $pc_t = \log(P_t/C_t)$ is the log price to cash-flow ratio. The solution, derived in Appendix A.3, implies that the coefficients on these state variables take the form

$$\begin{aligned}
\mathbf{A}_s' &= -\xi\left[\mathbf{1}'(\mathbf{I} - \mathbf{\Phi}_s) - (\mathbf{1}'\mathbf{\Sigma}_s\mathbf{1})\mathbf{\Gamma}'\right]\left[(\mathbf{I} - \kappa_1\mathbf{\Phi}_s) - \kappa_1\xi\mathbf{\Sigma}_s\mathbf{1}\mathbf{\Gamma}'\right]^{-1} \\
\mathbf{A}_x' &= -\left[\left(\xi^2(\mathbf{1}'\mathbf{\Sigma}_s\mathbf{1}) + \sigma_a^2 + \kappa_1\xi(\mathbf{A}_s'\mathbf{\Sigma}_s\mathbf{1})\right)\mathbf{1}'\right](\mathbf{I} - \kappa_1\mathbf{\Phi}_x)^{-1} \\
\mathbf{A}_\delta' &= -\mathbf{1}'(\mathbf{I} - \kappa_1\mathbf{\Phi}_\delta)^{-1}
\end{aligned}$$

where the leverage risk effect is captured by the term

$$\mathbf{\Gamma}' = \bar{x}\xi(\xi - 1)\mathbf{1}'\Phi.$$

The elements of $\mathbf{A}_x$, and $\mathbf{A}_\delta$ are all negative, implying that an increase in the risk-free rate or an increase in the price of risk $x_t$ at any frequency reduces the price-cash flow ratio, as either event increases the rate at which future payouts are discounted. The size of these effects depends on the persistences of the risk-free rate and risk price processes, as captured by $\mathbf{\Phi}_\delta$ and $\mathbf{\Phi}_x$, with more persistent shocks translating into larger effects.

The signs of the elements of $\mathbf{A}_s'$ depend on the values of $\mathbf{\Gamma}$. As discussed above, were $\mathbf{\Gamma} = 0$, the elements of $\mathbf{A}_s$ would also be negative, yielding a counterfactual negative correlation between the earnings share and $pc_t$, with the correlation approaching zero as the persistence of the earnings share components approach unity. In contrast, when the leverage risk effect

[20] If shocks to the earnings share improved shareholder fundamentals permanently, these shocks would drive prices up proportionally with earnings, leaving valuation ratios unaffected and the correlation zero.

is active, we have $\boldsymbol{\Gamma} > 0$, lessening or reversing this counterfactual negative correlation.

As shown in Appendix A.3, the model's log equity premium is given by

$$\log \mathbb{E}_t \left[ R_{t+1}/R_{f,t} \right] = \left( \Psi + \sigma_a^2 \right) x_t - \Psi \boldsymbol{\Gamma}' \tilde{\mathbf{s}}_t \tag{14}$$

where

$$\Psi \equiv \xi(\mathbf{1}'\boldsymbol{\Sigma}_s \mathbf{1}) + \kappa_1 \xi(\mathbf{A}_s' \boldsymbol{\Sigma}_s \mathbf{1})$$

is a measure of average earnings share risk, which arises directly through cash flows (first term), as well as the covariance of the earnings share with the *pc* ratio (second term). Equation (14) shows that the equity premium is the sum of two terms: (i) product of the price of risk and the average total cash flow risk, including both earnings share and output risk, and (ii) time variation in the risk premium due to e.g., higher earnings share risk when $\tilde{\mathbf{s}}_t$ is low, via the leverage risk effect.

# 3 Data

We next describe our data sources, with full details available in Appendix A.1.

Our data consist of quarterly observations spanning the period 1952:Q1 to 2017:Q4. We construct all of our data series at the level of the US corporate sector. This choice stands in contrast to previous work, which has compared trends in aggregate measures of output and the labor share against trends in the value of public equity.[21] A weakness of this existing approach is that the Bureau of Economic Analysis (BEA) data on output and labor share are not limited to the publicly traded sector and cover a far broader swath of the economy. This creates the potential for confounding compositional effects over time if, for example, publicly traded firms have experienced different shifts in their profit share or productivity compared to non-public firms. Moreover, Koh et al. (2020) find that versions of the profit and labor share based on these economic aggregates are highly sensitive to how "ambiguous" income is classified as either labor or capital income, posing additional measurement challenges.

In contrast, we construct all of our series at the same level — the US corporate sector — to expressly avoid these compositional effects. Our data also have the additional advantage of "unambiguously" classifying labor and capital income, using the terminology of Koh et al. (2020), thereby avoiding any need to classify the ambiguous portion of national income. At the same time, a consequence of our approach is that our measure of equity values include equity in non-public firms. Although the vast majority of equity value is accounted for by

[21] See e.g., GLL, Farhi and Gourio (2018).

public firms, our equity measure is broader than is used in most existing work, and may exhibit slight differences as a result.[22]

For our estimation, we use observations on six data series: the log share of domestic output accruing to earnings (the earnings share), denoted $ey_t = e_t - y_t$; a measure of a short term real interest rate as a proxy for the log risk-free rate, denoted $r_{f,t}$; a professional survey forecast of average future real risk-free rates over the next 40Q, denoted $\bar{r}^{40}_{f,t}$; growth in output for the corporate sector as measured by growth in corporate net value added, denoted $\Delta y_t$; the log market equity to output ratio for the corporate sector, denoted $py_t = p_t - y_t$; and a proxy for the market risk premium, denoted $rp_t$.

Our motivation behind this choice of series is as follows. The series $ey_t$, $r_{f,t}$ and $\Delta y_t$ pin down the values of $s_t, \delta_t, \Delta a_t$ in the model at each date. The series $\bar{r}^{40}_{f,t}$ ensures that the model correctly allocates between high and low frequency components of the risk free rate to match the expected persistence of the risk-free rate, in addition to its level. The series $py_t$ ensures that the model is able to fully explain movements in market equity in each period, while the risk premium estimate $rp_t$ provides additional discipline for the risk price process.

Turning to the data, our earnings share measure $ey_t$ is equal to the ratio of total corporate earnings to domestic net value added. To compute total corporate earnings, we combine corporate domestic after-tax profits from the National Income and Product Accounts (NIPA) with data on corporate foreign direct investment income from the BEA's International Transaction Accounts. The domestic after-tax profits represent domestic net value added, net of domestic labor compensation, taxes, and interest payments. Foreign earnings represent equity income from directly held foreign subsidiaries. Because the foreign income data is only available from 1982 on, we impute this series over the full sample using NIPA data on total foreign income (direct and indirect) as a proxy, which provides an extremely close fit over the overlapping period (see Appendix A.1 for details).

For our other observables, our real risk-free rate measure $r_{f,t}$ is the 3-month Treasury Bill (T-Bill) yield net of expected inflation, computed using an ARMA(1, 1) model. Our real risk-free rate forecast $\bar{r}^{40}_{f,t}$ is the mean Survey of Professional Forecasters (SPF) expectation of the annual average 3-month T-Bill return over the next ten years (BILL10) less the mean SPF expectation of annual Consumer Price Index (CPI) inflation over the same period (CPI10).[23] Our equity-output ratio $py_t$ is computed as the ratio of the market value of corporate equity from the Flow of Funds to corporate net value added from NIPA. By pricing equity values

[22]The Flow of Funds separately tracks public and private ("closely held") equity for the UScorporate sector from 1996:Q4 onward. Over this subsample, public equity makes up 83% of total corporate equity on average. The remaining 17% is a mix of 11.5% of private S-corporation equity, and 6.5% of private C-corporation equity.

[23]We thank our discussant Annette Vissing-Jorgenson for this suggestion.

using cash flows net of interest payments, we effectively assume that the market value of debt is equal to the present value of interest payments, allowing us to focus directly on equity rather than enterprise values.

Finally, for our observable measure of the risk premium ($rp_t$), we use the SVIX-based estimate derived by Martin (2017), who documents that a wide range of representative agent asset pricing theories fails to explain the high frequency variation in the risk premium implied by options data, even if they are broadly consistent with the lower-frequency variation suggested by variables like the price-dividend ratio or $cay_t$ (Lettau and Ludvigson (2001)).[24] Since our model allows for mixture processes, the risk premium we estimate is capable of accounting for both these higher- and lower-frequency components of the risk premium.

Importantly, other than indirectly in the calibration of $\xi$ below, we do not use NIPA payout data in our estimation. This choice is motivated by the observation that payouts are a function of current and future earnings, as well as transitory factors that affect the timing with which they are paid out, subject to an intertemporal budget constraint. These two sources of variation have very different implications for future payouts. A rise in payouts due to a persistent rise in earnings implies that an increase in payouts today will continue to forecast larger payouts in the future, paid by a more profitable corporate sector. In contrast, a rise in payouts holding profitability fixed implies that the corporate sector is exhausting resources it could have paid out at a later date, forecasting *smaller* payouts in the future.

As a result, variation driven purely by the timing of payouts adds noise to the fundamental forecasts of profitability and its associated payouts that are our key objects of interest. This problem can be severe when estimating a model of equity pricing, since observed variation in the timing of payouts is large and subject to extreme swings due to temporary factors such as changes in tax law that are likely unrelated to economic fundamentals.[25] Thus, direct use of these data as cash flows would require extensive investigation to align what is being measured with the desired theoretical input. For these reasons, we consider earnings to be a better indicator of future payouts and fundamental equity value than current payouts, and verify ex-post that our model reproduces the path of net dividends in Section 5.3.

**Measurement Issues.** In addition to the "ambiguous income" issue discussed above, Koh et al. (2020) and Atkeson (2020) note that long term trends in factor shares appear to

[24]Martin (2017) uses options data to compute a lower bound on the equity risk premium, then argues that this lower bound is tight and is therefore a good measure of the true risk premium on the stock market.

[25]For a recent example, see NIPA Table 4.1, which shows an unusually large increase in 2018:Q1 in net dividends received from the rest of the world by domestic businesses, generating a very large decline in net payout. BEA has indicated that these unusual transactions reflect the effect of changes in the US tax law attributable to the Tax Cut and Jobs Act of 2017 that eliminated taxes for US multinationals on repatriated profits from their affiliates abroad.

depend heavily on the accounting treatment of intangible investment. We provide evidence in Appendix A.10 that our results on the role of factor shares should be robust to these concerns for two reasons. First, this critique largely applies to factor shares of *gross* value added. In this paper, however, we consider factor shares of *net* value added, which as noted by Koh et al. (2020) are much less sensitive to these assumptions, and deliver more conservative measures of the long-term rise in profit shares. Second, to the extent that various accounting treatments differ, we show that these differences mostly appear in the first half of the sample. Over the post-1989 subsample where we estimate the largest role for factor shares, all measures of the earnings share display very similar changes. As a result, we believe our construction of the earnings share series, and resulting estimates on the role of factor shares, are robust to these important concerns about accounting treatments.

We also test the influence of multinational profit shifting on our core data measures. Our main analysis uses BEA data on the US corporate sector that may be affected by changes in the formal location of corporate headquarters or profits. For example, Guvenen, Mataloni Jr, Rassier and Ruhl (2021) document that US firms appear to shift their profits to their foreign subsidiaries to reduce their tax burden. Because we include both domestic and foreign subsidiary profits in our main profit share measure, this should not influence our main results. However, these findings could matter for how profits are apportioned between domestic and foreign sources and for the interpretation of why the foreign profit share may have changed. Specifically, some of the rise in profit share that we attribute to foreign earnings in Section 5.6 may actually represent increases in the domestic profit share that have been shifted overseas for accounting purposes.

Relatedly, Bertaut, Bressler and Curcuru (2021) show that some US firms change their headquarter location for tax reasons, for example by using corporate inversions. Because firms formally relocating away from the US would drop out of the US Corporate sector according to the BEA, these switches could influence our data series. Since taxes are paid on corporate profits, however, the firms with the greatest incentive to relocate are those with higher profits, who should therefore have selected *out* of our US sample over time. This should only bias our measured rise in the profit share — and its contribution to the growth in equity values — downward. Beyond this intuition, we directly check in Appendix A.11 whether our key BEA measures line up with equivalents constructed from Compustat, which may be less affected by these nominal changes in location. Appendix Figure A.7 shows that ratios of market equity to earnings from our BEA data line up closely with those from Compustat, and confirms that if anything our earnings share measures are conservative.[26]

[26] While we would ideally like to validate our measure of the profit share, this would require a measure of value added to use in the denominator. However, value added is not directly computable using Compustat

# 4 Estimation

The model just described consists of a vector of primitive parameters

$$\boldsymbol{\theta} = \left(\omega, g, \sigma_a^2, \operatorname{diag}(\boldsymbol{\Phi}_s)', \operatorname{diag}(\boldsymbol{\Phi}_x)', \operatorname{diag}(\boldsymbol{\Phi}_\delta)', \operatorname{diag}(\boldsymbol{\Sigma}_s)', \operatorname{diag}(\boldsymbol{\Sigma}_x)', \operatorname{diag}(\boldsymbol{\Sigma}_\delta)', \bar{s}, \bar{\delta}, \bar{x}, \right)',$$

With the exception of a small group of parameters, discussed below, the primitive parameters are freely estimated using Bayesian methods with flat priors.

To estimate our model, we relate our state vector $\mathbf{z}_t$ to our observable series $\mathcal{Y}_t = \left(ey_t, r_{ft}, \bar{r}_{f,t}^{40}, py_t, \Delta y_t, rp_t\right)'$ using the linear measurement equation

$$\mathcal{Y}_t = \mathbf{H}_t \mathbf{z}_t + \mathbf{b}_t \tag{15}$$

where the full structure for $\mathbf{H}_t$ and $\mathbf{b}_t$ can be found in Appendix A.4. Since our model has more shocks in $\boldsymbol{\varepsilon}$ than observable series in $\mathcal{Y}_t$, the model is able to explain all of the variation in our observable series, allowing us to estimate (15) without measurement error. We note that the coefficient matrix $\mathbf{H}_t$ and vector $\mathbf{b}_t$ depend on $t$ because some of our observable data series are not available for the full sample. In particular, the sample for the real rate forecast $\bar{r}_{f,t}^{40}$ spans 1992:Q1 - 2017:Q1, with one observation every 4Q, while the SVIX risk premium $rp_t$ spans 1996:Q1 - 2012:Q1 quarterly observations, both of which are shorter than our full sample period 1952:Q1 - 2017:Q4. As a result, the state-space estimation uses different measurement equations to include these equations for these respective series when the relevant data are available, and exclude them when they are missing (see Appendix A.4 for details). Combined, (7) and (15) describe the full state space system used for estimation.

We estimate the parameters of the model as follows. Given a vector of primitive parameters $\theta$, we construct our state space system using (7) and (15). We then use the Kalman filter to compute the log likelihood $L(\theta)$, which is equivalent to the posterior under our flat priors, up to a restriction that ensures the correct ordering of our low and high frequency processes.[27] To sample draws of $\theta$ from the parameter space $\Theta$ we use a random walk Metropolis-Hastings (RWMH) algorithm (see Appendix A.4 for further details).

Given our parameter draws, we employ the simulation smoother of Durbin and Koopman (2002) to compute one draw of the latent states $\{\mathbf{z}_t^j\}$ for $t = 1, \ldots, T$ for each parameter draw

data, which does not separate out expenses on intermediate goods. Since mismeasurement of the $E/Y$ ratio should lead to corresponding mismeasurement of the $ME/E$ ratio, we validate this second metric instead.

[27] The latent state space includes components that differ according to their degree of persistence. With flat priors, a penalty to the likelihood is required to ensure that the low frequency component has greater persistence than the higher frequency component. This is accomplished using a prior density that is equal to zero if $\phi_{(\cdot),LF} \leq \phi_{(\cdot),HF}$ for any relevant component of the state vector, and is constant elsewhere.

$\{\theta^j\}$, yielding a distribution of latent state paths that characterize the model's uncertainty over its latent state estimates. Given our lack of measurement error, each latent state path perfectly matches the growth in market equity $\Delta p_t$ over time and at each point in time, a property we exploit when calculating our growth decompositions below.

**Calibrated Parameters** We calibrate, rather than estimate, four parameters. The first three are the average growth rate of net value added $g$, the average log profit share $\bar{s}$, and the average real risk-free rate $\bar{\delta}$. Since these represent the means of our observable series, we take the conservative approach of fixing them equal to their sample means. We do this to avoid a potential estimation concern: because some of our series are very persistent, the estimation might otherwise have a wide degree of freedom in setting steady state values that are far from the observed sample means. At quarterly frequency, we obtain the values $g = 0.552\%$, $\bar{s} = -2.120$ (corresponding to a share in levels of 12.01%), and $\bar{\delta} = 0.281\%$.

The final calibrated parameter is $\xi = \frac{\overline{S}}{\overline{S}-\omega}$, which relates payout growth to earnings growth according to (10), and can also be pinned down directly by sample means. Since $C_t = (S_t - \omega)Y_t$, we can rearrange and take sample averages of both sides to obtain $\omega = \bar{S} - \overline{\frac{C}{Y}}$. Computing $\overline{S}$ as the mean of the total profit to domestic output ratio and $\overline{\frac{C}{Y}}$ as the mean of the payout to output ratio observed in the data yields the value $\xi = 2.002$. We further test and validate this parameter and its implications for cash flows in Sections 5.3 and 5.7.

# 5 Results

## 5.1 Parameter Estimates

We begin with a discussion of the estimated parameter values and latent states. Table 1 presents the estimates of our primitive parameters based on the posterior distribution obtained with flat priors. A number of results are worth highlighting.

First, the persistence parameters of the low frequency components of the state variables are of immediate interest, since they determine the role of each latent variable in driving market equity values over longer periods of time. Table 1 shows that the earnings share and risk price contain highly persistent components, with median estimates of $\phi_{s,LF} = 0.993$ and $\phi_{x,LF} = 0.988$, respectively. In contrast, the low frequency risk-free rate process is substantially less persistent, with median value $\phi_{\delta,LF} = 0.964$. In part because more persistent processes have stronger influence on equity values in our model, we will find that the risk-free rate plays the smallest role in driving equity values among these components.

For a more complete look at the estimated persistence of our stochastic processes, Figure

**Table 1: Parameter Estimates**

| Variable | Symbol | Median | 5% | 95% | Mode |
|---|---|---|---|---|---|
| Risk Price Mean | $\bar{x}$ | 6.0466 | 4.7683 | 7.6212 | 5.8676 |
| Risk Price (HF) Pers. | $\phi_{x,HF}$ | 0.6943 | 0.5451 | 0.7986 | 0.6781 |
| Risk Price (HF) Vol. | $\sigma_{x,HF}$ | 2.1854 | 1.6106 | 2.9669 | 2.1724 |
| Risk Price (LF) Pers. | $\phi_{x,LF}$ | 0.9882 | 0.9809 | 0.9946 | 0.9886 |
| Risk Price (LF) Vol. | $\sigma_{x,LF}$ | 0.6232 | 0.3736 | 0.9823 | 0.6295 |
| Risk-Free (HF) Pers. | $\phi_{\delta,HF}$ | 0.8473 | 0.7704 | 0.8938 | 0.8413 |
| Risk-Free (HF) Vol. | $\sigma_{\delta,HF}$ | 0.0017 | 0.0015 | 0.0019 | 0.0017 |
| Risk-Free (LF) Pers. | $\phi_{\delta,LF}$ | 0.9639 | 0.9514 | 0.9790 | 0.9630 |
| Risk-Free (LF) Vol. | $\sigma_{\delta,LF}$ | 0.0010 | 0.0007 | 0.0013 | 0.0010 |
| Factor Share (HF) Pers. | $\phi_{s,HF}$ | 0.8846 | 0.8334 | 0.9194 | 0.9035 |
| Factor Share (HF) Vol. | $\sigma_{s,HF}$ | 0.0530 | 0.0472 | 0.0577 | 0.0527 |
| Factor Share (LF) Pers. | $\phi_{s,LF}$ | 0.9930 | 0.9780 | 0.9988 | 0.9873 |
| Factor Share (LF) Vol. | $\sigma_{s,LF}$ | 0.0152 | 0.0084 | 0.0264 | 0.0168 |
| Productivity Vol. | $\sigma_a$ | 0.0152 | 0.0142 | 0.0164 | 0.0154 |
| Forecast Mean Adjustment | $\nu$ | 0.0018 | -0.0006 | 0.0042 | 0.0020 |

*Notes:* The table reports parameter estimates from the posterior distribution. All parameters are reported at quarterly frequency. The sample spans the period 1952:Q1-2017:Q4.

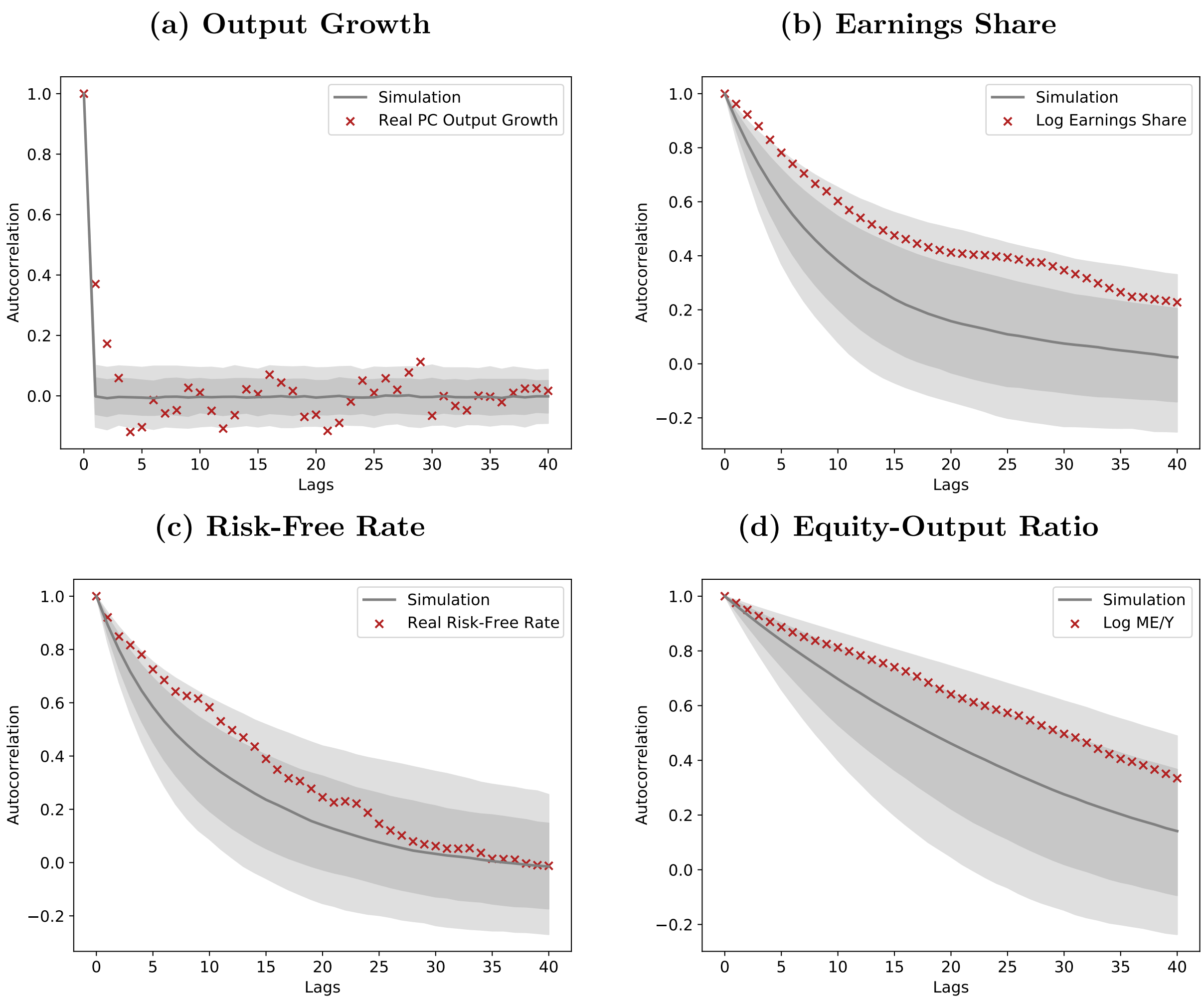
3 compares the autocorrelations of the latent states in the model and data.[28] To account for small sample bias, the model autocorrelations are obtained from 10,000 simulations, each the length of the data sample, taken from 10,000 equally spaced parameter draws from our MCMC estimation.[29] Our model-generated autocorrelations generally match their data equivalents well, especially at the longer lags that are more important for asset prices.

In both model and data, the autocorrelations of output growth hover around zero, consistent with our i.i.d. assumption, whereas the autocorrelations for the earnings share, the risk-free rate, and the log ME-to-output ratio start decline gradually as the lag order increases, consistent with persistent but stationary processes. Panels (b) and (d) show that the model underestimates the persistence of the both the earnings share and the equity-output ratio at long horizons. Since the strength of the earnings share's effect on valuations increases with its persistence, this implies that our results on the role of the earnings share are, if anything, conservative. Last, Panel (c) shows that the autocorrelations of the risk-free rate converge to zero by quarterly lag 40 in both model and data, and in particular display much less persistence than the equity-output ratio.

[28]We include all four observable series that are available over the full sample. We omit the SPF real rate forecast and the SVIX risk premium, both of which are available only on a much shorter sample, and are therefore unsuitable for computing longer autocorrelations.

[29]"Equally spaced" means sampled at regular intervals from the Markov Chain. Because the parameter draws in the original Markov Chain are highly serially correlated, this resampling dramatically speeds up computation time with little loss of fidelity in characterizing the distribution of parameters.

**Figure 3: Observable Autocorrelations**




*Notes:* The figure compares the data autocorrelations for the observable variables available over the full sample, compared to the same statistics from the model. For the model equivalents, we use 10,000 evenly spaced parameter draws from our MCMC chain, and for each compute the autocorrelations from a simulation the same length as the data. The center line displays the simulation median, while the dark and light gray bands represent 66.7% and 90% credible sets, respectively. The sample spans the period 1952:Q1-2017:Q4.

Turning to the risk price, Table 1 shows that our median estimate of the average risk price is only $\bar{x} = 6.047$. Because shareholders in our model consume corporate cash flows that are much more volatile than aggregate consumption, our model is able to reproduce a large equity risk premium without high levels of aversion to risk or ambiguity.[30]

Next, returning to the discussion in Section 2.1, the correlation between the *pc* ratio and the earnings share components depends on the strength of the leverage risk effect, which is in turn determined by our model parameters. Our median estimate is $A_s' = (-0.07, -1.80)$,

[30] We note that this estimate may be influenced by our parsimonious investment process, and might differ in an environment with a time-varying investment-output ratio correlated with output growth.

with the entries corresponding to the low and high frequency components, respectively. Thus, while the high frequency component of the earnings share is still negatively correlated with the *pc* ratio, the correlation becomes effectively zero for the low frequency component. This correlation is closer to the data pattern displayed in Figure 2, but again implies a conservative estimate of the influence of the earnings share on risk premia.

Last, we check whether our linear processes generate implausible or unlikely values for our latent states. For the earnings share process $S_t$, the estimated parameters imply that earnings shares in excess of unity never occur, meaning that the probability is indistinguishable from zero. For example, at the parameter mode, the unconditional distribution of the earnings share is more than 13 standard deviations away from unity.

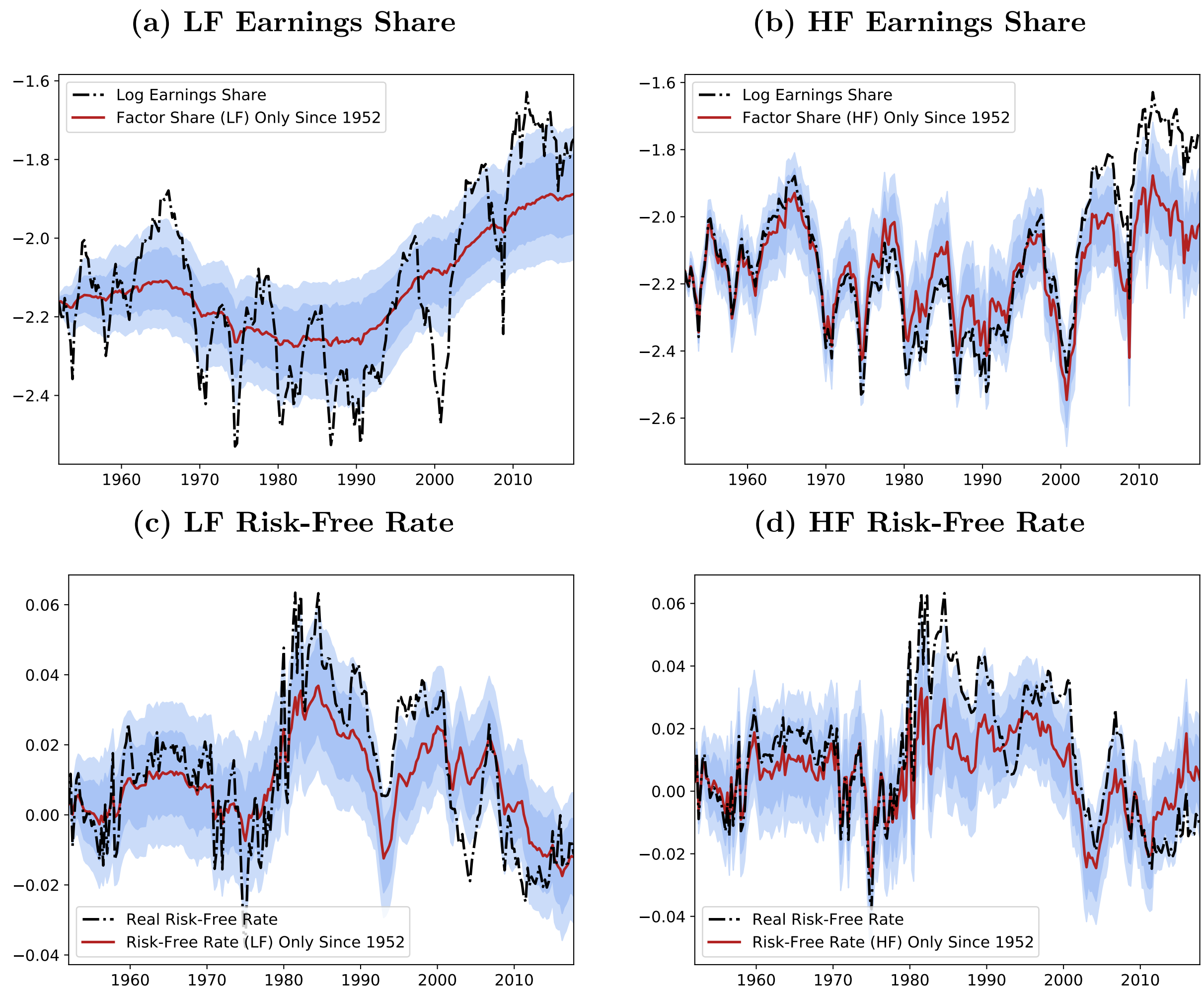
For the risk price process $x_t$, the estimated parameters imply that a negative risk price can occur, with unconditional probability equal to 12.6% at the parameter mode. As argued in GLL, this occurs because explaining the level and volatility of asset prices requires a risk price that is both low (since investors face substantial cash flow risk) and volatile (since *pc* ratios exhibit large movements at high frequency). In a nonlinear model, this could be obtained with a non-negative but highly skewed risk price process, as in GLL. However, in the linear specification required for tractable time series estimation, this combination of high volatility and low mean makes some probability of negative values unavoidable. These negative values do not cause any extreme behavior in our linearized model, but simply imply that agents sometimes price assets in a highly confident or overoptimistic manner. Regardless, our latent state point estimates do not imply negative risk premia at any point in our sample (see Appendix Figure A.4).

## 5.2 Latent State Estimates

Figure 4 displays our model's decomposition of the earnings share $s_t$ and real risk-free rate $\delta_t$ into their low and high frequency components. Each panel plots the observable series, alongside the variation attributable to a single frequency subcomponent, holding the other fixed. In all of our decomposition figures throughout the paper, the red (solid) line shows the median outcome over our parameter and latent state estimates, while the shaded areas are 66.7% and 90% credible sets accounting for both parameter and latent state uncertainty.

Panels (a) and (b) show the time-variation in the log earnings share $ey_t$ over our sample, along with the portion of this variation attributable to each estimated factors share component $s_{LF,t}$ and $s_{HF,t}$. Beginning with the data series itself, we observe that the earnings share undergoes major variation over the sample period. From a starting point in levels of 11.5% in 1952:Q1, the earning share rises to 15.3% in 1966:Q1, before falling to a low of 8.0% in

**Figure 4: Latent State Components**




*Notes:* The figure exhibits the observed earnings share and real risk-free rate series along with the model-implied variation in the series attributable to their low and high frequency components. The red center line corresponds to the median of the distribution of outcomes, accounting for both parameter and latent state uncertainty, while the dark and light blue bands correspond to 66.7% and 90% credible sets, respectively. The sample spans the period 1952:Q1-2017:Q4.

1974:Q3. After remaining low for nearly 15 years, the earnings share undergoes a dramatic rise over the last three decades in the sample, from 8.9% in 1989:Q1 to a high of 19.6% in 2011:Q4, before ending at 17.4% in 2017:Q4.

Our model decomposes this overall series into a low and high frequency component. This decomposition is central to our asset pricing results, since forward looking equity prices in our model respond strongly to movements in the low frequency component, while movements in the high frequency component are too transitory to have a large effect. As a result, it is critical that this series accurately tracks the true data series, and is not distorted by the model's need to match asset prices. Reassuringly, Figure (a) shows that the estimated low frequency component accurately tracks the slow moving trend in the earnings share series.

Panels (c) and (d) similarly show the evolution of the risk-free rate over time, along with the portion of this variation attributable to the estimated low and high frequency components. From the data series, it is clear that, although real rates are low today, they are not unusually so by historical standards, with real rates at similarly low levels at several points in the 1950s and late 1970s. Further, the series appears far from a unit root, with even these swings reverting relatively quickly. This stands in sharp contrast to the time series for nominal interest rates, which features a highly persistent trend in inflation, demonstrating the importance of using real rates. The low frequency component $\delta_{LF,t}$ captures the underlying trend, as well as driving most of the variation in 10-year real risk-free rate forecasts used to match the SPF forecasts, while the high frequency component $\delta_{HF,t}$ largely captures transitory fluctuations, as well as some sharp movements in the early 1980s and 2000s.

## 5.3 Dynamics of Cash Flows

A crucial intermediate step between the evolution of the earnings share and the resulting implications for asset prices is the transmission from earnings to cash flows (corporate payouts). As mentioned above, the forward looking nature of equity valuations imply this relationship is particularly important at low frequencies.

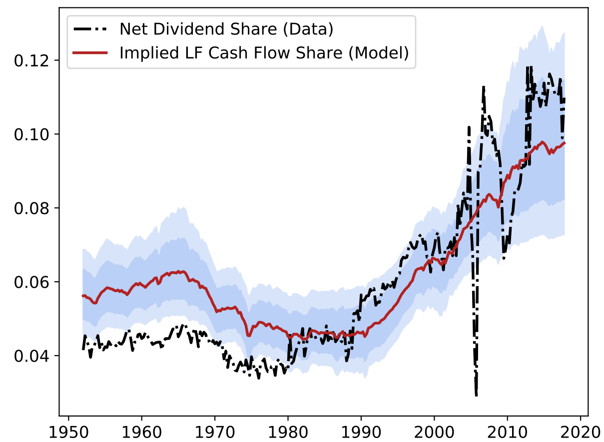
In the model, this low frequency component of the payout-output ratio can be defined as the portion of its variation driven by the low frequency component of the earnings share:

$$cy_{LF,t} = \overline{cy} + \xi \tilde{s}_{LF,t}. \tag{16}$$

In the data, the low-frequency component of payouts is less obviously defined. However, a simple and effective decomposition can be obtained by dividing total payouts into its two subcomponents: net dividends and net repurchases. Appendix Figure A.3 shows that net dividends form a stable and highly persistent series that accounts for the vast majority of the long-term movement in payouts, both over the full sample and since 1989. In contrast, net repurchases are highly volatile but much less persistent, explaining little long-term variation. In light of this, we use the ratio of net dividends to output as our empirical counterpart to the low frequency component of the corporate payout share in the model.

With our low frequency components of the payout share defined in model and data, we now display them in Figure 5. This figure shows a remarkably good fit between the two series at low frequencies, particularly over the period since 1989, and one that if anything understates net dividend growth over the full sample and our main post-1989 subsample.

**Figure 5: Low Frequency Cash Flow Share, Model vs.Data**




*Notes:* Net Dividend Share is the ratio of net dividends to net value added for the corporate sector (source: Flow of Funds). Implied LF Cash Flow Share is equal to $cy_{LF,t}$ in equation (16). The red line represents the median outcome over our estimates, while the dark and light blue bands represent 66.7% and 90% confidence intervals, respectively.

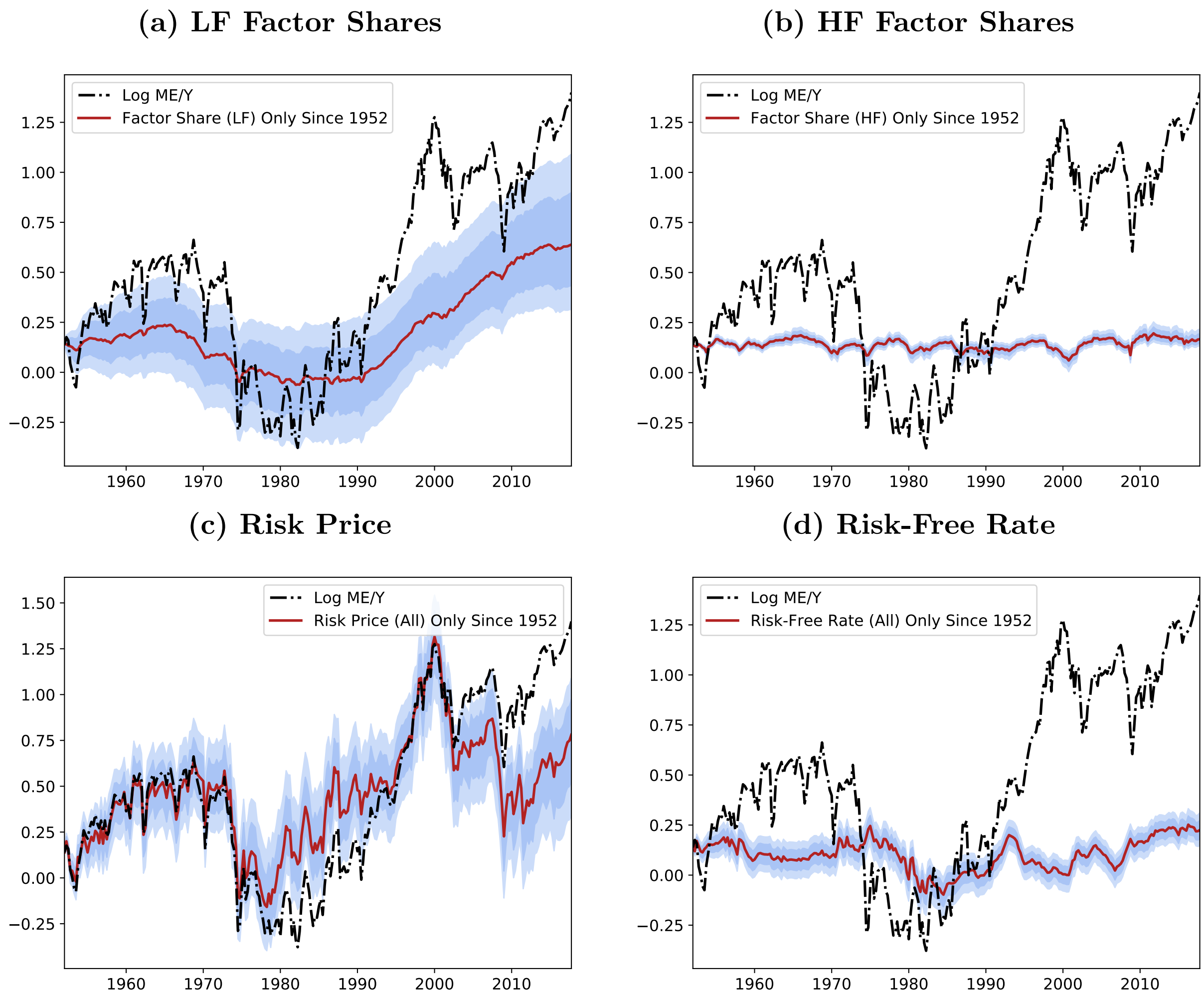
These results imply that while (10) is clearly a parsimonious approximation of the actual path of cash flows, abstracting from changes in financial leverage through debt, cyclical variation in the investment share of output, or changes in payout policy, it is highly effective at capturing the core variation in cash flows over our sample, providing strong support for our ultimate findings for equity values. Further details on payouts in the model and data can be found in Appendix A.7.

## 5.4 Dynamics of Equity Values

With the estimation of the latent states and analysis of implied cash flows complete, we next present their contribution to the evolution of market equity over our sample, displayed in Figure 6. Each panel displays the log equity-to-output (ME/Y) ratio $py_t$, alongside the variation in $py_t$ attributable to an individual latent component, holding the others fixed at their initial value in 1952:Q1. We note that this decomposition is additive, so that the contributions in the four panels, if demeaned by the average value of the data, would add exactly to the true demeaned data series.

Beginning with the top row, Panel (a) shows that the low frequency factor shares component $s_{LF,t}$ explains much of the low frequency trend in the $py$ ratio, particularly over the last three decades of the sample. In contrast, the high frequency component $s_{HF,t}$ explains

## Figure 6: Market Equity-Output Ratio Decomposition



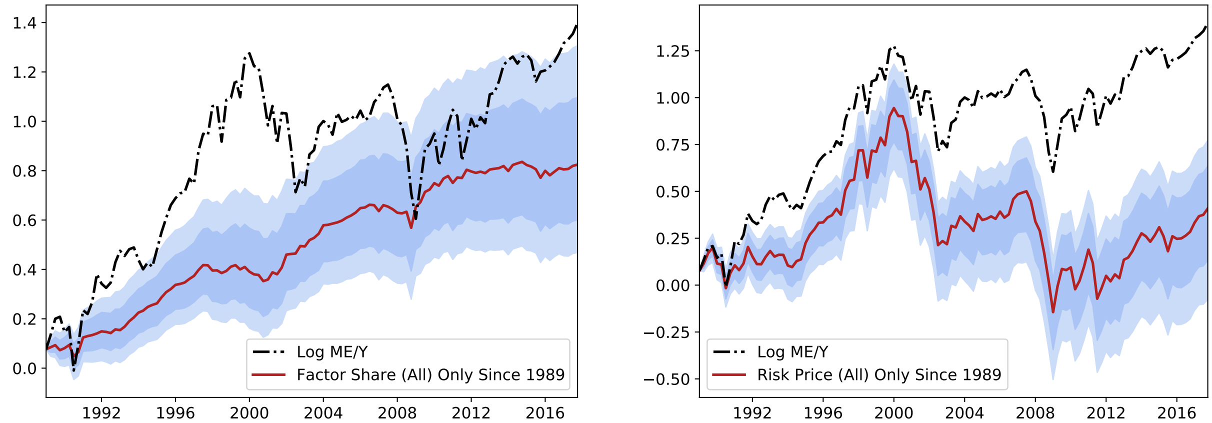

*Notes:* This figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to certain latent components. The top row displays the contribution of the low and high frequency components of the earnings share $s_{LF,t}$ and $s_{HF,t}$, while the bottom row displays the total contribution of the orthogonal risk price $x_t$ and the real risk-free rate $\delta_t$. The red center line corresponds to the median of the distribution of outcomes, accounting for both parameter and latent state uncertainty, while the dark and light blue bands correspond to 66.7% and 90% credible sets, respectively. The sample spans the period 1952:Q1-2017:Q4.

much less of the variation in equity values. This weak effect is not due to a lack of variability of the $s_{HF,t}$ series, which Figure 4 Panel (b) shows is highly volatile, and explains a large portion of earnings share dynamics. Instead, it occurs because more transitory movements in profits have weaker effects on forward-looking asset prices, demonstrating the importance of estimating our latent state processes at multiple frequencies.

Moving to the bottom row, Panel (c) displays the combined contribution of the risk price components $x_{LF,t}$ and $x_{HF,t}$. These secular variations in the risk price drive most variation in

**Figure 7: Market Equity-Output Ratio Decomposition: 1989 - 2017 Subsample**

**(a) Factor Shares** **(b) Risk Price**




*Notes:* This figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to certain latent components over the subsample 1989:Q1 - 2017:Q4. The left panel displays the combined contribution of the earnings share $\tilde{\mathbf{s}}_t$ while the right panel displays the combined contribution of the risk price components $\tilde{\mathbf{x}}$. The red center line corresponds to the median of the distribution of outcomes, accounting for both parameter and latent state uncertainty, while the dark and light blue bands correspond to 66.7% and 90% credible sets, respectively.

valuations at at high and medium frequencies, as well as much of the lower-frequency trend in the first half of the sample. In particular, this component explains nearly all of the large short-run swings in equity values over our sample, including the technology boom/bust, and the crash following the 2008 financial crisis.

Since our risk price process effectively serves as a residual in our estimation, explaining any variation in equity values not explained by cash flows, we note that its contribution need not literally reflect changes in risk attitudes, but could account for a number of alternative forces. These could include structural changes or frictions in financial markets, change in firm leverage, variation in the composition of stock market participants, or news about future output growth or earnings shares. However, the key result from Panel (c) is that movements in the risk price — regardless of its ultimate interpretation — fail to explain much of the rise in equity valuations over the last three decades, with little upward trend in this series between the late 1980s and the end of the sample.

Last, Panel (d) shows the combined contribution of the risk-free rate components $\delta_{LF,t}$ and $\delta_{HF,t}$. Our estimates attribute a modest role to risk-free rate variation in explaining equity valuations over our sample. This finding, which stands in sharp contrast to alternative works in the literature, is due to our relatively low estimated persistence for the risk-free rate processes. We return to this discussion in Section 5.8.

**Table 2: Growth Decomposition**

| Contribution | 1952-2017 | 1952-1988 | 1989-2017 |
|---|---|---|---|
| **Total** | **1405.81%** | **151.23%** | **477.34%** |
| Factor Share $s_t$ | 20.50% | -21.09% | 43.96% |
| | [8.33%, 35.52%] | [-51.68%, 6.91%] | [22.31%, 70.19%] |
| Orth. Risk Price $x_t$ | 22.72% | 25.33% | 17.68% |
| | [7.59%, 35.10%] | [-4.66%, 57.20%] | [-8.71%, 39.73%] |
| Risk-Free Rate $\delta_t$ | 3.24% | -15.65% | 13.80% |
| | [0.31%, 6.34%] | [-26.30%, -6.12%] | [9.77%, 18.47%] |
| Real PC Output Growth | 53.54% | 111.41% | 24.57% |

*Notes:* The table presents the growth decompositions for the real per-capita value of market equity. The row "Total" displays the total growth in market equity over this period, in levels. The remaining rows report the share of this overall growth explained by each component, obtained by measuring the difference in implied growth between the data and a counterfactual path in which that variable is held fixed at its initial value for the relevant subsample. To ensure an additive decomposition, we measure the share of total growth explained in logs. The reported statistics are means over shares computed from 10,000 equally spaced parameter draws from our MCMC chain. Below each set of means in brackets are the 5th and 95th percentiles over the same distribution, providing a 90% credible set that accounts for both parameter and latent state uncertainty. Under our assumption that output growth is a random walk with drift, its contribution is known exactly and requires no confidence interval. Further decompositions into low and high frequency components, can be found in Appendix Table A.1. The sample spans the period 1952:Q1-2017:Q4.

To clarify the contributions since 1989 — the portion of the sample with extremely high growth in both equity valuations and earnings shares — Figure 7 plots the contributions of factor shares and the risk price, respectively, for this subsample. This figure shows that the prime driver of valuations over this period is the factor share process. Movements in the risk price explain much of the cyclical variation, but fail to capture the overall upward trend.

## 5.5 Growth Decompositions

In this section, we summarize the contributions of the different components over our sample. As for Figure 6, we compute the contribution of each component by the counterfactual growth in equity values that would have occurred allowing that component to vary while holding all others fixed at their initial values. By construction, these components sum to 100% of the observed variation in equity values, since our latent state estimates perfectly match at each point in time the observed log market equity-to-output ratio, $py_t$, as well as output growth $\Delta y_t$.

Table 2 presents the decompositions for the total change in the log of real market equity $p_t$, either over the whole sample or over the period before or since 1989.[31] The year 1989

[31] The growth decompositions for the log level of real market equity $p_t$ are computed by adding back the

separating the two subperiods is chosen visually due to the sharp change in the growth of profit shares and equity values around this date. However, we note that a major advantage of our approach is that our quarter-by-quarter estimates allow decompositions of any possible subperiod, making this choice much less important than for steady state analyses that depend crucially on how the different eras are defined.[32]

Our estimates indicate that about 44% of the market increase since 1989 and 21% over the full sample is attributable to the sum of the two factor share components $s_{LF,t}$ and $s_{HF,t}$, with the vast majority coming from the low frequency component (see Appendix Table A.1 for contributions by frequency). In contrast, we find that growth in the real value of corporate output has been a far less important driver of equity values since 1989, explaining just 25% of the increase in equity values since 1989. We note that this number represents the *total* contribution of output growth, including its deterministic trend. The roles of the other components are smaller over the post-1989 period, with the decline in the risk price contributing 18%, and declining interest rates contributing 14%.

These patterns may be contrasted against the previous subsample, from 1952 to 1988, when economic growth accounted for 111% of the rise in the stock market, while factor share movements contributed negatively to the market's rise. These findings underscore a striking aspect of post-war equity markets: in the longer 37 year subsample for which equity values grew comparatively slowly, economic growth propelled the market, while factor shares played a negative role. But the market experienced more than three times greater growth in value in the much shorter period from 1989 to 2017, when factor share shocks reallocated rewards to shareholders even as economic growth slowed.

Combining these periods, we find that output growth explains only 54% of total log growth in market equity over the full sample, despite being the only non-stationary variable in our model, demonstrating the importance of non-output factors even over a horizon as long as 65 years. Factor shares explained 21% of the full sample rise, while the declining risk price contributed 23%, and real interest rates contributed a much smaller 3%.

## 5.6 Sources of Earnings Share Variation

In this section, we turn to the drivers of the earnings share itself. To begin, we briefly present an accounting breakdown of corporate output. Starting with gross value added, the

growth $\Delta y_t$ in real output (net value added) to the growth $\Delta py_t$. Since $\Delta y_t$ is deflated by the implicit price deflator for net value added, the decomposition for $p_t$ pertains to the value of market equity deflated by the implicit NVA price deflator.

[32]Decompositions for these alternative subperiods are straightforward to compute given our estimates of the contribution series, which are publicly available at `http://www.dlgreenwald.com/uploads/4/5/2/8/45280895/gll_public_data.zip`.

BEA removes depreciation to yield net value added, which we use as our measure of output $Y_t$. From this, a fraction $\tau_t$ of $Y_t$ is devoted to taxes and interest payments (as well as a catchall of "other" charges against earnings). We refer to $\tau_t$ simply as the "tax and interest" share for brevity. The remaining $1-\tau_t$ is divided between labor compensation and domestic after-tax profits (domestic earnings, $E_t^D$).[33] We denote labor's share of domestic value added net of taxes and interest as $L_t^D$, so that $E_t^D = (1-\tau_t)(1-L_t^D)Y_t$. Finally, firms receive some earnings from their foreign subsidiaries $E_t^F = F_t Y_t$, where $F_t$ is the ratio of foreign earnings to domestic output, yielding the total earnings decomposition.

$$E_t = E_t^D + E_t^F = \underbrace{\Big((1-\tau_t)(1-L_t^D) + F_t\Big)}_{S_t} Y_t.$$

Because we are working entirely within the corporate sector, this decomposition is entirely among "unambiguous" components of income, and therefore sidesteps the important critique raised by Koh et al. (2020) that many measures of factor shares depend heavily on how various "ambiguous" components of income, such as proprietor's income, are classified.

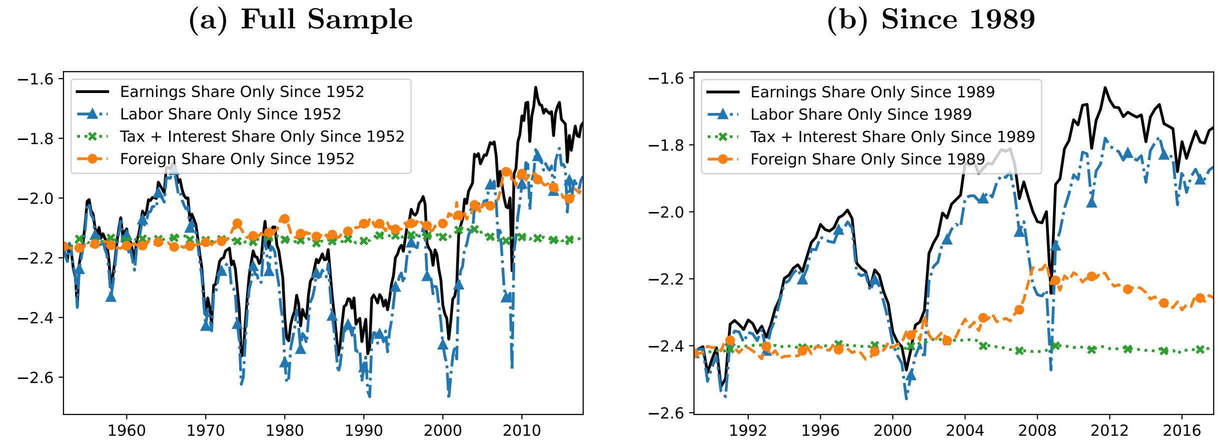
To decompose the contributions of the various components described above we once again compute counterfactual series allowing a single component ($\tau_t$, $L_t^D$, or $F_t$) to vary, while holding the others fixed at their initial values. The resulting series are displayed in Figure 8. Panel (a) shows that movements in the domestic labor share $L_t^D$ explain the vast majority of variation in the earnings share. At lower frequencies, an upward trend in the foreign earnings share has also contributed substantially to the rise in $S_t$. Combined, the tax and interest shares play close to zero role.[34] Panel (b) repeats this decomposition on the 1989 - 2017 subsample that has featured rapid increases in equity valuations. This panel shows movements in the domestic labor share have been the dominant driver of the earnings share over this period, with the foreign share playing a smaller role, and the tax and interest share again playing close to zero role.

Taken together, these results imply that the declining domestic labor share has played

[33]We use the BEA corporate sector labor compensation data to measure $L_t^D(1-\tau_t)Y_t$. Some researchers have questioned whether the BEA adequately accounts for all of employee compensation in the form of restricted stock or stock options (e.g., Koh et al. (2020), Eisfeldt, Falato and Xiaolan (2018)). If no equity-based compensation were actually captured by the BEA labor compensation data, then $S_t^D$ should be interpreted as the traditional cash compensation share, and fluctuations in $S_t$ potentially influenced by any factor that drives the traditional cash compensation share. The precise interpretation of why $S_t^D$ or $S_t$ varies, although important and interesting in its own right, is not central to our investigation. Whatever the reason for a changing $S_t$, our empirical methodology can investigate the extent to such fluctuations have added to the rapid growth in the market value of corporate equity over the post-war period.

[34]This result is partially due to the separate impacts of the tax and interest shares largely canceling out due to a strong negative correlation, perhaps due to the tax deductibility of interest payments.

**Figure 8: Role of Components in Earnings Share**




*Notes:* The figure decomposes the corporate earnings share $S_t$ into contributions from changes in the domestic labor share $S^D$, the tax and interest share $Z$, and the foreign share $F$. Each series shows the result of allowing a single component to vary, while the others are held fixed at their initial values for that period (1952 or 1989). The sample spans the period 1952:Q1-2017:Q4.

the largest role in the sustained rise in the corporate earnings share. Combined with our results earlier in the section showing that the earnings share has been the main driver of valuations since 1989, this suggests that much of stock market gains over this period have come at the expense of US labor compensation.

## 5.7 Asset Pricing Moments

Up to this point, our model estimates have decomposed the contributions of various forces over our data sample. At the same time, the observed sample is only a single realization. In this section, we use the model to compare the observed asset pricing moments over our sample to the unconditional distributions of these moments implied by the model.

The results can be seen in Table 3. The columns labeled "Unconditional" report averages across 10,000 simulations of the model, evaluated at 10,000 equally spaced parameter draws from our MCMC chains, each using a sample length equal to that of our historical sample. Next, the columns labeled "Fitted," compute moments using draws of the estimated latent states conditional on the actual historical sample, computed using the disturbance smoother. Finally, the columns labeled "Data" report the actual sample moments of our observed data series. Note that the "Fitted" and "Data" moments are identical by construction for the risk-free rate, earnings growth, and earnings share growth, because we use these series as observables and fit their behavior exactly over the sample with no measurement error.

For series not matched by construction, Table 3 shows that the fitted moments are close to

### Table 3: Asset Pricing Moments

| | Unconditional | | Fitted | | Data | |
|---|---|---|---|---|---|---|
| Variable | Mean | StD | Mean | StD | Mean | StD |
| Log Equity Return | 5.920 | 17.866 | 7.994 | 16.906 | 8.856 | 15.844 |
| Log Risk-Free Rate | 1.130 | 1.729 | 1.124 | 1.946 | 1.124 | 1.946 |
| Log Excess Return | 4.791 | 17.969 | 6.871 | 16.893 | 7.389 | 16.562 |
| Log Price-Payout Ratio | 3.488 | 0.424 | 3.317 | 0.364 | 3.437 | 0.465 |
| Log Earnings Growth | 2.216 | 8.704 | 2.819 | 11.911 | 2.819 | 11.911 |
| Log Payout Growth | 2.205 | 16.867 | 3.444 | 22.017 | 4.115 | 33.459 |
| Log Earnings Share Growth | -0.011 | 8.323 | 0.624 | 10.459 | 0.624 | 10.459 |
| Log Payout Share Growth | -0.022 | 16.669 | 1.249 | 20.814 | 1.920 | 32.186 |

*Notes:* All statistics are computed for annual (continuously compounded) data and reported in units of percent. For annualization, returns, earnings, and payouts are summed over the year in levels. Log growth of earnings, payouts, the earnings share, the payout share, and the price-payout ratio are computed using these annual sums of earnings and payouts, as well as Q4 equity prices from each year. "Unconditional" numbers are averages across 10,000 simulations of the model of the same size as our data sample. "Fitted" numbers use the estimated latent states fitted to observed data in our historical sample. The sample spans the period 1952:Q1-2017:Q4.

the data. Importantly, the model's operating leverage effect allows it to match the fact that both the mean and volatility of growth in the log payout share $c_t - y_t$ are substantially higher compared than the corresponding moments for growth in the log earnings share $e_t - y_t$.[35] At the same time, the fact that our payout growth slightly understates the data suggests that our calibration for $\xi$ is conservative, and that our model is not overstating the impact of the leverage effect. This slight understatement of cash flow growth leads the fitted log excess return (6.9%) to slightly understate its data counterpart (7.4%).

Turning to risk premia, Table 3 shows that the model's unconditional average log excess equity return is 4.8% per annum. This moment represents the mean risk premium implied by our parameter estimates, reflecting compensation for bearing risk in the stock market. By contrast, the mean *fitted* excess stock market return is 6.9%, which reflects not only ex-ante risk compensation, but also the effect of unexpected realizations of shocks over our sample.

This difference between our fitted and unconditional mean excess returns implies that high returns to holding equity in the post-war period have been driven in large part by a highly unusual sample, one characterized by a long string of factor share shocks that redistributed rewards from productive activity toward shareholders. Our estimates imply that roughly 2.1pp per annum of the post-war mean log return on stocks in excess of a T-Bill rate is attributable to these and other realized shocks, leading the realized excess return

[35]This volatility is scaled upward by exactly $\xi = 2.002$ in the model's quarterly simulations, but differ from this precise ratio in Table 3 due to annualization.

to overstate the true equity risk premium by 43%. These findings provide a cautionary tale for the common practice of using the sample mean excess return as an estimate of the average equity risk premium, even over this 66-year post-war period.[36]

## 5.8 Interest Rate Persistence

Our results differ in important ways from contemporaneous papers such as Farhi and Gourio (2018) and Corhay et al. (2018). While these papers find a crucial role for falling interest rates in driving the increase in asset prices over recent decades, we find that interest rates account for only 14% of stock market growth since 1989. Moreover, while these papers both conclude that risk premia have risen over this period, Panel (a) of Figure A.4 shows that we estimate risk premia to have fallen to historically low levels by the end of our sample.

We believe these opposing results are mostly due to the different estimation approaches behind them. While we estimate our model directly on the time series, allowing for shocks to enter with a variety of estimated persistences, Farhi and Gourio (2018) and Corhay et al. (2018) measure changes across steady states, in which parameters can change only permanently. As a result, these papers interpret the observed drop in risk-free rates as a permanent shift, causing major changes in how long-term cash flows are discounted, and leading to a huge increase in market value. Since the implied increase in market value from falling risk-free rates is even larger than the actual increase to be explained, these models infer that risk premia must have risen to match the realized growth in asset prices.

In contrast, our model views changes in interest rates as far from permanent, with a median estimate of the quarterly persistence of the low frequency component of interest rates of 0.964. As a result, investors in our model did not believe that interest rates would remain permanently high in the 1980s, nor do they expect them to remain permanently low today, strongly dampening the effect of the fall in rates on the value of market equity. This smaller direct effect from interest rates allows us to match the observed rise in asset prices in an environment with falling risk premia. In particular, we show in Appendix A.8 that our model implies equity premia that have been falling for decades, and that by the end of 2017 had reached lows previously only seen during the culminations of the tech boom in 2000 and the twin housing/equity booms in 2006.

We view our approach, and therefore our findings, as strongly preferred by the data. Recall that, because we include the mean SPF expectation of the average real 3-Month T-Bill return over the next ten years as an observable, our estimates of the risk-free rate process match this forecast in each period it is available. Since we also match the current

[36] Avdis and Wachter (2017) arrive at a similar conclusion using a different estimation methodology.

real short rate, this means that our model is able to reproduce the expected persistence of real interest rates, as perceived by forecasters at each point in time. This strong discipline on our persistence parameters and latent states allows us to precisely estimate the risk-free rate persistence, which we can bound far from unity. This can be seen visually in Figure 3 as the sample autocorrelation of our real risk-free rate series decreases rapidly with the lag order, falling by half within the first 15 quarters, and falling close to zero at the 10-year horizon — a pattern inconsistent with a process dominated by permanent changes. Our model is able to match this pattern well, and does not understate the autocorrelation at long horizons.[37]

Last, we investigate how well our model is able to explain variation in long real bond yields in the data. While our definition of the risk-free rate component is restricted to the path of short rates, it is possible that time varying term premia beyond the scope of our model could be influencing the value of other long-maturity assets like equities. In this case, the model would attribute such movements to changes in the risk price, leading to a potentially different interpretation of our results. Nonetheless, while we do not directly target long bond yields in our estimation or construction of the model, we show in Appendix A.9 that the model provides a good fit for the evolution of 10-year TIPS yields at low frequencies, providing further support for the persistence of our estimated interest rate process.

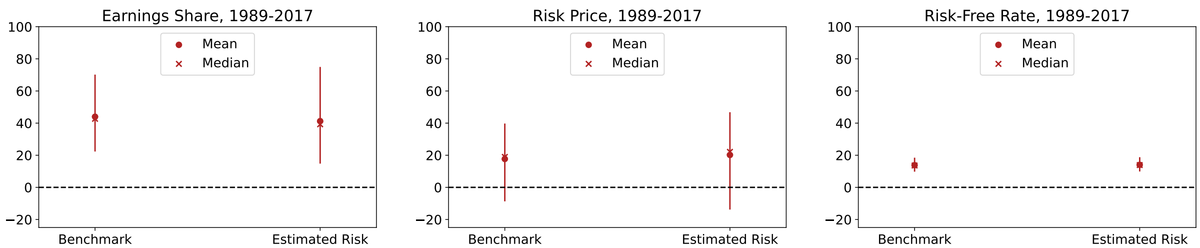
A natural caveat to these results is that they are based on the backward-looking statistical properties of risk-free rates in the historical data, during which time interest rate fluctuations did not appear statistically to be permanent. If the economy has instead undergone a structural break, with interest rates changing much more persistently than in the past, our model might understate the effect of such a change. Regardless, this type of misattribution would primarily affect the estimated contribution of the risk price, which effectively serves as our residual, and should be largely orthogonal to our core results on the role of factor shares, which are heavily disciplined by their own data series.

# 6 Robustness

To close our results, we examine the robustness of our main finding on the role of factor shares to our key modeling and calibration assumptions.

**Figure 9: Model Comparison, Contributions to $p_t - y_t$.**

**(a) Earnings Share** **(b) Risk Price** **(c) Risk-Free Rate**




*Notes:* These figures plot the growth decompositions for the real value of market equity under alternative model specifications, with each panel corresponding to a different fundamental component, and the different lines in each panel corresponding to alternative models. Growth decompositions are obtained by measuring the difference in implied growth between the data and a counterfactual path in which that variable is held fixed at its initial value for the relevant subsample. To ensure that these shares add up to 100%, these rows measure the share of total growth explained in logs. Each model reports the mean and median in red, while the red error bars span from the 5th to 95th percentiles.

## 6.1 Robustness: The Leverage Risk Effect

For our first set of robustness checks, we relax the tight parametric link between factor shares and risk premia that stems from our microfounded leverage risk effect. To do so, we solve an "Estimated Risk" model in which we set $\mathbf{\Gamma} = 0$ — shutting down the leverage risk effect — but instead allow the price of risk to load on the earnings share according to

$$x_t = \bar{x} + \mathbf{1}'\tilde{\mathbf{x}}_t + \lambda \mathbf{1}'\tilde{\mathbf{s}}_t.$$

We view this as a parsimonious way of flexibly capturing the strength of the earnings share's influence on risk premia, with $\lambda < 0$ delivering the negative correlation implied by Figure 2 and obtained in the Benchmark model from the leverage risk effect. However, unlike our Benchmark model, in which the strength of this link is pinned down by theory, our Estimated Risk specification freely estimates the strength of this effect through $\lambda$, and could eliminate it entirely if preferred by the data.

The results are displayed graphically in Figure 9, which compares the Benchmark and Estimated Risk models for our main results: the contributions to the rise in market equity valuations since 1989. The figure shows that this more flexible specification delivers a decom-

[37] These findings are not directly comparable to those in Bianchi, Lettau and Ludvigson (2016), who find evidence of a low frequency component in interest rates driven by monetary policy, since the monetary policy component they uncover is correlated with risk-premium variation, whereas we identify only the mutually uncorrelated components of risk-free rate and equity premium variation.

position nearly identical to the Benchmark model. Although the error bars on the Estimated Risk model are slightly wider, likely due to the inclusion of an additional free parameter, the point estimates and general ranges are highly similar. To be more precise, the Estimated Risk model delivers average contributions of (41%, 20%, 14%) for the earnings share, risk price, and risk-free rate, respectively, compared to equivalent values of (44%, 18%, 14%) for the Benchmark model. Because the model was free to choose an arbitrary strength for the influence of the earnings share on risk premia, these nearly identical results provide strong quantitative support for our specification the leverage risk effect.

## 6.2 Robustness: Cash Flows vs.Risk Premia

For our final set of results, we decompose the contribution of the earnings share into the components driven by the change in cash flows vs. the change in the risk premium, then consider robustness to alternative degrees of earnings share persistence. We begin with the workhorse decomposition of Campbell and Shiller (1989) for the price-payout ratio:

$$pc_t = \text{const} + \mathbb{E}_t \sum_{j=0}^{\infty} \kappa_1^j \Delta c_{t+j+1} - \mathbb{E}_t \sum_{j=0}^{\infty} \kappa_1^j r_{t+j+1}.$$

Since the log ME/Y ratio $py_t$ is equal to the sum $pc_t + cy_t$, we can apply a single restriction — our loglinear approximation for the cash flow share of output (10) — to obtain[38]

$$py_t = \text{const} + \underbrace{\xi s_t + \xi \mathbb{E}_t \sum_{j=0}^{\infty} \kappa_1^j \Delta s_{t+j+1}}_{\text{direct cash flow component}=py_t^{CF}} + \mathbb{E}_t \sum_{j=0}^{\infty} \kappa_1^j \Delta y_{t+j+1} - \mathbb{E}_t \sum_{j=0}^{\infty} \kappa_1^j r_{t+j+1}. \tag{17}$$

The term in braces represents the contribution of the earnings share process $s_t$ to the log ratio of market equity to output, directly through cash flows, while ignoring any influence on risk premia. In our structural model, this component, which we denote $py_t^{CF}$ is given by

$$py_t^{CF} = \xi \left\{ \bar{s} + \mathbf{1}' \Big[ \mathbf{I} - \mathbf{1}'(\mathbf{I} - \Phi_s)(\mathbf{I} - \kappa^1 \Phi_s)^{-1} \Big] \tilde{\mathbf{s}}_t \right\}. \tag{18}$$

We can therefore compare the growth of $py_t^{CF}$ and $py_t$ to compute the contribution of growth in the log ME/Y ratio explained by the direct cash flow effect.

The results are displayed in Table 4. Under our Benchmark model estimates, the direct change in cash flows explains 26.38% of the rise in the $py$ ratio since 1989. Since the total

[38]We thank our discussant, Valentin Haddad, for this helpful suggestion.

**Table 4: Comparison, Share of ME/Y Explained (1989 - 2017)**

| | | AR(1) Models (by $\phi_s$) | | | |
|---|---|---|---|---|---|
| | Bench. | 0.980 | 0.990 | 0.995 | 1.000 |
| Cash Flow Contribution | 26.38% | 28.71% | 45.08% | 62.68% | 102.15% |

*Notes:* This table displays the share of the growth in the log market equity to output ratio explained by the implied contributions of the earnings share via cash flows, defined as $(py^{CF}_{2017:Q4} - py^{CF}_{1989:Q1})/(py_{2017:Q4} - py_{1989:Q1})$, where $py$ is the log ratio of market equity to output, and $py^{CF}$ is the direct cash-flow component.

contribution of earnings share changes is estimated at 58.28%, these results imply that around 45% of our overall earnings share contribution in the Benchmark model is due to the direct influence on cash flows, with the remaining share due to the influence on risk premia through the leverage risk effect.[39]

While these results already imply that the direct cash flow component is an important driver of market equity over this period, with a magnitude similar to total economic growth, we believe these results understate the true cash flow contribution, as the Benchmark model appears to understate the autocorrelation process in the data. This can be seen in Figure 3 Panel (b), which shows that small sample autocorrelations from our model simulations lie below the true sample autocorrelations on average at all lag orders, including the longest.

This downward bias in persistence estimates is common in statistical applications, and is not straightforward to correct in our baseline model.[40] Instead, we provide results from a simpler parametric specification to demonstrate the strength of the cash flow effect at plausible levels of bias-corrected persistence. In place of our full structural model, we approximate $s_t$ by a simpler AR(1) process

$$s_{t+1} = (1 - \phi_s)\bar{s} + \phi_s s_t + \varepsilon_{s,t+1}$$

which implies $\mathbb{E}_t \Delta s_{t+j+1} = -(1 - \phi_s)\phi^j s_t$. Substituting, solving for the geometric sum, and omitting all terms not entering our "direct cash flow component" above, we obtain the

[39] These numbers differ from those in Table 2 because they decompose growth in the ratio of market equity to output ($py$) instead of the growth in real per-capita market equity ($p$).

[40] Because the model has freedom to allocate between the low and high frequency components, manually increasing the persistence of the low frequency component, or even both components, results in the model assigning more variation to the high frequency component, leaving the overall results unchanged.

following expression for $py_t^{CF}$ under the AR(1) specification:

$$py_t^{CF} = \xi \left[ 1 + \left( \frac{1 - \phi_s}{1 - \kappa_1 \phi_s} \right) \right] s_t.$$

For given parameter choices of $\phi_s$, $\xi$, and $\kappa_1$, we can thus directly evaluate the contribution of the earnings share over time, without taking a stand on the remaining blocks of the model. We obtain $\xi$ and $\kappa_1$ directly from the data, using $\xi = 2.002$, as explained above, and calibrating $\kappa_1 = \exp(\overline{pc}) / \exp(\overline{pc} + 1)$, where we obtain $\overline{pc} = 4.823$ as the average of our log price-to-payout ratio from our sample.[41]

The implied contributions since 1989:Q1 are reported in Table 4 for a range of possible persistences: $\phi_s \in \{0.98, 0.99, 0.995, 1.000\}$. Appendix Figure A.9 shows that the implied $py_t^{CF}$ series are reasonable, and track the true $py_t$ series well over time, even at the higher persistence values, providing support for the specification. Table 4 shows that the direct cash flow contribution is nontrivial even for a persistence of 0.980, while the assumption of a unit root would explain more than 100% of the rise in the log ME/Y ratio over this period.

The question then becomes which of these persistence values is most appropriate. A bootstrap bias corrected AR(1) estimate yields $\phi_s = 0.990$ (see Appendix Section A.6.1 for details), which corresponds to nearly half of the rise in the log ME/Y ratio being explained through the direct influence of the earnings share on cash flows. For equity values, however, the autocorrelation at the first lag is not as important as the autocorrelation at longer lags. Appendix Figure A.10 reproduces the simulated autocorrelation plots from Figure 3 for these AR(1) models, showing that even more persistent processes, such as $\phi = 0.995$, or even a unit root, provide a better fit of the observed longer autocorrelation pattern for $s_t$. Although the empirical autocorrelations in the data decay substantially with the lag order, our simulation results indicate that this is an endemic feature of sample autocorrelations given our sample size, even for extremely persistent processes. For more formal evidence, an augmented Dickey-Fuller test also fails to reject the presence of a unit root with p-value 0.164. From our results in Table 4, a persistence of 0.995 or more would imply direct cash flow contributions in excess of 60% over the 1989 - 2017 period.

Overall, this analysis implies that under minimal assumptions — that investment is proportional to output, not earnings, and that the earnings share follows a simple AR(1) process — the data are consistent with a strong direct effect of the earnings share on the value of market equity over the last three decades, and that our benchmark estimates if anything provide a lower bound on this contribution.

[41] This value of $\kappa_1$ differs slightly from our baseline results, in which $\kappa_1$ is pinned down by the equilibrium $pd$ ratio in the model.

# 7 Conclusion

In this paper, we investigate the causes of rising equity values over the post-war period. We do this by estimating a flexible parametric model of how equities are priced that allows for influence from a number of mutually uncorrelated latent components, while at the same time inferring what values those components must have taken over our sample to explain the data. The identification of mutually uncorrelated components and the specification of a log linear model allow us to precisely decompose the observed market growth into distinct component sources explaining 100% of the variation in equity values over our sample and at each point in time.

We confront our model with data on equity values, output, the earnings share of output, interest rates, and a measure of the conditional equity premium implied by options data. We find that the high returns to holding equity over the post-war era have been attributable in large part to an unpredictable string of factor share shocks that reallocated rewards away from labor compensation and toward shareholders. Indeed, our estimates suggest that at least 2.1pp of the post-war average annual log equity return in excess of a short-term interest rate is attributable to this string of reallocative shocks, rather than to genuine compensation for bearing risk. This estimate implies that the sample mean log excess equity return overstates the true risk premium by 43%.

Factor share shocks alone would have driven a 116% increase in the value of real per-capita market equity since 1989, explaining 44% of actual log growth over this period. Equity values were modestly boosted since 1989 by persistently declining interest rates and a decline in the price of risk, which contributed 14% and 18%, respectively. But growth in the real value of aggregate output contributed just 25% since 1989 and just 54% over the full sample. By contrast, economic growth was overwhelmingly important for rising equity values from 1952 to 1988, where it explained over 100% of the market's rise. Still, this 37 year period generated less than half the growth in equity wealth created in the 29 years since 1989. In this sense, factor shares, far more than economic growth, have been the preponderant measure of fundamental value in the stock market over the last three decades.

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# Appendix: For Online Publication

## A.1 Data Description

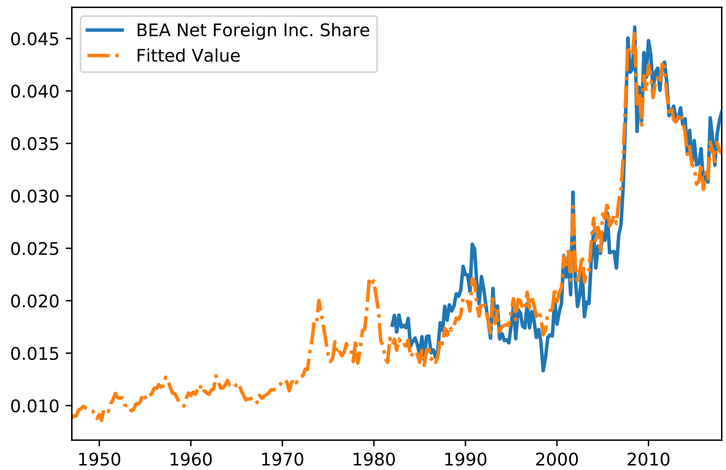
**Corporate Equity.** Corporate equity is obtained from the Flow of Funds Table B103, series code LM103164103, nonfinancial corporate business; corporate equities; liability. Unadjusted transactions estimated by Federal Reserve Board (Capital Markets and Flow of Funds Sections), using data from the following commercial sources: cash mergers and acquisitions data from Thompson Financial Services SDC database; public issuance and share repurchase data from Standard and Poor's Compustat database; and private equity issuance data from Dow Jones Private Equity Analyst and PriceWaterhouseCoopers Money tree report. Level at market value is obtained separately as the sum of the market value of the nonfinancial corporate business (FOF series LM103164103) and the financial corporate business (FOF series LM793164105). Source: Federal Reserve Board.

**Foreign Earnings.** Total earnings is the sum of domestic after-tax profits from NIPA and earnings of US multinational enterprises on their overseas operations. Total earnings are defined as as share of domestic net-value-added for the corporate sector. (See the next subsection for the sources of domestic data.)

$$\begin{aligned} E_t &= S_t Y_t \\ &= \left(S_t^D Z_t + F_t\right) Y_t. \end{aligned}$$

In the above, $F_t$ is the foreign profit share of domestic output. The measure of foreign profits in the numerator of $F_t$ is based on data from Table 4.2 of the US International Transactions in Primary Income on Direct Investment, obtained from BEA's International Data section. We refer to this simply as corporate "direct investment." Specifically, these data are from the "income on equity" row 2 of Direct investment income on assets, asset/liability basis. Note that US direct investment abroad is ownership by a US investor of at least 10 percent of a foreign business, and so excludes household portfolio investment. This series is available from 1982:Q1 to the present. To extend this series backward, we first take data on net foreign receipts from abroad (Corporate profits with IVA and CCAdj from BEA NIPA Table 1.12. (A051RC) or from Flow of Funds (FOF) Table F.3 (FA096060035.Q less corporate profits with IVA and CCAdj, domestic industries from BEA NIPA Table 1.14 (A445RC)), which is available from the post-war period onward. This series includes portfolio investment income of households as well as direct investment, but its share of domestic net-value-added for the corporate sector is highly correlated with the foreign direct investment share of net-value-added. We regress the direct investment share of net-value-added on the foreign receipts share of domestic net-value-added and then use the fitted value from this regression as the measure of $F_t$ in data pre-1982. Because the portfolio income component is relatively small, the fit of this regression is high, as seen in Figure A.1, which compares the fitted series with the actual series over the post-1982 period.

**Figure A.1: Net Foreign Income Share: Data vs. Fitted value**




*Notes:* The sample spans the period 1952:Q1-2018:Q2.

**Domestic Variables: Corp. Net Value Added, Corp. Labor Compensation, Corp. After-Tax Profits, Taxes, and Interest.** Define domestic corporate earnings $E_t^D$ as

$$E_t^D = S_t^D (1 - \tau_t) NVA_t,$$

which is equivalent to

$$E_t^D = \left[ 1 - \underbrace{\frac{LC_t}{ATP_t + LC_t}}_{\text{Labor share of labor+profit}} \right] (1 - \tau_t) NVA_t.$$

Data for the net value added ($NVA$) comes from NIPA Table 1.14 (corporate sector series codes A457RC1 and A438RC1). We use per capita real net value added, deflated by the implicit price deflator for net value added. After tax profits (ATP) for the domestic sector come from NIPA Table 1.14 (corporate sector series code: W273RC1). Corporate sector labor compensation (LC) for the domestic sector is from Table 1.14 (series code A442RC). The doestic after-tax profit share ($ATPS$) of $NVA$ is identically equal to

$$\begin{aligned} ATPS &= \frac{ATP}{ATP+LC}\frac{ATP+LC}{NVA} = \underbrace{\frac{ATP}{ATP+LC}}_{=S_t^D}\frac{NVA-(\text{taxes and interest})}{NVA} \\ &= S_t^D\left[1-\underbrace{\left(\frac{\text{taxes and interest}}{NVA}\right)}_{=\tau_t}\right] \\ &= S_t^D Z_t, \end{aligned}$$

where $S_t^D$ is the domestic after-tax profit share of combined profit plus labor compensation, "taxes and interest" is the sum of taxes on production and imports less subsidies (W325RC1), net interest and miscellaneous payments (B471RC1), business current transfer payments (Net) (W327RC1), and taxes on corporate income (B465RC1). Source: Bureau of Economic Analysis.

**Net Dividends Plus Net Repurchases (Equity Payout).** Net dividends minus net equity issuance is computed using flow of funds data. Net dividends ("netdiv") is the series named for corporate business; net dividends paid (FA096121073.Q). Net repurchases are repurchases net of share issuance, so net repurchases is the negative of net equity issuance. Net equity issuance ("netequi") is the sum of Equity Issuance for Nonfinancial corporate business; corporate equities; liability (Table F.103, series FA103164103) and Equity Issuance for domestic financial sectors; corporate equities; liability (Table F.108, series FA793164105). Since netdiv and netequi are annualized, the quarterly payout is computed as payout=(netdiv-netequi)/4. The units are in millions of dollars. Source: Federal Reserve Board.

**Price Deflators.** Implicit price deflator and GDP deflator. A chain-type price deflator for the nonfinancial corporate sector (NFCS) is obtained implicitly by dividing the net value added of nonfinancial corporate business by the chained real dollar net value added of nonfinancial corporate business from NIPA Table 1.14. This index is used to deflate net value added of the corporate sector. There is no implicit price deflator available for the whole corporate sector, so we use deflator for the non-financial corporate sector instead. The GDP deflator is used to construct a real returns and a real interest (see below). GDPDEF is retrieved from FRED. Our source is the Bureau of Economic Analysis.

**Interest Rate.** The nominal risk-free rate is measured by the 3-Month Treasury Bill rate, secondary market rate. We take the (average) quarterly 3-Month Treasury bill from FRED (code: TB3MS). A real rate is constructed by subtracting the fitted value from a regression of GDP deflator inflation onto lags of inflation from the nominal rate. Our source is the board of governors of the Federal Reserve System and the Bureau of Economic Analysis.

**Risk Premium Measure.** Our measure of the risk premium comes from Martin (2017). This paper uses option data to compute a lower bound on the equity risk premium, then argues that this lower bound is in fact tight, and a good measure of the true risk premium on the stock market. We obtain this series from the spreadsheet `epbound.xls` on Ian Martin's website, which corresponds to the value

$$EPBound_{t\to T} = 100 \times \left( R_{f,t} SVIX^2_{t\to T} - 1 \right)$$

which is equivalent to the bound on the annualized net risk premium, in percent. To translate these measures to our model's quarterly frequency, we use the risk premium measure computed over the next three months. We then convert this variable into a log return, average it over the quarter, and label it $rp_t$.

**Survey Data on Expected Average Risk-Free Rate** For the average short-term expected nominal interest rate, we use the mean forecast from the Survey of Professional Forecasters for variable BILL10, which is the 10-year annual-average forecast for returns on 3-month Treasury bills. We subtract from this the mean forecast for variable CPI10, the 10-year annual-average forecast of inflation, to obtain a survey forecast for the average real rate over the next ten years.

## A.2 A Stylized Model of Workers and Shareholders

We consider a stylized limited participation endowment economy in which wealth is concentrated in the hands of a few asset owners, or "shareholders," while most households are "workers" who finance consumption out of wages and salaries. The economy is closed. Workers own no risky asset shares and consume their labor earnings. There is no risk-sharing between workers and shareholders. A representative firm issues no new shares and buys back no shares. Cash flows are equal to output minus a wage bill,

$$C_t = Y_t - w_t N_t,$$

where $w_t$ equals the wage and $N_t$ is aggregate labor supply. The wage bill is equal to $Y_t$ times a time-varying labor share $\alpha_t$,

$$w_t N_t = \alpha_t Y_t \implies C_t = (1 - \alpha_t)\, Y_t. \tag{A.1}$$

We rule out short sales in the risky asset:

$$\theta_t^i \geq 0.$$

Asset owners not only purchase shares in the risky security, but also trade with one another in a one-period bond with price at time $t$ denoted by $q_t$. The real quantity of bonds is denoted $B_{t+1}$, where $B_{t+1} < 0$ represents a borrowing position. The bond is in zero net supply among asset owners. Asset owners could have idiosyncratic investment income $\zeta_t^i$, which is idependently and identically distributed across investors and time. The gross financial assets of investor $i$ at time $t$ are given by

$$A_t^i = \theta_t^i \,(V_t + C_t) + B_t^i.$$

The budget constraint for the $i$th investor is

$$\begin{aligned} C_t^i + B_{t+1}^i q_t + \theta_{t+1}^i V_t &= A_t^i + \zeta_t^i \\ &= \theta_t^i \,(V_t + C_t) + B_t^i + \zeta_t^i, \end{aligned} \tag{A.2}$$

where $C_t^i$ denotes the consumption of investor $i$.

A large number of identical nonrich workers, denoted by $w$, receive labor income and do not participate in asset markets. The budget constraint for the representative worker is therefore

$$C^w = \alpha_t Y_t. \tag{A.3}$$

Equity market clearing requires

$$\sum_i \theta_t^i = 1.$$

Bond market clearing requires

$$\sum_i B_t^i = 0.$$

Aggregating (A.2) and (A.3) and imposing both market clearing and (A.1) implies that aggregate (worker plus shareholder) consumption $C_t^{Agg}$ is equal to total output $Y_t$. Aggregating over the budget constraint of shareholders shows that their consumption is equal to the capital share times aggregate consumption $C_t^{Agg}$:

$$C_t^S = C_t = \underbrace{(1-\alpha_t)}_{KS_t} C_t^{Agg}.$$

A representative shareholder who owns the entire corporate sector will therefore have consumption equal to $C_t^{Agg} \cdot KS_t$. This reasoning goes through as an approximation if workers own a small fraction of the corporate sector even if there is some risk-sharing in the form of risk-free borrowing and lending between workers and shareholders, as long as any risk-sharing across these groups is imperfect. While individual shareholders can smooth out transitory fluctuations in income by buying and selling assets, shareholders as a whole are less able to do so since purchases and sales of any asset must net to zero across all asset owners.

## A.3 Model Solution

**Perturbation Details.** The derivation of the perturbed stochastic discount factor (11) follows here. We seek a perturbation of the terms determining the risk exposure of the SDF, the nonlinear expression $\tilde{m}_{t+1} = -x_t \Delta c_{t+1}$. Our perturbation includes terms linear in both the state vector $\mathbf{z}_t$, the shock vector $\boldsymbol{\varepsilon}_{t+1}$, and interactions between the two, while omitting all other higher-order terms. While our solution could handle terms quadratic in $\mathbf{z}_t$, they would be irrelevant since the term $\mu_t$ would implicitly offset them in all states. On the other hand, terms quadratic in $\boldsymbol{\varepsilon}_{t+1}$ would influence the solution, but would break our solution methodology.

To derive the perturbed stochastic discount factor, we first express $\hat{m}_t$ in terms of the current period's states and next period's shocks:

$$\hat{m}_{t+1} = -x_t \left\{ \log\Big[\exp\Big(\bar{s} + \mathbf{1}' \left(\boldsymbol{\Phi}_s \tilde{\mathbf{s}}_t + \boldsymbol{\varepsilon}_{s,t+1}\right)\Big) - \omega\Big] - \log\Big[\exp\Big(\bar{s} + \mathbf{1}'\tilde{\mathbf{s}}_t\Big) - \omega\Big] + g + \varepsilon_{y,t+1} \right\}.$$

Evaluating the derivatives of this expression with respect to shocks and states, we obtain

$$\begin{aligned}
\frac{\partial \hat{m}_{t+1}}{\partial \tilde{\mathbf{x}}_t} &= -\Delta c_{t+1} \mathbf{1}' \\
\frac{\partial \hat{m}_{t+1}}{\partial \tilde{\mathbf{s}}_t} &= -x_t \left\{ \left( \frac{S_{t+1}}{S_{t+1} - \omega} \right) \mathbf{1}' \Phi - \left( \frac{S_t}{S_t - \omega} \right) \mathbf{1}' \right\} \\
\frac{\partial \hat{m}_{t+1}}{\partial \boldsymbol{\varepsilon}_{s,t+1}} &= -x_t \left( \frac{S_{t+1}}{S_{t+1} - \omega} \right) \mathbf{1}' \\
\frac{\partial \hat{m}_{t+1}}{\partial \varepsilon_{y,t+1}} &= -x_t \\
\frac{\partial^2 \hat{m}_{t+1}}{\partial \boldsymbol{\varepsilon}_{s,t+1} \partial \tilde{\mathbf{x}}_t} &= -\mathbf{1} \left( \frac{S_{t+1}}{S_{t+1} - \omega} \right) \mathbf{1}' \\
\frac{\partial^2 \hat{m}_{t+1}}{\partial \boldsymbol{\varepsilon}_{s,t+1} \partial \tilde{\mathbf{s}}_t} &= x_t \mathbf{1} \left( \frac{S_{t+1}}{S_{t+1} - \omega} \right) \left( \frac{\omega}{S_{t+1} - \omega} \right) \mathbf{1}' \boldsymbol{\Phi}_s
\end{aligned}$$

We therefore approximate

$$\begin{aligned}
\hat{m}_{t+1} \simeq & -\bar{x} g - g \mathbf{1}' \tilde{\mathbf{x}}_t - \bar{x} \xi \mathbf{1}' (\boldsymbol{\Phi}_s - I) \tilde{\mathbf{s}}_t - \bar{x} \xi \mathbf{1}' \boldsymbol{\varepsilon}_{s,t+1} - \bar{x} \varepsilon_{y,t+1} \\
& - \tilde{\mathbf{x}}_t' \mathbf{1} \xi \mathbf{1}' \boldsymbol{\varepsilon}_{s,t+1} + \tilde{\mathbf{s}}_t' \boldsymbol{\Phi}_s' \bar{x} \xi (\xi - 1) \mathbf{1}' \boldsymbol{\varepsilon}_{s,t+1} - \tilde{\mathbf{x}}_t' \mathbf{1} \varepsilon_{y,t+1} \\
= & \cdots - \big( x_t \xi - \bar{x} \xi (\xi - 1) (\mathbf{1}' \boldsymbol{\Phi}_s \tilde{\mathbf{s}}_t) \big) \mathbf{1}' \boldsymbol{\varepsilon}_{s,t+1} - x_t \varepsilon_{y,t+1}
\end{aligned}$$

where the omitted terms are known at time $t$, and whose exact values are thus irrelevant to our solution as they will be directly offset by the implicitly defined $\mu_t$. Combining this result with the identity

$$\mathbb{E}_t[s_{t+1}] = \bar{s} + \mathbf{1}' \boldsymbol{\Phi}_s \tilde{\mathbf{s}}_t$$

and rearranging yields (11).

**Price-Payout Ratio** This section derives the coefficients of the main asset pricing equation (13). To begin, define for convenience the variables

$$\begin{aligned}
u_{t+1} &= \log(PC_{t+1} + 1) - pc_t \\
q_{t+1} &= m_{t+1} + \Delta c_{t+1}
\end{aligned}$$

so that $m_{t+1} + r_{t+1} = u_{t+1} + q_{t+1}$. Applying the log linear approximation to $\log(PC_{t+1} + 1)$ and substituting in our guessed functional form (13) yields

$$\begin{aligned}
u_{t+1} &= \log(PC_{t+1} + 1) - pd_t \\
&= \kappa_0 + \kappa_1 \Big( A_0 + \mathbf{A}_s' \tilde{\mathbf{s}}_{t+1} + \mathbf{A}_x' \tilde{\mathbf{x}}_{t+1} + \mathbf{A}_\delta' \tilde{\boldsymbol{\delta}}_{t+1} \Big) - \Big( A_0 + \mathbf{A}_s' \tilde{\mathbf{s}}_t + \mathbf{A}_x' \tilde{\mathbf{x}}_t + \mathbf{A}_\delta' \tilde{\boldsymbol{\delta}}_t \Big) \\
&= \kappa_0 + (\kappa_1 - 1) A_0 + \mathbf{A}_s' \left( \kappa_1 \boldsymbol{\Phi}_s - \mathbf{I} \right) \tilde{\mathbf{s}}_t + \mathbf{A}_x' (\kappa_1 \Phi_x - \mathbf{I}) \tilde{\mathbf{x}}_t + \mathbf{A}_\delta' (\kappa_1 \boldsymbol{\Phi}_\delta - \mathbf{I}) \tilde{\boldsymbol{\delta}}_t \\
&\qquad + \kappa_1 \mathbf{A}_s' \boldsymbol{\varepsilon}_{s,t+1} + \kappa_1 \mathbf{A}_x' \boldsymbol{\varepsilon}_{x,t+1} + \kappa_1 \mathbf{A}_\delta' \boldsymbol{\varepsilon}_{\delta,t+1}.
\end{aligned}$$

Now turning to $q_{t+1}$, we can expand the expression to yield

$$q_{t+1} = -\delta_t - \mu_t + g + \xi \mathbb{E}_t \Delta s_{t+1} + (1 - \gamma_{s,t}) \xi \mathbf{1}' \boldsymbol{\varepsilon}_{s,t+1} + (1 - x_t) \varepsilon_{y,t+1}$$

where

$$\begin{aligned}
\gamma_{s,t} &= x_t - \bar{x}(1 - \xi)(\mathbb{E}_t[s_{t+1}] - \bar{s}) \\
&= \bar{x} + \mathbf{1}' \tilde{\mathbf{x}}_t - \bar{x}(1 - \xi) \mathbf{1}' \Phi \tilde{\mathbf{s}}_t \\
&= \bar{x} + \mathbf{1}' \tilde{\mathbf{x}}_t + \boldsymbol{\Gamma}' \tilde{\mathbf{s}}_t.
\end{aligned}$$

for

$$\boldsymbol{\Gamma}' = -\bar{x}(1 - \xi) \mathbf{1}' \boldsymbol{\Phi}_s.$$

Next, we apply our fundamental asset pricing equation $0 = \log \mathbb{E}_t \left[ q_{t+1} + u_{t+1} \right]$, which under lognormality implies

$$0 = \mathbb{E}_t[q_{t+1}] + \mathbb{E}_t[u_{t+1}] + \frac{1}{2} \mathrm{Var}_t(q_{t+1}) + \frac{1}{2} \mathrm{Var}_t(u_{t+1}) + \mathrm{Cov}(q_{t+1}, u_{t+1}).$$

These moments can be calculated as

$$\begin{aligned}
\mathbb{E}_t[q_{t+1}] &= -\delta_t - \mu_t + g - \xi \mathbf{1}' (\mathbf{I} - \boldsymbol{\Phi}_s) \tilde{\mathbf{s}}_t \\
\mathbb{E}_t[z_{t+1}] &= \kappa_0 + (\kappa_1 - 1) A_0 + \mathbf{A}_s' \left( \kappa_1 \boldsymbol{\Phi}_s - \mathbf{I} \right) \tilde{\mathbf{s}}_t + \mathbf{A}_x' (\kappa_1 \Phi_x - \mathbf{I}) \tilde{\mathbf{x}}_t + \mathbf{A}_\delta' (\kappa_1 \boldsymbol{\Phi}_\delta - \mathbf{I}) \tilde{\boldsymbol{\delta}}_t \\
\mathrm{Var}_t(q_{t+1}) &= (1 - \gamma_{s,t})^2 \xi^2 (\mathbf{1}' \boldsymbol{\Sigma}_s \mathbf{1}) + (1 - x_t)^2 \sigma_a^2 \\
\mathrm{Var}_t(z_{t+1}) &= \kappa_1^2 \Big( \mathbf{A}_s' \boldsymbol{\Sigma}_s \mathbf{A}_s' + \mathbf{A}_x' \boldsymbol{\Sigma}_x \mathbf{A}_x + \mathbf{A}_\delta' \boldsymbol{\Sigma}_r \mathbf{A}_\delta \Big) \\
\mathrm{Cov}_t(q_{t+1}, z_{t+1}) &= \kappa_1 \xi (1 - \gamma_{s,t}) \mathbf{A}_s' \boldsymbol{\Sigma}_s \mathbf{1}
\end{aligned}$$

Substituting, we obtain

$$
\begin{aligned}
0 = & -\bar{\delta} + g + \kappa_0 + (\kappa_1 - 1)A_0 + \frac{1}{2}\Big((1-2\bar{x})\xi^2(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1}) + (1-2\bar{x})\sigma_a^2\Big) \\
& + \frac{1}{2}\kappa_1^2\Big(\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{A}_s' + \mathbf{A}_x'\boldsymbol{\Sigma}_x\mathbf{A}_x + \mathbf{A}_\delta'\boldsymbol{\Sigma}_r\mathbf{A}_\delta\Big) + \kappa_1\xi(1-\bar{x})\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{1} \\
& + \Big[-\xi\mathbf{1}'(\mathbf{I} - \boldsymbol{\Phi}_s) + \mathbf{A}_s'(\kappa_1\boldsymbol{\Phi}_s - \mathbf{I}) - \xi^2(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1})\boldsymbol{\Gamma}_s' - \kappa_1\xi(\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{1})\boldsymbol{\Gamma}_s'\Big]\tilde{\mathbf{s}}_t \\
& + \Big[\mathbf{A}_x'(\kappa_1\Phi_x - \mathbf{I}) - \xi^2(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1})\mathbf{1}' - \mathbf{1}'\sigma_a^2 - \kappa_1\xi\mathbf{1}'\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{1}\Big]\tilde{\mathbf{x}}_t \\
& + \Big[\mathbf{A}_\delta'(\kappa_1\boldsymbol{\Phi}_\delta - \mathbf{I}) - \mathbf{1}\Big]\tilde{\boldsymbol{\delta}}_t.
\end{aligned}
$$

Applying the method of undetermined coefficients now yields the solutions

$$
\begin{aligned}
\mathbf{A}_s' &= \Big[\xi\mathbf{1}'(\mathbf{I} - \boldsymbol{\Phi}_s) - \xi^2(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1})\boldsymbol{\Gamma}_s'\Big]\Big[(\kappa_1\boldsymbol{\Phi}_s - \mathbf{I}) + \kappa_1\xi\boldsymbol{\Sigma}_s\mathbf{1}\boldsymbol{\Gamma}_s'\Big]^{-1} \\
\mathbf{A}_x' &= \Big[\Big(\xi^2(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1}) + \sigma_a^2 + \kappa_1\xi(\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{1})\Big)\mathbf{1}'\Big](\kappa_1\Phi_x - \mathbf{I})^{-1} \\
\mathbf{A}_\delta' &= \mathbf{1}'(\kappa_1\boldsymbol{\Phi}_\delta - \mathbf{I})^{-1}
\end{aligned}
$$

while the constant term must solve

$$
\begin{aligned}
0 = & -\bar{\delta} + g + \kappa_0 + (\kappa_1 - 1)A_0 + \frac{1}{2}\Big((1-2\bar{x})\xi^2(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1}) + (1-2\bar{x})\sigma_a^2\Big) \\
& + \frac{1}{2}\kappa_1^2\Big(\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{A}_s' + \mathbf{A}_x'\boldsymbol{\Sigma}_x\mathbf{A}_x + \mathbf{A}_\delta'\boldsymbol{\Sigma}_r\mathbf{A}_\delta\Big) + \kappa_1\xi(1-\bar{x})\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{1}.
\end{aligned}
\tag{A.4}
$$

### A.3.1 Equilibrium Selection

The parameters $\kappa_0$ and $\kappa_1$ determine the steady state *pc* (price-payout ratio), which depends on $A_0$. But since $\kappa_0$ and $\kappa_1$ are both themselves nonlinear functions of $A_0$, the equilibrium condition (A.5) is also nonlinear, leading to the possibility that multiple solutions, or no solution, exists. In fact, we confirm that both of these outcomes can occur in our numerical solutions. However, our numerical results indicate that, when there is more than one solution there are at most two, and one can be discarded because it is economically implausible.

To see this, consider the log risk premium, given in equilibrium by

$$
\mathbb{E}[r_{t+1}] - r_{f,t} = -\frac{1}{2}\text{Var}(r_{t+1}) - \text{Cov}(m_{t+1}, r_{t+1}).
$$

In the case where there are two solutions, one solution typically has a plausible level for the steady state *pc*, and implies that higher *pc* ratios (which take different values depending on where in the posterior distribution of model parameters we evaluate the function) coincide with lower risk premia $\mathbb{E}[r_{t+1}] - r_{f,t}$ and a lower absolute covariance with the SDF (i.e., a less negative $\text{Cov}(m_{t+1}, r_{t+1})$). This solution is economically reasonable. By contrast, when there is a second solution, we have always found that it is characterized by values for $pc$ that are higher than the economically reasonable solution (e.g., a value for $PC = \exp(pc)$ of nearly 3,000 at the posterior mode). Instead, these solutions are an artifact of the loglinear approximation we use, when applied too far from the reasonable region of the state space. In summary, since the higher $pc$ solution typically implies extreme values and unreasonable behavior of $pc$, we select between these solutions by enforcing that the equilibrium chosen always chooses the lower $pc$ solution.

### A.3.2 Expected Returns

Combining the relations

$$\begin{aligned} 0 &= \log \mathbb{E}_t[M_{t+1}R_{t+1}] \\ &= \mathbb{E}_t[m_{t+1}] + \mathbb{E}_t[r_{t+1}] + \frac{1}{2}\text{Var}_t(m_{t+1}) + \frac{1}{2}\text{Var}_t(r_{t+1}) + \text{Cov}_t(m_{t+1}, r_{t+1}) \\ -r_{f,t} &= \log \mathbb{E}_t[M_{t+1}] \\ &= \mathbb{E}_t[m_{t+1}] + \frac{1}{2}\text{Var}_t(m_{t+1}) \end{aligned}$$

and rearranging, we obtain

$$\begin{aligned} \log \mathbb{E}_t[R_{t+1}/R_{f,t}] &= \mathbb{E}_t[r_{t+1}] + \frac{1}{2}\text{Var}_t(r_{t+1}) - r_{f,t} \\ &= -\text{Cov}_t(m_{t+1}, r_{t+1}). \end{aligned}$$

Since

$$\begin{aligned} r_{t+1} &= \text{const}_t + \underbrace{\kappa_1\Big(\mathbf{A}_s'\boldsymbol{\varepsilon}_{s,t+1} + \mathbf{A}_x'\boldsymbol{\varepsilon}_{x,t+1} + \mathbf{A}_\delta\boldsymbol{\varepsilon}_{\delta,t+1}\Big)}_{pc\text{ growth}} + \underbrace{\xi\mathbf{1}'\boldsymbol{\varepsilon}_{s,t+1} + \varepsilon_{a,t+1}}_{\text{cash flow growth}} \\ m_{t+1} &= \text{const}_t - \gamma_{s,t}\mathbf{1}'\boldsymbol{\varepsilon}_{s,t+1} - x_t\varepsilon_{a,t+1} \end{aligned}$$

we obtain

$$\text{Cov}_t(m_{t+1}, r_{t+1}) = -\gamma_{s,t}\big(\kappa_1\mathbf{A}_s' + \xi\mathbf{1}'\big)\boldsymbol{\Sigma}_s\mathbf{1} - x_t\sigma_a^2$$

Substituting for $\gamma_{s,t}$ and rearranging yields (14).

### A.3.3 Forecasting Real Rates

This section derives our 40Q average real rate forecast in the model. As an intermediate step, note that for a given matrix $A$, the geometric sum, assuming it converges, is equal to

$$\sum_{j=0}^{\infty} A^j = I + A \sum_{j=0}^{\infty} A^j$$

which implies

$$\sum_{j=0}^{\infty} A^j = (I - A)^{-1}.$$

Similarly the partial sum can be obtained as

$$\begin{aligned}\sum_{j=0}^{N-1} A^j = \sum_{j=0}^{\infty} A^j - \sum_{j=N}^{\infty} A_j = \sum_{j=0}^{\infty} A^j - A^N \left( \sum_{j=0}^{\infty} A^j \right) = (I - A)^{-1} - (I - A)^{-1} A^N \\ = (I - A)^{-1}(I - A^N).\end{aligned}$$

Applying this to our interest rate forecast, we have

$$\bar{r}_{f,t}^N = \frac{1}{N} \sum_{j=0}^{N-1} \mathbb{E}_t \delta_{t+j} = \bar{\delta} + \frac{1}{N} \mathbf{1}' \sum_{j=0}^{N-1} \mathbb{E}_t \tilde{\boldsymbol{\delta}}_{t+j}.$$

Since our law of motion for $\delta$ implies $\mathbb{E}_t \tilde{\boldsymbol{\delta}}_{t+j} = \boldsymbol{\Phi}_\delta^j \tilde{\boldsymbol{\delta}}_t$, we can substitute to obtain

$$\begin{aligned}\bar{r}_{f,t}^N &= \bar{\delta} + \frac{1}{N} \mathbf{1}' \left( \sum_{j=0}^{N-1} \boldsymbol{\Phi}_\delta^j \right) \tilde{\boldsymbol{\delta}}_t \\ &= \bar{\delta} + \frac{1}{N} \mathbf{1}' (I - \boldsymbol{\Phi}_\delta)^{-1} (I - \boldsymbol{\Phi}_\delta^N) \tilde{\boldsymbol{\delta}}_t\end{aligned}$$

where the last line follows from our partial geometric sum formula above.

### A.3.4 Bond Pricing

We can represent our model in the form

$$\log M_{t+1} = -\delta_t - \frac{1}{2} \Lambda_t' \Sigma \Lambda_t - \Lambda_t' \varepsilon_{t+1}$$

where

$$\Lambda_t = \Lambda_0 + \Lambda_1 \mathbf{z}_t$$
$$= \underbrace{\begin{bmatrix} \xi\bar{x} \\ 0 \\ 0 \\ \bar{x} \end{bmatrix}}_{\Lambda_0} + \underbrace{\begin{bmatrix} -\bar{x}\xi(\xi-1)\mathbf{1}'\mathbf{\Phi}_s & \xi\mathbf{1}' & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \mathbf{1}' & 0 & 0 \end{bmatrix}}_{\Lambda_1} \underbrace{\begin{bmatrix} \tilde{\mathbf{s}}_t \\ \tilde{\mathbf{x}}_t \\ \tilde{\boldsymbol{\delta}}_t \\ \Delta a_t \end{bmatrix}}_{\mathbf{z}_t}.$$

To price a zero-coupon bond of maturity $n$, we guess that the log bond price $p_{n,t}$ takes the functional form

$$p_{n,t} = A_n + B_n' \mathbf{z}_t.$$

This guess is trivially verified for $n = 0$, with $p_{0,t} = 0$ implying the initialization $A_0 = 0$, $B_0' = 0$. To prove by induction, assume the claim holds for $n$. Then we have

$$\begin{aligned}
p_{n+1,t} &= \log E_t \exp\left\{-\delta_t - \frac{1}{2}\Lambda_t'\Sigma\Lambda_t - \Lambda_t'\varepsilon_{t+1} + A_n + B_n'\Phi_z z_t + B_n'\varepsilon_{t+1}\right\} \\
&= \log E_t \exp\left\{-\delta_t - \frac{1}{2}\Lambda_t'\Sigma\Lambda_t + (B_n' - \Lambda_t')\varepsilon_{t+1} + A_n + B_n'\Phi_z z_t\right\} \\
&= \log E_t \exp\left\{-\delta_0 - \delta_1' z_t - \frac{1}{2}\Lambda_t'\Sigma\Lambda_t + (B_n' - \Lambda_t')\varepsilon_{t+1} + A_n + B_n'\Phi_z z_t\right\} \\
&= -\delta_0 - \delta_1' z_t - \frac{1}{2}\Lambda_t'\Sigma\Lambda_t + \frac{1}{2}B_n'\Sigma B_n - B_n'\Sigma\Lambda_t + \frac{1}{2}\Lambda_t'\Sigma\Lambda_t + A_n + B_n'\Phi_z z_t \\
&= -\delta_0 - \delta_1' z_t + \frac{1}{2}B_n'\Sigma B_n - B_n'\Sigma\Lambda_0 - B_n'\Sigma\Lambda_1 z_t + A_n + B_n'\Phi_z z_t \\
&= \left(-\delta_0 + \frac{1}{2}B_n'\Sigma B_n - B_n'\Sigma\Lambda_0 + A_n\right) + \left(-\delta_1' - B_n'\Sigma\Lambda_1 + B_n'\Phi_z\right) z_t
\end{aligned}$$

which implies

$$\begin{aligned}
A_{n+1} &= -\delta_0 + \frac{1}{2}B_n'\Sigma B_n - B_n'\Sigma\Lambda_0 + A_n \\
B_{n+1}' &= -\delta_1' - B_n'\Sigma\Lambda_1 + B_n'\Phi_z.
\end{aligned}$$

This both completes the proof and provides the recursion used to compute long-term real bond prices in our model.

## A.4 Estimation Details

This section provides additional details on our state space specification and estimation procedure.

**Measurement Equation.** To construct measurement equation, we relate our observed series to the model's primitive parameters and latent state variables using the following system of equations:

$$
\begin{aligned}
ey_t &= \mathbf{1}'\mathbf{s}_t \\
r_{f,t} &= \mathbf{1}'\boldsymbol{\delta}_t \\
\bar{r}^{40}_{f,t} &= \frac{1}{40}\sum_{j=0}^{40-1}\mathbb{E}_t\delta_{t+j} = \bar{\delta} + \frac{1}{40}\mathbf{1}'(I-\boldsymbol{\Phi}_\delta)^{-1}(I-\boldsymbol{\Phi}_\delta^{40})\tilde{\boldsymbol{\delta}}_t + \nu \\
py_t &= pc_t + cy_t \\
&= \overline{py} + (\mathbf{A}_s' + \xi')\widetilde{\mathbf{s}}_t + \mathbf{A}_\delta'\widetilde{\boldsymbol{\delta}}_t + \mathbf{A}_x'\widetilde{\mathbf{x}}_t \\
\Delta y_t &= g + \Delta a_t \\
rp_t &= \left(\Psi + \sigma_a^2\right)\left(\bar{x} + \mathbf{1}'\tilde{\mathbf{x}}_t\right) - \Psi\boldsymbol{\Gamma}'\tilde{\mathbf{s}}_t.
\end{aligned}
$$

where $cy_t = c_t - y_t$, and $\overline{py} = A_0 + \bar{c} + \boldsymbol{\xi}'\bar{\mathbf{s}}$.[42] For the real risk-free rate forecast ($\bar{r}^{40}_{f,t}$), the parameter $\nu$ allows for an average difference between the model and data forecasts due to the different inflation series used (CPI for the forecasts vs. the GDP deflator for the model), as well as any average bias among the forecasts.[43]

**Time Variation in State Space.** To begin, we provide additional details on time variation in our state space measurement equation (15). Because our risk premium measure $rp_t$ is not available over the full sample, we use a time varying measurement equation to accommodate the missing values. In periods when our measure of the risk premium is available

[42] We note that $\Delta a_t$ is exactly pinned down by the observation equation for $\Delta y_t$.

[43] A full derivation of the formula for $\bar{r}^{40}_{f,t}$ can be found in Appendix A.3.3.

(1996:Q1 to 2012:Q1), equation (15) takes the form

$$\begin{bmatrix} ey_t \\ r_{ft} \\ \bar{r}^{40}_{f,t} \\ py_t \\ \Delta y_t \\ rp_t \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} \mathbf{z}_t + \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix}$$

where $\mathbf{H}_2$ and $\mathbf{b}_2$ are the rows of the measurement matrix and constant vector that compute the implied value for $rp_t$, and $\mathbf{H}_1$ and $\mathbf{b}_1$ are the respective values for the other observables. In periods when data on $rp_t$ is not available, equation (15) instead takes the form

$$\begin{bmatrix} ey_t \\ r_{ft} \\ \bar{r}^{40}_{f,t} \\ py_t \\ \Delta y_t \end{bmatrix} = \mathbf{H}_1 \mathbf{z}_t + \mathbf{b}_1.$$

A completely analogous procedure is used for the SPF forecast variable $\bar{r}^{40}_{f,t}$, since these forecasts are also not available over the entire sample. During periods when the forecast data are available, we expand the measurement equation (15) to include an additional row, while for periods when the data are not available, we omit this row.

**MCMC Details.** We next describe the procedure used to obtain the parameter draws. First, because some of our variables are bounded by definition (e.g., volatilities cannot be negative), we define a set of parameter vectors satisfying these bounds denoted $\Theta$. We exclude parameters outside of this set, which formally means that we apply a Bayesian prior

$$p(\boldsymbol{\theta}) = \begin{cases} \text{const} & \text{for } \boldsymbol{\theta} \in \Theta \\ 0 & \text{for } \boldsymbol{\theta} \notin \Theta \end{cases}$$

Our restrictions on $\Theta$ are as follows: all volatilities ($\sigma$) and the average risk price $\bar{x}$ are bounded below at zero. All persistence parameters ($\phi$) are bounded between zero and unity.

With these bounds set, we can evaluate the posterior by

$$\pi(\boldsymbol{\theta}) = L(y|\boldsymbol{\theta}) p(\boldsymbol{\theta}).$$

so that the posterior is simply proportional to the likelihood over $\Theta$ and is equal to zero outside of $\Theta$.

To draw from this posterior, we use a Random Walk Metropolis Hastings algorithm. We initialize the first draw $\boldsymbol{\theta}_0$ at the mode, and then iterate on the following algorithm:

1. Given $\boldsymbol{\theta}_j$, draw a proposal $\boldsymbol{\theta}^*$ from the distribution $\mathcal{N}(\boldsymbol{\theta}_j, c\boldsymbol{\Sigma}_\theta)$ for some scalar $c$ and matrix $\boldsymbol{\Sigma}_\theta$ defined below.

2. Compute the ratio
$$\alpha = \frac{\pi(\boldsymbol{\theta}^*)}{\pi(\boldsymbol{\theta}_j)}.$$

3. Draw $u$ from a Uniform [0, 1] distribution.

4. If $u < \alpha$, we accept the proposed draw and set $\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}^*$. Otherwise, we reject the draw and set $\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j$.

For the covariance term, we initialize $\boldsymbol{\Sigma}_\theta$ to be the inverse Hessian of the log likelihood function at the mode. Once we have saved 10,000 draws, we begin updating $\boldsymbol{\Sigma}_\theta$ to be the sample covariance of the draws to date, following Haario, Saksman, Tamminen et al. (2001), with the matrix re-computed after every 1,000 saved draws. For the scaling parameter $c$, we initialize it at 2.4/length($\boldsymbol{\theta}$) as recommended in Gelman, Stern, Carlin, Dunson, Vehtari and Rubin (2013). To target an acceptance rate for our algorithm of 25%, we adapt the approach of Herbst and Schorfheide (2014) in updating

$$c_{new} = c_{old} \cdot \left(0.95 + 0.1 \frac{\exp(16(x - 0.25))}{1 + \exp(16(x - 0.25))}\right)$$

after every 1,000 saved draws, where $c_{old}$ is the pre-update value of $c$.

With these methods in place, we compute our estimation results in ten independent chains, each containing 550,000 draws of $\boldsymbol{\theta}$. We discard the first 50,000 draws from each chain as burn-in, leaving 5,000,000 parameter draws. Since these draws are highly serially correlated, we increase computational efficiency in most applications by using every 500th draw, leaving a total of 10,000 draws over which our margins of parameter uncertainty are computed.

## A.5 Extension: Persistent Output Growth

Following Bansal and Yaron (2004) and the large subsequent literature, we can consider a model with persistent output growth. In this case, the output growth technology becomes

$$\Delta y_{t+1} = \bar{g} + \mathbf{1}'\tilde{\mathbf{g}}_t + \varepsilon_{a,t+1}, \qquad \varepsilon_{a,t+1} \overset{\text{iid}}{\sim} N(0, \sigma_a^2)$$
$$\tilde{g}_{t+1} = \mathbf{\Phi}_g \tilde{\mathbf{g}}_t + \boldsymbol{\varepsilon}_{g,t+1}, \qquad \boldsymbol{\varepsilon}_{g,t+1} \overset{\text{iid}}{\sim} N(0, \mathbf{1}'\mathbf{\Sigma}_g \mathbf{1})$$

As before, we can define

$$u_{t+1} = \log(PC_{t+1} + 1) - pc_t$$
$$q_{t+1} = m_{t+1} + \Delta c_{t+1}$$

so that $m_{t+1} + r_{t+1} = u_{t+1} + q_{t+1}$. Applying the log linear approximation to $\log(PC_{t+1} + 1)$ and substituting in our guessed functional form, which now includes a loading on $\tilde{\mathbf{g}}_t$, yields

$$\begin{aligned} u_{t+1} &= \log(PC_{t+1} + 1) - pd_t \\ &= \kappa_0 + \kappa_1 \Big(A_0 + \mathbf{A}_s'\tilde{\mathbf{s}}_{t+1} + \mathbf{A}_x'\tilde{\mathbf{x}}_{t+1} + \mathbf{A}_\delta'\tilde{\boldsymbol{\delta}}_{t+1} + \mathbf{A}_g'\tilde{\mathbf{g}}_{t+1}\Big) - \Big(A_0 + \mathbf{A}_s'\tilde{\mathbf{s}}_t + \mathbf{A}_x'\tilde{\mathbf{x}}_t + \mathbf{A}_\delta'\tilde{\boldsymbol{\delta}}_t + \mathbf{A}_g'\tilde{\mathbf{g}}_t\Big) \\ &= \kappa_0 + (\kappa_1 - 1)A_0 + \mathbf{A}_s'\left(\kappa_1 \mathbf{\Phi}_s - \mathbf{I}\right)\tilde{\mathbf{s}}_t + \mathbf{A}_x'(\kappa_1 \Phi_x - \mathbf{I})\tilde{\mathbf{x}}_t + \mathbf{A}_\delta'(\kappa_1 \mathbf{\Phi}_\delta - \mathbf{I})\tilde{\boldsymbol{\delta}}_t + \mathbf{A}_g'(\kappa_1 \mathbf{\Phi}_g - \mathbf{I})\tilde{\mathbf{g}}_t \\ &\quad + \kappa_1 \mathbf{A}_s'\boldsymbol{\varepsilon}_{s,t+1} + \kappa_1 \mathbf{A}_x'\boldsymbol{\varepsilon}_{x,t+1} + \kappa_1 \mathbf{A}_\delta'\boldsymbol{\varepsilon}_{\delta,t+1} + \kappa_1 \mathbf{A}_g'\boldsymbol{\varepsilon}_{g,t+1}. \end{aligned}$$

Ae before we can expand $q_{t+1}$ to yield

$$q_{t+1} = -\delta_t - \mu_t + \bar{g} + \mathbf{1}'\tilde{\mathbf{g}}_t + \xi \mathbb{E}_t \Delta s_{t+1} + (1 - \gamma_{s,t})\xi \mathbf{1}'\boldsymbol{\varepsilon}_{s,t+1} + (1 - x_t)\varepsilon_{a,t+1}$$

to account for the fact that expected cash flow growth now depends on current growth. As before,

$$\gamma_{s,t} = \bar{x} + \mathbf{1}'\tilde{\mathbf{x}}_t + \mathbf{\Gamma}'\tilde{\mathbf{s}}_t$$
$$\mathbf{\Gamma}' = -\bar{x}(1 - \xi)\mathbf{1}'\mathbf{\Phi}_s.$$

Next, we apply our fundamental asset pricing equation $0 = \log \mathbb{E}_t\left[q_{t+1} + u_{t+1}\right]$, which under lognormality implies

$$0 = \mathbb{E}_t[q_{t+1}] + \mathbb{E}_t[u_{t+1}] + \frac{1}{2}\text{Var}_t(q_{t+1}) + \frac{1}{2}\text{Var}_t(u_{t+1}) + \text{Cov}(q_{t+1}, u_{t+1}).$$

These moments can be calculated as

$$
\begin{aligned}
\mathbb{E}_t[q_{t+1}] &= -\delta_t - \mu_t + \bar{g} + \mathbf{1}'\tilde{\mathbf{g}}_t - \xi \mathbf{1}'(\mathbf{I} - \mathbf{\Phi}_s)\tilde{\mathbf{s}}_t \\
\mathbb{E}_t[z_{t+1}] &= \kappa_0 + (\kappa_1 - 1)A_0 + \mathbf{A}_s' \left(\kappa_1 \mathbf{\Phi}_s - \mathbf{I}\right) \tilde{\mathbf{s}}_t + \mathbf{A}_x'(\kappa_1 \Phi_x - \mathbf{I})\tilde{\mathbf{x}}_t + \mathbf{A}_\delta'(\kappa_1 \mathbf{\Phi}_\delta - \mathbf{I})\tilde{\boldsymbol{\delta}}_t \\
&\quad + \mathbf{A}_g'(\kappa_1 \mathbf{\Phi}_g - \mathbf{I})\tilde{\mathbf{g}}_t \\
\mathrm{Var}_t(q_{t+1}) &= (1 - \gamma_{s,t})^2 \xi^2 (\mathbf{1}'\mathbf{\Sigma}_s \mathbf{1}) + (1 - x_t)^2 \sigma_a^2 \\
\mathrm{Var}_t(z_{t+1}) &= \kappa_1^2 \Big( \mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{A}_s' + \mathbf{A}_x' \mathbf{\Sigma}_x \mathbf{A}_x + \mathbf{A}_\delta' \mathbf{\Sigma}_r \mathbf{A}_\delta + \mathbf{A}_g' \mathbf{\Sigma}_g \mathbf{A}_g \Big) \\
\mathrm{Cov}_t(q_{t+1}, z_{t+1}) &= \kappa_1 \xi (1 - \gamma_{s,t}) \mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1}
\end{aligned}
$$

Substituting, we obtain

$$
\begin{aligned}
0 = -\bar{\delta} + g + \kappa_0 + (\kappa_1 - 1)A_0 &+ \frac{1}{2}\Big( (1 - 2\bar{x})\xi^2 (\mathbf{1}'\mathbf{\Sigma}_s \mathbf{1}) + (1 - 2\bar{x})\sigma_a^2 \Big) \\
&+ \frac{1}{2}\kappa_1^2 \Big( \mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{A}_s' + \mathbf{A}_x' \mathbf{\Sigma}_x \mathbf{A}_x + \mathbf{A}_\delta' \mathbf{\Sigma}_r \mathbf{A}_\delta + \mathbf{A}_g' \mathbf{\Sigma}_g \mathbf{A}_g \Big) + \kappa_1 \xi (1 - \bar{x}) \mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1} \\
&+ \Big[ -\xi \mathbf{1}'(\mathbf{I} - \mathbf{\Phi}_s) + \mathbf{A}_s' \left(\kappa_1 \mathbf{\Phi}_s - \mathbf{I}\right) - \xi^2 (\mathbf{1}'\mathbf{\Sigma}_s \mathbf{1}) \mathbf{\Gamma}_s' - \kappa_1 \xi (\mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1}) \mathbf{\Gamma}_s' \Big] \tilde{\mathbf{s}}_t \\
&+ \Big[ \mathbf{A}_x'(\kappa_1 \Phi_x - \mathbf{I}) - \xi^2 (\mathbf{1}'\mathbf{\Sigma}_s \mathbf{1}) \mathbf{1}' - \sigma_a^2 - \kappa_1 \xi \mathbf{1}'(\mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1}) \Big] \tilde{\mathbf{x}}_t \\
&+ \Big[ \mathbf{A}_\delta'(\kappa_1 \mathbf{\Phi}_\delta - \mathbf{I}) - \mathbf{1} \Big] \tilde{\boldsymbol{\delta}}_t. \\
&+ \Big[ \mathbf{1}' + \mathbf{A}_g'(\kappa_1 \mathbf{\Phi}_g - \mathbf{I}) \Big] \tilde{\mathbf{g}}_t
\end{aligned}
$$

Applying the method of undetermined coefficients now yields the solutions

$$
\begin{aligned}
\mathbf{A}_s' &= \Big[ \xi \mathbf{1}'(\mathbf{I} - \mathbf{\Phi}_s) - \xi^2 (\mathbf{1}'\mathbf{\Sigma}_s \mathbf{1}) \mathbf{\Gamma}_s' \Big] \Big[ (\kappa_1 \mathbf{\Phi}_s - \mathbf{I}) + \kappa_1 \xi \mathbf{\Sigma}_s \mathbf{1} \mathbf{\Gamma}_s' \Big]^{-1} \\
\mathbf{A}_x' &= \Big[ \Big( \xi^2 (\mathbf{1}'\mathbf{\Sigma}_s \mathbf{1}) + \sigma_a^2 + \kappa_1 \xi (\mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1}) \Big) \mathbf{1}' \Big] (\kappa_1 \Phi_x - \mathbf{I})^{-1} \\
\mathbf{A}_\delta' &= \mathbf{1}'(\kappa_1 \mathbf{\Phi}_\delta - \mathbf{I})^{-1} \\
\mathbf{A}_g' &= \mathbf{1}'(\mathbf{I} - \kappa_1 \mathbf{\Phi}_g)^{-1}
\end{aligned}
$$

while the constant term must solve

$$
\begin{aligned}
0 = -\bar{\delta} + g + \kappa_0 + (\kappa_1 - 1)A_0 &+ \frac{1}{2}\Big( (1 - 2\bar{x})\xi^2 (\mathbf{1}'\mathbf{\Sigma}_s \mathbf{1}) + (1 - 2\bar{x})\sigma_a^2 \Big) \\
&+ \frac{1}{2}\kappa_1^2 \Big( \mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{A}_s' + \mathbf{A}_x' \mathbf{\Sigma}_x \mathbf{A}_x + \mathbf{A}_\delta' \mathbf{\Sigma}_r \mathbf{A}_\delta + \mathbf{A}_g' \mathbf{\Sigma}_g \mathbf{A}_g \Big) + \kappa_1 \xi (1 - \bar{x}) \mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1}
\end{aligned}
\tag{A.5}
$$

In the case where $\tilde{\mathbf{g}}_t$ is a scalar (has only a single component), the solution becomes

$$A_g = \frac{1}{1 - \kappa_1 \phi_g}.$$

which is the coefficient on the long-run consumption growth component in the original specification of Bansal and Yaron (2004) in the limiting case as the EIS approaches infinity. The addition of risk to consumption growth through $\mathbf{\Sigma}_g$ would also lead to a slightly different scaling for $\mathbf{A}_x$ that would imply different recovered levels of the state process $\tilde{\mathbf{x}}_t$, but is unlikely to change the overall contribution of risk aversion to our *pc* series.

As a result, persistent output growth or long-run risk in our model would behave just as in a version of the long-run risk model with a very high EIS, with changes in the expected growth rate leading to changes in the *pc* ratio. To the extent that we abstract from this feature, the model will instead use variation in $\tilde{\mathbf{x}}_t$, which is effectively computed as a residual, to absorb this variation.

## A.6 Additional Details

### A.6.1 Bootstrap Bias Correction

The bootstrap bias corrected estimate for the AR(1) model of the log earnings share is computed as follows. First, we run the regression

$$s_t = a + \phi s_{t-1} + \varepsilon_{s,t} \tag{A.6}$$

to obtain the estimates $\hat{a}^{OLS}, \hat{\phi}^{OLS}$. We then bootstrapping many samples from the data generating process

$$s_t^j = \hat{a}^{OLS} + \hat{\phi}^{OLS} + \tilde{\varepsilon}_{s,t}^j$$

where residuals $\tilde{\varepsilon}_{s,t}^j$ are drawn with replacement from $\{\varepsilon_{s,t}\}$. For each $j$ in 100,000 simulations, we repeat the regression (A.6) to obtain estimates $\hat{a}^j, \hat{\phi}^j$, which we average to obtain the estimates $\hat{a}^{boot}, \hat{\phi}^{boot}$. The approximate bias is computed as $\hat{\phi}^{OLS} - \hat{\phi}^{boot}$, implying that the corrected persistence estimator is obtained as

$$\hat{\phi}^* = \hat{\phi}^{OLS} + \left(\hat{\phi}^{OLS} - \hat{\phi}^{boot}\right).$$

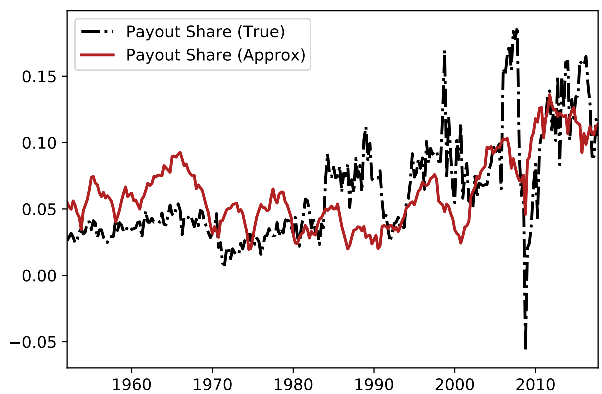
For example suppose that every time we run OLS there is a downward bias of 0.05. The true data has persistence 0.95, so the OLS on historical data measures 0.9, while OLS on data simulated using our OLS estimate of 0.9 yields an average estimate of 0.85. In this case the corrected coefficient is $\hat{\phi}^* = 0.9 + (0.9 - 0.85) = 0.9 + 0.05 = 0.95$.

## A.7 Additional Detail: Model-Implied Cash Flows

This section provides additional detail on the comparison of implied cash flows in the model to payouts actually observed in the data. Because our model obtains implied cash flows from earnings, using (3), or its log-linear approximation (10), rather than using actual payout data, it is important to check that these series are sufficiently close.

Figure A.2 displays the model-implied cash flow share of output against the actual payout share from the data. The figure shows that the series line up moderately well, particularly toward the end of the sample, and that the model-implied series is able to reproduce a large rise in payouts over the second half of the sample. At the same time, the two series display nontrivial discrepancies at higher frequences.

**Figure A.2: Cash Flow Share, Implied vs.Data**

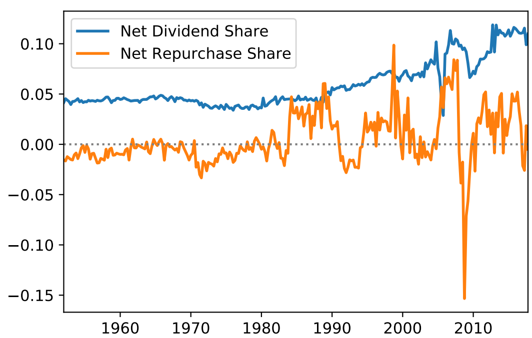



*Notes:* This figure compares the implied cash flow series used by our model $C_t = (S_t - \omega)E_t$, compared to the true corporate payout series in the data. The implied series takes $S_t$ and $E_t$ directly from the data, and uses the calibrated value $\omega = 0.0601$ consistent without our calibration of $\xi$.

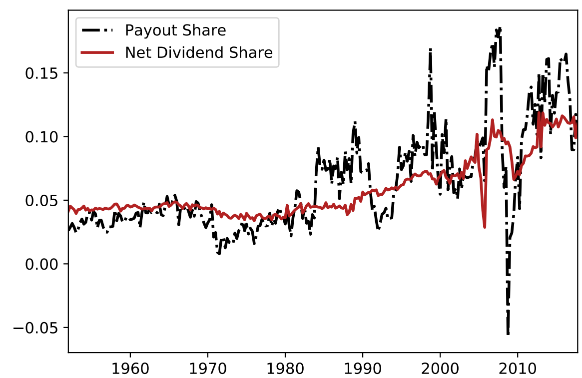
In our model, low frequency variation in cash flows are ultimately much more important for asset valuation. To focus in on the lower-frequency trend in cash flows, we split our payout data into its two component series: net dividends, and net repurchases (i.e., the negative of net equity issuance), displayed in Figure A.3. Figure A.3a display the raw series, showing that the net dividend share is always positive and exhibits slow and persistent dynamics, while the net repurchase share is close to zero on average and fluctuates wildly at high frequencies. Although repurchases have become an increasingly important form of payout, they largely cancel out with issuance of new equity, yielding a series without a major discernible trend. As a result, Panel (b) shows that we can effectively treat the net dividend share as a good measure of the low frequency trend in payouts, with the exception of the transitory downward spike in net dividends during the financial crisis.

**Figure A.3: Payout Data Components**

**(a)** Net Dividend vs. Net Repurchase Shares




**(b)** Payout Share vs. Net Dividend Share




*Notes:* Panel (a) separately displays the two components of total payout: the net dividend share (equal to (net dividends) / (net value added) for the corporate sector) and the net repurchase share ($(-1)\times$ (net equity issuance) / (net value added) for the corporate sector). Panel (b) compares the net dividend share as just defined to the payout share (payouts / (net value added) for the corporate sector, where payouts are net dividends minus net equity issuance). Source: Flow of Funds.

With the net dividend share as a proxy for the low frequency trend in the payout share in the data, we can compare it to its counterpart in the model. This can be computed using (10) as

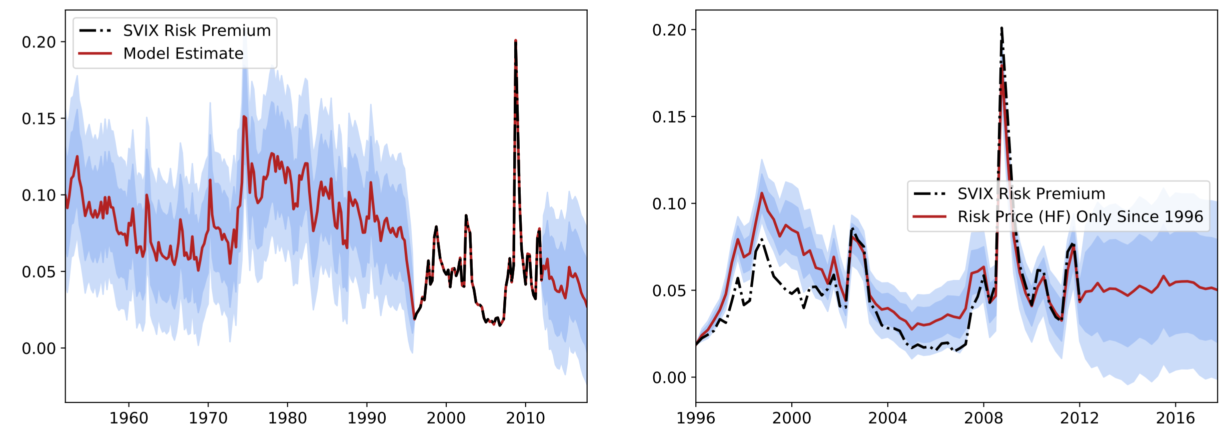
$$cy_{LF,t} = \overline{cy} + \xi \tilde{s}_{LF,t}. \tag{A.7}$$

The resulting series is displayed alongside the net dividend share in Figure 5. Unlike the implied series for $cy_t$ in Figure A.2, which can be computed directly from the data given $\xi$, computing $cy_{LF,t}$ depends on the decomposition of the earnings share into its low and high frequency components, and therefore on the model's parameter and latent state estimates. This leads to uncertainty in our estimate, characterized by the blue error bands, while the median is plotted in red.

Figure 5 shows that the model's implied low frequency component cash flows delivers an excellent fit of the net dividend share in the data. The fit is particularly good over the subsample since 1989 on which our main results are based, and does not overstate the growth in payouts. The main discrepancy between the series is the transitory downward spike in the data during the financial crisis, which is not really representative of the low frequency

**Figure A.4: Estimated Risk Premium and Risk Price Component**

**(a) Full Sample** **(b) Contribution of HF Risk Price**




*Notes:* Panel (a) plots the estimated risk premium over the sample along with the risk premium implied by the SVIX, available for the subperiod 1996:Q1-2012Q1. Panel (b) plots the component of the risk-premium driven only by the high frequency orthogonal risk price along with the risk preimium implied by the 3-month SVIX. The label "Only Since" followed by a date describes a counterfactual path where a single component is allowed to vary, while all other components of the risk premium were held fixed from that date on. The red center line corresponds to the median of the distribution of outcomes, accounting for both parameter and latent state uncertainty, while the dark and light blue bands correspond to 66.7% and 90% credible sets, respectively. The period 1996:Q1-2012:Q1 in Panel (a) lacks bands because the our estimation procedure ensures that the risk premium matches the data exactly for each quarter of this subsample. The sample spans the period 1952:Q1-2017:Q4.

trend, and in fact was more severe in the net dividends data than in the overall payout data, likely due to a slowdown in equity issuance at this time.

In summary, these results provide strong support for our model's approximate cash flow series (10). Although the implied and data series differ at high frequencies, these discrepancies largely reflect the timing of payouts, and should not play a huge role in equity pricing. At the same time, the model is highly effective at capturing the underlying trend in payouts through its low frequency component, providing an excellent fit for the data.

## A.8 Dynamics of the Equity Premium

In addition to decomposing the growth in market equity, our model also estimates a time series for the equity risk premium, shown in Figure A.4. Panel (a) plots our overall estimated risk premium, which is affected by both the orthogonal risk price component $x_t$ and by $s_t$ through the leverage risk effect. Panel (b) shows our estimate of the equity premium variation that is attributable to only the high frequency component of the of the orthogonal risk price component, $x_{HF,t}$. Both panels superimpose the equity premium implied by the three-month

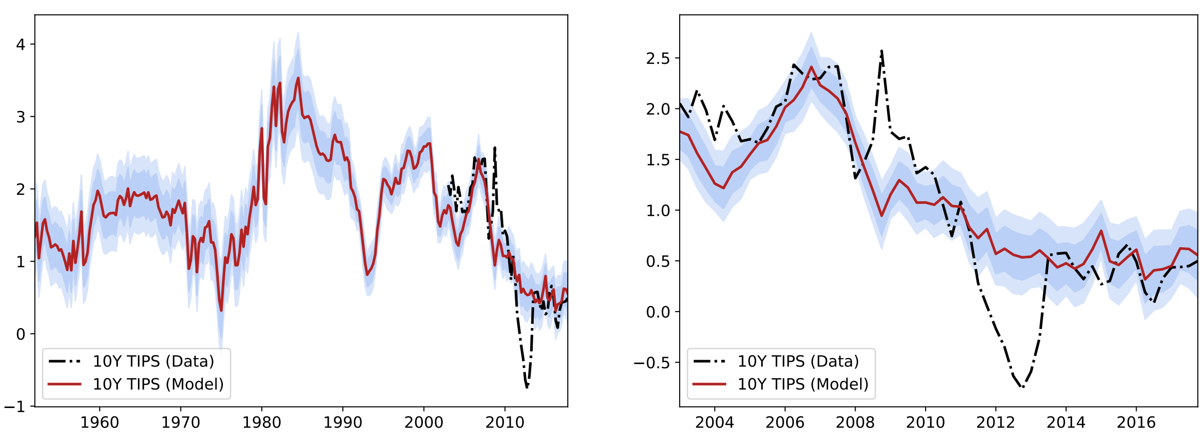
SVIX over the subperiod for which the latter is available, from 1996:Q1-2012:Q1.

Two points are worth noting. First, with the exception of the spike upward during the financial crisis of 2008-2009, Panel (a) shows that the estimated equity premium has been declining steadily over the past several decades and is quite low by historical standards at the end of the sample. By 2017:Q4, the estimates imply that the equity premium reached the record low values it had attained previously only in two episodes: at the end of the tech boom in 1999-2000, and at the end of the twin housing/equity booms in 2006. Second, Panel (b) shows that the estimation assigns to the high frequency orthogonal risk price component, $x_{HF,t}$, virtually all of the variation in the risk premium implied by the options data, while the remaining variation is ascribed to both the lower frequency component of the risk price, and to the earnings share via the leverage risk effect. The overall risk premium is therefore influenced by a trending low frequency component and a volatile high frequency component, consistent with the findings of Martin (2017).

## A.9 Model-Implied Real Bond Rates

**Figure A.5: 10-Year TIPS Yields, Model vs. Data**

**(a) Full Sample** **(b) TIPS Sample**




*Notes:* These plots compare the yields on ten-year real bonds in model and data. The data measure is obtained from the Federal Reserve Board of Governors (FRED code: FII10). The model value is obtained using the bond pricing formulas in Appendix A.3.4. The red line displays the mean estimate taken over 10,000 equally spaced parameter draws, while the light and dark blue bands represent 67% and 90% confidence intervals, respectively. The left panel displays the full sample period 1952:Q1-2017:Q4, while the right panel displays the 2003:Q1-2017:Q4 subsample on which the TIPS data is available.

In this section, we compare the implied rates on long real bonds (TIPS) in the model and data. Figure A.5 displays 10-year real bond yields in the model alongside 10-year TIPS yields in the data (details on this computation can be found in Appendix A.3.4). To allow for the fact that TIPS are computed using CPI inflation while our real rates are computed using the GDP deflator, as well as for the possibility that 10-year TIPS may include term or liquidity premia on average, we add a constant to our model-implied TIPS rate, equal to 0.61%, so that our model-implied and actual TIPS rates have the same mean over the subsample over which TIPS data are available (2003:Q1 - 2017:Q4). Panel (a) displays the full sample, while Panel (b) zooms in on the 2003:Q1 - 2017:Q4 TIPS subsample.

Comparing the series in Figure A.5 shows that the general trajectory of the model-implied TIPS yields closely matches the data. Model and data yields display similar overall declines over the sample, equal to 1.22% and 1.55% from 2003:Q1 (the start of the TIPS sample) to 2017:Q3, respectively. Similarly, model and data yields exhibit falls of 1.65% and 1.85% from the 2006:Q3 (the TIPS peak, outside of a brief spike during the financial crisis) to 2017:Q4, respectively. Although Panel (b) shows some deviations between model and data — in particular a failure of the model to capture a dip in rates between 2012 and 2013 — these results show that the model explains most movements in real long-term bonds at the 10-year horizon, despite the fact that these data are not a target of the estimation.

## A.10 Intangible Accounting

As pointed out by Koh et al. (2020), the BEA has changed its treatment of intangible investments over time, which can influence measured factor shares. In particular, while intangible investments were considered intermediate expenses prior to 1999 (and "expensed" or removed from gross or net value added), they are now considered investments, meaning that value added (and hence profits or operating surplus) must be increased to maintain the accounting identity that value added is equal to factor payments plus investments.

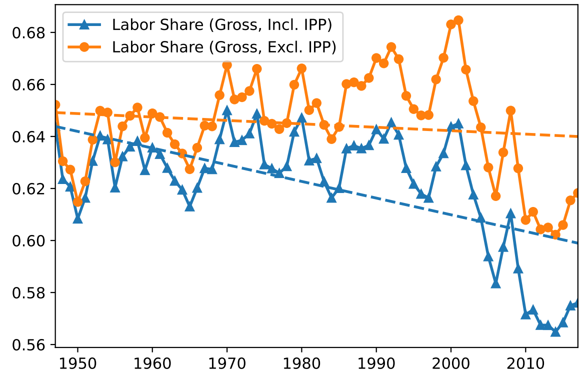
Our data sample, covering the entire corporate sector, differs slightly from that used by Koh et al. (2020) and Atkeson (2020), who consider the nonfinancial corporate sector only. However, we are able to reproduce the core data patterns in our sample, with the key patterns displayed in Figure A.6. To begin, Panel (a) shows that labor's share of gross NVA including and excluding IPP, defined by

$$\text{Labor Share (Gross Incl. IPP)} = \frac{\text{Labor Compensation}}{\text{Gross Value Added}}$$

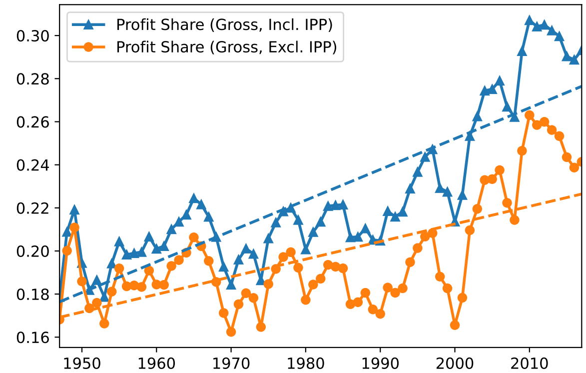
$$\text{Labor Share (Gross Excl. IPP)} = \frac{\text{Labor Compensation}}{\text{Gross Value Added} - \text{IPP Investment}}$$

where labor compensation and gross value added (GVA) come from NIPA Table 11.4, and

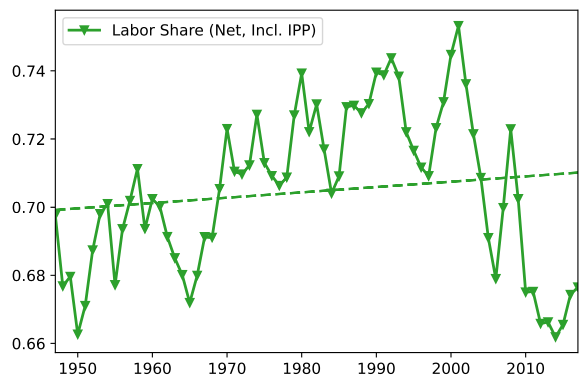
## Figure A.6: Factor Shares Accounting for Investment



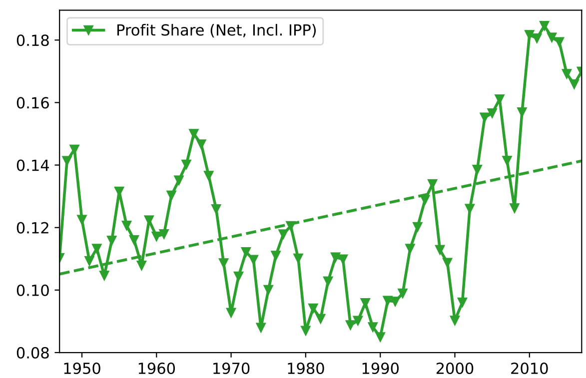

(a) Labor Share



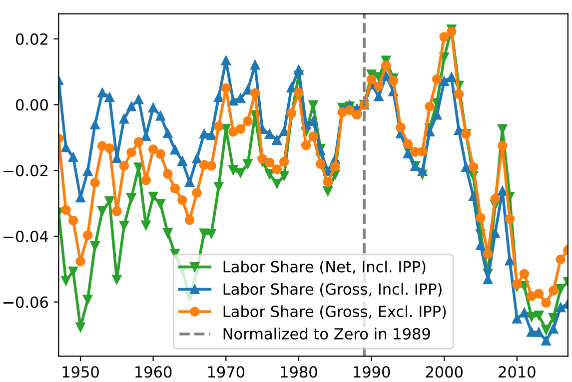

(b) Profit Share



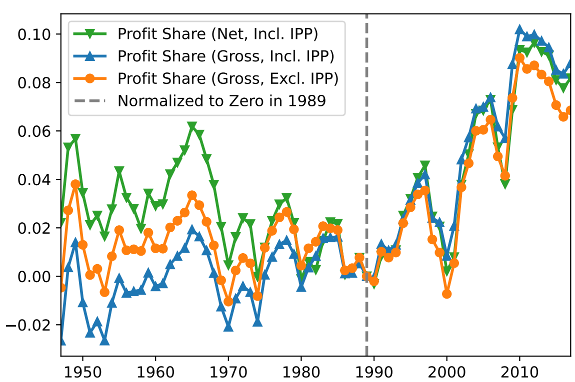

(c) Labor Share




(d) Profit Share




(e) Labor Share




(f) Profit Share

IPP investment is defined as intangible corporate investment from BEA Fixed Asset Table 4.7. As can be seen, removing IPP investment from the denominator reduces and nearly eliminates the downward trend in the labor share of gross NVA over this period.

Panel (b) displays a similar comparison for the profit share, using the definitions

$$\text{Profit Share (Gross Incl. IPP)} = \frac{\text{Domestic After-Tax Profit + Foreign Profit}}{\text{Gross Value Added}}$$

$$\text{Profit Share (Gross Excl. IPP)} = \frac{\text{Domestic After-Tax Profit + Foreign Profit - IPP}}{\text{Gross Value Added} - \text{IPP Investment}}.$$

Note in this case that the IPP investment must be removed from both the numerator and denominator. As can be seen, the choice of whether to include or expense IPP also has an important influence on profit share growth, with the "Excl. IPP" version displaying a lower trend over the sample.

Since our definition of the profit share does not remove IPP, one therefore might be concerned that we are taking the least conservative measure, which potentially overstates the rise in the profit share over time. However, this concern is much less serious when considering that our paper does not actually use shares of gross net value added (i.e., the series plotted in Panels (a) and (b)), but rather shares of *net value added*, which is equal to gross value added minus depreciation. As noted in Koh et al. (2020), shares of net value are much less sensitive to this definitional choice, since IPP investment and depreciation take similar magnitudes, largely canceling out. To show this, Panels (c) and (d) display shares of net value added, defined by

$$\text{Labor Share (Net Incl. IPP)} = \frac{\text{Labor Compensation}}{\text{Net Value Added}}$$

$$\text{Profit Share (Net Incl. IPP)} = \frac{\text{Domestic After-Tax Profit + Foreign Profit}}{\text{Net Value Added}}.$$

These panels show that the labor share of net value added does not contain this potentially spurious downward trend over the sample, and looks closer to the gross version excluding (i.e., expensing) IPP. Similarly, the profit share of net value added displays trend growth closer to the gross version excluding IPP, implying that our definition should not be aggressively overstating growth in the profit share.

Last, we note that these definitional differences are much smaller over the second half of the sample, over which we obtain our sharpest results. Panels (e) and (f) display the three definitions for each share (gross including IPP, gross excluding IPP, and net including IPP), with a constant removed so that each series is equal to zero in 1989. These figures show that the definitions are largely irrelevant to the path of labor and profit shares since 1989, during the period when we claim the largest influence of changes in factor shares. While these figures do indicate a more substantial influence of these definitions on the evolution of profit shares earlier in the sample, we note that our chosen series (the profit share of net value added) rises the least over the full sample, and in that sense can be considered the most conservative option.

In summary, while the question of how to deal with IPP investment remains an important one, our choice to use shares of net value added should be robust to these definitional concerns, which in any case have little influence in the post-1989 sample where we find the largest role of factor shares.

## A.11 Robustness: BEA vs. Compustat

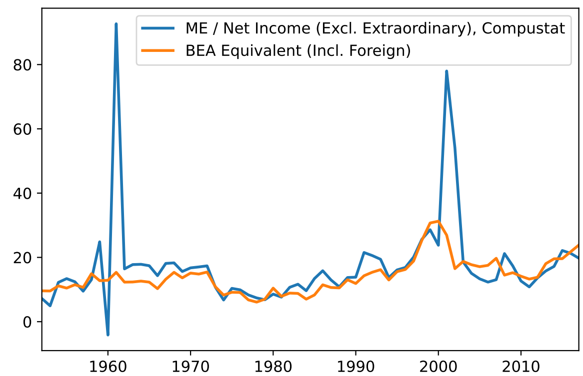
This section compares the ratio of market equity to profits in our BEA data for the US corporate sector to the corresponding measures in merged CRSP/Compustat data. We compute the value of market equity as the closing share prices times the number of shares outstanding in CRSP monthly data. We then merge the CRSP data with Compustat's Fundamentals Annual table using the provided linking table, and keep the last observation of each year to form an annual panel. We then sum over market equity and our various profit measures in each year, and divide to form our ratios. The profit measures we consider are: (i) earnings, equal to "Profits After Tax" in the BEA data, and "Net Income before Extraordinary Items" (code: IB) in the Compustat data; (ii) EBITDA, equal to "Net Operating Surplus" in the BEA data and "EBITDA" in the Compustat data (code: EBITDA); and (iii)

Figure A.7 displays the resulting comparison. For each row, the left panel compares the series in levels while the right panel normalizes both the BEA and Compustat series to be equal to zero in 1989, to more easily contrast the growth since this date. The top row uses earnings as the denominator, measured as profits after tax in the BEA data (including foreign earnings, as described in the main text), and as net income before extraordinary expenses ("IB") in Compustat. The series match very closely outside of some large spikes in the Compustat data, displaying very similar growth since 1989.

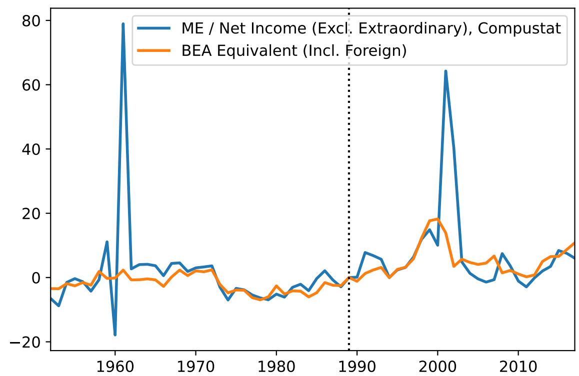
The second row uses EBITDA as the denominator ("Net Operating Surplus" in the BEA data, "EBITDA" in Compustat), while the third row uses pretax income ("Corporate Profits" in the BEA data, "PI" in Compustat). Both BEA series include foreign earnings as described in the main text. These series also match closely, and lack the extreme spikes in the Compustat data. To the extent that they differ, the ratio of ME to profit rises slightly more in the BEA data since 1989. This implies that the profit share likely rises by less in the BEA data compared to the Compustat data over this period, implying that our measures of the profit share are likely conservative.

## Figure A.7: Earnings Share and Valuations

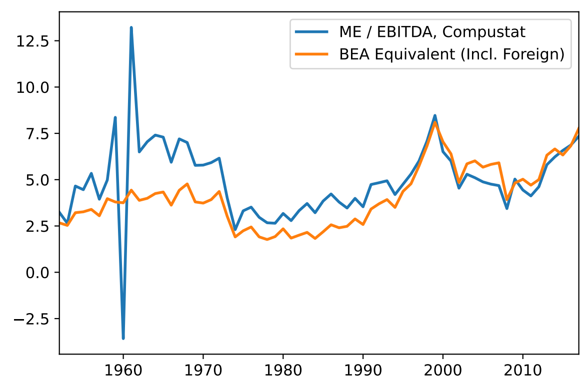
### (a) ME/Earnings



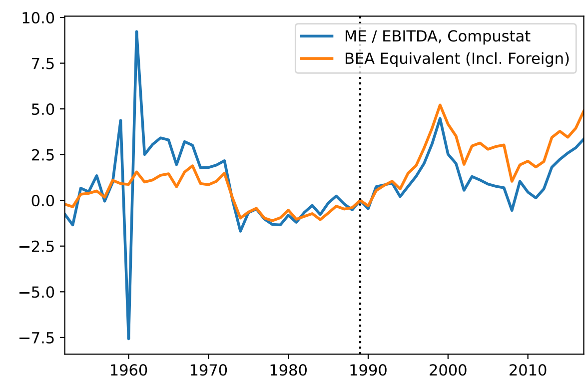

### (b) ME/Earnings (Norm. 1989 = 0)



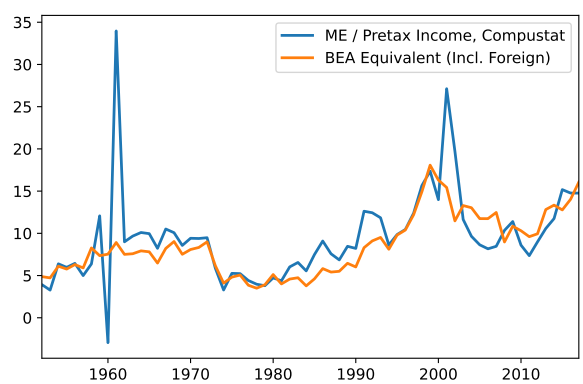

### (c) ME/EBITDA



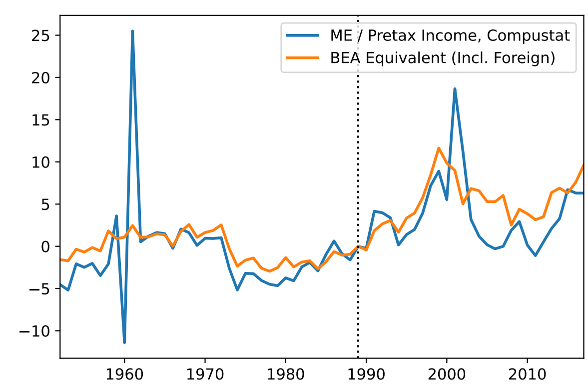

### (d) ME/EBITDA (Norm. 1989 = 0)




### (e) Pretax Income




### (f) Pretax Income (Norm. 1989 = 0)




*Notes:* The sample spans the period 1952:Q1-2017:Q4.

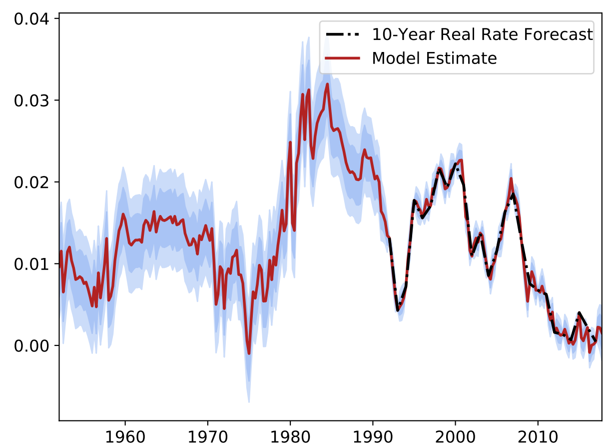
## A.12 Additional Tables and Figures

**Table A.1: Growth Decomposition**

| Contribution | 1952-2017 | 1952-1988 | 1989-2017 |
|---|---|---|---|
| **Total** | **1405.81%** | **151.23%** | **477.34%** |
| Factor Share $s_t$ | 20.50% | -21.09% | 43.96% |
| | [8.33%, 35.52%] | [-51.68%, 6.91%] | [22.31%, 70.19%] |
| $s_{LF,t}$ | 19.36% | -19.28% | 40.21% |
| | [6.48%, 35.09%] | [-52.72%, 12.96%] | [16.97%, 67.62%] |
| $s_{HF,t}$ | 1.14% | -1.81% | 3.75% |
| | [-0.21%, 2.88%] | [-6.14%, 1.72%] | [0.67%, 7.97%] |
| Risk Price $x_t$ | 22.72% | 25.33% | 17.68% |
| | [7.59%, 35.10%] | [-4.66%, 57.20%] | [-8.71%, 39.73%] |
| $x_{LF,t}$ | 22.63% | 25.79% | 17.54% |
| | [7.46%, 35.17%] | [-4.27%, 58.45%] | [-9.00%, 39.96%] |
| $x_{HF,t}$ | 0.08% | -0.46% | 0.14% |
| | [-2.27%, 2.46%] | [-7.49%, 6.09%] | [-3.29%, 3.60%] |
| Risk-Free Rate $\delta_t$ | 3.24% | -15.65% | 13.80% |
| | [0.31%, 6.34%] | [-26.30%, -6.12%] | [9.77%, 18.47%] |
| $\delta_{LF,t}$ | 3.27% | -13.18% | 12.06% |
| | [-0.79%, 7.31%] | [-26.75%, 0.30%] | [6.46%, 17.77%] |
| $\delta_{HF,t}$ | -0.03% | -2.47% | 1.73% |
| | [-1.13%, 1.13%] | [-6.97%, 1.14%] | [-0.14%, 4.22%] |
| Real PC Output Growth | 53.54% | 111.41% | 24.57% |

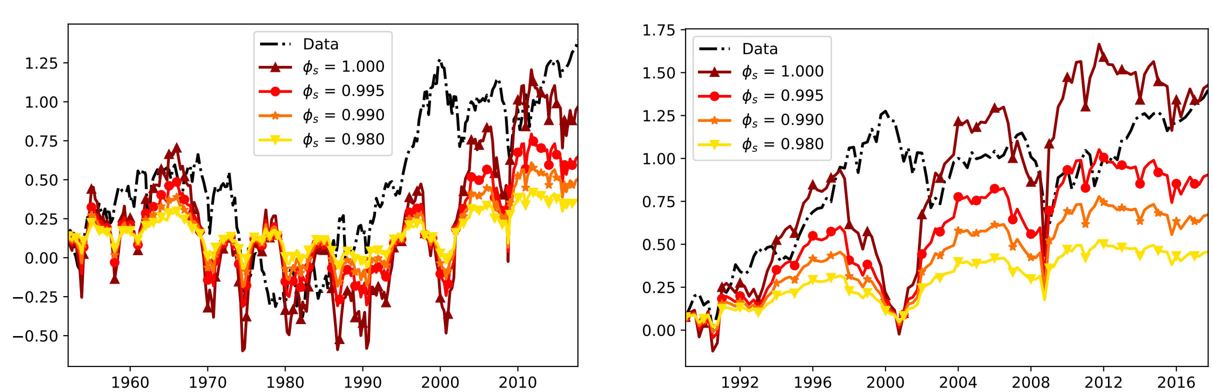
*Notes:* The table presents the growth decompositions for the real per-capita value of market equity. The row "Total" displays the total growth in market equity over this period, in levels. The remaining rows report the share of this overall growth explained by each component, obtained by measuring the difference in implied growth between the data and a counterfactual path in which that variable is held fixed at its initial value for the relevant subsample. To ensure an additive decomposition, we measure the share of total growth explained in logs. The reported statistics are means over shares computed from 10,000 equally spaced parameter draws from our MCMC chain. Below each set of means in brackets are the 5th and 95th percentiles over the same distribution, providing a 90% confidence interval. The sample spans the period 1952:Q1-2017:Q4.

# Figure A.8: Model-Implied Risk-Free Rate Forecast




# Figure A.9: Implied Contributions, AR(1) Models

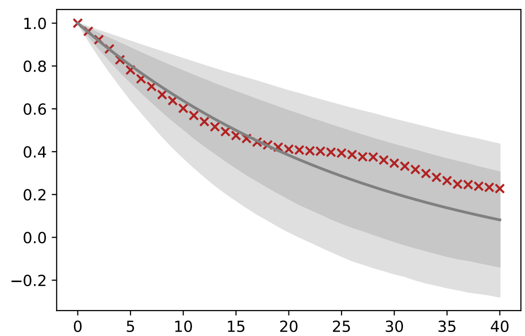
**(a) Full Sample** **(b) Since 1989**



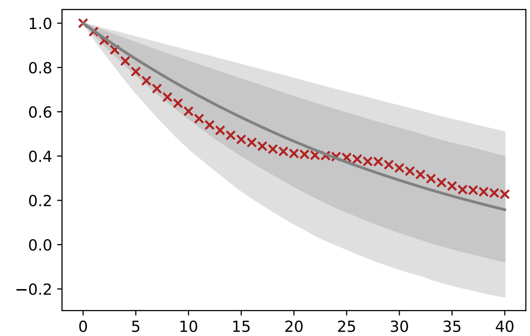

*Notes:* This figure superimposes the implied contributions of the earnings share to the log market equity to output ratio via cash flows measured as in equation (17), over the earnings share data. Each implied contribution is computed by adding the difference in the right hand side of (17) to the initial value of $py_t$ at the start of the relevant subsample.

## Figure A.10: Simulated Autocorrelations, AR(1) Models

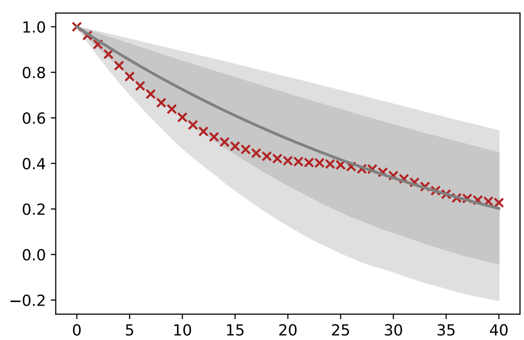
**(a)** $\phi_s = 0.98$



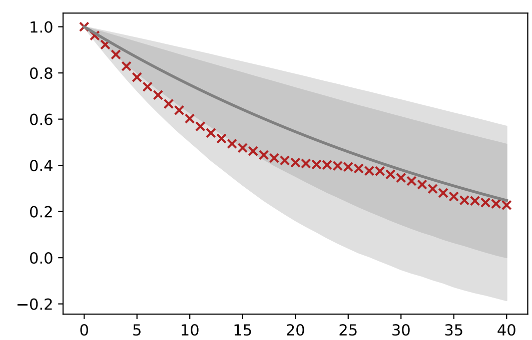

**(b)** $\phi_s = 0.99$




**(c)** $\phi_s = 0.995$




**(d)** $\phi_s = 1$




*Notes:* The figure compares the data autocorrelations for the observable variables available over the full sample, compared to the same statistics from the simplified AR(1) models described in Section 6. For the model equivalents, we compute autocorrelations from each of 10,000 simulations the same length as the data, drawn with the persistence parameter found in that panel's title. The center line corresponds to the mean of these autocorrelations, while the dark and light gray bands represent 66.7% and 90% credible sets, respectively. The sample spans the period 1952:Q1-2017:Q4.