How Things Break

Solids fail through the propagation of cracks, whose speed is controlled by instabilities at the smallest scales

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Galileo was seventy-two years old, his life nearly shattered by a trial for heresy before the Inquisition, when he retired in 1635 to Florence to construct the *Dialogues Concerning Two New Sciences*. His first science is the study of the forces that hold objects together, and the conditions that cause them to fall apart, the dialogue taking place in a shipyard, triggered by observations of craftsmen building the Venetian fleet. The second science concerns ``local motions:'' laws governing the movement of projectiles. The two subjects Galileo founded have fared differently. One is a respectable branch of mechanical engineering, while the other is a core subject that physicists learn at the beginning of their education. Although now, as in Galileo's time, ship-builders need good answers to questions about the strength of materials, the subject has never yielded easily to basic analysis. Galileo identified the main difficulty: *``one cannot reason from the small to the large, because many mechanical devices succeed on a small scale that cannot exist in great size.[1]''* Nearly three hundred years elapsed after Galileo wrote these lines before science reached the atomic scale and began to answer the questions he had posed on the origins of strength, and the relation between large and small.

Despite the tremendous development of solid state physics in this century, attention physicists paid to how things break has been slight. In part, the subject seems too hard. Cracks form at the atomic scale, extend to the macroscopic level, are irreversible, and travel far from equilibrium. Many of the tools with which solid state physics was built do not work: for example, there is no perfect lattice left in which to calculate the quantum mechanical motion of electrons, and cracks move so quickly that even basic quantities such as temperature are ill-defined near their tips. There is also the embarrassment of explaining to colleagues that one is working on failure. The strength of solids calculated from an excessively idealized starting point comes out completely wrong; it is not determined by performance under ideal conditions, but instead by the survival of the most vulnerable spot under the most adverse of conditions.

Failure of Perfect Solids

Here is how a perfect solid would break.

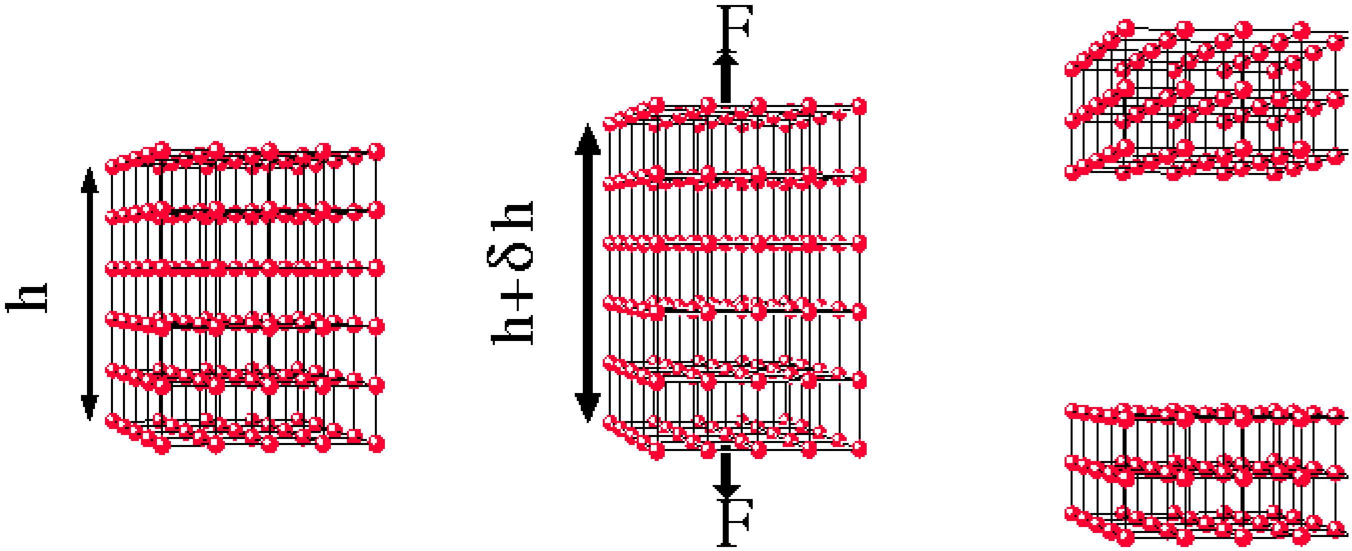


Figure: A flawless solid would break as a single unit, but only carefully prepared fibers of glass and metals have ever been made to fail in this way.

Take a block of material, of height *h*, and cross-sectional area *A*, pulled by a force *F* (Figure 1). The block separates into halves when its atoms are pulled beyond the breaking point. To estimate the force *F* needed to reach this goal, recall that Young's modulus *Y* relates the stress Σ on a body to its extension δh through

$$\Sigma = F/A = (\delta h/h)Y \qquad (1)$$

Suppose that the block snaps when the atoms move apart by 20%; the critical stress Σ to make this happen is

$$\Sigma = Y/5 \qquad (2)$$

A glance at Table 1 shows that the estimate in Equation (2) fails by around two orders of magnitude. It is natural to dismiss this discrepancy as a result of the crude approximations used to obtain (2), but enough effort has been put into carrying out much more sophisticated quantum-mechanical versions of the calculations to show that the estimate is really quite good, and that the error lies elsewhere.

An engineer and a physicist compete to find the best material to build a house. The engineer chooses brick because she knows it is what everyone else uses. The physicist decides to conduct some basic research. Turning to the periodic table, he finds the element with the highest bonding strength and melting point, and first proposes diamond. Trying to find something cheaper, he next proposes a vitreous mixture of silicon and oxygen, since the raw materials are abundant and safe and form strong bonds. All is well until someone throws the first stone. In fact, the relation between bonding energies and strength of materials is far from direct; physicists had best respect the practical experience of engineers until they can really explain why one should not build glass houses.

Material	Young's Modulus Y (10^{11}dyne/cm^2)	$Y/5$ (10^{11}dyne/cm^2)	Theoretical Strength (10^{11}dyne/cm^2)	Practical Strength (10^{11}dyne/cm^2)
Iron	16	3	3	.085
Copper	19	4	3	.049
Silicon	18	4	3	.062
Glass	7	1	4	.002

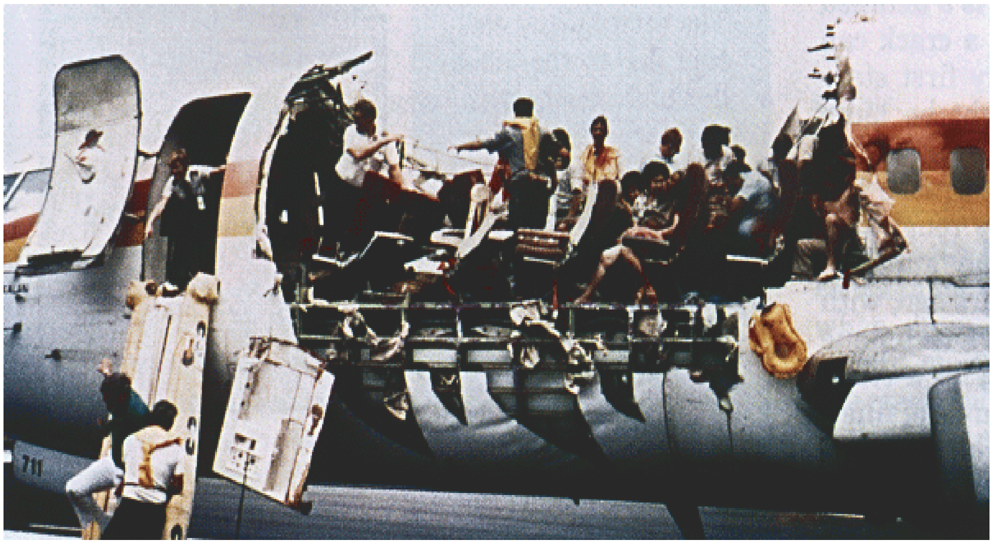
Table 1: The practical and theoretical strengths of materials differ by orders of magnitude. The theoretical strengths are obtained from realistic models of atomic bonding, but can be achieved experimentally only in carefully prepared thin whiskers of material[2].

Introduction of Cracks

Flaws in materials determine strength, so one must move from an ideal material to one in which a flaw occupies the center of attention. This task was first carried out in 1913 by Inglis. He considered a large plate of elastic material with an elliptical hole. Pulling upon the sheet with a uniform stress (force per area) Σ far from the hole, he found that stresses near the narrow end of the hole were much larger than Σ by a factor $2\sqrt{l/\rho}$, where l is the length of the hole, and ρ its radius of curvature. Just as a lightning rod

generates huge electric fields, so a slit creates enormous tensions near its tip. If a flaw is sufficiently thin, it need not be particularly long to pose a threat to the body in which it lives. According to Table 1 brittle materials fail at stresses one hundred times smaller than one at first expects. Suppose, as Griffith did in 1921, that materials are plagued with slits, whose tips reach a destructive stress while the rest of the body lies safely below it. Taking $\rho = 1$Å, and l=1 μm gives $\sqrt{l/\rho} \sim 100$. This argument explains

the practical strength of brittle solids, since it is nearly impossible to prepare materials without micron-sized flaws at the surface, ready to spring into action at stresses smaller than expected [2, 3]. Notice that there is no requirement of a critical density of flaws. A single one will do. Therefore, for structures of great importance, such as airplanes or nuclear containment vessels, arguments based upon statistical likelihood of flaws are unable to guarantee safety, and case-by-case examination of structures is essential. In addition, structures must be designed with special care to avoid making growth of flaws more likely.



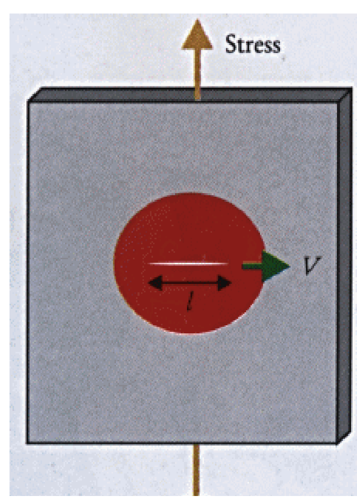
BOX 1: Large advances in the understanding of fracture have tended to follow great public disasters. In 1919, a molasses tank 50 feet high and 90 feet wide burst in Boston, killing twelve people and several horses. The court auditor concluded that ``the only rock to which he could safely cling was the obvious fact that at least one-half of the scientists must be wrong."[4] The most important case in this century occurred during the second World War. War-time production demands led to the Liberty freighter, the first all-welded ship. Of the nearly 4700 welded ships launched by the end of the war, over 200 suffered catastrophic failure, some splitting in two while lying at anchor in port, and over 1200 suffered some sort of severe damage due to fractures. The discipline of fracture mechanics emerged from these catastrophes. The ships were redesigned, eliminating for example sharp corners on hatches, and systematic procedures were developed by which to test the fracture resistance of materials. Following the war, failure by fracture cursed the early airline industry. Ill-placed rivet holes destroyed several of Britain's Comet aircraft, and played a role in moving the center of civilian aircraft production to the United States. Aircraft are now subject to a systematic program of inspection, acknowledging that every structure has flaws, but that those past a certain size are intolerable. Procedures have continued to evolve in response to accidents, most recently after an incident where part of the top of an Aloha Airlines jet peeled off during flight.

Box 2: A crack of length *l* grows at rate *v* in a plate. There are three important energies:

Potential Energy: The potential energy decreases as the crack extends, and since the size of the region where this happens scales as l^2 , the potential energy released scales as $-l^2$. It also scales as the applied stress stress squared.

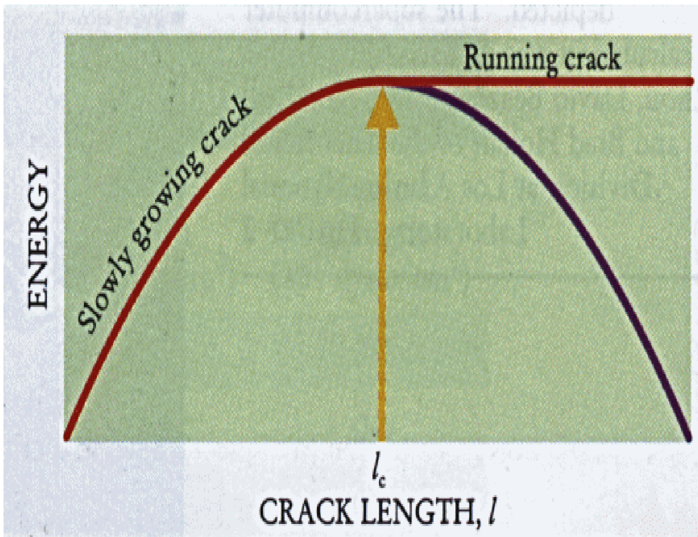
Fracture Energy: Making the crack move forward requires breaking bonds, creating new surfaces, and generating heat; the energy required scales as the length of the crack, *l*.

Kinetic Energy: The total kinetic energy due to the motion of the crack scales as l^2v^2 , since the amount of mass which moves as the crack opens scales as l^2 .



For cracks which move very slowly, only potential and fracture energies are important, and the sum of these as a function of *l* is shown in the sketch at the lower right. Since potential energy decreases as l^2 and fracture energy increases as *l*, for very small cracks the fracture energy is always larger, and the energy increases with *l*. This is a fortunate fact, or else all solid objects would be completely unstable to the slightest mechanical stress. But eventually the potential energy overwhelms the fracture energy, at the critical crack length l_c , called the Griffith point, and from here on, more energy is released than consumed by crack extension. Now extension is rapid and spontaneous. Since the sum of fracture and potential decreases as $(l-l_c)^2$ for $l > l_c$, and energy is conserved by converting potential to kinetic energy, one easily finds that the velocity of the crack must be

$$v(t) = v_{max}(1 - \frac{l_c}{l}) \qquad (3)$$



The critical stress needed to snap a body with a crack of size *l* scales as $\sqrt{l}$. Like the results of many other scaling arguments, Equation (3) is better than one has any right to expect. Fifteen years of careful mathematical work, documented in the book by L. Ben Freund[5], extracts the same formula from a remarkably general boundary-value problem of classical elasticity. In the rigorous formulation v_{max} turns out to be the Rayleigh wave speed.

Brittle and Ductile Materials

Many of the greatest successes of solid state theory have flowed from explaining qualitative properties of solids. Why are some materials conductors and others insulators? Electron band theory provides an answer. Why are some transparent and others opaque? Calculations for interaction of matter with light show why. The most important qualitative fact in mechanical properties of solids is that some are brittle, and shatter in response to a blow, while others are ductile, and the blow merely causes them to deform. Why?

This question is nothing but the question of what makes a crack grow, in a new guise. Take a slab of material, make a saw cut in it, and pull. In a brittle material, the tip of the saw cut spontaneously sharpens down to atomic dimensions, and like a knife blade one atom wide slices its way forward[3]. In a ductile material the tip of the saw cut blunts, broadens and flows, so that great effort is required to make it progress.

There is no completely satisfactory answer to the question of why some materials are brittle and others are ductile; the manufacturers of atoms seem to omit this property when writing down their technical specifications. The most well developed attack on the problem considers stationary, atomically sharp cracks in otherwise perfect crystals, and asks what happens when slowly increasing stresses are inflicted upon them. James Rice and Robb Thomson showed in 1974[6] how to estimate whether the crack will move forward in response, or whether instead a crystal dislocation will pop out of the crack tip, causing it to become blunt. Figure 2 shows results of an exceptionally large computer simulation in which an elliptical crack is placed into copper, one of the most pliable of metals. The tip of the crack spawns clouds of dislocations, appearing as stringy white vortex cores, which travel off into the crystal in unexpected directions, and provide strong impediments to further motion.

Brittleness and ductility are not in fact inherent in the atoms which make up a solid. Most solids have a definite temperature at which they make a transition from brittle to ductile behavior. For silicon, this temperature is around 500 C[3]. This transition is not as well understood as the more familiar equilibrium phase changes.

Crack Dynamics

Cracks would cause no one any trouble it they never moved, so it is natural to investigate their dynamics in some detail. The first calculations along these lines were carried out by Neville Mott, in response to the Liberty ship disasters, and created an amazingly successful scaling theory, described in Box 2.

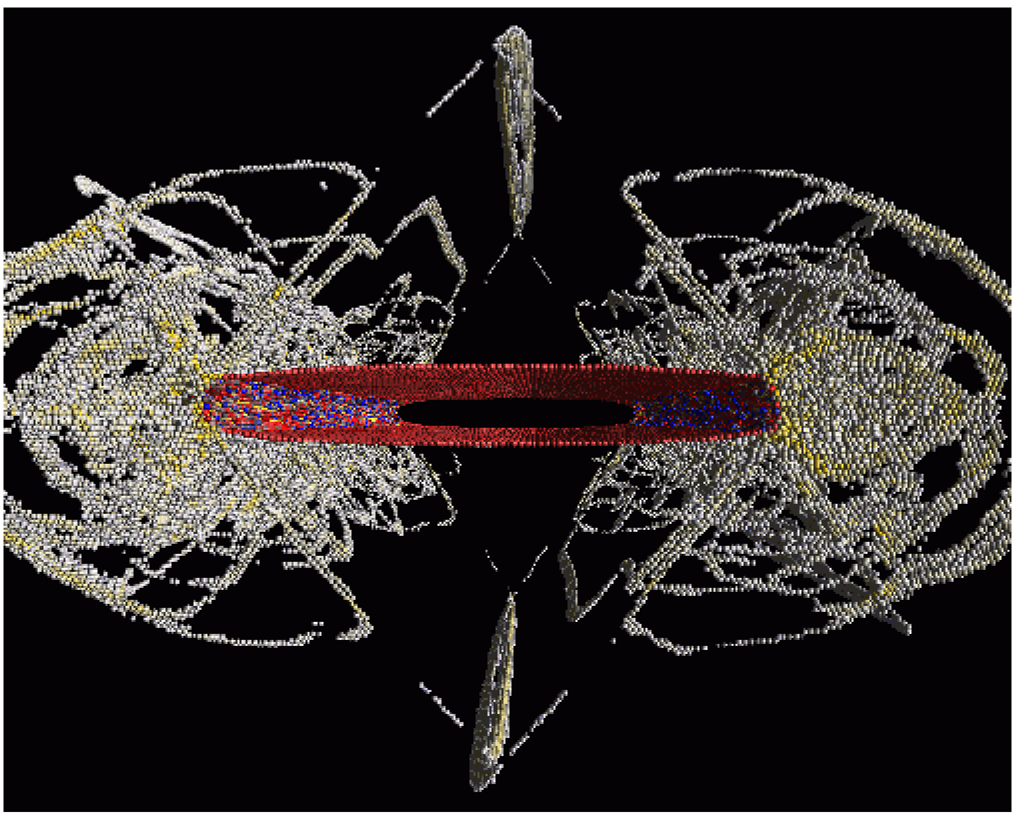


Figure: Thirty-five million atom simulation of a ductile material. The red atoms outline an elliptical crack in a 0.1 micron thick sheet of copper which is place under tension in the vertical direction. As the crack attempts to propagate horizontally, it emits clouds of dislocations, shown in white, some of which have collided directly overhead the crack. Only the atoms at the surface of the crack, or within the cores of dislocations are depicted. The calculation was performed by Shujia Zhou, David Beazley, Peter Lomdahl, and Brad Holian of the Theoretical Division, using the SPaSM molecular dynamics code on a Cray T3D computer in the Advanced Computing Laboratory at Los Alamos.

The scaling theory stood up remarkably well to increasingly sophisticated mathematical improvement. Its only defect was that it never agreed with experiment[7]. All equations of motion for cracks predicted that they should accelerate up to the Rayleigh wave speed, the speed at which sound travels over a flat surface, or earthquakes travel over the surface of the earth. Experiments dating back as far as 1937[8] showed that cracks in glass went at most half this speed. In a field where the main goal was to keep large tankers from splitting in half, the question of precisely how fast a crack ran across the hull seemed rather esoteric. But if the goal is a detailed understanding of the conditions under which a crack can move, getting the velocity right is a necessary first step.

One hint that the motion of cracks might be more complicated than that of particles moving in straight lines came from examining the new surfaces cracks leave behind them. The surfaces often have visibly rough features, as shown in Figure 3, which develop only after the crack has traveled some distance. Several years ago, with Harry Swinney, and Steve Gross, we developed a technique which made it possible to measure the velocity of a crack twenty million times per second, tens of thousands of times in succession, and to within an accuracy of around twenty meters per second[9]. The method involved depositing a very thin layer of aluminum upon a Plexglas or glass sample, and then monitoring its resistance as a crack ran through it. The great detail in these data clearly showed that crack motion in brittle materials could pass through a number of distinct phases.

Birth: Samples are prepared with a notch sawed in one side of a long sample. Long sharp initial notches turn into rapidly running cracks at low stresses, while short blunt notches begin to run when the energy density in the strip of material ahead of them is as much as ten times greater. In almost all cases, cracks accelerate in less than a microsecond to a substantial fraction of the sound speed, at least 200 meters per second.
Childhood: The early phases of crack motion involve calm and efficient progress through the sample. The new surfaces left behind the crack are smooth and mirrorlike, as shown on the lower right of Figure 4; the crack velocity is smoothly and slowly increasing as shown at the left of Figure 4. For long sharp initial cracks, the entire sample is severed in this fashion
Crisis: However, cracks which pass beyond a critical threshold in velocity begin to buck and plunge, as shown at upper left of Figure 4. They leave increasingly rough surface in their wake, shown in Figure 3, and their velocity undulates at frequencies of hundreds of kilohertz.

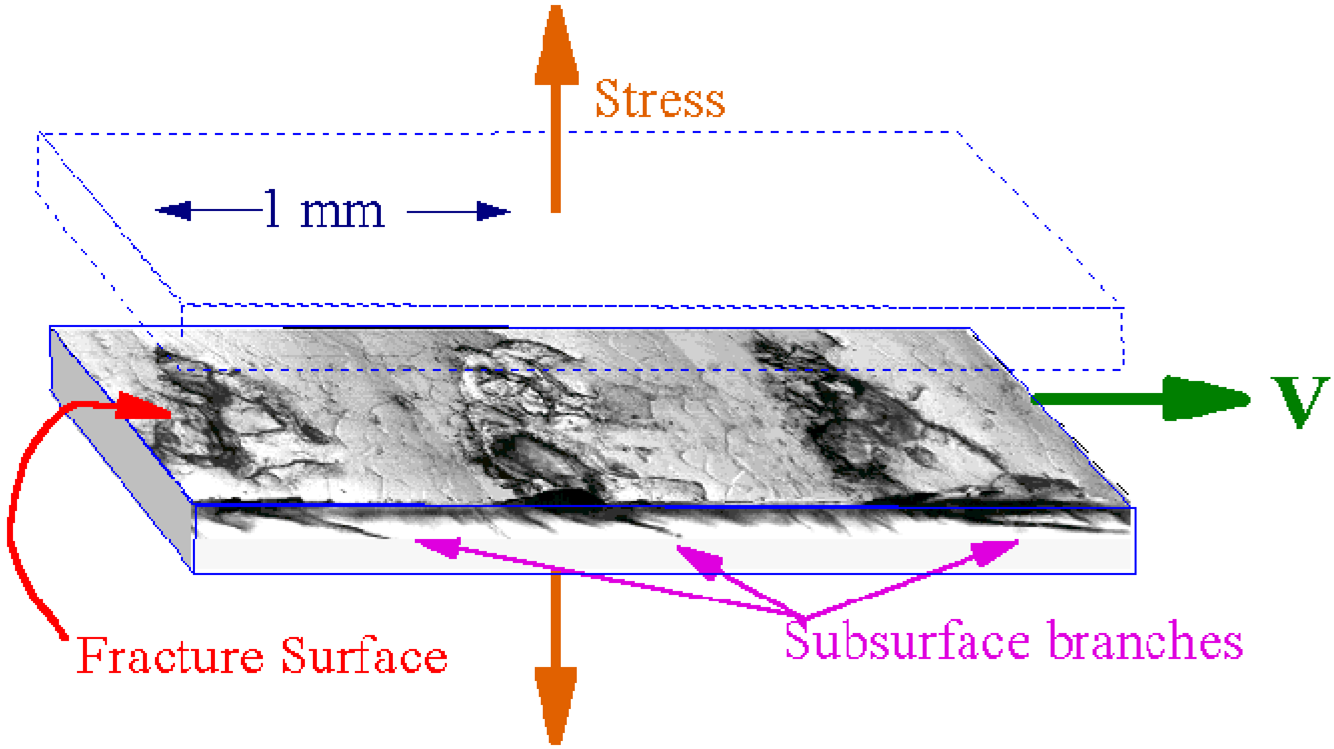


Figure: Once cracks travel faster than a critical velocity in Plexiglas, the fracture surface acquires visible roughness with a wavelength of roughly a millimeter, resulting from the violent process of creating subsurface branches. The amplitude of the surface roughness is two orders of magnitude smaller than the depth of the subsurface branches.

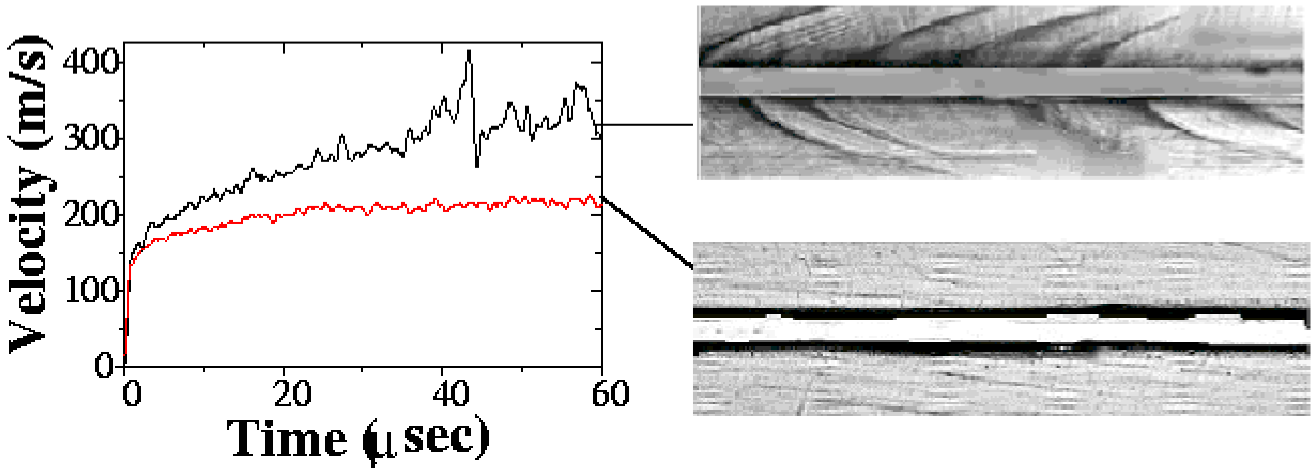


Figure: Depending upon the force with which they are pulled, cracks in Plexiglas either travel calmly, leaving smooth surface beneath, as shown by the velocity trace at left, and the side view of the crack at right, or beyond a critical velocity move at a wildly undulating speed, leaving a thicket of small branches penetrating the surface behind them.

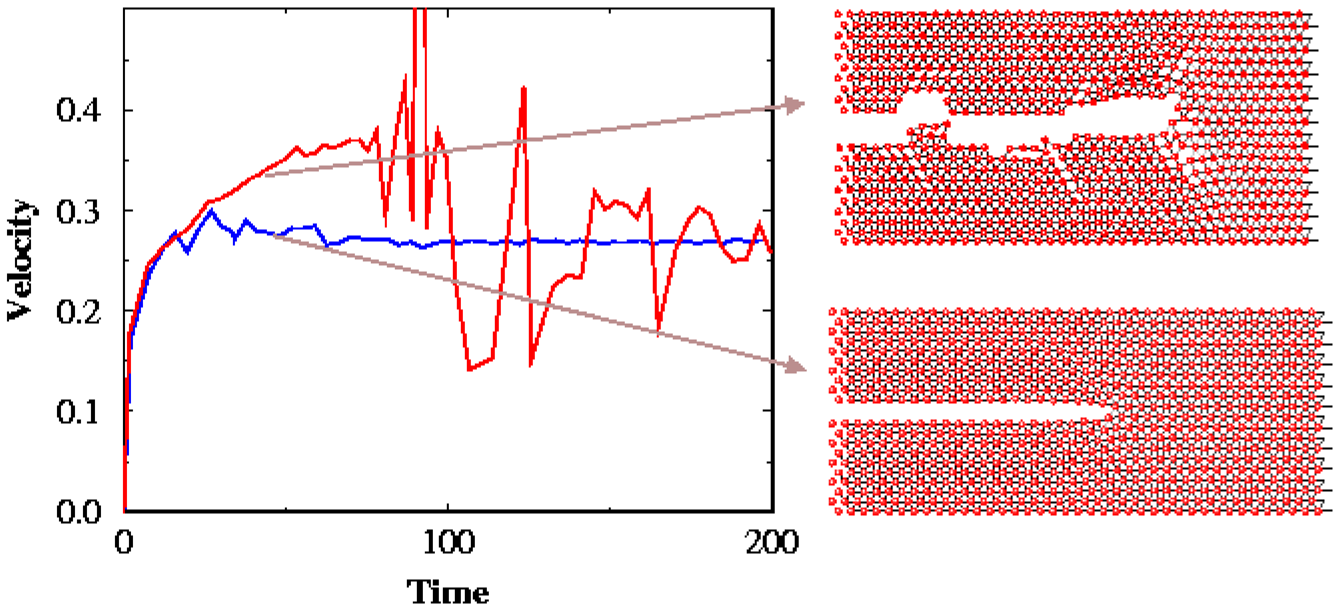


Figure: Computer simulations in a simple model at the atomic scale display a transition between smoothly moving cracks and a violent branching instability that is surprisingly similar to experiment. Just as in experiments, the transition is a function of the energy stored per unit length to the right of the crack.

Thus, cracks in brittle materials suffer a dynamical instability, which makes them unable to accelerate up to the high velocities predicted by classic theories of dynamic fracture.

Origin of Dynamical Instability

Lurking behind the theories of dynamical fracture have always been certain puzzling contradictions. Elisabeth Yoffe carried out the first detailed calculation of dynamical fracture[5], and pointed out that cracks are strongly influenced by special relativity - not as they approach the speed of light, but that of sound. Stresses in the neighborhood of the crack adopt a universal form in the neighborhood of the tip, and this universal singularity contracts in the direction of rapid motion. She observed that at around sixty percent of the sound speed, lobes developed in the stress field surrounding the crack which might be expected to force it to deviate from a straight line.

Moving cracks are even more prone to instability than Yoffe's calculation shows. Emily Ching, Hiizu Nakanishi, and James Langer[10] have pointed out that if one looks out in front of a crack moving at any speed, and asks in what direction the stresses act most strongly to tear material apart, the answer is that the largest stresses are straight ahead of the crack, but at right angles to its direction of motion. According to this calculation, cracks should always move perpendicular to themselves, and stable motion should be impossible.

Thus, from the viewpoint of classical elasticity, assuming that cracks are stable leads to an equation of motion which they do not obey, and probing stability more deeply makes it seem puzzling that cracks are able to propagate at all.

These difficulties have partly been answered by calculations at the atomic scale. There is a very special set of forces between atoms, discovered by Leonid Slepyan[11], which makes it possible find analytical solutions for cracks moving in lattices. The behavior of cracks in these models has several surprising features, but all of them are mirrored in the experiments. These features are[12]

> *Birth:* There is a range of velocities at which steady crack motion is forbidden. The range starts at zero, and lasts until around 20% of the sound speed, after which crack motion becomes possible.
> *Childhood:* Above the forbidden band follows a range of velocities for which steady stable crack motion is allowed and perfectly stable. At exactly the same externally applied stress, however, a stationary crack could also be stable.
> *Crisis:* Above a critical velocity, steady crack motion becomes unstable.

Careful investigation of solutions of these models shows both how to defeat the instabilities lurking behind continuum theory, as well as how the crack tip disintegrates when pressed too hard. For a range of low velocities, steady moving crack solutions are completely stable. As the crack speeds up, the relativistic contraction discovered by Yoffe becomes more and more important, until eventually horizontal bonds above the crack line begin to snap. Whether the crack arrives at this point depends, of course, on how hard it is being pulled, but once it happens, perfect steady motion along a line becomes impossible. Simulations, such as shown in the upper right of Figure 5 showed that the crack might decide to build tree-like patterns of subsurface cracks once steady motion became impossible.

Having seen fracture trees in simulation[13], we set out to find them in experiment. The first try involved an ill-considered attempt to sand down a piece of Plexiglas that nearly set a milling machine on fire (JF takes no responsibility for MM's fine efforts in the laboratory), but soon we did better[14], as shown in the upper right of Figure 4. So extensive does the network of branches in Plexiglas become, that they explain the inability of cracks to accelerate to the predicted limiting speed[15]. Once instability sets in, pulling more on a crack simply makes it dig in its heels harder, generating that much more subsurface damage, but scarcely leading to any more acceleration. In some simulations, as shown on the left side of Figure 5, pulling harder on a crack can actually slow it down. Over ninety percent of the energy being fed to the tip of a crack can be consumed by subsurface instability.

The Key and the Glass

Engineering fracture mechanics has had enormous success improving the safety of structures in this century. Attempts to understand the mechanism of fracture at an atomic level have not yet had a comparable impact. The main reason is not hard to find.

Structural materials in common use have evolved from a process of trial and error which has occupied thousands of years[2]. At a microscopic level, they are incredibly complex. For example, Plexiglas, which in Figures 4 and 5 we blithely compare with a triangular lattice, is actually composed of molecules a million units long tangled about one another in an amorphous web. Iron only becomes useful after the addition of subtle impurities in elaborate industrial processes. The most widely used structural material of all - wood - obtains marvelous mechanical properties in ways that humans have not yet learned to imitate.

Green twigs bend and dry twigs snap, but while the dislocations shown in Figure 2 provide an explanation for the ductility of copper crystals, they help little with something as non-crystalline as Plexiglas, let alone a tree. Almost all of solid state physics rests upon calculations carried out in crystals, but whereas the perfect crystal makes a wonderful electrical conductor, it makes a lousy brick. The largest remaining challenge for physics in the study of breakage is therefore to bridge the gap between idealized calculations, and the rich diversity of the real world. Computer simulations have an important role to play[16, 17], and can treat an imposing number of atoms, but conceptual understanding of how to reason from the small to the large will play an equally important role. The computer can treat 100 million atoms for a few times 10^{-12} seconds, but we need to understand 10^{23} atoms on time scales of minutes or years.

Wigner remarked that solid state physics ``deals in a scientific way with those subjects with which we must deal in our everyday experience. For example, we are never afraid when dropping a key that it will fly to pieces, as a glass would[18]." This first fact that children learn about solids seems however to be one of the last that scientists will be able to explain. A microscopic picture of the strength of solids has begun to break through, but much more remains to be learned.

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References

1 Galileo, *Two New Sciences, Including Centers of Gravity and Force of Percussion* (University of Wisconsin Press, Madison, 1974).

2 J. E. Gordon, *The New Science of Strong Materials, or Why You Don't Fall through the Floor*, 2nd ed. (Princeton University Press, Princeton, 1976).

3 B. Lawn, *Fracture of Brittle Solids*, 2nd ed. (Cambridge University Press, New York, 1993).

4 C. F. E. Tipper, *The Brittle Fracture Story* (Cambridge University Press, Cambridge, 1962).

5 L. B. Freund, *Dynamic Fracture Mechanics* (Cambridge University Press, Cambridge, 1990).

6 R. Thomson, Solid State Physics **39**, 1 (1986).

7 K. Ravi-Chandar and W. G. Knauss, International Journal of Fracture **26**, 141 (1984).

8 H. Schardin, L. Mucke, W. Struth, and W. a. Rhein, The Glass Industry **36**, 133 (1955).

9 J. Fineberg, S. Gross, M. Marder, and H. Swinney, Physical Review **B45**, 5146 (1992).

10 E. S. C. Ching, H. Nakanishi, and J. S. Langer, Physical Review Letters **76**, 1087 (1996).

11 L. I. Slepyan, Soviet Physics Doklady **26**, 538 (1981).

12 M. Marder and S. Gross, Journal of the Mechanics and Physics of Solids **43**, 1 (1995).

13 M. Marder and X. Liu, Physical Review Letters **71**, 2417 (1993).

14 E. Sharon, S. Gross, and J. Fineberg, Physical Review Letters **74**, 5096 (1995).

15 E. Sharon and J. Fineberg, Physical Review Letters **76**, 2117 (1996).

16 F. F. Abraham, D. Brodbeck, R. A. Rafey, and W. E. Rudge, Physical Review Letters **73**, 272 (1994).

17 A. Nakano, R. K. Kalia, and P. Vashishta, Physical Review Letters **73**, 2336 (1994).

18 E. Wigner, Scientific Monthly **42**, 40 (1936).

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