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How to Choose Parameters of $\bar{X} - VSSI$ Control Chart with Adaptive Parameters

R. C. Leoni[1], A. F. B. Costa[1], J. W. J. Silva[2,3,*], N. A. S. Sampaio[3], R. B. Ribeiro[2,4]

[1]Universidade Estadual Paulista - UNESP-FEG, Guaratinguetá, SP, Brazil
[2]Faculdades Integradas Teresa D'Ávila, FATEA, Lorena, SP, Brazil
[3]Associação Educacional Dom Bosco, AEDB, Resende, RJ, Brazil
[4]Faculdade de Tecnologia de São Paulo, FATEC, Cruzeiro, SP, Brazil

Abstract This paper describes how to plan and estimate the optimal parameters of an adaptive chart to monitor a process average using ($\bar{X} - VSSI$) variable sample size and interval. The $\bar{X} - VSSI$ chart was chosen because it is a scheme with great potential for practical application and only requires knowledge of the sample size and the time between sample selection. Markov chains were used to evaluate the chart performance based on the average time between the process uncontrolled and the signal generated by the chart. Two functions written in R language that assist the user in the design of an adaptive control chart $\bar{X} - VSSI$ are exhibited.

Keywords Statistical Process Control (SPC), Adaptive Control Chart, Markov Chain, R Language

1. Introduction

Control charts are used to monitor production processes aiming to signal deviations in relation to the target of a quality characteristic. Detection of small or moderate deviations by means of charts proposed by Shewhart [1] is time consuming and therefore several types of charts have been proposed. Some authors [2-10] introduced the adaptive control charts which are named this way because they do not present all their fixed parameters. This type of chart construction forecasts that at least one of its parameters may vary. These parameters are: the control limits, the sample size and the time interval in which a sample is collected.

In adaptive control charts it is common to use Markov chains to evaluate the chart performance according to the set of chosen parameters [6, 10, 11]. In order to assess the statistical properties, it is used the subjacent idea of dividing the variation interval of monitored statistics in a finite set of states, where the transient states of chain are in the chart control region and the absorbent state is in the established region as out of control.

Adaptive charts are not available in traditional statistical software, despite showing better performance than the charts with fixed parameters. Adaptive parameters determination is not a trivial task; thus, this article proposes the use of a free software to plan and estimate the optimal parameters of an adaptive chart for $\overline{X}$ with sample size and interval being variable ($\overline{X} - VSSI$). The average number of samples until the moment in which the chart indicates the out of control condition (ARL) and the average time between the instant in which the process is changed and the time in which the graph indicates the out of control condition (ATS) are the performance measures used as a reference for the parameters choice. The ARL (Average Run Length) is the average number of points that must be plotted before a point indicates an out-of-control condition, the ATS (Average Time to Signal) represents the average time needed for the control scheme to detect a situation outside of control from the beginning of the process.

The $\overline{X} - VSSI$ chart was chosen because it is a scheme with great potential for practical application, for its use requires only to know the sample size and the time between samples selection after the optimal parameters are established. The statistical properties of control chart are optimized considering the approach presented by Zimmer [11], that is, a Markov chain is used to establish the parameters keeping under control the statistical risk type I and type II [12].

The rest of the paper is organized as follows: section 2 presents the $\overline{X} - VSSI$ control chart. In section 3, it is described the procedure to evaluate the performance of a $\overline{X} - VSSI$ chart using Markov chains. In Section 4, two functions written in R language that assist the user in the design of an adaptive control chart $\overline{X} - VSSI$ are exhibited. Finally, conclusions and future research directions

* Corresponding author:
jwjsilva@gmail.com (J. W. J. Silva)
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complete the article.

Two functions written in R language that assist the user in the design of an adaptive control chart $\bar{X}-VSSI$ are exhibited.

2. $\bar{X}-VSSI$ Control Chart

Reynolds [2] was the first to consider the adaptive design of control chart varying the time interval in which a sample is collected. Later there appeared a large number of studies aiming to vary the other control chart parameters, being confirmed that this technique generally increases the chart power on detection of special reasons that modify the average of quality characteristic (variable) to be monitored [9, 10, 12, 13].

The $\bar{X}-VSSI$ control chart is adaptive with respect to the sample size and the time interval in which a sample is collected. This chart was used by several researchers [3, 4, 9, 10] for monitoring the $\bar{X}$ statistics of a process.

In a control chart with sample size and interval being variable (see Figure 1) the sample size and time interval in which a sample is taken can vary depending on the information from the most recent sample collected. In this kind of chart, random samples of different sizes are collected in intervals of variable length according to the function:

$$(n(i),h(i))=\begin{cases}(n_2,h_1) & if \quad w<Z_{i-1}<k\\ (n_1,h_2) & if \quad -w<Z_{i-1}<w\\ (n_2,h_1) & if \quad -k<Z_{i-1}<-w\end{cases} \tag{1}$$

where i=1,2,…, is the number of the sample; $n(i)$ is the size of the i^{th} sample $(n_1<n_2)$; $h(i)$ is the time practiced to remove the i^{th} sample $(h_1<h_2)$; k and w are boundaries that define control regions; Z_i is the control statistics.

$$Z_i=(\bar{x}_i-\mu_0).\left(\sigma_0/\sqrt{n(i)}\right)^{-1} \tag{2}$$

where $\bar{x}_i$ is the sample average of the *i-th* subgroup; μ_0 and σ_0 are the mean and standard deviation of the process when in control.

The choice between the pairs $(n(i),t(i))$ depends on the position of the last point (Z_{i-1}) marked on the chart. For a $\bar{X}-VSSI$ chart, one can divide the control area into three regions mutually exclusive and exhaustive, as follows (see Figure 1):

- Region within the alarm limits: $I_1=[-w,w]$.
- Region between the control and alarm limits: $I_2=[-k,-w)\bigcup(w,k]$.
- Region outside the control limits: $I_3=[-\infty,-k)\bigcup(k,\infty]$.

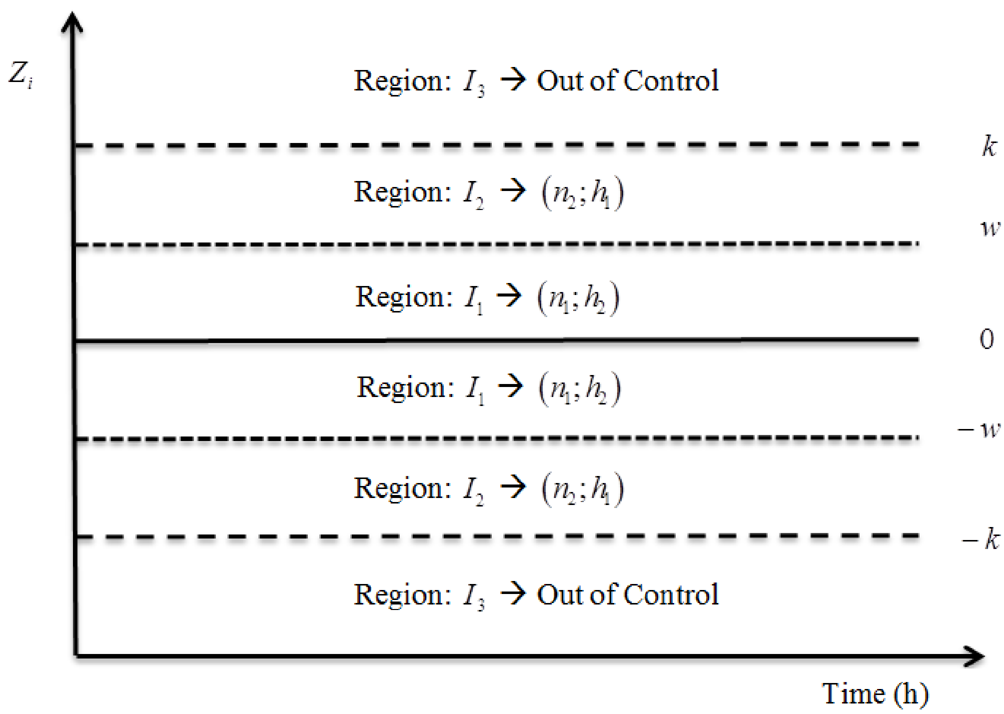

Figure 1. Control regions of a chart with variable sample size and interval

If the statistic Z_i is marked within region $I_1 = [-w, w]$, the control (or inspection) is relaxed using the pair (n_1, h_2), otherwise, if the current point Z_i lies within region $I_2 = [-k, -w) \bigcup (w, k]$, control will be stricter using the pair (n_2, h_1).

3. $\bar{X} - VSSI$ Chart Performance

The statistical performance of a control chart can be evaluated by calculating the ARL or ATS statistics. Depending on the process operating condition, there is the ARL when the process is in control (ARL_0), that is, the expected number of samples between two successive false alarms and the ARL for process out of control (ARL_δ), which represents the expected number of samples between the occurrence of special cause that alters the monitored parameter and the signal triggered by the graph. The symbol δ represents the displacement degree occurred in the process average and can be calculated by the expression: $\delta = (\mu_1 - \mu_0)/\sigma_0$ where μ_1 represents the new mean baseline after the process is out of adjustment [8]. Similarly, there is the ATS when the process is in control (ATS_0), representing the average time between two successive false alarms and the ATS for process out of control (ATS_δ), representing the expected time between the occurrence of special cause and the signal triggered by the graph.

3.1. *Markov Chains for* $\bar{X} - VSSI$ Chart

It is possible to calculate the ARL and ATS statistics using Markov chains. One observes the expected number of transitions before the monitored statistics to be in absorbing state of the chain. The Markov chain, proposed by Zimmer [11], was used in this article to evaluate the ARL in control and out of control, ARL_0 and ARL_δ, respectively. Each transition probability is calculated as a probability of the statistics to be within one of the regions of the control range (I_1, I_2 or I_3). In this chain, there are two transient states and an absorbing state that corresponds to the process out of control.

The matrix of transition of chain states which represents the operation of the process in control (P_0) can be divided into four submatrices:

$$P_0 = \begin{bmatrix} Q_0 & R_0 \\ 0 & I \end{bmatrix} \tag{3}$$

where Q_0 is the transition matrix between transient states; R_0 is the transition matrix of transient states to absorbing state; 0 is the matrix that affirms the impossibility of going from an absorbing to a transient state and *I is* the identity matrix.

In a Markov chain, the element (*i,j*) of the matrix $[I - Q_0]^{-1}$ represents the average number of visits to the *j* transient state before reaching the absorbing state, since the process started in *I* state. Each transition probability in control is calculated as the probability of a point of the monitored statistic being situated within one of the regions of the control interval. Therefore, ARL_0 is calculated by:

$$ARL_0 = \{b\}^T [I - Q_0]^{-1} \{1\} \tag{4}$$

where $\{b\}^T$ is a vector with initial probabilities; *I* is the identity matrix; $\{1\}$ is a unit vector and Q_0 is a transition matrix.

$$Q_0 = \begin{bmatrix} \Phi(w) - \Phi(-w) & 2[\Phi(k) - \Phi(w)] \\ \Phi(w) - \Phi(-w) & 2[\Phi(k) - \Phi(w)] \end{bmatrix} \tag{5}$$

where $\Phi(.)$ denotes the standard normal cumulative function; *k* and *w* are the limits that define the chart control region.

The average time so that the chart produces a false alarm is:

$$ATS_0 = \{b\}^T [I - Q_0]^{-1} \{h\} \tag{6}$$

where $\{h\}$ is a vector with the sampling intervals.

The process transition matrix operating out of control is given by:

$$P_\delta = \begin{bmatrix} Q_\delta & R_\delta \\ 0 & I \end{bmatrix} \tag{7}$$

In order to calculate the measures of performance ARL_δ and ATS_δ it is used:

$$ARL_\delta = \{b\}^T [I - Q_\delta]^{-1} \{1\} \tag{8}$$

and

$$ATS_\delta = \{b\}^T [I - Q_\delta]^{-1} \{h\} \tag{9}$$

being the transition matrix given by:

$$Q_\delta = \begin{bmatrix} Q_{\delta 11} & Q_{\delta 12} \\ Q_{\delta 21} & Q_{\delta 22} \end{bmatrix} \tag{10}$$

where:

$$Q_{\delta 11} = \Phi\left(w - \delta\sqrt{n_1}\right) - \Phi\left(-w - \delta\sqrt{n_1}\right);$$

$$Q_{\delta 21} = \Phi\left(w - \delta\sqrt{n_2}\right) - \Phi\left(-w - \delta\sqrt{n_2}\right);$$

$$Q_{\delta 12} = \left[\Phi\left(k-\delta\sqrt{n_1}\right)-\Phi\left(w-\delta\sqrt{n_1}\right)\right] + \left[\Phi\left(-k-\delta\sqrt{n_1}\right)-\Phi\left(-w-\delta\sqrt{n_1}\right)\right];$$

$$Q_{\delta 22} = \left[\Phi\left(k-\delta\sqrt{n_2}\right)-\Phi\left(w-\delta\sqrt{n_2}\right)\right] + \left[\Phi\left(-k-\delta\sqrt{n_2}\right)-\Phi\left(-w-\delta\sqrt{n_2}\right)\right].$$

The vector with initial probabilities $\{b\}^T$ is defined according to the initial conditions of operation in the process:

$$\{b\}^T = \left\{ \frac{\Phi(w)-\Phi(-w)}{\Phi(k)-\Phi(-k)} \quad \frac{2\left[\Phi(k)-\Phi(w)\right]}{\Phi(k)-\Phi(-k)} \right\} \tag{11}$$

This article considers the condition known as Steady-State, that is, it is assumed that the process starts in control and, at some future time, there is a special question that causes a shift in the target value of the monitored statistics.

3.2. Optimum Statistical Design for the $\bar{X}-VSSI$ Chart

The planning of a control chart can be formalized as an optimization problem in which the decision variables are the chart parameters. Figure 2 illustrates the objective function and constraints that define the optimal set of parameter of chart $\bar{X}-VSSI$.

Objective Function:

$$\min ATS\left(n_1, n_2, h_1, h_2, w, k \mid \delta\right)$$

Subject to:

$$ATS\left(n_1, n_2, h_1, h_2, w, k \mid \delta = 0\right) = ATS_0;$$
$$E(n) = n_0;$$
$$E(h) = h_0;$$
$$0 < w < k;$$
$$h_{\min} \le h_1 \le h_2 \le h_{\max};$$
$$n_{\min} \le n_1 \le n_2 \le h_1.r_{insp}$$

Figure 2. Objective function and constraints for the control chart parameters $\bar{X}-VSSI$

In Figure 2, n_1 and n_2 are the sample sizes; h_1 and h_2 are the time intervals between sample collection; *w* and *k* are chart control limits; δ is the displacement degree occurred in the process mean; ATS_0 is the average time between two successive false alarms; n_0 is the expected value of the sample size collected with the process in control; h_0 is the expected time to collect a sample with process in control and r_{insp} is the quantity of pieces (a part, a component, etc.) which can be inspected per time unit considered in h_0.

To illustrate that the optimization problem is reduced to find the pair (n_1,n_2) that minimizes the objective function, consider, with no generality loss, that $E(h) = h_0 = 1$ time unit (e.g. 1 hour, 0.5 hours, etc.) and ARL_0=370.4. Therefore, ATS_0=ARL_0=370.4 and *k*=3.

The expected value of the sample size with the process in control, $E(n) = n_0$, is given by:

$$E(n) = n_0 = \frac{\Phi(w)-\Phi(-w)}{\Phi(k)-\Phi(-k)} n_1 + \frac{2\left[\Phi(k)-\Phi(w)\right]}{\Phi(k)-\Phi(-k)} n_2 \tag{12}$$

A pair of samples (n_1, n_2) is selected; since (n_1, n_2), n_0 and *k* are known, *w* can be inferred directly from the expression (12).

The shorter ideal sampling interval (h_1) is:

$$h_1 = \frac{n_2}{r_{insp}} \tag{13}$$

where r_{insp} is the amount of parts (a part, component, etc.) which can be inspected per time unit considered in $E(h) = h_0$. For example, if $r_{insp} = 60$ given that $h_0 = 1$ hour, it is assumed that it is possible to inspect 60 pieces every hour. For more details, see Celano [14, 15].

Once defined h_0, h_1, *w* and *k*, one can obtain h_2 through the expected time to collect a sample:

$$E(h) = h_0 = \frac{\Phi(w)-\Phi(-w)}{\Phi(k)-\Phi(-k)} h_1 + \frac{2\left[\Phi(k)-\Phi(w)\right]}{\Phi(k)-\Phi(-k)} h_2 \tag{14}$$

The optimization problem is finally reduced to find the pair (n_1, n_2) that minimizes the objective function.

Section 4 presents an application example in which one reveals values that the pair (n_1, n_2) should be used. For this, one used the software R [18] as a tool to calculate the optimal parameters of an $\bar{X}-VSSI$ chart.

4. Example

In this section, one proposes two functions (see Appendix) developed for use in the R software that evaluate the $\bar{X}-VSSI$ control chart performance and solve the optimization problem shown in Figure 2.

The first function, called ***VSSI***, evaluates the control chart performance by calculating the ATS_δ when provided by the user: n_1, n_2, *n*, *delta* (δ), h_0 and *r_insp*.

The second function, ***VSSI.optimum***, solves the optimization problem shown in Figure 2. This function requires as input: n_0, *delta* (δ), h_0, *r_insp* and a value for nmax that refers to the highest size of sample allowable for collection.

To illustrate the use of functions, consider the example shown in Costa [8]. A packaging milk line has an average value of 1000 mL and a standard deviation estimated to be 4.32 mL. It is done the monitoring of the average of the process by inspecting samples size $n_0 = 5$ at every time unit. Suppose that such a unit is equal to h_0=60 minutes. In this example, the parameters designed for the control chart are fixed, that is, the sample size, the sampling interval and the limits do not alter after they are estimated. In order to use the $\bar{X}-VSSI$ control chart, it is necessary to calculate the control limits (w and k) and the sampling scheme (n_1, h_2) and (n_2, h_1).

Selecting, for instance, $n_1 = 2$; $n_2 = 8$; $n_0 = 5$; $\delta = 1.0$; ARL_0=370.3983; h_0=60 and *r_insp*=60, the ***VSSI*** function provides the parameters shown in Figure 3.

In this example, $\delta = 1.0$ means that the process average went from $\mu_0 = 1000$ (in control) to $\mu_1 = \mu_0 + \delta * \sigma_0 = 1000 + 1*4.32 = 1004.32$ (out of control).

Consider the case where $\delta = 2.0$. The Figure 4 illustrates the results obtained with the function ***VSSI***. It is observed that the ATS is lower ($ATS_{\delta=2} < ATS_{\delta=1}$), then, when major deviations occur in the process mean, the chart performance is better.

However, an optimum scheme to monitor this process is the one presenting the best performance, that is, the lowest ATS_δ. By means of ***VSSI. optimum*** function, one can obtain parameters that minimize ATS_δ. Figure 5 shows the best schemes for the cases shown in Figures 3 and 4.

In this case, the user who wants to control the average value of a process considering a displacement possibility presented here, has simply to build the $\bar{X}-VSSI$ control chart with the parameters shown in Figure 5. Other $\bar{X}-VSSI$ charts can be easily constructed by modifying input values of functions ***VSSI*** and ***VSSI.optimum***.

5. Conclusions

It was displayed in this article, how it evaluates the control chart performance of $\bar{X}-VSSI$ by means of Markov chains and, mainly, how to get the parameters that minimize the ATS. For this, two functions written in the language setting to R were created in order to minimize the ATS and present the best parameters to be used in constructing the $\bar{X}-VSSI$ control chart. Adaptive schemes are recognized as being more efficient than the control charts schemes with fixed parameters. Nevertheless, the use of adaptive schemes for control graphs is not common in practice, because the traditional statistical software do not display routines for these types of charts. Thus, with the programs presented here, the user has a tool in which it is possible to plan the $\bar{X}-VSSI$ control chart use in order to monitor the average value of a feature of desired quality.

```
> VSSI(n1=2,n2=8,n0_FSR=5,delta=1,ARL0=370.3983,h0=60,r_insp=60)
    ATSd        n1        n2        h1        h2     delta
 93.5959    2.0000    8.0000    8.0000  112.0000    1.0000
      ATS0        ARL0          k          w         n0          h0      r_insp
22223.8980    370.3983     3.0000     0.6724     5.0000     60.0000     60.0000
```

Figure 3. Parameters obtained with the function VSSI (n_1=2, n_2=8, n0_FSR=5, delta=1, ARL_0=370.3983, h_0=60, r_insp=60). Note: $h_0$should be set in minutes

```
> VSSI(n1=2,n2=8,n0_FSR=5,delta=2,ARL0=370.3983,h0=60,r_insp=60)
    ATSd        n1        n2        h1        h2     delta
 63.1409    2.0000    8.0000    8.0000  112.0000    2.0000
      ATS0        ARL0          k          w         n0          h0      r_insp
22223.8980    370.3983     3.0000     0.6724     5.0000     60.0000     60.0000
```

Figure 4. Parameters obtained with function VSSI (n_1=2, n_2=8, n0_FSR=5, delta=2, ARL_0=370.3983, h_0=60, r_insp=60)

```
> VSSI.otimo(n0_FSR=5,delta=1,ARL0=370.3983,h0=60,r_insp=60,nmax=40)
    ATSd      n1      n2      h1       h2   delta       ATS0      ARL0       k       w      n0       h0   r_insp
 91.5370  3.0000  7.0000  7.0000 113.0000  1.0000 22223.8980  370.3983  3.0000  0.6724  5.0000  60.0000  60.0000
> VSSI.otimo(n0_FSR=5,delta=2,ARL0=370.3983,h0=60,r_insp=60,nmax=40)
    ATSd      n1      n2      h1       h2   delta       ATS0      ARL0       k       w      n0       h0   r_insp
 60.6035  4.0000  6.0000  6.0000 114.0000  2.0000 22223.8980  370.3983  3.0000  0.6724  5.0000  60.0000  60.0000
```

Figure 5. Parameters obtained with the function VSSI.optimum (n0_FSR, delta, ARL_0, h_0, r_insp, nmax)

Appendix

Source code to evaluate the performance and choose an optimum statistical design for the control chart $\bar{X}-VSSI$ in R environment.

It is presented hereafter two functions called ***VSSI*** and ***VSSI.optimum.*** To use them, simply copy them in the R environment and follow the application example.

```
# Function: VSSI
# function that evaluates the X̄ −VSSI chart performance by means of a Markov chain

rm(list=ls(all=TRUE))
VSSI <- function(n1,n2,n0_FSR,delta,ARL0,h0,r_insp) {
k0<- qnorm(1-(1/(2*ARL0)))
time<- h0
h0<- 1
b_vector<- matrix(c(1,0), nrow=1, ncol=2) #vetor {b}
fi_k0<- pnorm(k0)
w0<- qnorm((fi_k0*(n2-n0_FSR)/(n2-n1)+0.5*(n0_FSR-n1)/(n2-n1)))
h1<- n2/r_insp
h2<- h0*(pnorm(k0)-pnorm(-k0))/(pnorm(w0)-pnorm(-w0))-h1*(2*(pnorm(k0)-pnorm(w0)))/(pnorm(w0)-pnorm(-w0))
# In control - Transition Probabilities
p0_o_o <- pnorm(w0)-pnorm(-w0)
p0_o_ab <- 2*(pnorm(k0)-pnorm(w0))
p0_ab_o <- p0_o_o
p0_ab_ab <- p0_o_ab
# steady state probabilities
p1<- p0_ab_o/(p0_o_ab+p0_ab_o)
p2<- p0_o_ab/(p0_o_ab+p0_ab_o)
# Matriz de transição
P <- matrix(c(p0_o_o, p0_o_ab, 1-p0_o_o-p0_o_ab, p0_ab_o, p0_ab_ab, 1-p0_ab_o-p0_ab_ab , 0, 0, 1), nrow = 3, ncol=3,
byrow=TRUE,dimnames = list(c("O", "A or B", "OOC"),c("O", "A or B", "OOC")))
# fundamental Markov matrix
Qo<- matrix(c(p0_o_o, p0_o_ab, p0_ab_o, p0_ab_ab), nrow=2, ncol=2, byrow=TRUE,
dimnames = list(c("O", "A or B"), c("O", "A or B")))
Id <-   matrix(c(1,0,0,1), nrow=2, ncol=2, byrow=TRUE)
# unit vector
one<- matrix(c(1,1), nrow=2, ncol=1)
# [(I-Q0)^-1]
Id_Qo<- solve (Id-Qo)
# vector {n}
n_vector<- matrix(c(n1,n2), nrow=2, ncol=1)
#Out of control - Transition Probabilities
pd_o_o<- pnorm(w0-delta*sqrt(n1))-pnorm(-w0-delta*sqrt(n1))
pd_o_ab<- pnorm(k0-delta*sqrt(n1))-pnorm(w0-delta*sqrt(n1))+pnorm(-w0-delta*sqrt(n1))-pnorm(-k0-delta*sqrt(n1))
pd_ab_o<- pnorm(w0-delta*sqrt(n2))-pnorm(-w0-delta*sqrt(n2))
pd_ab_ab<- pnorm(k0-delta*sqrt(n2))-pnorm(w0-delta*sqrt(n2))+pnorm(-w0-delta*sqrt(n2))-pnorm(-k0-delta*sqrt(n2))
#State transition matrix
Pd<- matrix(c(pd_o_o, pd_o_ab, 1-pd_o_o-pd_o_ab, pd_ab_o, pd_ab_ab, 1-pd_ab_o-pd_ab_ab , 0, 0, 1), nrow = 3, ncol=3,
byrow=TRUE,
dimnames = list(c("O", "A or B", "OOC"), c("O", "A or B", "OOC")))
# fundamental Markov matrix
Qd<- matrix(c(pd_o_o, pd_o_ab, pd_ab_o, pd_ab_ab), nrow=2, ncol=2, byrow=TRUE,
```

Source code to evaluate the performance and choose an optimum statistical design for the control chart $\overline{X} - VSSI$ in R environment (Continuation).

```
dimnames = list(c("O", "A or B"), c("O", "A or B")))
Id_Qd<- solve(Id-Qd) # [(I-Qd)^-1]
h_vector<- matrix(c(h2,h1), nrow=2, ncol=1) # vetor {h}
p1 <-(pnorm(w0)-pnorm(-w0))/(pnorm(k0)-pnorm(-k0))
p2 <-(2*(pnorm(k0)-pnorm(w0)))/(pnorm(k0)-pnorm(-k0))
b_vector<- matrix(c(p1,p2), nrow=1, ncol=2) #vetor {b} SS
ATS <- b_vector %*% Id_Qd %*% h_vector      #b*[(I-Q0)^-1]*h
#transformation for the user unit of time in minutes
h0<- h0 * time; h1 <- h1 * time; h2 <- h2 * time; ATS <- ATS * time; ATS0 <- ARL0*h0
parameters<- c(ATS,n1,n2,h1,h2,delta,ATS0,ARL0,k0,w0,n0_FSR,h0,r_insp)
names(parameters) <- c("ATSd","n1","n2","h1","h2","delta","ATS0","ARL0","k","w","n0","h0","r_insp")
parameters<- round(parameters,4)
parameters }
```

```
# Application example of    function VSSI
# n1 - sample size 1
# n2 - 1sample size 2
# n0_FSR - expected value (average) for the sample size (process in control)
# delta - shift degree    in the process mean
# ARL0 – the expected number of samples between two successive false alarms
# h0 – expected time to collect a sample (process in control).
NOTE: launch the value of ho in minutes.
# r_insp - quantity of parts which can be inspected per   time unit considered in h0.

VSSI(n1=2,n2=8,n0_FSR=5,delta=1,ARL0=370.3983,h0=60,r_insp=60)
```

```
# Function: VSSI.optimum
#    function that chooses the optimal parameters of the  X̄ - VSSI    chart

VSSI.optimum<- function(n0_FSR,delta,ARL0,h0,r_insp,nmax){
LI=n0_FSR-1 ; LS=n0_FSR+1
n1opt=1 ; n2opt=2*n0_FSR-n1opt
ATSopt=VSSI(n1opt,n2opt,n0_FSR,delta,ARL0,h0,r_insp)[1]
for (n1 in 1:LI) {
for (n2 in LS:nmax) {
x1<- 0.5*n1+0.5*n2
x2<- n0_FSR
if (identical(all.equal(x1, x2), TRUE)) {
result<- VSSI(n1,n2,n0_FSR,delta,ARL0,h0,r_insp)[1]
if    (result<ATSopt) {
n1opt=n1 ; n2opt=n2
ATSopt=as.numeric(VSSI(n1,n2,n0_FSR,delta,ARL0,h0,r_insp)[1]) }}}}
print(VSSI(n1opt,n2opt,n0_FSR,delta,ARL0,h0,r_insp)) }
```

```
# Application Example of function VSSI.optimum
# nmax – maximum permissible value for the sample size
# Note: This function depends on the previous one. In order to use the function VSSI.optimum copy also the function VSSI
in the R software desktop .
VSSI.optimum(n0_FSR=5,delta=1,ARL0=370.3983,h0=60,r_insp=60,nmax=40)
```

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