Human Capital, Bankruptcy and Capital Structure*

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ABSTRACT

In a setting in which firms can choose their capital structures, we derive the optimal compensation contract when employees are averse to holding their own human capital risk, but equity holders are not averse to holding this risk. In the absence of other frictions, all firms will be unlevered and instead will hold cash. In the presence of corporate taxes, optimal debt levels are consistent with levels observed, implying that human capital risk is of the same order of importance as taxes in the capital structure decision. Our model provides a number of new predictions on the cross sectional distribution of firm leverage. Consistent with existing empirical evidence, it implies that persistent unexplained idiosyncratic differences in leverage across firms should exist. Ceteris paribus, firms with more leverage should pay higher wages. This result represents an heretofore unexplored empirical implication of the risk sharing model of the firm.

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1 Introduction

Ever since Modigliani and Miller (1958) first showed that capital structure is irrelevant in a frictionless economy, financial economists have puzzled over what the frictions are that make the capital structure decision so important in reality. The most important friction yet identified (by Modigliani and Miller themselves) is corporate taxes: Because dividends are subject to corporate taxation while interest payments are not, firms can potentially realize significant tax savings by maintaining high levels of debt. However, in practice, firms maintain only modest levels of debt. As Miller (1988) pointed out in a 30 year retrospective on his own work:

"In sum, many finance specialists, myself included, remain unconvinced that the high-leverage route to corporate tax savings was either technically unfeasible or prohibitively expensive in terms of bankruptcy or agency costs." (p. 113)

Miller goes on to argue that corporate debt levels resulted from sub-optimal decision making, and points to two innovations that were happening at the time of the retrospective – the growth in junk bond markets and an explosion in the number of LBOs – as evidence of managers changing behavior and moving towards more "optimal" debt levels. However, subsequent developments have not borne out Miller's prediction. In a recent study, Graham (2000) finds (p. 1903) that "...even extreme estimates of distress costs do not justify observed debt policies." Why, then, do firms appear to have too little debt?

Clearly, an opposing friction must exist. However, economists have struggled to identify it. One candidate are (direct) bankruptcy costs: High levels of debt increase the probability of bankruptcy, so if there are deadweight costs associated with bankruptcy, these costs will be a disincentive to issue debt. However, in an important paper on bankruptcy costs, Haugen and Senbet (1978) argue that these costs cannot exceed the cost of negotiating around them — debt holders bear the costs of bankruptcy, so they have incentives to recapitalize the firm outside of bankruptcy, and thus avoid these costs. Hence, deadweight bankruptcy costs cannot exceed the cost of renegotiating, which significantly limits their potential role as an effective counterweight to the large benefit of the tax shield.

A simple "back of the envelope" calculation shows the significance of this point: Suppose costs of 30% are incurred in the event of bankruptcy (a very high number, in light of Haugen and Senbet (1978)), and there is a 5% chance of bankruptcy occurring. Then the expected cost of bankruptcy at the time of the capital structure decision is 1.5% of the financial distress value – substantially less than 1.5% of the value of the firm at the time of the decision. Graham (2000) estimates the tax benefit of debt to be in the range of 10% of value

at the time of the capital structure decision, so taxes are at least an order of magnitude more important than bankruptcy costs.

Largely in response to Haugen and Senbet's critique, Titman (1984) argues that potentially the indirect costs of bankruptcy could explain existing debt levels — costs precipitated by the bankruptcy filing that affect other stakeholders (besides debt and equity holders) such as customers. Although an extensive literature documenting and studying the indirect costs of bankruptcy has developed since Titman's insight, researchers have nevertheless still struggled to identify an indirect cost of bankruptcy as important as taxes.¹ In this paper we argue that a largely ignored indirect cost of bankruptcy, the cost borne by the firm's employees, is potentially as important as taxes.

An interesting characteristic of the existing literature on bankruptcy costs is the apparent disconnect between the costs that researchers study and the ones identified in the popular press. During a corporate bankruptcy a major focus of the popular press is the human cost of bankruptcy, yet these costs have received minimal attention in the research literature. It is not difficult to understand why. In an efficient labor market there should be no human costs associated with bankruptcy. If employees are being paid their competitive wage, it should be relatively costless to find a new job at the same wage. For substantial human costs of bankruptcy to exist, employees must be entrenched — they must incur costs associated either with not being able to find an alternative job, or with taking another job at substantially lower pay. At first blush, such entrenchment seems difficult to reconcile with optimizing behavior: Even if labor markets are inefficient, why do shareholders ignore this inefficiency, and instead overpay their employees? It would appear to be relatively costless to lower wages to their competitive levels, especially at times when the firm is facing the prospect of bankruptcy.²

In this paper we argue that this intuition is wrong. In an economy with perfectly competitive capital and labor markets, one should *expect* large human costs of bankruptcy, and it is precisely these indirect costs that limit the use of corporate debt. We begin with the insight of Harris and Holmström (1982) on the form of optimal employment contracts in perfect capital and labor markets. In a setting without bankruptcy, they show that the optimal employment contract guarantees job security (employees are never fired), and pays employees a fixed wage that never goes down, but rises in response to good news about em-

¹see, for example, Andrade and Kaplan (1998)

²One possible explanation is the existence of firm specific human capital (see Neal (1995)). Yet in an efficient labor market it is not clear that employees are necessarily paid for their investments in human capital. Furthermore, even if they are, in a competitive economy like the United States it is hard to argue that most employees' skills are not easily transferable, or that wages could not be lowered during financial distress.

ployee ability. The intuition behind their result is that, while employees are averse to their own human capital risk, this risk is idiosyncratic, so equity holders can costlessly diversify it away by investing in many firms. Optimal risk sharing then implies that the shareholders will bear all of this risk by offering employees a fixed wage contract. The problem with this contract is that employees cannot be forced to work under it: Employees who turn out to be better than expected will threaten to quit unless they get a pay raise. This imperfection leads to the optimal employment contract Harris and Holmström (1982) identified.³

In Harris and Holmström (1982), firms carry no debt, and equity holders do not have limited liability. To credibly commit to the terms of the contract, equity holders guarantee the fixed wage by implicitly promising to make the wage payments even when the firm could not — that is, the equity holders have unlimited personal liability. In principle, there is no reason why the optimal equity contract requires limited liability. However, such contracts would be very difficult to trade in anonymous markets. Without the ability to trade, equity holders would no longer be able to diversify costlessly, and so the underlying assumption that they are not averse to human capital risk would be difficult to support. Hence, imposing the restriction that equity has limited liability is important.

Our first objective in this paper is to derive the optimal compensation contract in a setting that includes both limited liability equity and debt. We show that the optimal employment contract in this setting is similar to that in Harris and Holmström (1982): Unless the firm is in financial distress, wages never fall, and rise whenever employees turn out to be better than expected. However, at the point where the firm cannot make interest payments, the employee takes a temporary pay cut. If the financial health of the firm improves, wages return to their contracted level. If it deteriorates further, and the firm cannot make interest payments even with the temporary wage concessions, it is forced into bankruptcy. In bankruptcy the firm can abrogate its contracts — employees can be terminated and new, more productive, employees can be hired to replace them. As a result, entrenched employees are forced to take a wage cut and earn their current market wage, either with the current firm or with a new firm.

The form of this optimal employment contract has important implications for optimal capital structure. Note that most employees are likely to become entrenched — because their pay can only be increased, eventually it will be increased by too much. Because such employees are destroying value (the value of the firm would go up were they replaced), investors in the firm actually benefit from a bankruptcy filing because they can effectively fire such employees or lower their wages to competitive levels. Thus, ex post the effect of

³Several other papers in labor economics have studied optimal wages when the firm is risk neutral but the workers are risk averse. See, for example, Holmström (1983), Bester (1983), or Thomas and Worrall (1988).

the optimal contract is to create a *benefit* to investors of filing for bankruptcy. Because this cost is an indirect cost (at the time of bankruptcy it accrues to employees and not investors), Haugen and Senbet's critique does not apply — investors do not an incentive to avoid bankruptcy.

The implications of the optimal labor contract on the level of debt occur ex ante. The amount of risk sharing between investors and employees depends on the level of debt – under the optimal labor contract, higher debt levels imply less risk sharing. Thus, in the absence of taxes, the optimal level of debt is zero — with no frictions other than an incomplete market in human capital, all firms will be unlevered. In fact, it is optimal for firms in our model to have negative leverage, that is, maintain cash balances. When corporate taxes are introduced into the model, a theory of optimal capital structure emerges that can resolve the apparent paradox present in the data: Firms maintain only modest levels of debt relative to the measured levels of the direct costs of bankruptcy. In fact, in some cases firms will optimally maintain cash balances even when there is a tax disadvantage to do so (that is, investors would be better off investing the cash themselves).

Our model predicts a number of determinants of firm leverage that have heretofore have not been investigated. Perhaps most interesting, given the empirical evidence, is our result that large purely idiosyncratic components to the capital structure of a firm should exist. Because the capital structure decision trades off the risk benefits of debt and the corporate tax shield, firms that happen to have more risk averse employees will have lower levels of debt. But because such firms have lower levels of debt, they will represent attractive employment opportunities for relatively more risk averse employees. That is, the effect is reinforcing. Ultimately, heterogeneity in risk aversion in the labor market should result in persistent heterogeneity in the average risk aversion of employees and in capital structure choices amongst otherwise identical firms. The existence of persistent heterogeneity in firms capital structure has puzzled financial economists;⁴ the effects studied in this paper have the potential to resolve this puzzle.

An important new empirical prediction that arises from our model is a relation between leverage and wages. Ceteris paribus, higher wages should be associated with lower leverage. Consequently, firms with unexplained persistently higher levels of debt should pay lower wages. Furthermore, the relation between wages and leverage is a direct consequence of the risk sharing model of the firm, first postulated by Knight (1921). Thus, the relation between wages and leverage is a hitherto unexplored test of the importance of this model of the firm.

By imposing the additional assumption that capital is less risky than labor, we derive another empirical implication of the model: Labor intensive firms should have lower leverage

⁴see Lemmon et al. (2006)

than capital intensive firms. Capital intensive firms tend to be larger (especially if accounting numbers are used as a measure of firm size), so our model implies that a cross-sectional relation between debt levels and firm size should exist — large firms will be more highly levered. This prediction is supported by the existing empirical evidence — Titman and Wessels (1988), Rajan and Zingales (1995) and Fama and French (2002) all document a positive cross-sectional relation between leverage and firm size. Interestingly, the model also predicts a positive relation between firm size and wages. This relation has also been documented empirically, and it regarded as a puzzle by labor economists (see Brown and Medoff (1989)).

The rest of the paper is organized as follows. In the next section we review the related literature. In Section 3 we describe the model and derive the optimal labor contract in our setting. We show that in the absence of taxes, firms should not have positive levels of debt. In Section 4 we derive the empirical implications on the firm's leverage of the optimal contract. We then parameterize the model and illustrate its implications. Although a thorough empirical test of the model is beyond the scope of this paper, in Section 5 we discuss a number of existing studies that bear directly on the implications of the model. Section 6 concludes the paper.

2 Review of the Literature

Motivated by the Haugen and Senbet (1978) critique, Titman (1984) introduces the idea of indirect costs of bankruptcy. Titman (1984) argues that other stakeholders not represented at the bargaining table at a bankruptcy filing, such as customers, can suffer material costs resulting from the bankruptcy. Because the claimants who are at the bargaining table, the debt and equity holders, do not incur these costs, they have no incentive to negotiate around them. Hence such costs could be substantial. We argue in this paper that one such cost, the human cost of bankruptcy, that has received limited attention in the literature, is potentially the single most important indirect cost of bankruptcy.

Several papers have analyzed the interaction between capital structure choice and the firm's employees' compensation and their incentives. Like us, Chang (1992) analyzes the optimal contract between investors and employees, but with a very different focus; he does not model either the ability of the employees or the role of labor markets. In his model, employees are risk averse and thus should be given a constant wage. However, in some states of the world, value-enhancing restructurings should be undertaken, which are costly for the employees. It is assumed that the employees' contract cannot be made contingent on such restructuring events and, as a consequence, employees always try to avoid restructuring.

Investors therefore finance the firm with both equity and debt. If the firm defaults on the debt, then investors are in charge and can force a restructuring. In a related paper, Chang (1993) focuses on the interaction between payout policy, capital structure and compensation contracts. Managers value control, so they must be motivated to pay out capital to the investors. The employee's compensation is therefore linked to the payout to equityholders. However, the optimal payout level may change over time. Such a change is only feasible if control is transferred from the management to investors. This transfer is achieved by issuing the right amount of debt ex ante so that bankruptcy occurs in those states when new information about the optimal payout level is likely to be available. Thus, our paper shares a key insight with both Chang (1992) and Chang (1993), namely, the triggering of bankruptcy allows value enhancing recontracting.

Berkovitch, Israel, and Spiegel (2000) also study the relation between managerial compensation and capital structure. However the focus of that paper is different. In that paper compensation policy is designed to incentevise managers to exert costly effort; risk differences between employees and investors are ignore. We do the opposite, we ignore incentive issues and concentrate on risk. Interestingly, like us, that paper derives the empirical prediction at leverage and wages should be positively correlated in the cross section.

In an early contribution, Baldwin (1983) models a firm that undertakes a capital investment. Ex-post, employees can appropriate the return to capital, because capital costs have been sunk. Issuing a sufficient amount of debt may mitigate this hold-up problem. If higher wages are demanded from a highly levered firm bankruptcy occurs which is assumed to be costly for workers. Perotti and Spier (1993) emphasize a similar role of debt. In their model equity holders may issue junior debt and thereby create an underinvestment incentive. This can then be used to obtain wage concessions from employees to restore incentives to invest. Stulz (1990) analyzes a firm where shareholders cannot observe either the firm's cash flows or the employee's investment decisions. Management always wants to invest as much as possible. Because shareholders know this, they will not always fully satisfy the employee's demand for capital. Therefore the employee cannot take all positive NPV projects when the firm's cash flows are low and its investment opportunities are good, and will overinvest when the firm's cash flows are high and its investment opportunities are poor. It is shown that it is optimal for investors to design a capital structure consisting of debt and equity to reduce the costs of over- and underinvestment.

More recently, Cadenillas, Cvitanić, and Zapatero (2004) model a firm with a risk averse manager, who is subject to moral hazard. It is assumed that the manager receives stock as his only source of compensation. Equityholders can choose to lever the firm, thereby changing the manager's compensation. When choosing the optimal leverage, they take into

account that the employee applies costly effort and selects the level of volatility, both of which affect expected returns. DeMarzo and Fishman (2006) derive the optimal labor contract in a similar moral hazard setting and show that it requires the firm to issued debt.

e need to review entrenchment literature properly — I have done some work, but honestly I just ran out of energy.

This paper is closely related to the literature that examines how management entrenchment affects capital structure. Zwiebel (1996) provides a formal model of an employee's capital structure choice when ownership is separated from control. In this paper, an entrenched employee determines the firm's capital structure, recognizing that he can only be replaced if the firm is taken over or if the firm goes bankrupt. Because the employee derives extra utility from keeping his job, he wishes to avoid being fired. This can be done by issuing debt. By doing that, the employee commits not to undertake negative NPV projects, and thereby makes a hostile takeover unprofitable. In equilibrium, managers with low abilities issue debt, and therefore do not take on negative NPV projects. This allows them to avoid both hostile takeovers and bankruptcy. Novaes and Zingales (1995) and Jung et al. (96) derive similar results in a similar setting.

Morellec (2004) extends the model by Zwiebel (1996), and derives a continuous-time model of an entrenched employee who may find it optimal to issue debt to avoid a hostile takeover. He allows for a tax advantage of debt, so that there exists an optimal debt level even in the absence of agency problems. The paper shows how the employee's capital structure choice deviates from the firm value maximizing capital structure. Subramanian (2002) also analyzes a firm where the employee makes capital structure and investment decisions, taking his personal bankruptcy costs and risk aversion into account. In each period, the employee's income is derived by a bargaining process with the equityholders.

Our analysis differs in several important ways from the previous literature on managerial capital structure choice discussed above. The existing literature takes managerial entrenchment as exogenous, relying on specified managerial characteristics, such as empire building preferences or effort aversion, that destroys shareholder value, and cannot be eliminated by appropriate compensation contracts. Instead we derive managerial entrenchment as an optimal response to managerial risk aversion and our main contribution is that this optimal response *itself* has capital structure implications. That is, the level of risk employees face determines the likelihood of employee entrenchment which then determines the firm's leverage. We analyze this role of capital structure without relying on moral hazard or asymmetric information and solve for the optimal employees' compensation under fairly mild contracting restrictions. Because we have no moral hazard in our model, and we assume that both labor markets and capital markets are competitive, *ex ante* the employee captures all the economic

rents and makes the capital structure choice that maximizes his utility. Consequently there is no inefficiency associated with entrenchement in our model — the only friction is the inability of employees to insure their human capital, which is not a focus of the prior literature on entrenchment and capital structure.

Berens and Cuny (1995) provide an important alternative explanation for low leverage ratios in the absence of insignificant bankruptcy costs. They point out that interest payments can only be deducted up to the amount of current income. For a growing firms with relatively low current cash flows, there is little to shield, so the usefulness of debt is limited. Their point is significant even for firms with relatively modest growth rates. For example, using historical estimates and assuming a zero real growth rate (so all growth in cashflows results from inflation), Berens and Cuny (1995) show that the optimal debt ratio of riskless firm is 40%.

Tserlukevich (2005) expands the analysis of Berens and Cuny (1995) by explicitly modeling corporate growth options when real investment is irreversible. This model can explain many of the empirical stylized facts on capital structure (e.g. low and mean reverting leverage ratios, a negative relation between past profits and current leverage, as well as between past stock returns and current leverage). Although it is likely that Berens and Cuny (1995) insight explains part of the reason firms limit their use of debt, it cannot be the full story; Graham (2000) provides evidence that firms could increase leverage substantially before the effective corporate tax rates start to decrease. Thus, even relative to their low initial earnings, growth firms still seem to under-utilize debt.

In a recent paper, Hennessy (2005) develops a model of indirect bankruptcy costs that, like us, relies on the ability to abrogate contracts in bankruptcy but his focus is different. He assumes the input quality delivered by the firm's suppliers is unobservable. Incentives must therefore be provided through implicit contracts where bonus payments or refunds from the supplier are discretionary. If the firm issues too much debt, then the supplier can no longer be induced to produce optimal quality. The credibility of both firms declines and profits fall.

This paper is also related to the literature in labor economics that focuses on the risk-sharing role of the firm. For example, Gamber (1988) includes bankruptcy in a setting similar to Harris and Holmström's, derives as an implication that real wages wages should respond more to permanent shocks than temporary shocks and finds empirical support for this result. More recently, Guiso et al. (2005) test this implication using firm level wage data and also find strong support for the risk-sharing role of the firm. Our paper adds to this literature by deriving another testable implication of this model of the firm — that leverage and wages should be inversely related.

3 Optimal Labor Contract

In this section, we derive the optimal contract for a risk-averse employee working for a firm with risk-neutral investors. We extend the results of Harris and Holmström (1982) by both allowing for debt, limited liability equity, and personal bankruptcy if the manger's wage drops below zero. We also derive our results in continuous, rather than discrete, time.

The economy consists of a large number of identical firms, all of which begin life at time 0 and last forever. Each firm requires two inputs to operate, capital in the amount K and an employee who is paid a wage c_t and produces at t, the fully observable (and contractible) value, $KR + \phi_t$. R is certain, it is the risk free pre tax return on capital and ϕ_t is uncertain but fully observable; it is the productivity of the employee. Firms make their capital structure decision once, at time 0, by raising the capital required by issuing debt, D, and equity $K - D \ge 0$. The debt is perpetual and will turn out to be riskless (the firm will always be able to meet its interest obligations), so it has a coupon rate of r, the risk free rate of interest. The firm must pay corporate taxes at rate τ on earnings after interest expense, so the interest tax shield is $Dr\tau$, where D is the amount of debt outstanding. There are no personal taxes in the model. For simplicity, after taxes, capital earns the risk free return, so $R \equiv \frac{r}{1-\tau}$. Thus, the firm produces after tax cash flows of $(\frac{Kr}{1-\tau} - Dr + \phi_t - c_t)(1-\tau) + Dr$ at time t, Dr of which is paid out as interest on debt, and the rest is paid out as a dividend, δ_t , given by

$$\delta_t = Kr - Dr(1 - \tau) + (\phi_t - c_t)(1 - \tau). \tag{1}$$

Because the firm will always be able to meet its interest obligations, the level of debt remains fixed forever. Let $\beta \equiv e^{-r}$.

We assume that capital markets are perfectly competitive. The only source of risk in the model is uncertainty in the employee's output which we assume is idiosyncratic to the employee and thus the firm. Consequently investors can diversify this risk away so the return on all invested capital is the risk free rate, r. We assume that the capital investment is irreversible and that there is no depreciation.

Bankruptcy occurs at the stopping time T when the firm cannot meet its cash flow obligations. At that point in time we assume all contracts can be unilaterally abrogated, so that the firm is no longer bound by the employee's labor contract. Hence at time T firm hires a new employee who immediately puts the capital to productive use. Because there are no costs of bankruptcy, the firm is restored to its initial state (and hence its initial value) and thus can meet its interest obligations, which explains why the firm's debt is riskless (and perpetual).

A bankruptcy filing therefore creates value in our model. For simplicity we assume that

the equity holders are able to hold onto their equity stake and hence capture this value. In fact the assumption that equity holders remain in control reflects the reality of Chapter 11 bankruptcy protection in the U.S.⁵, but most of the results in this paper remain valid even when debt holders capture some or all of this value.

Because of our assumption that the firm can unilaterally abrogate all contracts in bankruptcy it will not make payments after a bankruptcy filing to any fired employee. Thus, the
firm cannot commit to severance payments, or to a corporate pension after a bankruptcy
filing. The first assumption reflects reality — bankrupt firms rarely make severance payments; the second assumption is consistent with the growth of defined contribution pension
plans. In addition we also assume that a firm cannot make severance payments to a fired
employee prior to bankruptcy. Although allowing such payments in our simple model would
be Pareto improving, they are suboptimal in a world with moral hazard where the employee
can secretly lower his productivity, thereby triggering the firing event and thus initiating the
severance payments. We will comment further on the implications of this assumption in the
conclusion.

There is a large, but finite supply of employees with time separable expected utility and a rate of time preference equal to the risk free rate: $E_t \left[\int_t^\infty \beta^s u(c_s) \, ds \right]$, and $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Following Harris and Holmström (1982) we assume that employees are constrained to consume their wages — they cannot borrow or lend⁶ and can only earn wage based compensation, in particular they cannot be paid in the form of securities issued by the firm. This is not a strong assumption with regards to equity or stock options — such compensation decreases risk sharing in our setting and so employees would not want them. More generally, because we place no restriction of the form of the wage contract, it includes the possibility of a contract that matches the payoff on any corporate security prior to bankruptcy, so such security based compensation is feasible in our setting. The important restriction this assumption imposes is that it rules out compensation contracts that survive bankruptcy. For example, we do not allow employees to hold corporate debt. As we discuss in the conclusion of this paper, the fact that we do not see employees holding corporate debt probably derives from the associated moral hazard.

 $^{^5}$ Equity holders often maintain control even in countries without Chapter 11 protection, see Strömberg (2000)

⁶As Harris and Holmström explain, if employees could borrow without an option to declare personal bankruptcy, the first best contract where the employee earns a fixed wage forever is achievable, so as in Harris and Holmström (1982) this constraint is binding. However, unlike Harris and Holmström (1982), in our setting the savings constraint is also binding — employees have an incentive to save to partially mitigate the effects of a bankruptcy filing. Relaxing this assumption would significantly complicate the analysis and would not change the form of optimal contract but it would affect the tradeoff between the benefits of tax shield and the amount of insurance.

To derive the optimal labor contract, we solve for the contract that maximizes the employee's expected utility subject to the constraints that the firm operates in a competitive capital and labor market. We begin by assuming that the (stochastic) point at which the firm declares bankruptcy is independent of the labor contract and that after a bankruptcy filing the employee's competitive wage is zero. We then derive the optimal contract and show that under the optimal contract, both assumptions are satisfied.

Because capital markets are competitive, the market value of equity at time t, V_t , is the present value of all future dividends,

$$V_{t} = E_{t} \left[\int_{t}^{T} \beta^{s-t} \delta_{s} ds + \beta^{T-t} V_{T} \right],$$

$$= E_{t} \left[\int_{t}^{T} \beta^{s-t} \left((K - D)r + (\phi_{s} - c_{s})(1 - \tau) + Dr\tau \right) ds + \beta^{T-t} V_{T} \right],$$

$$= E_{t} \left[(K - D) \left(1 - \beta^{T-t} \right) + \beta^{T-t} V_{0} + \int_{t}^{T} \beta^{s-t} \left((\phi_{s} - c_{s})(1 - \tau) + Dr\tau \right) ds \right]$$
(2)

where we use the fact that at the point of bankruptcy, T, the firm is restored to its initial state and hence, $V_T = V_0$. The initial value of equity must equal the value of the capital supplied, $V_0 = K - D$, so

$$V_t = K - D + E_t \left[\int_t^T \beta^{s-t} \left((\phi_s - c_s)(1 - \tau) + Dr\tau \right) ds \right],$$
 (3)

and so at time 0 we have

$$E_0 \left[\int_0^T \beta^t ((\phi_t - c_t)(1 - \tau) + Dr\tau) dt \right] = 0.$$
 (4)

Firms compete to hire finitely many managers of a given ability in a competitive labor market. As a result, the firm cannot pay the employee less than his market wage (because otherwise he would quit and work for another firm). Because the manager is hired at his market wage, so the value of equity cannot exceed the time 0 value of equity, $V_t \leq V_0$, $\forall t$ (because if it did, the manager would have to make less than his market wage). Hence,

$$E_{\tau} \left[\int_{\tau}^{T} \beta^{t-\tau} ((\phi_t - c_t)(1 - \tau) + Dr\tau) dt \right] \le 0, \qquad \forall \tau \in [0, T].$$
 (5)

Prior to bankruptcy the firm must be able to meet its interest obligation each period. Thus, because the dividend received by shareholders can never be negative, the employee's wages cannot exceed the total cash generated by the firm less the amount required to service the debt, i.e.

$$c_t \le \phi_t + r \left[\frac{K}{1 - \tau} - D \right]. \tag{6}$$

At time 0, the optimal contract maximizes the employee's utility subject to the above three constraints:

$$\max_{c} E_0 \left[\int_0^T \beta^t u(c_t) \, dt \right] \tag{7}$$

s.t.
$$E_0 \left[\int_0^T \beta^t ((\phi_t - c_t)(1 - \tau) + Dr\tau) dt \right] = 0,$$
 (8)

$$E_{\tau} \left[\int_{\tau}^{T} \beta^{t-\tau} ((\phi_t - c_t)(1-\tau) + Dr\tau) dt \right] \leq 0, \quad \forall \tau \in [0, T],$$
 (9)

$$(\phi_t - c_t)(1 - \tau) - r[K - D(1 - \tau)] \le 0, \quad \forall t \in [0, T].$$
 (10)

where we have used the assumption that the employee earns zero after date T. Note that, while the first two constraints are identical to those in Harris and Holmström (1982), the last, reflecting the presence of debt, is new. Before we derive the form of the optimal contract, we first need to define the employee's market wage at time t, $c^*(\phi, t)$, given ability ϕ_t .

Definition 1 Let $\{c_s : t \leq s \leq T\}$ solve

$$\max_{c} E_{t} \left[\int_{t}^{T} \beta^{s} u(c_{s}) ds \right]$$

$$s.t. \quad E_{t} \left[\int_{t}^{T} \beta^{s} ((\phi_{s} - c_{s})(1 - \tau) + Dr\tau) ds \right] = 0,$$

$$E_{\tau} \left[\int_{\tau}^{T} \beta^{s-\tau} ((\phi_{s} - c_{s})(1 - \tau) + Dr\tau) ds \right] \leq 0, \quad \forall \tau \in [t, T],$$

$$(\phi_{s} - c_{s})(1 - \tau) - r \left[K - D(1 - \tau) \right] \leq 0, \quad \forall s \in [t, T].$$

Then the employee's market wage at time t is

$$c^*(\phi, t) \equiv c_t$$

that is, it is the market wage at time t sets the value of the firm's equity equal to K-D.

The market wage is therefore the wage the employee would earn were he hired by an otherwise identical new firm at time t. It is also the wage the firm would have to pay a new employee were he hired at time t.

The following proposition derives the optimal labor contract for an employee hired at time 0 under the assumption that the future distribution of ϕ_t depends only on its current value and time:

Proposition 1 The optimal contract for an employee hired at time 0 is unique and given at all dates prior to bankruptcy by

$$c_t(\phi^t) = \min \left\{ \phi_t + r \left[\frac{K}{1 - \tau} - D \right], \max_{0 \le s \le t} \left\{ c^*(\phi_s, s) \right\} \right\}, \tag{11}$$

where

$$\phi^t \equiv \{\phi_s; 0 \le s \le t\},\$$

The proof of the proposition can be found in the appendix. Note that the form of the optimal contract is similar to that in Harris and Holmström (1982). If the firm can meet its interest obligations, wages never fall, and they rise in response to positive shocks in employee ability. The main difference occurs when the firm cannot meet its interest obligations and is in *financial distress*. In these states the employee takes a temporary pay cut so that the firm's interest obligations can be met, hence avoiding bankruptcy. Define the "no-distress" wage at date t, \bar{c}_t by

$$\overline{c}_t = \max_{0 \le s \le t} \left\{ c^*(\phi_s, s) \right\},\,$$

Financial distress occurs when the firm's revenues, less the no-distress wage, do not cover the interest owed:

$$\frac{Kr}{1-\tau} + \phi_t - \overline{c}_t \le Dr,$$

or equivalently when $\phi_t < \phi^*$, where

$$\phi^* \equiv \overline{c}_t - \left[\frac{K}{1 - \tau} - D \right] r. \tag{12}$$

The firm pays zero dividends when it is in distress, and the manager receives all cash left over after making the debt payments, i.e., in financial distress

$$c_t = \frac{Kr}{1-\tau} + \phi_t - Dr,$$

$$\leq \overline{c}_t.$$

If the employee gives up all his wages and the firm still cannot make interest payments, it is

forced into bankruptcy. This occurs when

$$Kr + \phi(1-\tau) - Dr(1-\tau) = 0,$$
 (13)

or equivalently, when

$$\phi_t = \underline{\phi} \equiv \left[D - \frac{K}{1 - \tau} \right] r. \tag{14}$$

Note that the time of bankruptcy is independent of the labor contract. Furthermore, when the employee loses his job in bankruptcy, he cannot find another job at a positive wage because $\phi = \underline{\phi} \leq 0$ and $c^*(\phi, D) \leq \phi$ for any D. Hence the employee chooses not to work and gets zero forever (effectively, the reservation wage in this model). Thus we are justified in making both of these assumptions. Note also that if D < 0, so the firm is holding cash, (13) reduces to the statement that the cash flow of the firm, including the interest received on the cash, is less than zero.

We are now ready to derive the first implication of this labor contract for the capital structure choice of the firm.

Proposition 2 In the absence of corporate taxes, it is suboptimal to choose a positive level of debt.

Proof: Consider two debt levels satisfying $0 \le D_1 < D_2$. First note that when $\tau = 0$, the firm's cash flow is independent of the level of debt. Let T_1 and T_2 be the point of bank-ruptcy with debt levels D_1 and D_2 respectively. Note that $T_1 > T_2$. Because the firm's cash flow is independent of the level of debt, it could offer a wage contract that pays the optimal wage when $D = D_2$ until T_2 , and nothing thereafter, when the debt level is D_1 . However, this is not optimal because it is dominated by a contract that pays strictly positive wages between T_2 and T_1 . Thus, the employee is strictly better off with debt level D_1 than with debt level D_2 . This is true for any $0 \le D_1 < D_2$, so a positive level of debt is suboptimal.

An interesting feature of our model is that, in the absence of corporate taxes, all firms have an incentive to hold cash:

Corollary 1 In the absence of corporate taxes, the optimal level of cash is infinite.

Proof: The proof is straightforward and follows the same logic as the proof of Proposition 2.

The intuition behind this result is simple. In the absence of taxes, investors are indifferent between investing in productive capital and holding cash. In each case they earn the same return r. However, by holding an infinite amount of cash, they can drive the probability of bankruptcy to zero, thereby achieving perfect insurance. Of course, once corporate taxes are introduced, cash earns a lower after tax return, providing a disincentive to holding cash.

4 Implementing the Optimal Contract

So far, we have demonstrated that in the absence of taxes the inability of employees to fully insure their own human capital risk implies that firms will have preference for equity. Of course, in reality, the tax deductibility of interest creates a strong incentive to issue debt. In this section we derive testable implications of this tradeoff.

To model human capital risk, we necessarily have to make a number of restrictive assumptions, for example, we need an explicit assumption on preferences. These assumptions effectively rule out the possibility of the model quantitatively matching the data. That is not our objective. Instead by making a number of restrictive assumptions and choosing a set of realistic parameter values we are able to demonstrate that human capital risk alone can effectively counterbalance taxes to produce realistic debt-to-equity ratios.

Our strategy is as follows. We first solve explicitly for the optimal contract offered by the firm to the employee. Because we assume that the supply of capital is infinite, but the number of employees is finite, firms that do not choose a level of debt that maximizes the employee's utility will not be able to hire an employee. Consequently, all firms that are in business pick a level of debt that maximizes the employee's utility. Consequently, we derive an explicit expression for the employee's indirect utility function as a function of the level of debt, and then optimize this function to find the optimal debt level.

4.1 Wage Contract

To derive a closed form expressions for firm value and employee utility requires making further restrictive assumptions. The first is that ϕ_t follows a random walk:

$$d\phi_t = \sigma \, dZ. \tag{15}$$

With this assumption, the variance remains constant and the value of the firm does not explicitly depend on t, so the optimal labor contract can now be written in the more compact

form:

$$c_{t} = c(\phi_{t}, \overline{\phi}_{t})$$

$$= \min \left\{ \phi_{t} + r \left[\frac{K}{1 - \tau} - D \right], c^{*}(\overline{\phi}_{t}) \right\}, \quad \text{where}$$
(16)

$$\overline{\phi}_t \equiv \max_{0 \le s \le t} \phi_t. \tag{17}$$

Furthermore, because the value of equity, V_t , does not depend on t, we will henceforth write $V(\phi, \overline{\phi}) \equiv V_t$. Whenever the employee is paid his competitive wage, the value of equity is K - D. This occurs whenever his current ability equals his running maximum ability level, i.e.,

$$V(\phi, \phi) = K - D \tag{18}$$

for all ϕ . At other times $V(\phi, \overline{\phi}) < K - D$, that is, the value of equity is either equal to the value when the employee is hired, or it is lower. The implication is that the value of the firm can never exceed the value if the human capital is replaced, the opposite of what q theory predicts about physical capital. There, the value of the firm is never lower than the replacement value of physical capital.

For simplicity we restrict attention to the case when the firm does not start in financial distress, that is, $\phi_0 > \phi^*$. In addition, to ensure that $c_0 > 0$ we assume that

$$\phi_0 > \frac{\sigma}{\sqrt{2r}} - \frac{Dr\tau}{1 - \tau} \tag{19}$$

The following proposition (with proof in the appendix) derives an explicit expression for the value of the firm's equity:

Proposition 3 The value of the firm's equity at time t is

$$V(\phi_t, \overline{\phi}_t) = \begin{cases} H(\overline{\phi}_t) e^{\sqrt{2r} \phi_t/\sigma} + M(\overline{\phi}_t) e^{-\sqrt{2r} \phi_t/\sigma} + \frac{(\phi_t - c^*(\overline{\phi}_t))(1-\tau)}{r} + K - D(1-\tau) & \text{if } \phi_t \ge \phi^* \\ Q(\overline{\phi}_t) e^{\sqrt{2r} \phi_t/\sigma} + G(\overline{\phi}_t) e^{-\sqrt{2r} \phi_t/\sigma} & \text{if } \phi_t < \phi^* \end{cases}$$

and the functions $H(\cdot), M(\cdot), Q(\cdot)$, and $G(\cdot)$ are given in the appendix. The competitive market wage, $c^*(\overline{\phi})$, is uniquely defined implicitly via

$$c^*(\overline{\phi}) \equiv \left\{ c \left| \Delta(\phi, D, c) = 0, \overline{\phi} + \frac{Dr\tau}{1 - \tau} - \frac{\sigma}{\sqrt{2r}} \le c < \overline{\phi} + \frac{Dr\tau}{1 - \tau} \right. \right\},\,$$

where

$$\Delta(\phi_t, D, c) \equiv \left(2\sqrt{2}\left(\frac{D-K}{1-\tau}\right)r^{3/2} + \left(e^{-\frac{\sqrt{2r}c}{\sigma}} - e^{\frac{\sqrt{2r}c}{\sigma}}\right)\sigma\right)e^{\frac{\sqrt{2r}\left(\left(\frac{K}{1-\tau}-D\right)r+\phi_t\right)}{\sigma}} - \sigma - \sqrt{2r}\left(\phi_t - c + \frac{Dr\tau}{1-\tau}\right) + e^{\frac{2\sqrt{2r}\left(\left(\frac{K}{1-\tau}-D\right)r+\phi_t\right)}{\sigma}}\left(\sigma - \sqrt{2r}\left(\phi_t - c + \frac{Dr\tau}{1-\tau}\right)\right).$$

To plot the value of equity, we use the parameters listed in Table 1. Our intention here is not to calibrate the model — it is far too simple to capture all the complexities of actual capital structure decisions. For one thing, direct bankruptcy costs do exist, so the constraint that the value of equity is restored to its initial value at the point of bankruptcy is clearly unrealistic. However, to evaluate whether the effects we study are economically important, we attempt to pick parameters that are economically realistic. We use a risk aversion coefficient of 2, consistent with values derived from experiments and a tax rate of 20%, which is less than U.S. corporate income tax rate to compensate for the tax advantage of equity at the personal level.

Because capital is riskless, another important parameter is the fraction of revenue attributable to labor versus capital. We pick an initial $\phi_0 = \overline{\phi} = 1$ and K = 50. With r = 3%, this implies that the revenue attributable to capital is Kr = 1.5. So at these parameter values, the revenue attributable to labor is two thirds the revenue attributable to capital. At first glance this choice might seem at odds with the empirical estimate of labor's share of income of about 75%, but that estimate is derived from the national income accounts and is unlikely to be representative of labor's share of revenue of a publicly traded corporation. The reason firms choose to go public is access to capital markets, so capital intensive firms are much more likely to go public. In any event, in the next section we will investigate the sensitivity of the optimal debt-equity ratio at different parameter choices.

Variable	Symbol	Value
Capital	K	50
Initial ϕ	$\overline{\phi}$	1
Risk Aversion	γ	2
Interest Rate	r	3%
Tax Rate	au	20%
Standard Deviation	σ	20%

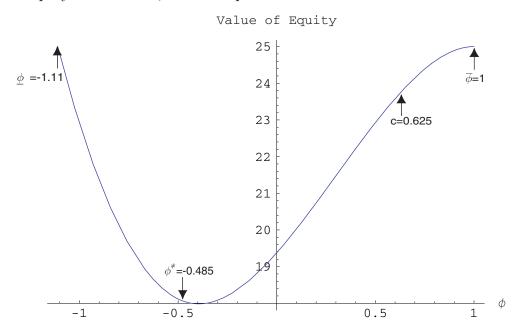
Table 1: Parameter Values

Figure 1 plots the value of equity under the optimal wage contract as a function of the

⁷See, for example, Krueger (1999).

employee's ability for the parameter values listed in Table 1 and a debt-to-equity ratio of 1.06 (we will show presently that this level of debt is optimal). The value of equity equals the initial equity investment at inception and at bankruptcy, that is, at any point a new employee is hired and paid their market wage. At all other points the value of equity is below the amount of the initial equity investment. Equity holders still get a fair market return because when the employee is hired, she is hired a wage below her ability — c = 0.625 in this case, and her initial ability is $\overline{\phi} = 1$. This difference, plus the tax shield, generates a positive cash flow (dividend) to equity holders that compensates for the drop in the value of equity and guarantees equity holders the competitive market expected return.

Figure 1: Value of Equity: The plot shows the value of equity as a function of employee ability (ϕ) between $\phi = -0.96$ and $\overline{\phi} = 1$. The parameters values are listed in Table 1 with a debt-to-equity ratio of 1.06, which is optimal.



4.2 Employee's Utility

Given the wage schedule derived in the previous section, we can calculate the employee's expected utility,

$$J(\phi, \overline{\phi}) \equiv E\left[\int_0^\infty e^{-rt} u(c_t) dt \middle| \phi_0 = \phi\right],$$

where c_t follows the optimal policy derived above. To derive a closed-form expression for J, we assume that the manager's preferences are given by

$$u(c) = -e^{-\gamma c} \tag{20}$$

The following proposition (with proof in the appendix) derives an explicit expression for the employee's derived utility function:

Proposition 4 The employee's expected utility at time t is

$$J(\phi_t, \overline{\phi}_t) = \begin{cases} A(\overline{\phi}_t) e^{\sqrt{2r} \phi_t/\sigma} + B(\overline{\phi}_t) e^{-\sqrt{2r} \phi_t/\sigma} - \frac{e^{-\gamma c^*(\overline{\phi})}}{r} & \text{if } \phi_t \ge \phi^* \\ C(\overline{\phi}_t) e^{\sqrt{2r} \phi_t/\sigma} + F(\overline{\phi}_t) e^{-\sqrt{2r} \phi_t/\sigma} - \frac{e^{-\gamma(\phi_t - \underline{\phi})}}{r - \frac{\gamma^2 \sigma^2}{2}} & \text{if } \phi_t < \phi^* \end{cases}$$

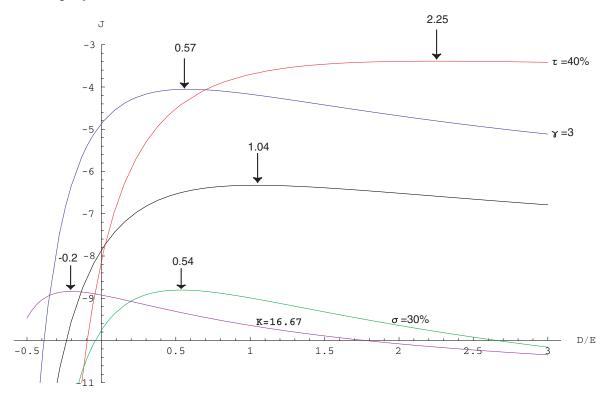
where the functions $A(\cdot)$, $B(\cdot)$, $C(\cdot)$, and $F(\cdot)$ are given in the appendix.

The black line in Figure 2 is the derived utility function, J, as a function of the debt-to-equity ratio for the parameters in Table 1. Note the utility is maximized when the debt-to-equity ratio is 1.06, the ratio we used to generate Figure 1. For this set of parameters human capital risk can act as a counterbalance to taxes and deliver a realistic debt-to-equity ratio.

To illustrate the cross-sectional implications of our model, Figure 2 also plots the derived utility function for different parameter values. Each colored line is the derived utility function with parameter values given in Table 1 with one parameter changed — this parameter takes the value indicated on each curve. As the plot makes clear, the model is also capable of generating large cross-sectional dispersion in debt-to-equity ratios. If the tax rate is doubled to 40%, the optimal debt-equity ratio rises to 2.25. On the other hand, if either the uncertainty of the firm's cash flows or the risk aversion of the employee is increased by 50%, the optimal debt-equity ratio is cut approximately in half. Similarly, if the labor intensity of the firm is increased by reducing the amount of capital to 16.67, so that only one third of revenue is attributable to capital, the debt-equity ratio drops to -0.2, that is, the firm holds no debt and about 25% of the initial enterprise value of the firm in cash despite the fact that cash is tax disadvantaged; the firm must pay tax on the interest earned whereas investors do not (because there are no personal income taxes in this model).

To further investigate the cross-sectional implications of our model on the debt-to-equity ratio, in the next section we explicitly derive the optimal level of debt by maximizing J.

Figure 2: **Employee's Derived Utility:** The black curve shows the employee's utility, J, as a function of the debt-to-equity ratio for the parameters in Table 1. The colored curves show the employees utility with just the indicated parameter changed to the value indicated on the curve. The arrows mark the maximum value of each function, that is, the optimal debt-to-equity ratio.



4.3 The Optimal Level of Debt

The the optimal level of debt maximizes the employee's derived utility function with respect to D. Writing J as an explicit function of D, $J(\phi, \overline{\phi}, D)$, the optimal level of debt therefore solves

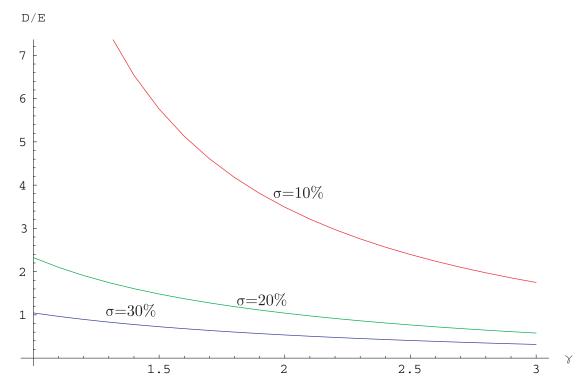
$$\frac{\partial}{\partial D}J(\overline{\phi},\overline{\phi},D)=0.$$

This equation is relatively straightforward to solve numerically, the only complication being that $c^*(\overline{\phi}, D)$ is only defined implicitly by (43).

We begin by exploring the relation between risk aversion and leverage. Figure 3 plots the optimal debt-to-equity ratio as a function of the level of employee risk aversion, γ for three different levels of uncertainty in employee productivity, σ . It confirms what is intuitively clear in our model — leverage is related to employee willingness to bear risk. Firms with more risk averse employees optimally have lower levels of leverage as do firms with higher

levels of uncertainty in labor productivity; firms that have employees who value human capital insurance more (either because these employees are more risk averse, or they have greater levels of uncertainty in their productivity) optimally respond by reducing debt and thus giving up tax shields to enhance risk sharing.

Figure 3: Optimal D/E as a Function of Employee Risk Aversion: The plot shows the optimal debt-to-equity ratio as a function of the level of risk aversion, γ , at three different levels of uncertainty in the productivity of labor, σ . The values of the remaining parameters are listed in Table 1.

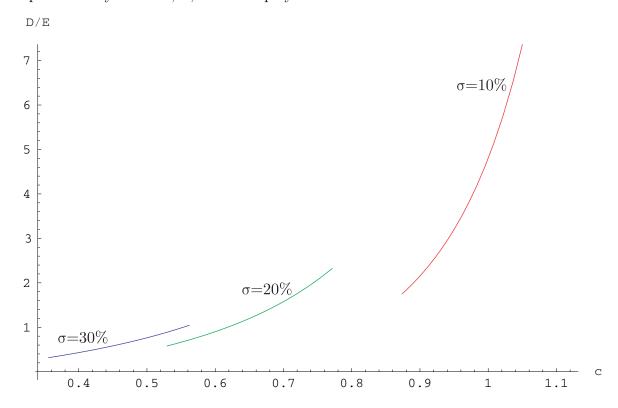


These results suggest two empirical implications of our model. All else equal, firms with more idiosyncratic uncertainty should hold less debt as should firms with more risk averse employees. The relation between leverage and employee risk aversion is, to our knowledge, an inference unique to this model which has not been investigated.

At first blush risk aversion might appear to be an unlikely driver of cross-sectional variation in firm leverage. The corporations that comprise most studies have thousands of employees; if differences in risk aversion amongst employees are uncorrelated with each other, the average risk aversion of a typical employee across firms will be about the same. However, an important implication of our model is that differences in risk aversion are unlikely to be uncorrelated within a firm. To understand why, first note from Figure 3 that the firm's optimal leverage is related to risk-aversion of its employees. This implies that it is not

optimal for all (otherwise identical) firms to have the same leverage in an economy in which employees have different levels of risk aversion. Less risk averse employees are better off working for firms with higher leverage and more risk averse employees are better off working for firms with lower leverage. Hence an implication of our model is that there should be cross sectional variation in firm leverage that is attributable to cross sectional variation in the average risk aversion of the firm's employees. Furthermore, because new hires will select firms based on their leverage (and offered wages), they will prefer to work for firms with employees that have similar levels of risk aversion. Hence such firms will preferentially hire such employees and so differences in risk aversion, and hence leverage, should persist.

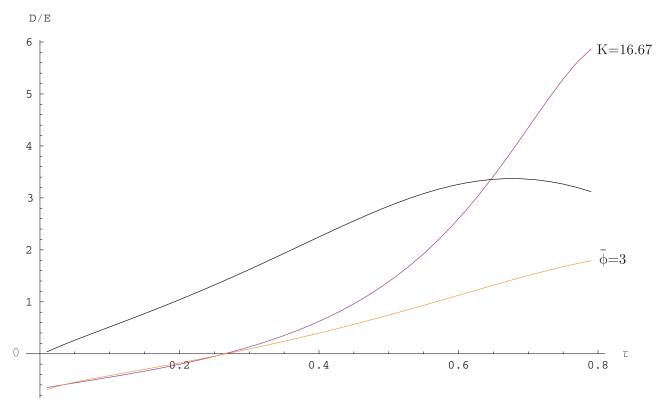
Figure 4: **Firms with Higher Leverage Pay Higher Wages:** The plot shows the cross sectional distribution of wages, c, and debt levels for firms that vary in their employee risk aversion (as plotted in Figure 3). Each line corresponds to different levels of uncertainty in the productivity of labor, σ , of the employee.



Because employee risk aversion is unobservable, its role in capital structure cannot be directly tested. However, as Figure 4 demonstrates, the relation between wages and leverage can be used as an indirect test of the importance of employee risk aversion in explaining cross-sectional variation in firm capital structure. As is evident from the plot, higher leverage is associated with higher wages, even after controlling for other sources of wage differentials

such as cash flow uncertainty. Thus, wages should have explanatory power in explain firm leverage. Empirically, controlling for other sources of wage differentials is difficult, but this result has the potential to explain at least some of the large unexplained persistent cross sectional variation in leverage within industries documented in Lemmon et al. (2006).

Figure 5: **Optimal D/E:** The black curve is the optimal debt-equity ratio for a given tax rate for the parameters in Table 1. The purple curve is the same plot, but in this case K is reduced to 16.67 while for the orange curve $\bar{\phi}$ is increased to 3, implying in both cases that the fraction of revenue attributable to labor goes from 40% (black curve) to 66% (purple curve).



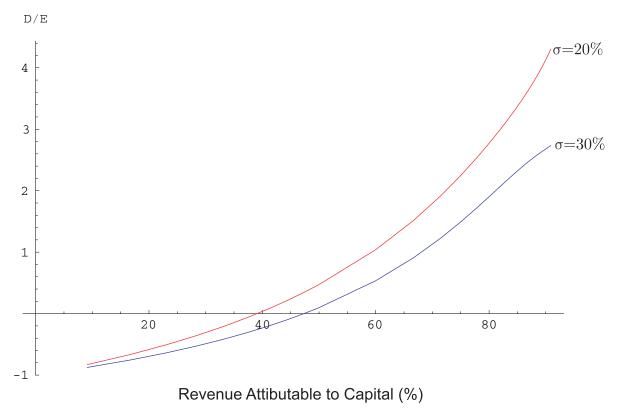
The black curve in Figure 5 plots the optimal debt-to-equity ratio as a function of the tax rate for the parameters in Table 1. At reasonable tax rates, economies with higher tax rates generate higher tax shields and so employees respond by accepting high leverage ratios. However, because the employee captures the benefits of the tax shield, contractual wages in economies with very high taxes are high, which increases the uncertainty in compensation implying that employees bear more risk. When tax rates are very high the cost of bearing this risk overcomes the benefit of the tax shield, so firms in economies with very high tax rates can have lower leverage ratios than similar firms in economies with slightly lower tax rates.

The purple and orange curves in Figure 5 illustrate an important implication of our model — that cross-sectional variation in the ratio of labor to capital also has the potential to explain cross-sectional variation in firms' leverage. The colored curves illustrate the effect of raising the fraction of revenue attributable to labor from 40% to 66% by increasing the productivity of labor (orange curve) or decreasing the amount of capital required (purple curve) to run the firm. Because the marginal product of labor is risky while the marginal product is capital is riskless the effect is to increase risk, so for reasonable tax levels, firms maintain a lower debt-equity ratio. Indeed, such firms hold cash in economies with taxes rates less than about 27%. However, in economies with high tax rates, firms with a higher productivity of labor (orange curve) have significantly less leverage. The reason why is that this productivity is compensated with higher wages. Hence employees at such firms have much more to lose, so they place a higher value on employment insurance, which limits the firm's use of debt. Firms where the amount of capital is reduced (purple curve) pay even lower wages than firms at the original level of capital (black curve) because these firms generate less cash that needs to be shielded. In economies with low tax rates, the increased risk of bankruptcy is the dominant effect, so these firms maintain lower leverage levels. But in economies with very high tax rates, the tax shields become very valuable, and because wages are lower than in the other two cases employees face less risk, so these firms optimally take on more leverage.

Figure 6 plots the optimal debt-to-equity ratio as a function of the fraction of revenues attributable to capital (for a tax rate of 20%). To keep the productivity of labor constant (at 1) the amount of capital, K, is varied from 3 to 333 corresponding to a variation in fraction of revenue attributable to capital from 9% to 91%. Consistent with what we found in Figure 5 at reasonable tax rates, labor intensive firms have lower levels of debt, something that is, at least anecdotally, characteristic of the economy. Furthermore, because capital intensive firms tend to be large (especially if accounting numbers are used as a measure of firm size), this also delivers the empirical implication that larger firms have higher leverage, which is consistent with the empirical evidence.⁸

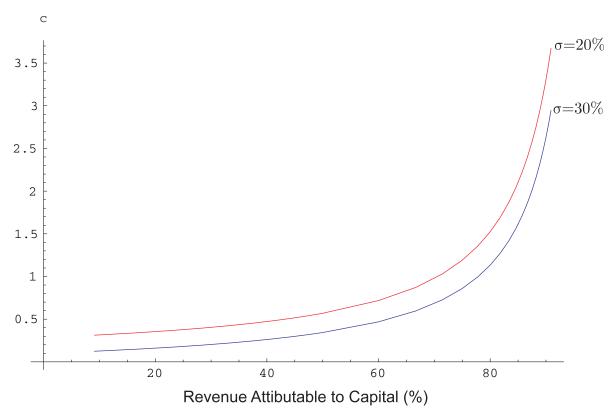
⁸See, for example, Rajan and Zingales (1995)

Figure 6: **Firm Size and Debt Levels:** The plot shows the optimal debt-to-equity ratio as a function of the amount of capital K, expressed as a percentage of revenue attributable to capital (K is varied from 3 to 333). Each line corresponds to an economy with different levels of cash flow uncertainty, σ . The values of the remaining parameters are listed in Table 1.



An interesting question is what the cross-sectional variation in the capital versus labor intensity of firms implies about wages. Ceteris paribus, labor intensive industries have a higher probability of bankruptcy, so there is less scope for risk sharing and one would expect higher wages in these industries. However, firms in these industries endogenously respond by holding less debt (or even cash), thus decreasing the probability of bankruptcy. Figure 7 shows that the endogenous response is enough to reverse the initial effect — holding the initial productivity of labor fixed, capital intensive firms and hence larger firms pay higher wages. This relation between firm size and wages is a robust characteristic of the data and is regarded as a puzzle by labor economists (see Brown and Medoff (1989)).

Figure 7: Physical Capital Intensive Firms Pay Higher Wages: The plot shows the cross sectional distribution of wages, c, (at optimal debt levels) for different levels of physical capital (K is varied from 3 to 333). Each line corresponds to an economy with differing levels of cash flow uncertainty, σ . The values of the remaining parameters are listed in Table 1.



It is important to emphasize that the relation between leverage and capital intensity depends on our assumption about the relative risks of labor and capital. If we had instead assumed that capital was risky and labor riskless, these inferences would be reversed. However, there are good reasons to suppose that in general, labor is indeed riskier than capital. First note that the risk of interest is *idiosyncratic* risk — there is no obvious reason to risk share systematic risk. Clearly, capital uncertainty is likely to have large systematic components while labor uncertainty, because it depends on the employees own ability, is likely to be mainly idiosyncratic. Second, key employees, such as the CEO, can make idiosyncratic decisions that have large consequences for the firm. Finally, the fact that wages and firm size are positively correlated is, by itself, evidence that labor is riskier than capital. Furthermore, Rajan and Zingales (1995) also find indirect evidence of the same relation. They define a variable they call "tangibility" (the ratio of fixed assets to book value of assets) and find that is significantly positively related to leverage in almost every country they study. Because capital intensive firms are likely to have a higher percentage of tangible assets, this result is also indirect evidence that capital intensive firms have higher leverage implying that labor is riskier than capital.

5 Discussion

A thorough empirical evaluation of our model is beyond the scope of this paper. However, in this section we discuss existing empirical evidence that is consistent with the predictions of the model.

The strongest test of any model is a test of the predictions that are unique to the model. In our case, chief amongst these is the prediction that cross sectional differences in the risk aversion of employees across firms should exist and that these differences should drive persistent cross sectional variation in capital structure. Although existing research has not investigated risk aversion at the firm level, there is strong evidence of persistent cross sectional variation in capital structure as documented in Lemmon et al. (2006). Although the extent to which this cross sectional variation can be explained by risk aversion remains an open question, in principal it can be resolved. If cross sectional variation in risk aversion is driving cross sectional variation in capital structure, wages should have explanatory power in explaining this variation. Obtaining firm level wage data is clearly a challenge, however our results suggest that perhaps obtaining this data might be worth the effort.

An important implication of this paper is that employees should care about the firm's likelihood of bankruptcy, or to put it in layman's terms, the firm's "safety." However, in most

⁹Such data has been used in the labor literature, see, for example, Guiso et al. (2005)

cases employees do not have the financial background to understand the relation between leverage and bankruptcy, so other more readily interpretable variables are likely to explain capital structure decisions. One such variable is the firm's credit rating — although most employees are unlikely to be able to relate leverage levels to bankruptcy probabilities, rating agencies perform this mapping for them and publish their results. Hence, an implication of our model is that a firm's credit rating should be an independent determinant of its capital structure, an empirical result documented in Kisgen (2006).

Because the likelihood of entrenchment is greater in firms with less debt, our model predicts an inverse relation between leverage and entrenchment. Berger, Ofek, and Yermack (1997) and Kayhan (2003) both find that managers who appear to be more entrenched (long tenure, compensation has low sensitivity to performance, few outside directors, no large shareholder) have low leverage. Bebchuk and Cohen (2005) investigate the effect of managerial entrenchment on market valuation. Consistent with the predictions of our model, they find that firms with managers that are more likely to be entrenched display lower Q-ratios. They leave as a puzzle why shareholders would voluntarily engage in what they identify as suboptimal behavior. A contribution of our model is the insight that is is not necessarily suboptimal to let managers become entrenched, even if, ex post this entrenchment leads to lower Q-ratios.

There is also empirical evidence consistent with our assumption that in bankruptcy value is created by optimally either firing existing employees or resetting their wages to competitive levels. Gilson and Vetsuypens (1993) find that almost 1/3 of all CEOs are replaced after bankruptcy. Those who keep their job experience large salary cuts (35% or so). But when new outside managers are hired, they are paid 36% more than the fired managers suggesting that by firing old employees and hiring new ones at their market wage, value is created.

A key insight that emerges from our analysis is the role of bankruptcy in limiting the potential to write explicit or implicit contracts with managers and other employees. Although bankruptcy is likely the most important mechanism that allows firms to abrogate existing contracts, other mechanisms, such as takeovers, also exist. If a firm is merged into another company, it ceases to exist as a seperate legal entity, which is likely to allow firms to fully or partially abrogate implicit, and possibly also explicit contracts, or at least make the contracts harder to enforce. Consistent with this view, Pontiff, Shleifer, and Weisbach (1990) find that hostile takeovers are followed by an abnormally high incidence of pension asset reversions. These pension asset reversions account for approximately 11% of takeover gains. That hostile takeovers may create value gains ex post is widely recogognized. What this paper adds is that they also limit the risk sharing possibilities ex ante, which potentially might explain why the majority of firms have adopted anti-takeover provisions.

A risk-sharing view of capital structure is also in accordance with survey results reported by Graham and Harvey (2001). They find that the most important determinant of capital structure choice is financial flexibility and maintaining a good credit rating. By contrast, they find little evidence for asset substitution or asymmetric information as an important factor for capital structure choice. Clearly, firms with good credit rating and financial flexibility can offer more valuable human capital risk sharing to employees than firms with poor ratings and little financial flexibility, and hence might explain why managers focus on this determinants in particular.

6 Conclusion

According to the dominant corporate finance paradigm, capital structure choice is a tradeoff between the costs and benefits of debt. Although there is broad agreement amongst academics and practitioners on what the benefits of debt are, identifying the costs of debt remains one of the biggest puzzles in corporate finance. Most existing papers on capital structure require sizeable bankruptcy costs to act as a counterweight to the tax advantage of debt, but the empirical evidence does not support the notion that the firm (or its investors) bear substantial bankruptcy costs.

In contrast to the limited importance of bankruptcy costs borne by a firm's investors, the anecdotal evidence is that bankruptcy costs borne by employees of the firm are important. Yet, these bankruptcy costs have not received attention in the finance literature. Our analysis demonstrates that at reasonable parameter values, the bankruptcy costs borne by employees do in fact provide a first-order counterbalance to the tax benefit of debt.

Analyzing the human cost of bankruptcy generates a rich set of empirical predictions. First, we find that for reasonable parameters values, the model produces moderate leverage ratios, implying an apparent "underutilization" of debt tax shields if these costs are ignored. Second, the model predicts variation in the average risk aversion of employees across firms and that this variation should result in persistent variation in leverage ratios. Third, highly levered firms should pay higher wages to their employees. Forth, in our model capital intensive firms have higher optimal leverage ratios and pay higher wages. Finally, riskier firms choose lower leverage ratios.

An important simplifying assumption in our model is that we do not allow firms to make severance payments to fired employees prior to bankruptcy. Relaxing this assumption would complicate the analysis appreciably, but would not significantly change the results. Although the optimal contract would allow a firm to fire an employee prior to bankruptcy, it would still require that the firm continue to pay this employee the contracted wage. A new replacement

employee would be hired at a competitive wage, and the firm would now pays wages to current and all past employees. In all other respects the model inferences remain much the same. At the point of bankruptcy the firm stops making all wage payments (to both past and newly fired employees), so employees still continue to trade off the benefits of insurance against the benefits of the tax shield. Moreover, such a contract is Pareto improving only if moral hazard concerns are ignored. In reality, the benefits employees derive from being fired (they continue to earn an above market wage from their old employer and they can then supplement this income with a new job at the market wage) most likely explain why such contracts are uncommon (except at the highest levels).

Key to our results is the assumption that employment contracts do not survive bank-ruptcy. Given the costs imposed by bankruptcy process on the employees of the firm, it is perhaps surprising that in reality firms do not write employment contracts that survive the bankruptcy process. For example, one solution, that is in principle available, would be for firms to issue zero coupon senior perpetual debt to its employees. The only effect this debt would have would be in bankruptcy, when it ensures that the employees gain control of the firm because they hold the most senior claims. The most likely reason we do not see such contracts is the associated moral hazard — in this case employees would have an incentive to drive the firm into bankruptcy. Indeed, as DeMarzo and Fishman (2006) show, the existence of this kind of moral hazard is itself a determinant of firms' capital structures.

In deriving our results, we have made several simplifying assumptions. Relaxing these assumptions would lead to interesting extensions. Both dividend policy and dynamic capital structure decisions are exogenous in our model — the firm pays out all excess cash as dividends and never changes the level of debt. Allowing a manager to choose an optimal dynamic dividend policy, issue new or retire old debt and equity is likely to yield interesting new insights. After their contracts are determined, managers in our model would generally be reluctant to pay out dividends, preferring instead to pay down debt, because this decreases the chance of bankruptcy and therefore increases the human capital insurance that the labor contract provides. Yet, ex ante, the manager must commit to pay out some dividends, otherwise equity holders cannot realize a fair return on their investment. In addition they must maintain levels of debt that justify the tax shields implicit in the manager's compensation. Therefore, in principal, the compensation contract, dividend policy and dynamic capital structure policy should be jointly determined. At this stage, deriving a model that would endogenize all three decisions is daunting, however, such a model is likely to yield important insights that might explain the seemingly puzzling behavior documented in Welch (2004).

More generally, we believe that recognizing the interaction between labor and capital markets opens a new and exciting path for future research in corporate finance. Analyzing the

resulting implications could significantly improve our understanding of corporate behavior.

Appendix

A Proof of Proposition 1

We wish to prove that the optimal compensation policy is to set

$$c_t(\phi^t) = \min\left\{\phi_t + r\left[\frac{K}{1-\tau} - D\right], \max_{0 \le s \le t} \left\{c^*(\phi_s, s)\right\}\right\}.$$
 (21)

The proof of this proposition closely follows that of Proposition 1 in Harris and Holmström (1982). Following Harris and Holmström (1982) first note that the policy in (21) is feasible — it satisfies (8)–(10). We next show that this compensation policy maximizes the Lagrangian for program (7)–(10), that is, it satisfies the complementary slackness conditions. The Lagrangian can be written (after first multiplying the constraints (9) and (10) by the unconditional probability of the respective ϕ^{τ} , multiplying (10) by powers of β , and then collecting terms) as follows:

$$\max_{c_t} E_0 \int_0^T \beta^t \left[u(c_t) + \lambda^t ((\phi_t - c_t)(1 - \tau) + Dr\tau) + \mu_t ((\phi_t - c_t)(1 - \tau) - r \left[K - D(1 - \tau) \right] \right] dt,$$
(22)

where

$$\lambda^t \equiv \int_0^t \lambda_s(\phi^s) \, dt, \tag{23}$$

and $\lambda_s(\phi^s)$ is the Lagrange multiplier corresponding to Equation (9). The first order conditions take the form

$$u'(c_t) = \lambda^t - \mu_t. \tag{24}$$

Assume c_t is given by Equation (11), and define Lagrange multipliers

$$\lambda^t = u' \left(\max_{0 \le s \le t} \left\{ c^*(\phi_s, s) \right\} \right), \tag{25}$$

$$\lambda_t = \frac{d\lambda^t}{dt},\tag{26}$$

$$\mu_t = u' \left(\max_{0 \le s \le t} \left\{ c^*(\phi_s, s) \right\} \right) - u'(c_t).$$
 (27)

These immediately satisfy the first order condition, Equation (24). Since the maximum inside the bracket in Equation (25) is always increasing in t, Equation (26) immediately tells

us that

$$\lambda_t \begin{cases} \leq 0 & \text{when } c_t = c^*(\phi_t, t), \\ = 0 & \text{otherwise.} \end{cases}$$
 (28)

Also, since at all times

$$c_t \le \max_{0 \le s \le t} \left\{ c^*(\phi_s, s) \right\},\,$$

Equation (27) immediately tells us that

$$\mu_t \begin{cases} \leq 0 & \text{when } c_t = \max_{0 \leq s \leq t} \left\{ c^*(\phi_s, s) \right\}, \\ = 0 & \text{otherwise.} \end{cases}$$
 (29)

The contract defined by Equation (11), together with these Lagrange multipliers, thus maximizes the Lagrangian (by concavity, this is a consequence of its satisfying the first order conditions), and (together with the Lagrange multipliers) satisfies the complementary slackness conditions. By weak duality, the contract is also the solution to the original program, Equations (7)–(10).

The only remaining step is to show that (21) uniquely defines a single compensation policy. Assume that it does not, that is that two difference policies exist both satisfying (21). Because both policies are feasible a policy that combines the two policies by always taking the maximum wage of either policy at every point in time is feasible too. However, such a policy strictly dominates the other two policies, contracting the proof that any policy that satisfies (21) is optimal. Hence (21) uniquely defines single compensation policy.

B Proof of Proposition 3

By Ito's Lemma, when $\phi_t < \overline{\phi}_t$,

$$dV = V_{\phi} d\phi + \frac{1}{2} V_{\phi\phi} \sigma^2 dt. \tag{30}$$

In equilibrium, shareholders must earn a fair rate of return on their investment, implying that

$$E(dV) = (rV - \delta_t) dt,$$

where δ_t is the dividend payment, Combining these, we obtain a p.d.e. for $V(\phi, \overline{\phi})$:

$$\frac{1}{2}\sigma^2 V_{\phi\phi} - rV + \delta_t = 0. \tag{31}$$

From Equation (1), the dividend is given by

$$\delta_t = \begin{cases} Kr - Dr(1-\tau) + (\phi_t - c^*(\overline{\phi}))(1-\tau) & \text{if } \phi \ge \phi^*, \\ 0 & \text{otherwise.} \end{cases}$$
(32)

Equation (31) thus takes two different forms, depending on whether or not the firm is currently in financial distress:

$$\frac{1}{2}\sigma^{2}V_{\phi\phi} - rV + Kr - Dr(1-\tau) + (\phi - c^{*}(\overline{\phi}))(1-\tau) = 0 \quad \text{if } \phi \ge \phi^{*}, \qquad (33)$$

$$\frac{1}{2}\sigma^{2}V_{\phi\phi}^{f} - rV^{f} = 0 \quad \text{otherwise.} \qquad (34)$$

$$\frac{1}{2}\sigma^2 V_{\phi\phi}^f - rV^f = 0 \quad \text{otherwise.} \quad (34)$$

The notation V^f here is used to indicate the equity value when the firm is in financial distress. The general solutions to equations (33) and (34) are

$$V(\phi,\overline{\phi}) = H(\overline{\phi})e^{\sqrt{2r}\phi/\sigma} + M(\overline{\phi})e^{-\sqrt{2r}\phi/\sigma} + \frac{(\phi - c^*(\overline{\phi},D))(1-\tau)}{r} + K - D(1-\tau)(35)$$

$$V^{f}(\phi, \overline{\phi}) = Q(\overline{\phi})e^{\sqrt{2r}\phi/\sigma} + G(\overline{\phi})e^{-\sqrt{2r}\phi/\sigma}.$$
 (36)

To pin down the four unknown functions H, M, Q and G, we need four boundary conditions. The first, applying at the upper boundary $\phi = \overline{\phi}$, is 10

$$\frac{\partial}{\partial \overline{\phi}} \Big|_{\phi = \overline{\phi}} V(\phi, \overline{\phi}) = 0. \tag{37}$$

At the point the firm enters financial distress, ϕ^* , the values and derivatives must be matched, providing two additional boundary conditions,

$$V(\phi^*, \overline{\phi}) = V^f(\phi^*, \overline{\phi}), \tag{38}$$

$$V_{\phi}(\phi^*, \overline{\phi}) = V_{\phi}^f(\phi^*, \overline{\phi}). \tag{39}$$

Finally, at the point of bankruptcy (when the firm cannot meet its interest obligations even if the employee gives up all his wages), ϕ , the firm fires the employee and replaces him with an employee who puts the capital to full productive use, so

$$V^f(\phi, \overline{\phi}) = K - D. \tag{40}$$

¹⁰See Goldman, Sosin, and Gatto (1979).

These four boundary conditions are sufficient to pin down H, M, Q and G for any given specification of the wage function $c^*(\overline{\phi})$. However, we also want to determine the optimal wage function, c^* . This requires an additional condition, which is provided by Equation (18),

$$V(\overline{\phi}, \overline{\phi}) = K - D. \tag{41}$$

As written, the five equations (37)–(41), are enough in principle to determine H, M, Q, G and c^* , but applying them directly results in o.d.e.s for each function, due to the presence of the $\overline{\phi}$ derivative in Equation (37). To eliminate this derivative, we replace Equation (37) with another (equivalent) condition. To do this, note that since Equation (41) holds for all $\overline{\phi}$, we can differentiate it with respect to $\overline{\phi}$, obtaining

$$\frac{dV(\overline{\phi}, \overline{\phi})}{d\overline{\phi}} = \frac{\partial V(\phi, \overline{\phi})}{\partial \phi} \bigg|_{\phi = \overline{\phi}} + \frac{\partial V(\phi, \overline{\phi})}{\partial \overline{\phi}} \bigg|_{\phi = \overline{\phi}},$$

$$= 0.$$

Combining this with Equation (37) we obtain

$$\frac{\partial}{\partial \phi} \Big|_{\phi = \overline{\phi}} V(\phi, \overline{\phi}) = 0. \tag{42}$$

Replacing (37) with (42) has no effect on the solution, but is simplifies the derivation because we no longer have to consider any derivatives of H, M, Q, G or c^* . Using (41),(42),(38),(39) and (40) to solve for the coefficients and the optimal wage gives:

$$\begin{split} H(\overline{\phi}) &= \frac{\left(4\left(\frac{D-K}{1-\tau}\right)r^{3/2} + \sqrt{2}e^{-\frac{\sqrt{2r}c}{\sigma}}\sigma - \sqrt{2}e^{\frac{\sqrt{2r}c}{\sigma}}\sigma\right)e^{\frac{\sqrt{2r}\phi}{\sigma}} + 4\sqrt{r}(c - \frac{D\tau r}{1-\tau} - \overline{\phi})e^{\frac{\sqrt{2r}\phi}{\sigma}}}{\frac{4r^{3/2}}{1-\tau}\left(e^{\frac{2\sqrt{2r}\phi}{\sigma}} - e^{\frac{2\sqrt{2r}\phi}{\sigma}}\right)} \\ M(\overline{\phi}) &= \frac{\left(4\left(\frac{K-D}{1-\tau}\right)r^{3/2} - \sqrt{2}e^{-\frac{\sqrt{2r}c}{\sigma}}\sigma + \sqrt{2}e^{\frac{\sqrt{2r}c}{\sigma}}\sigma\right)e^{\frac{\sqrt{2r}(2\overline{\phi}+\phi)}{\sigma}} - 4\sqrt{r}(c - \frac{D\tau r}{1-\tau} - \overline{\phi})e^{\frac{\sqrt{2r}(\overline{\phi}+2\phi)}{\sigma}}}{\frac{4r^{3/2}}{1-\tau}\left(e^{\frac{2\sqrt{2r}\phi}{\sigma}} - e^{\frac{2\sqrt{2r}\phi}{\sigma}}\right) + 4\sqrt{r}(c - \overline{\phi} - \frac{D\tau r}{1-\tau})e^{\frac{\sqrt{2r}\phi}{\sigma}}}{\frac{4r^{3/2}}{1-\tau}\left(e^{\frac{2\sqrt{2r}\phi}{\sigma}} - e^{\frac{2\sqrt{2r}\phi}{\sigma}}\right)} \\ G(\overline{\phi}) &= \frac{\left(4\left(\frac{K-D}{1-\tau}\right)r^{3/2} - \sqrt{2}e^{-\frac{\sqrt{2r}c}{\sigma}}\sigma\right)e^{\frac{\sqrt{2r}(2\overline{\phi}+\phi)}{\sigma}} - 4\sqrt{r}(c - \frac{D\tau r}{1-\tau} - \overline{\phi})e^{\frac{\sqrt{2r}(\overline{\phi}+2\phi)}{\sigma}} + \sqrt{2}e^{\frac{\sqrt{2r}(c+3\phi)}{\sigma}}\sigma}}{\frac{4r^{3/2}}{1-\tau}\left(e^{\frac{2\sqrt{2r}\phi}{\sigma}} - e^{\frac{2\sqrt{2r}\phi}{\sigma}}\right)} \\ &= \frac{\left(4\left(\frac{K-D}{1-\tau}\right)r^{3/2} - \sqrt{2}e^{-\frac{\sqrt{2r}c}{\sigma}}\sigma\right)e^{\frac{\sqrt{2r}(2\overline{\phi}+\phi)}{\sigma}} - 4\sqrt{r}(c - \frac{D\tau r}{1-\tau} - \overline{\phi})e^{\frac{\sqrt{2r}(\overline{\phi}+2\phi)}{\sigma}} + \sqrt{2}e^{\frac{\sqrt{2r}(c+3\phi)}{\sigma}}\sigma}}{\frac{4r^{3/2}}{1-\tau}\left(e^{\frac{2\sqrt{2r}\phi}{\sigma}} - e^{\frac{2\sqrt{2r}\phi}{\sigma}}\right)} \\ &= \frac{\left(4\left(\frac{K-D}{1-\tau}\right)r^{3/2} - \sqrt{2}e^{-\frac{\sqrt{2r}c}{\sigma}}\sigma\right)e^{\frac{\sqrt{2r}(2\overline{\phi}+\phi)}}{\sigma}} - 4\sqrt{r}(c - \frac{D\tau r}{1-\tau} - \overline{\phi})e^{\frac{\sqrt{2r}(\overline{\phi}+2\phi)}}{\sigma}} + \sqrt{2}e^{\frac{\sqrt{2r}(c+3\phi)}{\sigma}}\sigma}}\right)}$$

and the wage is

$$c = c^*(\overline{\phi}, D)$$

where

$$c^*(\phi, D) \equiv \left\{ c \left| \Delta(\phi, D, c) = 0, \overline{\phi} + \frac{Dr\tau}{1 - \tau} - \frac{\sigma}{\sqrt{2r}} \le c < \overline{\phi} + \frac{Dr\tau}{1 - \tau} \right\} \right\}$$

and

$$\Delta(\phi, D, c) \equiv \left(2\sqrt{2}\left(\frac{D-K}{1-\tau}\right)r^{3/2} + \left(e^{-\frac{\sqrt{2r}c}{\sigma}} - e^{\frac{\sqrt{2r}c}{\sigma}}\right)\sigma\right)e^{\frac{\sqrt{2r}\left(\left(\frac{K}{1-\tau}-D\right)r+\phi\right)}{\sigma}} - \sigma - (43)$$

$$\sqrt{2r}\left(\phi - c + \frac{Dr\tau}{1-\tau}\right) + e^{\frac{2\sqrt{2r}\left(\left(\frac{K}{1-\tau}-D\right)r+\phi\right)}{\sigma}}\left(\sigma - \sqrt{2r}\left(\phi - c + \frac{Dr\tau}{1-\tau}\right)\right).$$

The proof that $\Delta(\phi, D, c)$ always has a unique root between $\overline{\phi} + \frac{Dr\tau}{1-\tau} - \frac{\sigma}{\sqrt{2r}}$ and $\overline{\phi} + Dr\tau$ is available on request. It follows because the firm does not start in financial distress and (19).

C Proof of Proposition 4

For any $\phi \leq \overline{\phi}$, the Bellman equation for the manager's value function, J, takes the form

$$\frac{1}{2}\sigma^2 J_{\phi\phi} - rJ + u(c) = 0. \tag{44}$$

The manager's pay, c, is given by

$$c = \begin{cases} c^*(\overline{\phi}) & \text{if } \phi \ge \phi^*, \\ \phi + r(K - D(1 - \tau)) = \phi - \underline{\phi} & \text{otherwise.} \end{cases}$$
 (45)

Equation (44) thus takes two different forms, depending on whether or not the firm is currently in financial distress:

$$\frac{1}{2}\sigma^2 J_{\phi\phi} - rJ - e^{-\gamma c^*(\overline{\phi})} = 0 \quad \text{if } \phi \ge \phi^*, \tag{46}$$

$$\frac{1}{2}\sigma^2 J_{\phi\phi}^f - rJ^f - e^{-\gamma(\phi - \underline{\phi})} = 0 \quad \text{otherwise.}$$
 (47)

The notation J^f here is used here to emphasize that J is being calculated when the firm is in financial distress. The general solutions to these p.d.e.s are

$$J(\phi, \overline{\phi}) = A(\overline{\phi})e^{\sqrt{2r}\phi/\sigma} + B(\overline{\phi})e^{-\sqrt{2r}\phi/\sigma} - \frac{e^{-\gamma c^*(\overline{\phi})}}{r}, \tag{48}$$

$$J^{f}(\phi, \overline{\phi}) = C(\overline{\phi})e^{\sqrt{2r}\phi/\sigma} + F(\overline{\phi})e^{-\sqrt{2r}\phi/\sigma} - \frac{e^{-\gamma(\phi-\underline{\phi})}}{r - \frac{\gamma^{2}\sigma^{2}}{2}}.$$
 (49)

To determine the functions A, B, C and F, we need four boundary conditions. The first boundary condition is

$$J^{f}(\underline{\phi}, \overline{\phi}) = \int_{0}^{\infty} e^{-rt} u(0) dt = -1/r.$$
 (50)

At the point of financial distress, ϕ^* , the values and slopes must match, yielding two additional boundary conditions:

$$J(\phi^*, \overline{\phi}) = J^f(\phi^*, \overline{\phi}), \tag{51}$$

$$J_{\phi}(\phi^*, \overline{\phi}) = J_{\phi}^f(\phi^*, \overline{\phi}). \tag{52}$$

The final boundary conditions are

$$\left. \frac{\partial}{\partial \overline{\phi}} \right|_{\phi = \overline{\phi}} J(\phi, \overline{\phi}) = 0, \tag{53}$$

$$\lim_{\phi, \overline{\phi} \to \infty} J(\phi, \overline{\phi}) = 0. \tag{54}$$

The first of these is analogous to Equation (37), and the second follows from the fact that, when $\overline{\phi}$ is very large, so is the manager's compensation, and

$$\lim_{c \to \infty} u(c) = 0.$$

These boundary conditions allow us to solve for the functions $A(\overline{\phi}), B(\overline{\phi}), C(\overline{\phi})$ and $F(\overline{\phi})$:

$$A(\overline{\phi}) = \int_{\overline{\phi}}^{\infty} \frac{\gamma \left(2e^{\frac{\sqrt{2r\phi}}{\sigma}} - e^{\frac{\sqrt{2r}(\phi - c^*(\overline{\phi}))}{\sigma}} - e^{\frac{\sqrt{2r}(\phi + c^*(\overline{\phi}))}{\sigma}} \right) \frac{\partial c^*(\overline{\phi})}{\partial \overline{\phi}}}{2e^{c^*(u)\gamma} \left(e^{\frac{2\sqrt{2r\phi}}{\sigma}} - e^{\frac{2\sqrt{2r}\phi}{\sigma}} \right) r} du$$
 (55)

$$B(\overline{\phi}) = \frac{1 - \frac{\sqrt{2r}}{\gamma\sigma} - 2e^{c\left(\gamma + \frac{\sqrt{2r}}{\sigma}\right)} + e^{\frac{2\sqrt{2r}c}{\sigma}} \left(1 + \frac{\sqrt{2r}}{\gamma\sigma}\right)}{2e^{c\gamma - \frac{\sqrt{2r}(\underline{\phi} - c)}{\sigma}} r \left(1 - \frac{2r}{\gamma^2\sigma^2}\right)} - e^{\frac{2\sqrt{2r}\underline{\phi}}{\sigma}} A(\overline{\phi})$$
(56)

$$F(\overline{\phi}) = \frac{\gamma \sigma \left(2\sqrt{2}e^{\frac{\sqrt{2r}\phi}{\sigma}}\gamma\sigma + e^{\frac{\sqrt{2r}(\phi-c)}{\sigma}-c\gamma}\left(2\sqrt{r} - \sqrt{2}\gamma\sigma\right)\right)}{2\sqrt{2}r\left(2r - \gamma^2\sigma^2\right)} - e^{\frac{2\sqrt{2r}\phi}{\sigma}}A(\overline{\phi}) \qquad (57)$$

$$C(\overline{\phi}) = -\frac{e^{-\frac{\sqrt{2r}(c+\phi)}{\sigma}}\gamma\sigma}{2e^{c\gamma}r\left(\sqrt{2r} + \gamma\sigma\right)} + A(\overline{\phi}) \qquad (58)$$

$$C(\overline{\phi}) = -\frac{e^{-\frac{\sqrt{2r}(c+\phi)}{\sigma}}\gamma\sigma}{2e^{c\gamma}r\left(\sqrt{2r} + \gamma\sigma\right)} + A(\overline{\phi})$$
(58)

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