Human Capital, Business Cycles and Asset Pricing

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Discussed by Urban Jermann

- Contribution: RBC model with human capital, study of asset pricing and business cycle implications

- Key model features:

  - Human capital accumulation, human capital and productivity shocks, labor-leisure-education trade-off, human capital in utility function

  - Time-to-plan friction for human capital and physical capital

  - Production economy, distinguishes between output, consumption and dividends

- Most prominent findings:
  - higher equity premium, upward sloping yield curve, cyclical variations in equity returns
  - matches business cycle statistics

Why is the equity premium larger than in standard model?

Time-to-plan

- Time-to-plan is a version of time-to-build

$$K_{t+1} = (1 - \delta_K) K_t + S_{1,t}$$

$$I_t = \phi_J^K S_{J,t} + \phi_{J-1}^K S_{J-1,t} ... + \phi_2^K S_{2,t} + \phi_1^K S_{1,t}$$

  - Time-to-build

$$\phi_j^K = \frac{1}{J}$$

  - Time-to-plan (here)

$$\phi_4^K = 0.01, \text{ and other } \phi_{j<4}^K = .33$$

Percentage Deviations From Unshocked Steady-State Paths
After a 1 Percent Unexpected Increase in the Level of Technology in Period 1

Model:* One-Period Time-to-Build; Time-to-Build; Time-to-Plan

Chart 1 **Output**

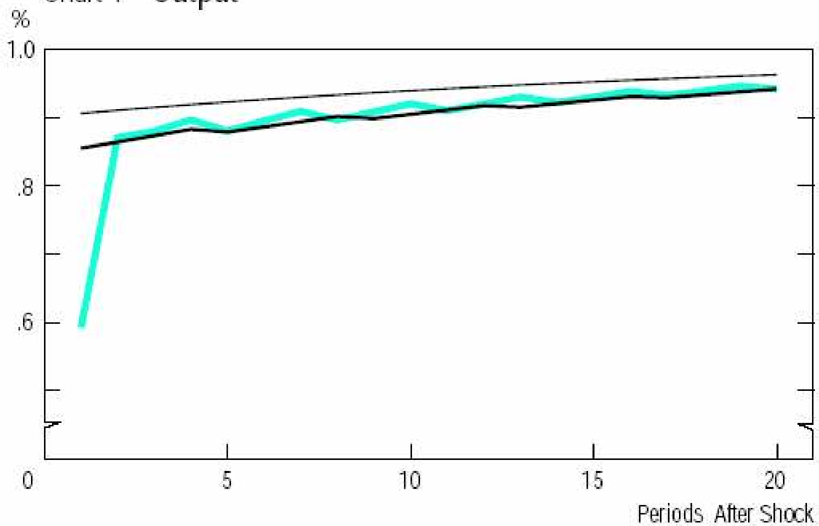


Chart 2 **Consumption**

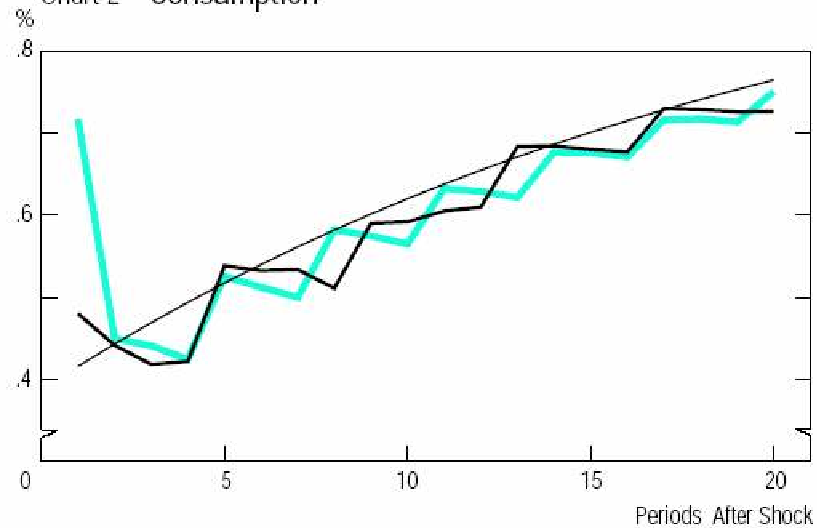


Chart 3 **Investment**

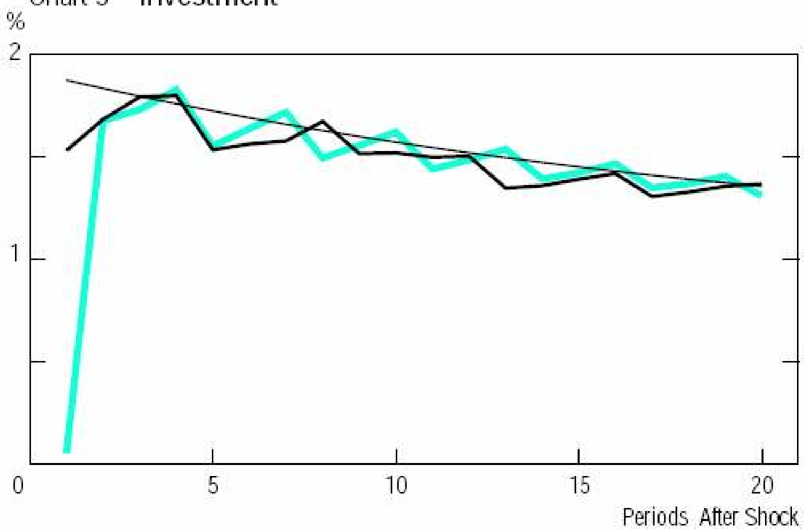
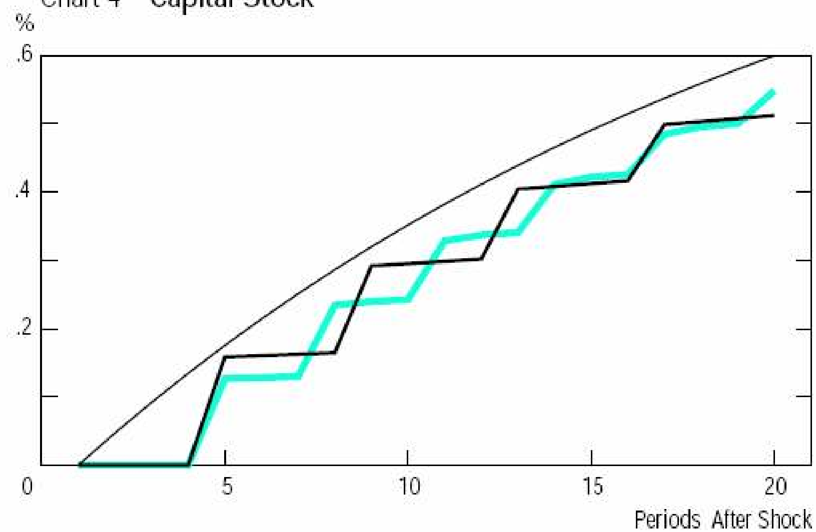


Chart 4 **Capital Stock**



Why is the equity premium larger than in standard model?

Human capital (shocks) in utility function

Assume "endowment" version of this model: no endogenous capital accumulation ($U = I = 0$) and no labor/leisure choice.

Then,

$$\begin{aligned} D_t &= \theta A_t^K K^\theta (H_t N)^{1-\theta}, \\ H_t &= A_t^H H_{t-1} \end{aligned}$$

and

$$D_{t+1}/D_t = \left(\frac{A_{t+1}^K}{A_t^K}\right) \cdot \left(A_{t+1}^H\right)^{1-\theta}$$

With Cobb-Douglas utility

$$V\left(C_t, H_t, L_t\right) = \left(\frac{1}{1-\gamma}\right)\left[C_t^{\alpha}\left(H_t L^{v}\right)^{1-\alpha}\right]^{1-\gamma},$$

and because with the given assumptions

$$D_t = \theta C_t$$

we have SDF

$$\beta \frac{V_1(C_{t+1}, H_{t+1}, L_{t+1})}{V_1(C_t, H_t, L_t)} = \beta \left(\left[\frac{A_{t+1}^K}{A_t^K}\right]\left(A_{t+1}^H\right)^{1-\theta}\right)^{\alpha(1-\gamma)-1}\left(A_{t+1}^H\right)^{(1-\alpha)(1-\gamma)}$$

that is

$$\beta \frac{V_1(C_{t+1}, H_{t+1}, L_{t+1})}{V_1(C_t, H_t, L_t)} = \beta \left(\frac{D_{t+1}}{D_t} \right)^{\alpha(1-\gamma)-1} \left(A^H_{t+1} \right)^{(1-\alpha)(1-\gamma)}$$

but if H_t is taken out of utility then we would have

$$\beta \frac{V_1(C_{t+1}, H_{t+1}, L_{t+1})}{V_1(C_t, H_t, L_t)} = \beta \left(\frac{D_{t+1}}{D_t} \right)^{\alpha(1-\gamma)-1}$$

- With IID lognormal A^H shocks, then

$$\log(R^K/R^f) = a \cdot \sigma^2,$$

$$\begin{aligned} \text{with shocks in utility} &: \quad a = \alpha\gamma + (1-\alpha) + \frac{(\gamma-1)(1-\alpha)}{1-\theta} \\ \text{without shocks in utility} &: \quad a' = \alpha\gamma + (1-\alpha) \end{aligned}$$

- With $1-\theta = .64$ and $\alpha\gamma + (1-\alpha) = 5$,

Assume $\alpha \in (.2 : .5)$, which implies $\gamma \in (9 : 21)$

$$\begin{aligned} \gamma &= 9 \quad \rightarrow \quad \frac{a}{a'} = 2.25 \\ \gamma &= 21 \quad \rightarrow \quad \frac{a}{a'} = 6 \end{aligned}$$

Calibration

- A lot of free parameters

$$\sigma_H, \psi_H, \varphi_{HK}, \rho, \nu, \kappa, \gamma$$

- Business cycles properties
  - Investment standard deviation relative to output std: here 1.87, US economy 2.93
  - Standard RBC better than "Standard" here (?) here 1.82, King+Rebelo (1999) 2.95

Countercyclical risk aversion and numerical methods (?)

- Paper uses log-linear approximations of first-order conditions

$$\lambda_t = a_c \cdot c_t + a_h \cdot h_t + a_l \cdot l_t$$

where $x_t \equiv \ln(X_t/X)$

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$$\begin{aligned} RRA_t &\equiv \frac{\partial \log(\partial V_t/\partial C_t)}{\partial \log C_t} \\ &= \frac{\partial}{\partial c_t}(a_c \cdot c_t + a_h \cdot h_t + a_l \cdot l_t) \\ &= a_c \end{aligned}$$