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Human Capital Life Cycle of Earnings Models: A Specific Solution and Estimation

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Introduction and Summary

The purpose of this chapter[*] is to consider human capital models of earnings behavior over an individual lifetime. A general class of life cycle models relating to individual earnings behavior is developed by considering alternative formulations of the basic Ben-Porath type model. An explicit solution to a specific formulation within this general class is considered in some detail. An empirical development of this explicit earnings function is estimated using data on a cohort of individuals surveyed at several points during their lifetime. The empirical estimates are discussed in detail. The estimated earnings function is then used to predict an individual's discounted present value of lifetime earnings.

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[*]This essay is one of three to be included in a proposed NBER volume entitled Economic Decision Making in a Life Cycle Context. The other two essays are written by James P. Smith and James Heckman. The volume is expected to be in draft form by September 1973.

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A General Class of Life Cycle Models Relating to Earnings Behavior Based on the Human Capital Concept

This section provides a set of alternative specifications of life cycle models relating to earnings behavior based on the human capital concept and couched in the calculus of variations optimization framework. The common element is that the models are concerned with earnings behavior over the life cycle and are logical extensions of the basic Ben-Porath (1967) model. The various formulations differ in their emphasis and the particular type of behavior question being studied. Some of the forms differ considerably in their qualitative predictions while others merely provide the ground for theoretical completeness. Some of the suggested specifications indicate problem areas which may be studied with the life cycle model, only one of which will be considered in detail here.

The life cycle of earnings model is developed by assuming the individual invests in himself with the objective of maximizing lifetime utility represented by an intertemporal utility function.[2/] The individual is assumed to have perfect knowledge of himself and the world and faces no uncertainties. It is assumed that the individual receives no income from physical assets and that he can produce new human capital according to a

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[1/] Specifications which affect the basic separability of consumption and investments and the homogeneity of human capital are qualitatively important. Specifications concerning homogeneity of direct inputs and loan market imperfections and subsidies within the separable case are qualitatively similar even though they may be important factors in individual decisions. That is, the former affect basic decision rules while the latter affect levels of state variables or enhance the completeness of the model while not changing the basic qualitative predictions.

[2/] This utility function does not contain activities involving time (or leisure) or the stock of human capital as arguments.

production function that requires only some portion of his existing human capital stock and an aggregate of direct inputs. Additions to his stock of human capital are that amount produced in eash period less the depreciation which occurs. From this set of assumptions several functions of interest are derived in parametric form.

Consider the formal model: the earning capacity of an individual is

$$Y^*(a) = R \cdot E(a)$$

where

$Y^*(a)$ is the individual's earning capacity at age a;

R is the constant return per unit per unit time on the stock of human capital;

E(a) is the total stock of human capital processed by the individual at age a. a is the length of time since the individual selected an investment pattern of began making his own decisions. He begins his own investment decisions at a = 0 and his working life ends at a = N. At any age, a, the individual has the choice of renting all or any proportion of his human capital stock in the labor market.

The individual attempts to maximize utility within his opportunity set. Three different components of the opportunity set are distinguished: endowment, market opportunities, and productive oppertunities. The time-distribution of endowments is an initial stock of human capital, E_0, held by the individual at time a = 0.[3] This initial stock of human capital can be translated into a stream of earnings which declines absolutely through the

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[3] It is assumed that physical assets do not enter the decision process. There are no earnings from physical assets or endowment of them.

life cycle approaching zero asymtotically if no investment is undertaken. The stock of human capital and thus earnings decline at the rate δ so that $E(a) = - E(a)$. Given that no investment is undertaken the individual would maximize his intertemporal utility function subject to the wealth constraint

$$W = \int_0^N e^{-rt} RE_t \, dt \quad .$$

The utility maximizing decision is obviously affected by the market opportunities for exchanges of funds for differing dates. If the individual has no loan market available, he must consume in each period what he earns. If the individual has available a "perfect" loan market for consumption purposes, he may borrow or loan unlimited funds at the constant rate r. In this latter case the individual is free to make interperiod transfers of consumption to maximize his utility subject to the above wealth constraint.

Now assume the individual does invest. Here we consider the productive opportunities. It is assumed that the only alternative use of the human capital stock is the process of producing more human capital according to the production function

$$q[K(a), D(a)] = K(a)^{\beta 1} D(a)^{\beta 2} \qquad (1)$$

where

$q[K(a), D(a)]$ is the output of new human capital produced in period a;

$K(a)$ is the amount of the existing stock of human capital used to produce more human capital in period a;

$D(a)$ is other educational inputs as an aggregate and purchased at the constant price P.

β is a Hicks neutral efficiency index of the individual's ability to produce human capital and assumed constant over the life cycle of the individual.

β_1 and β_2 are the respective production coefficients of K(a) and D(a), assumed constant over the life cycle.

The net change in the stock of human capital at any point in time is that amount produced less depreciation,

$$\dot{E}(a) = q(a) - \delta E(a) \quad . \qquad (2)$$

The individual has two decision variables, K(a) and D(a). In choosing to allocate another unit of human capital to production, K(a), at any point in time the individual must give up the corresponding market earnings it would have yielded. Thus RK(a) represents foregone earnings. The aggregate of other inputs, D(a), are purchased at the constant price P. Clearly, these investment decisions will be influenced by the existence of a loan market for financing purchases of D(a). That is, it will affect the production possibilities.

There are several possible sets of assumptions. First, there may be no market opportunities for borrowing or lending at all. The individual must finance current investment and consumption out of current market earnings. Utility is maximized by selecting the paths D(a) and K(a) which maximize the utility of the flow of earnings, or equivalently consumption, over the life cycle. Investment and consumption decisions are clearly not separable. Investment is constrained by earning capacity, i.e.,

$$R \cdot E(a) \geq R \cdot K(a) + P \cdot D(a) \quad .$$

Second, assume the existence of a perfect loan market for consumption purposes as defined above. Also assume that the loan markets for financing consumption and investment in direct educational inputs are perfectly separable so that funds borrowed for one purpose cannot be used for the other.[4/] Clearly, the loan market for consumption affects only intertemporal consumption decisions and the loan market for investment in human capital affects only productive possibilities and the decisions are separable. To avoid potential problems let us also assume that in the loan market for educational expenditures the individual can only borrow funds and cannot loan them.

Now assume that there is a perfect loan market for consumption but none for human capital investment. Purchased inputs must be financed by current market earnings but consumption may be financed by borrowing. Investment is constrained by earning capacity

$$R \cdot E(a) \geq R \cdot K(a) + P \cdot D(a) \quad .$$

The individual maximizes utility by selecting the paths K(a) and D(a) which maximize the present value of earnings net of educational expenditures,

$$NY(a) = R \cdot (E(a) - K(a)) - P \cdot D(a)$$

and then selecting the optimal consumption path subject to the wealth constraint

$$W = \int_0^N e^{-rt} C(t)\, dt = \int_0^N e^{-rt} NY(t)\, dt \; .$$

Clearly, here the rate of discounting net earnings is the rate for borrowing and lending in the consumption loan market.

If there is a separable loan market to finance purchased inputs, then the productive possibilities expand. Investment is not bounded by earning capacity if borrowing (but not lending) is allowed. The constraint is that

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[4/] This assumption captures the notion that financing investment in human capital is somehow different from financing other capital since human capital is embodied in the investor. No loan market is the extreme case but is analytically simplest.

the human capital input cannot exceed the total stock, that is

$$R \cdot E(a) \geq R \cdot K(a) \quad .$$

The case presented in the original Ben-Porath (1967) model is that the interest rates in both markets are equal. In this formulation the markets are undistinguishable and need not be separable to allow the consumption and investment decisions to be separable. Again utility is maximized by selecting among the expanded permissible paths of K(a) and D(a) for those which maximize the present value of net earnings discounted at the market rate.[5/] There exists of course many intermediate cases of imperfect loan markets.

This basic model may be extended by relaxing or altering the assumptions about the utility function, the opportunity set, or the external constraints faced by the individual. Different constructions may be used to concentrate analysis on particular issues.

For example, a simpler and clearer presentation of investment behavior and the resulting earnings function are permitted if analysis is restricted to the cases in which investment and consumption decisions are separable. This is the case when activities involving time (or leisure) and the stock of human capital do not enter the utility function. The loan markets for consumption and investment are separable,[6/] and the individual can borrow and lend unlimited sums for the purpose of consumption at a constant rate

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[5/] Other possibilities include separable loan markets with differing rates. The optimal decision paths may be different but the same basic optimizing principles apply.

[6/] Separable means that funds cannot be borrowed in one market and used for another.

as discussed above. Many interesting extensions may be considered even within this special case. The model may be formulated to allow parameters to vary with time in a known way. For example, the rate of depreciation of the capital stock may increase with age or the rental rate for human capital R and the price of the aggregate input P may vary exogenously in a known way. Secondly, these parameters may vary over time in an unknown way or may simply be unknown to the individual but randomly attained from some known distribution. For example, the end of working life N, the production parameter, or the prices R and P may be uncertain. Models of decision-making under uncertainty or risk can be developed to assertain the effect of such considerations.

The model may be extended to include an initial physical asset endowment and a timed stream of payments or debts. The individual decision functions would then include investments in physical assets. As long as the loan markets for investment are separable from the loan markets for consumption and a perfect loan market for consumption is available, the optimal consumption and investment decisions will be separable. The individual will maximize the present value of the stream of income from both physical and human capital.[7/] The loan markets for physical and human capital may or may not be separable but the existence of returns from physical capital to finance human capital investment and vice-versa will affect the patterns of investment and the resulting earnings. A timed stream of payments of debts may include such things as anticipated gifts, inheritance, subsidies and doles. An

[7/] The term earnings will refer to returns from human capital rented in the labor market while income will refer to both returns from physical and human capital.

individual may be given a payment (dole) as long as he is in fulltime schooling, he may receive a percent subsidy on all human capital investments or on direct expenditures (excluding foregone earnings) through a reduced P during fulltime schooling. These subsidies may or may not be accompanied by a commitment to pay a percent of earnings in the future (as in the Yale plan).

A rather severe assumption is that human capital is homogeneous in the sense that all units are perfect substitutes in the labor market both between individuals and over units held by an individual and thus rent at the same rate, R, per unit. Becker (1967, p. 3) says:

> The assumption of homogeneous human capital clearly differs in detail rather drastically from the usual emphasis on qualitative differences in education, training, and skills . . . these differences, while descriptively realistic and useful, are not required to understand the basic forces determining the distribution of earnings.

An individual's observed market earnings is the rental rate times the amount of human capital the individual chooses to allocate to the market at the time. This ignores the potentially fruitful area of differences between occupations. If human capital were considered a vector of skills rather than a homogeneous unit, then each occupation could be considered as requiring a different "mix" of these skills. Another important extension, then, is to consider a vector or human capital embodied in the individual and a vector of direct educational inputs rather than the homogenous aggregate of each. The vector of direct inputs D is a simple extension and is considered later. Considering a vector of human capital is a more complex matter. Each type of human capital has a different rental rate, and a different set of direct inputs. Since human capital is embodied in the individual each type must be used in the production of all others. The investment decision becomes much more complex.[8/]

Yet another way to develop the model is to consider the individual as endowed with a fixed amount of time at any point in the life cycle and faced with the decision of whether to use it, and the human capital which accompanies it, investing or in the market.[9/] The alternative used above considers the individual as possessing a stock of human capital and deciding where to allocate it. Once the time interpretation is specified and inferences concerning investment, earnings and wages are to be made, some destinction should be made between various lengths of the time unit, whether an hour, day or year. Under the time interpretation the meaning of "time in the market" is less clear with the possibility of on-the-job training as well as outside investment. For example, empirically a destinction should be made between investment made on-the-job and made off-the-job when speaking of time and wages. Whether considering an hour or a day or a week will not matter if all training is on-the-job. The observed wage per time unit will be the potential wage or earnings capacity less investment. Neither the potential wage nor the investment time can be identified but the time unit doesn't matter. However, if all investment occurs off-the-job the observed hourly wage will be the potential wage and a zero wage is observed for all training time units. In this latter case, time invested and potential

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[8/]The author is currently involved in the analysis of this problem.

[9/]This time interpretation introduces questions of whether time and human capital K enter the production function in the same way or not. This is discussed in Ben-Porath (1967).

wage can be identified but the time unit is crucial. Any combination of these is also possible but the whole problem is avoided if the problem is formulated as units of human capital.

Another potentially fruitful area for development is the fixed end points of the life cycle. The individual's investment decisions begin at a = 0 with the initial endowment of human capital E_0 and end at the end of working life a = N. Certainly, investment in the individual begins at birth or even before birth in the form of prenatal care during his mother's pregnancy. However, investment decisions are not made by the individual himself during the early part of his life. The investment decisions made for him by his parents or guardian are surely influenced by parents' attitudes toward education, income and financial position, number of other children and dependents in the family, educational attainment and health of the parents, and other socioeconomic variables. During some period the individual and his parents make decisions jointly before the individual begins to make his independent decisions. All during his early period, as well as later, some investment decisions are influenced by or even made by society. These decisions are in theform of public school quality and accessibility, teacher quality, school lunch program, and especially compulsory school attendance. An approximation to the point a = 0 is the age when compulsory school attendance ends. The fact that the individual may participate in his investment decisions prior to a = 0 does not matter. If E_0 is properly measured, it will reflect these decisions in summary form.

The model assumes that an individual selects optimal earnings and investments paths subject to $a^* \geq 0$. Students are legally constrained to stay in school until age 16 where a = 0. An individual could possibly increase his human wealth by leaving school before that time, i.e., $a^* < 0$. Individuals in this circumstance will not be investing optimally. The model will not predict well for them and the existence of such individuals in a sample will bias any parameter estimates.

Many questions cannot be considered in the models where consumption and investment decisions are separable. The primary specifications which make consumption and investment decisions inseparable are an imperfect loan market when the loan market for human capital investment is imperfect amount of investment in human capital as an argument in the utility function, and activities involving time as an input as an argument in the utility function. Anyone of these will make the decisions inseparable. There are many constructions of imperfections in the consumption loan markets. These include increasing cost of funds, limited funds, and even the perfect loan market for when the loan market for human capital investment is imperfect and inseparable. If the stock of human capital or investment itself enters the utility function then clearly the individual will not maximize utility by maximizing the present value of net earnings. Examples include the possibility that education increases efficiency in consumption (Michael, 1972) and that attending an educational institution may be partly consumption.

A much more complete analysis of the use of an individual's (or family's time) can be obtained by considering activities involving time that enter the utility function. Time may be considered an input into the production of ultimate consumption goods (Becker, 1965). The most obvious example is leisure activities. Time and the human capital accompanying it then have three alternative uses: the production of human capital, the production of consumption activity, and the acquisition of earnings (Stafford and Stephans, 1972). These models where consumption and investment decisions are not separable are more complex since they also involve many other issues affecting consumption behavior.

A Particular Solution

The particular specification of the general life cycle of earnings model used depends upon the questions to be studied, the degree of empirical complexity desired, and the data available. The particular specification presented here was chosen because it allows analysis of individual investment and earnings behavior while abstracting from consumption and leisure considerations, is capable of being fully solved analytically which illustrates the simultaneity of schooling and earnings while providing an exact functional form for earnings, and has a fairly straightforward empirical interpretation.

The solution presented in this paper is based upon the assumption of separable loan markets allowing unlimited borrowing and lending at a constant rate r for consumption but no loan market for human capital investment purposes. Direct educational expenditures must be purchased with current market earnings. Neither the stock of human capital nor activities involving time enter the utility function. Consumption and investment decisions are separable and the individual acts to maximize the present value of net earnings.

In the early period the individual specializes in the production of new human capital, using all of his earning capacity for investment. The period of specialization is

$$0 \leq a \leq a^*$$

where a* denotes the age at which the individual stops specializing and begins investing only a fraction of his earning capacity. Specialization ends when earning capacity ceases to be an effective constraint on investment. One implication of assuming no loan market for educational expenditures, and the only qualitative difference from the Ben-Porath perfect loan market case, is the prediction of positive labor force participation during the period of specialization. The individual supplies a constant fraction of his human capital to the market to finance expenditures for direct educational expenditures, i.e., $R \cdot K(a) = P \cdot D(a)$.

For the rest of the life cycle, after the period of specialization ends, $a^* \leq a \leq N$, the individual invests some fraction of his earning capacity in producing more human capital. Neither foregone earnings nor direct educational expenditures is a function of the initial stock of human capital E_o. Gross investment declines with age after the period of specialization reaching zero at the end of working life N. Earning capacity, observed earnings, and net earnings at any age after a* depend upon the stock of human capital and the investments at that age. The equations underlying these statements are given in Appendix A. See Figure 1 for a geometric representation.

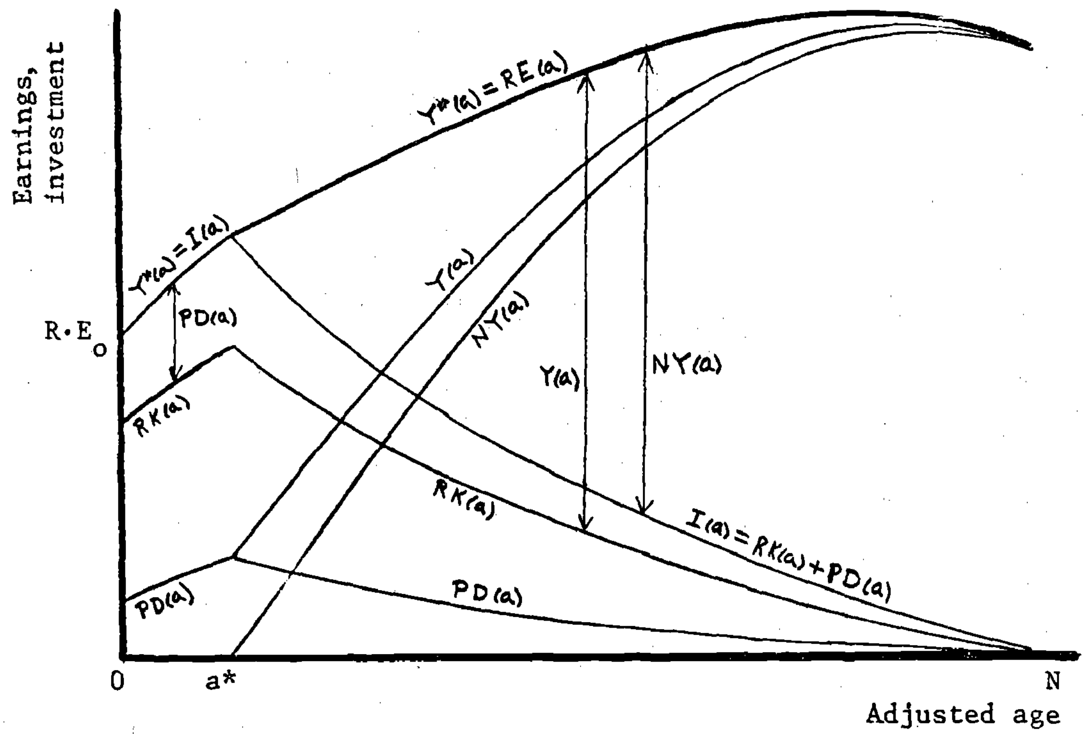

Figure 1. Earnings and investment paths and their relationship to one another

Note: The relative magnitudes of R·K(a) and P·D(a) depend upon R, P, β_2. R·K(a) is arbitrarily drawn larger than P·D(a).

The length of the period of specialization is endogenous to the model. It is determined by the characteristics of the individual and the optimization criteria. The optimum age to stop specilalzing in production and begin positive net earning is that point where the investment paths of the two regions cross. That is, the individual will invest according to the rule K(a) and D(a) for non-specialization except when he is constrained by his earning capacity during which period he will invest all of his earnings capacity. The solution for a* as a function of the parameters and initial endowment of human capital, but not age, is contained in an expression which cannot in general be solved in closed form for a*. This expression is presented in Appendix A. The expression containing the solution for a* must be considered simultaneously with either the observed or net earnings function to make any inferences about earnings. The expression allows inferences about the direction of effect of each characteristic on the length of the period of specialization. The length of the specialization period varies directly with N, R, and β and inversely with E_o, P, and r. The effect of all other characteristics is ambiguous.[10]

Effects of Parameters on Earnings

Consider the effect of the characteristics β, E_o, a, β_1, β_2, δ, r, N, P, and R on observed earnings and net earnings. Figure 1 illustrates[11] the position of the decision paths P•D(a) and R•K(a) and of the resulting

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[10]For a more detailed discussion of these effects, see Wallace and Ihnen, (see footnote 4).

[11]The exact shape of the curves drawn in the figures is arbitrary but their relative positions and direction of movement are drawn to be consistent with the solution to the model.

state variables $R \cdot E(a) = y^*(a)$, $Y(a)$, $NY(a)$, and total investment defined as

$$I(a) = R \cdot K(a) + P \cdot D(a) \tag{3}$$

for a given vector of characteristics.[12] The effects of characteristics are in terms of their effect on these paths.

The Initial Stock of Human Capital

The effect of E_o is clearly to increase investment during the period of specialization and thus raise potential earnings $Y^*(a)$, foregone earnings $R \cdot K(a)$ and direct educational expenditures $P \cdot D(a)$. Since the investment paths after the period of specialization, $a^* \leq a \leq N$, are not a function of E_o, their paths are unchanged. The result is that desired investment for $a^* \leq a \leq N$ intersects the maximum amount available, earning capacity, at an earlier point and the individual ends his period of specialization sooner but he has a larger earning capacity when he does end his specialization. His entire set of earnings functions are thus raised over the entire life cycle. The net result is that an individual with all other characteristics the same but a larger initial stock of human capital will have higher earnings over his lifetime and begin getting them sooner (see Figure 2). The individual's human wealth is clearly increased by an increase in E_o.

The Production Efficiency Parameter

The effect of β the production or efficiency parameter is on investment in both periods (Mincer, 1970, p. 12). First, during the

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[12] Either $R \cdot K(a)$ or $P \cdot D(a)$ may be larger but $R \cdot K(a)$ is drawn larger for illustrative purposes. Their relative magnitudes depend upon the relative magnitudes of β_1 and β_2 and P and R.

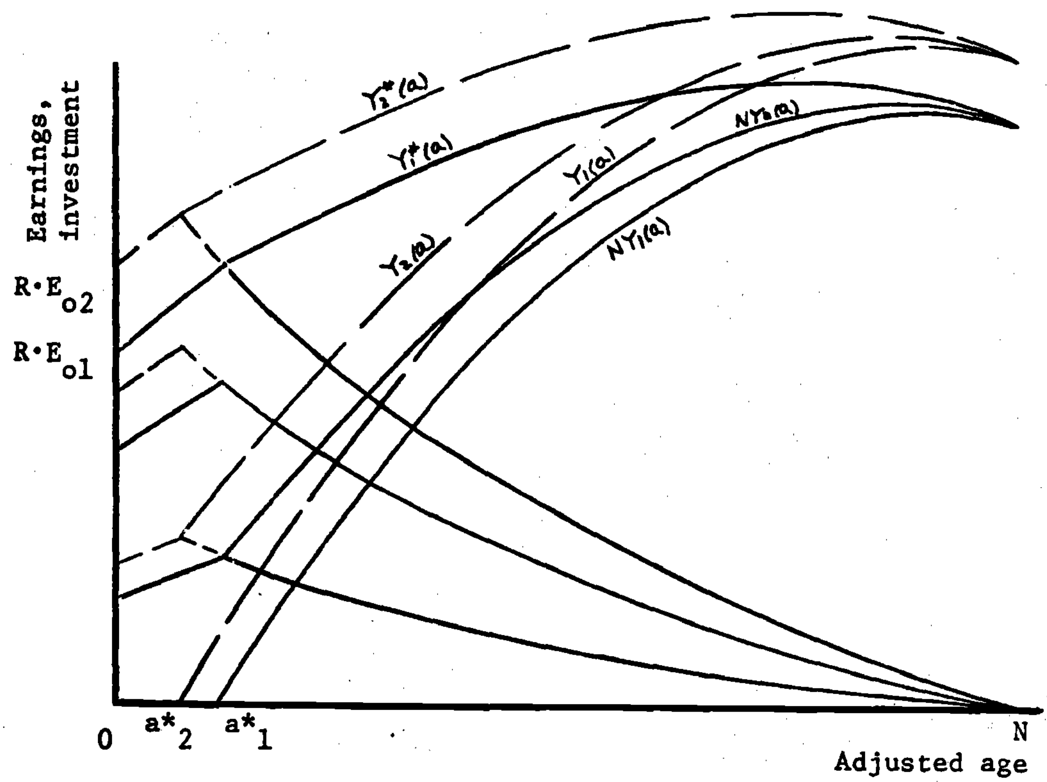

Figure 2. Changes in income and investment paths due to an increase in the initial stock of human capital

period of specialization investment the fixed maximum investment is more productive and earning capacity rises more sharply than before from E_o. Each type of investment increases correspondingly. After the end of specialization investment is still more productive and an individual invests more in both foregone earnings and direct expenditures. The investment path shifts up but still must decline monotonically to zero at $A = N$. The increased productivity in the earlier period tends to shorten the period of specialization while the increase in productivity in the later period tends to lengthen the period, but, as was mentioned earlier, the net effect is to lengthen the period of specialization. Both effects raise the earning capacity of the individual. In the period after specialization both earning capacity and investments are increased and the net effect on observed earnings and net earnings is less clear. Net earnings always begin from zero and with a larger β begin later and so must be lower for some initial period. After a "catch-up" period, net earnings are larger for a larger β. A similar statement is true for observed earnings. Therefore, the effect on earnings of more efficient production in this sense depends on the age of the individual, but the effect is to increase earnings after some "catch-up" period (see Figure 3). This "catch-up" period is very similar to the year of "overtaking" discussed by Mincer. Mincer describes this as the time it takes for a trained person to overtake an untrained one and speculates that it is rather short. This model supports this concept but restricts it to comparisons of persons with all characteristics except β the same. In this case a larger efficiency or ability parameter explains both the length of training or specialization and the higher earnings after the catch-up period.

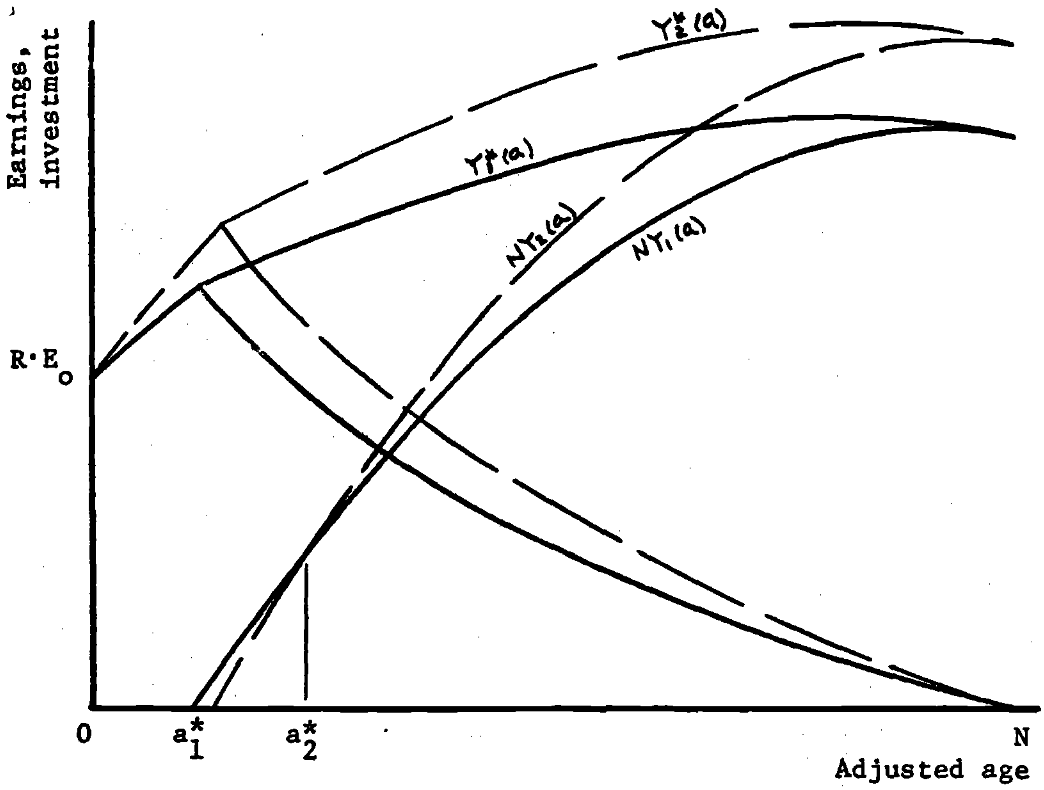

Figure 3. Changes in income and investment paths due to an increase in ability

Age

Age enters the earning function as an exponential function interacting with all other characteristics to determine the "shape" of investment and earnings paths over the life cycle. The earnings functions are the sum the exponential functions of age each weighted by functions of the parameters. Whether earnings rise or fall as age increases depends on which functions "dominate." Initially, earnings rise monotonically while the positive terms dominate and then decline monotonically for a period at the end of the working life.[13]

Rate of Depreciation, Deterioration or Obsolescence

The rate of depreciation, deterioration or obsolescence of the stock of human capital δ affects investment in both periods. In the period of specialization less human capital is available to invest since it depreciates at the rate δ. Earning capacity grows more slowly the higher the depreciation rate. After the period of specialization a higher rate of depreciation continues to have a negative effect on both foregone earnings and educational expenditure investments which decline to zero at some age and becomes a positive effect after that age. That is, a higher depreciation rate makes an individual place more importance on earnings and less on investment early in his planning horizon and more importance on investment later to keep earnings up. This might be termed a "leveling" effect on investment since a high depreciation rate shifts investment from early to later periods in the life cycle.

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[13]The existence of this decreasing portion is dependent upon the values of the characteristics especially the depreciation rate. If $\delta = o$, the earnings functions rise monotonically over the entire life cycle.

The Rate of Interest

The effect of the market rate of interest is to reflect the value of net earnings depending on how soon or late they appear. The importance of r is reflected in the objective function, to maximize the present value of net earnings over the life cycle. A higher rate of interest more heavily discounts future earnings and thus lowers investment after the period of specialization and shortens the period of specialization.

The Length of Working Life

Finally, the effect of the length of the working life or investment period is to shift horizontally the entire investment curves P•D(a) and R•K(a) left if N falls and right if N rises. Therefore, if the working life increases, both the length of the specialization period and investment after that period increase since the individual has longer to reap the benefits of his investments.

Factor Prices and Production Coefficients

Consider the effect of the factor prices R, P and the production coefficients β_1, β_2 where R is also the rental rate of human capital. Given the Cobb-Douglas form of the production function, the efficient ratio of investment is given by

$$\frac{P \cdot D(a)}{R \cdot K(a)} = \frac{\beta_2}{\beta_1} . \qquad (4)$$

What matters for investment purposes with respect to β_1, β_2 is their ratio. Any change in β_1 or β_2 results in a corresponding shift from the less productive to the more productive type of investment.

The effect of R and P is similar, but, since R also affects the return to additional units of human capital while P does not, the result is more complex. If the price paid for educational inputs, P, rises, there will be a substitution of foregone earnings for educational expenditures. Overall investment will fall after the end of the specialization period since the price of a factor of production has risen. If the rental rate and the price of human capital allocated to production, R, goes up, there will be a shift of investment from foregone earnings to educational expenditures. Both types of investment as well as overall investment will rise over the entire life cycle since investment becomes more profitable. If both P and R are increased in proportion, then relative amounts of inputs to investment will be unchanged but more of each type will be undertaken and thus total investment increased by the increase in earning capacity through R.

Changes in E_o and β for a Given Level of Schooling

A special case which is of interest is when the characteristics change in such a way that the period of specialization a* is unchanged. If one characteristic changes, there must be an offsetting change in another characteristic. For example, if only β and E_o are allowed to change, any combination satisfying the expression in Appendix A will yield the specified value of a*. E_o and β must move in the same direction to maintain the same value of a* since a rise in E_o shortens time in specialization while a rise in β lengthens it. It was shown earlier that a rise in either E_o or β tends to raise earnings but only after the "catch-up" period for β. When both increase in proportion to maintain a* the entire earning profiles Y(a) and NY(a) as well as earnings capacity rise over the entire life cycle. See Figure 4 for net earnings and Figure 5 for observed earnings. Consider the changes in two parts. First increase E_o from E_{o1} to E_{o2}. The result is to increase investment in the period of specialization only and to increase earnings everywhere. The period of specialization is lowered from a_1^* to a_2^* . Second, increase β just enough to raise a^* back up to a_1^*. This will raise the productivity of investment and thus investment and earnings in both periods complementing the effect of E_o. That is, persons with the same a^* but differing in E_o and β will have observed earnings and net earnings profiles which lie one wholly above the other. This result is verified in the empirical sections.

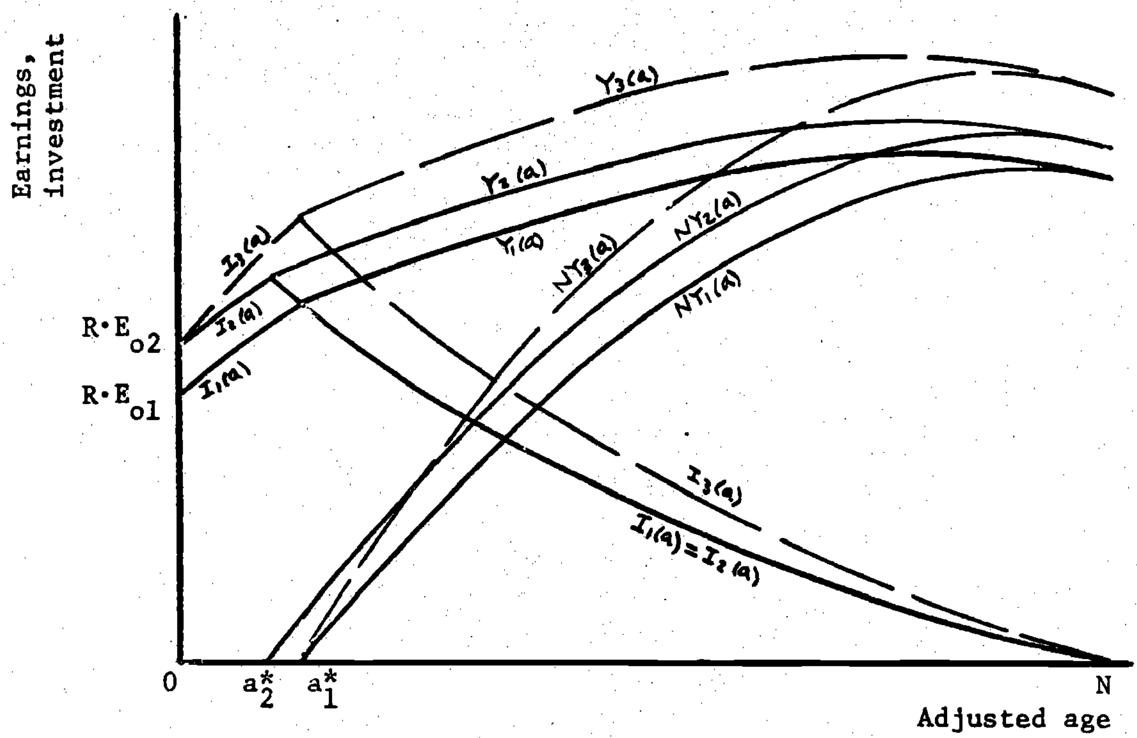

Figure 4. Changes in net income and investment paths due to changes in E_o and β which leave a^* unaltered

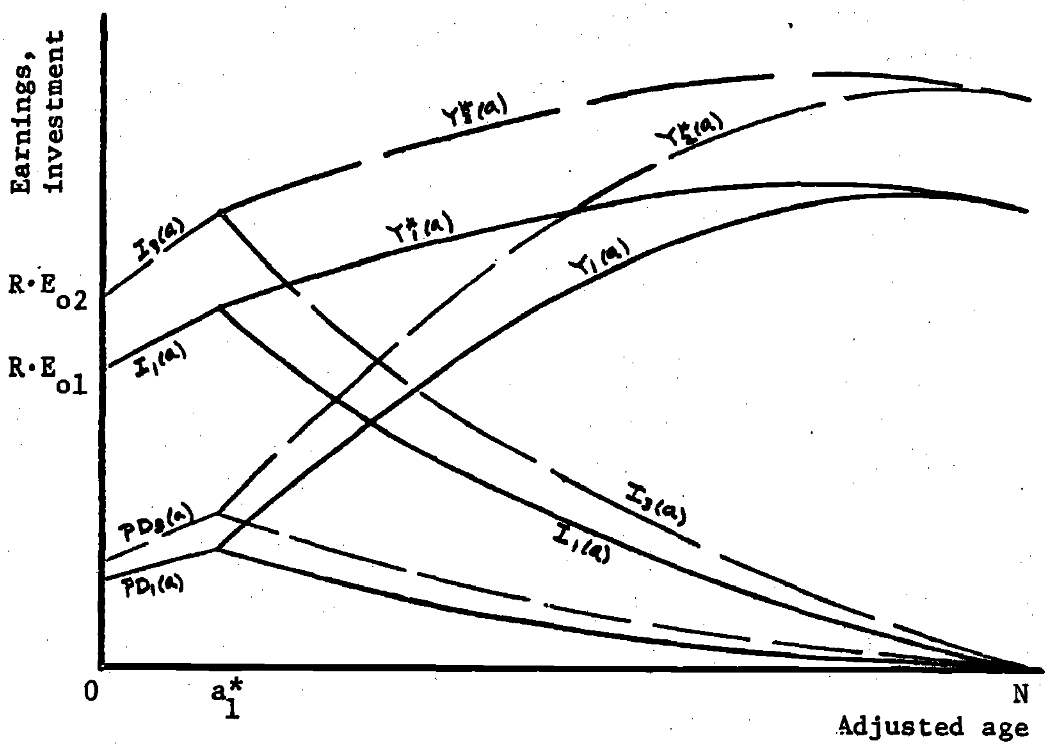

Figure 5. Changes in observed income and investment paths due to changes in E_o and β which leaves a* unaltered

Empirical Application Of The Model

The purpose of this section is to present and interpret an empirical approximation to the earnings function, to discuss empirical counterparts to theoretical entities and to use the model to study the effect of ability and schooling in determining the age-income profile. First, the model is interpreted in the context of a particular research problem and of data limitations. The data are discussed and model simplifications necessary for estimation are made. The model is estimated and results presented.

The research areas to which the model is applied depends not only upon the interests of the research but also upon available data. The model requires observation of several points along each individual's earnings profile for individuals with a wide range of parameters. Many of the parameters are not directly observable and must be approximated by variables available in the data set used. Other parameters must be assumed constant over all individuals for estimation purposes.

An area of research which is of current interest and to which the model applies particularly well is the study of the effect of ability and schooling upon the age-earning profile. The IRSS-Eckland and NBER-Thorndike samples contain information on ability measured by a test score, years of schooling, earnings on initial full-time job, earnings at the time of the survey or surveys, recalled earnings for other years and in one case, estimated earnings ten years in the future.

Interpretation of the Model for Empirical Purposes

The Efficiency Parameter as Ability

The meaning and effect of ability have been subjects of concern for many years and by many authors.[14] Definitions have ranged from the power to do something and the natural equipment to accomplish some small part of the meaner ambitions to such technical definitions as the "height" of marginal rates of return schedules. Clearly ability takes meaning only in the context of the task or objective to be performed. For the purpose of studying earnings functions ability may be defined in terms of producing earning capacity more or less efficiently.

A common use of the term ability is the ability to produce earnings. Does this mean observed earnings, net earnings or earning capacity? Clearly observed and net earnings are directly affected by current investment. One person may have more earning capacity than another at the same age but lower observed and net earnings due to larger investment. Even rankings by observed and net earnings may not be the same due to different relative productivities of human capital and direct purchased inputs represented by $(\frac{\beta_1}{\beta_2})$, all else equal, making foregone earnings different. Even earning capacity is not an ideal interpretation because it varies over the life cycle and is a

[14] For example Mincer (1970), Becker (1967), Hause (1971), Leibowitz (1973), Grilliches and Mason (1971).

function of past investment and investment by parents in the individual resulting in E_o the initial stock of human capital. Another common conception of ability is the rate at which an individual can or the rate at which an individual does accumulate earning capacity. The latter concept is represented in the model by $\dot{E}(a) - \delta E(a)$ the actual rate of accumulation of human capital. The former concept is the rate at which the individual could accumulate human capital if he used all of his human resources to do so, i.e., specialized in production. Both of these concepts depend upon the stock of human capital held in the individual at the time ability is measured and the rate of accumulation in either case is influenced by such factors as loan markets and subsidies which should not influence a "pure" definition of ability.

A definition of ability implied by this model and which is relatively free of the above defects is the efficiency with which the individual produces new human capital represented by the production parameters β, β_1, β_2. That is the index of ability is the relative efficiency with which an individual can use a given set of inputs to produce new human capital.

Consider more carefully the real world counterpart to β. β indexes the ability of individuals to use the same inputs more or less efficiently given β_1 and β_2. In the production sense β measures the effect of all inputs whose amounts are fixed and given for the individual. That is;

$$q\,[K(a), D(a)] = (\alpha A_1^{\alpha_1} A_2^{\alpha_2} A_3^{\alpha_3} \ldots A_n^{\alpha_n})\, K(a)^{\beta_1} D(a)^{\beta_2} \quad (3)$$

where $\beta = A_1^{\alpha_1} \cdot A_2^{\alpha_2} A_3^{\alpha_3} \ldots A_n^{\alpha_n}$ (4)

and the A_1 are those fixed inputs. The A_1 represents the effect of such factors as manual dexterity, general health, mental ability in several dimensions such as logic, comprehension, and speed of thinking, and such constraints as blindness.

If ability is interpreted as suggested above then it will be influenced by early family decisions and environment to the extent that these determine the fixed factors of human capital production represented in equation (4). Parents and society can influence the efficiency with which an individual can produce human capital,[15] and this is correspondingly reflected in the effect of family and socio-economic characteristics on measured ability. If these early decisions affect factors which are not later under the individuals influence by decision they affect his ability. If they affect factors which become the individuals decision functions later, PD or RKa, then their influence is reflected in the initial stock of human capital.

Initial Endowment as Early Family and Environmental Background

Early family and environmental background of the individual have been a part of studies of earnings function but usually are a "control" in the form of dummy intercept variables or separate analysis entirely. Some of the important factors in recent empirical works[16] are race, country, region of country, father's education, religion, and socio-economic class. The objective is often to control for these factors

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[15] These decisions can be considered as early as pre-natal care. This certainly has an effect on later physical and mental well being.

[16] For example see Chiswick (1968), Johnson (1970), Hause (see footnote 24).

while studying the effect of primary variables such as schooling or ability. For the purposes of this research, all of these factors are represented either in their effect on the initial stock of human capital. The initial endowment is the result or culminated effect of all such factors on the individual's history of investment up to the point where he can begin to make his own investment decisions. The point where he can begin to make his own investment decisions, $a = o$, is for the purpose of this analysis taken to be the time at which an individual can legally leave school, that is $a = 0$ at age 16 years. Before this time the individual's investment decisions are made for him by his parents or guardians and by public policy. These decisions are not necessarily those which are optimal for the individual.

Public policy dictates that the individual must be full time in school until he reaches 16 years of age. This might not be optimal for the individual since he may maximize the present value of the net income stream by leaving school before that time. General public policy and the local political process determine the degree and amount of direct educational expenditure subsidy received by the individual through the public school system and community education environment. At this point, race, country, region of the country, and local socioeconomic environment become important factors if there are systematic and significant differences in direct educational subsidies to individuals in different classifications.

Parents directly determine the individual's early investments through direct educational expenditures and through constraints on the amount of time and effort, and correspondingly human capital, the individual invests in training. Training outside of formal schooling, health care, educational material in the home, and direct exposure to parents skills in a loose apprenticeship manner are all part of investment. The parents' investment decisions are influenced by such factors as family income and wealth, number of dependents in the family, and socioeconomic status of the family as affecting attitudes toward investment in children. To the extent that these factors are significant and systematic, they influence the individual's later investment behavior through their effect on E_o and β.

These early family and environmental factors manifest their effect in both the initial endowment E_o and the ability index β. E_o represents the initial endowment of human capital which the individual will choose to allocate to investment or to the market. The production efficiency index β may be influenced by family and environmental factors to the extent that it represents the level of inputs which are fixed and not subject to individuals' decision and which are affected by family and environmental factors. That is, the individual may receive certain types of investments in him which he cannot make himself. These influences may be considered substantially complete and fixed at the time the individual begins to make his own investment decisions ($a = o$) concerning those factors of production that are not fixed. This indicates that ability defined as in this research should be measured as close as possible to the age when $a = 0$.

Time in Specialization as Schooling

Time in full-time schooling or training has been the primary subject of many authors. These models take the length of training as the basic source of heterogeniety of labor incomes. The models are formulated in terms of training periods which are completed before earnings begin and apply therefore strictly to schooling rather than to all occupational training. The length of time in schooling is the primary decision variable. When post-schooling investments are accounted for in the model, they are taken as given in each period. In terms of the model presented in this research, time in specialization in producing new human capital, a^*, corresponds approximately to time in full-time schooling or training. Both time in specialization and post-schooling investments are determined in this model by E_o and β. They are the result of individual choice in optimizing behavior and the determinants of that behavior are clearly specified. The end of schooling, a^*, enters the earnings function through the constant of integration in the solution of the model for post-schooling earning capacity. The end of full-time schooling is that age at which earning capacity ceases to be a constraint on investment. Time in schooling determines the individual's level of earning at each age but only indirectly through and in conjunction with β and E_o. Therefore, the effect of a^* or years of schooling is to indicate the end of specialization and the relationship between E_o and β which yield the same value of a*, other things constant.

Simultaneity of Earnings and Schooling

Earnings, either observed, Y, or net, NY, and time in specialization, a*, are endogenous variables determined within the system. The efficiency parameter representing ability β, the initial stock of human capital E_o representing the cumulative effect of family and environmental background and age a are exogenous variables which vary between individuals. All other characteristics, δ, r, β_1, β_2, R, P, and N, are assumed to be exogenous parameters which do not vary between individuals.

The observed and net earnings functions are given in Appendix A. Either of these earnings functions is determined simultaneously with a* whose solution is contained in the expression given in Appendix A. No closed form solution for a* is obtainable but the solution is a function of β and E_o. To study the earnings functions, a closed form solution for a* should be substituted into the equation for net or observed earnings making either a function of only the exogenous variables a, β, and E_o. But since no closed form solution for a* exists, one of the other exogenous variables involved in the solution for a* must be eliminated from the earnings functions, that is β or E_o.

To study the effect of ability, schooling, and their interaction on the age-earnings profile E_o is eliminated from the earnings functions by using the simultaneous constraint of a*. Substituting the solution for E_o from the expression containing the solution for a* into either the net or observed earnings function yields an earnings function having only ability, age, and schooling as arguments. The effect of E_o is automatically accounted for through β and a* subject to the simultaneity constraint. That is E_o would be redundant since a* and β are specified.

An Approximate Form of the Earnings Function

The earnings function must be put in a form which is amenable to estimation. Since the earnings reported in the survey questionnaires described in the next section correspond more closely to observed earnings than net earnings only the observed earnings function will be presented.

To reach an empirically viable form for the observed earnings function linear Taylor's series approximations for exponential functions of age and schooling are used.[17] Applying these approximations to the observed earnings function results in the empirical form

$$\begin{aligned} Y(a, \beta, a^*) = D_1 (\beta^c) + D_2(\beta^c) + D_3(\beta^c a^*) + D_4(\beta^c a^* a) \\ + D_5(\beta^c a^2) + D_6(\beta^c a^{*2}) + D_7(\beta^c a^2 a^*) \\ + D_8(\beta^c a a^{*2}) + D_9(\beta^c a^2 a^{*2}) \end{aligned} \quad (5)$$

where the coefficients D are defined in Appendix A.

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[17] One reason the approximation is necessary is to separate r and from the age variable. The general Maclaurin's series approximation used is $f(x) = f(0) + f'(0)\, x$.

The Empirical Specification

Several points should be noted on the relationship between empirical quantities and theoretical entities. First, what earnings are we speaking about? Earnings net of direct educational expenditures is the most obvious choice. However, the earnings appropriate for empirical analysis based on current data sets is observed earnings. We will concentrate on and study the estimating equation for observed earnings. Obviously any present value or rate of return calculations must be considered as upper bounds. Secondly, what are the appropriate time units of measurement for earnings? Should we use hourly, daily, weekly, yearly earnings? Any of the latter three units would seem appropriate, the smaller the time interval the more desirable the unit since the model is based on a continuous concept.

To consider the implications for hourly earnings we need more information. An individual is predicted, after the period of specialization, to invest a certain fraction of his human capital in producing more and to use the rest for obtaining observed market earnings. There are two interpretations of this investment activity. One is that this fraction is represented by time out of the labor force. All investment is done outside the labor market and the individual is paid for 100 per cent of the stock of human capital he brings to the labor market. This kind of situation is the one in which the wage rate is a measure of the stock of human capital up to a scalar multiple R.

An alternative interpretation is that individuals work a standard 40 hour week or standard work day consistent with employer's rules and carry out their investment activity on-the-job. The individual carries all of his human capital to the work place but uses some fraction of it to obtain earnings by being productive and the remainder for investment in on-the-job training. He is then paid only for that fraction actually productive to the employer. Payment may be different from the individual's product to the extent that some of the training is firm specific. See Becker [2]. Also the earnings will reflect a deduction of direct training costs and thus represent net earnings. In this paper this problem is not considered, and yearly earnings are used.[18]

The model is obviously based on the individual having full information and no uncertainty. The individual maximizes the present value of his net earnings stream based on full utilization of his human capital stock at any point in time for investment and/or earnings. This is obviously violated when the individual is unemployed. One way to handle this problem is to consider the estimation of full employment earnings. That is, what would the individual's earnings been if he had been fully employed.[19] To justify this we assume that the unemployment is totally unexpected by each individual, considered as transitory, and does not affect his future expectations or investment behavior.[20]

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[18] Only yearly earnings are available for all age points in the IRSS-Eckland sample and for all but one age point in the NBER-Thorndike sample.

[19] The weeks worked problem is handled similarly by Chiswick and Mincer [24], p. 5 of mimeograph. The formulation implicitly assumes elasticity of earnings with respect to weeks worked in unity.

[20] This obviously begs the question of weeks worked of labor force participation being a function of schooling and age and other economic variables. This is subject of study in itself. See J. Heckman, Jim Smith, and J. Mincer.

For the annual data available in the samples used here observed earnings should be weighted by the ratio of total weeks to weeks actually worked. Since the weeks worked data are available for only one year in each sample this approach is not used here. The alternative assumption used here is that a small incidence[21] of unemployment is expected to be experienced at random over the population independent of age, schooling mobility and is fully capitalized into investment decisions. Actual observed earnings are then used here.

Schooling and age for estimation purposes are measured as years of schooling and years of age beyond age 16, respectively. Earnings observed at age 40 for example are recorded as occurring at adjusted age 24. Adjusted schooling is measured years of schooling completed less the years of schooling at age 16 or 10 years.[22]

The ability index in the form β^c, where $c = 1/(1-\beta_1-\beta_2)$, enters the equation as a weight to the right-hand side of the earnings function and the elasticity of earnings with respect to β^c is unity. The production coefficients β, β_1, and β_2 and thus β^c have some inherent scale so that

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[21]Those experiencing unemployment during the questionnaire year are a small fraction of the total and the mean weeks worked was approximately 98 per cent.

[22]This definition makes comparisons and predictions relating to the 1960 Census of Population possible. An alternative definition considered was that adjusted schooling is the age of the individual when leaving the last year of full-time formal training. This definition does not allow calculation of the rate of return to completion of another year of schooling in the usual sense and has questionable interpretation since many individuals interrupted schooling to enter military service and received different subsidies before and after service. The results are not surprisingly different.

comparisons between individuals could be made if the scale were known and the parameters observable. For example a doubling of β would imply that twice as much human capital could be produced for the same inputs. However, only standardized measures of "ability" are available.

The measures of ability available for this research are based on examinations designed to measure reading comprehension, arithmetic skills, and in certain cases physical characteristics which are intended to predict success in some training or schooling activity. To the extent that the desire to predict the ability to acquire training more or less efficiently has guided the design of the examinations they will be a good proxy for β^c. However, to the extent that the tests measure the current stock of human capital the measure will be poor and will overstate the effect on earnings of ability. If the tests measure a component of skill level fixed to the individual and not subject to variation by decision then its variation correctly enters the measure of β^c.

Let us first consider measured ability as an unknown but exact (no random error) monotonic transformation of β^c. Measured ability is then interpreted as an ordinal index of β^c which preserves its ranking but does not preserve the , $\beta^c = 1$ property. We cannot say a doubling of the measured ability index indicates a doubling of β^c or earnings.[23]

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[23] We can however make statements about the effect of a change in measured ability, recognizing it as such, if all individuals have identical examinations.

If we consider a polynomial[24] approximation

$$\beta^c = m_1 + m_2 B + m_3 B^2 + \dots + m_{kh} B^k$$

to the monotonic transformation then the estimating equation becomes[25]

$$Y = D_1' + D_2' A + D_3' S + D_4' AS + D_5' B + D_6' BA + D_7' BS + D_8' BSA \qquad (6)$$

where A is adjusted age, S is adjusted schooling and B is measured ability. Age and schooling now enter both individually and in interaction with ability.[26]

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[24] A linear approximation is used here for simplicity of presentation. The actual degree of the polynomial used will be determined on mean squared error criterion.

[25] Note that $D_1' = m D_1$, $D_5' = n D_1$, etc. so that only the ratio $\frac{m}{n}$ can be estimated and there are four estimates of it and none of the di's.

The obvious alternative formulation is in terms of experience = A - S. The relationship becomes

$$Y = D_1' + (D_2' + D_3')\, S + D_4' S^2 + D_2' e + D_4' es + D_5' B + (D_6' + D_7')\, BS + D_8' S^2 + D_6' Be + D_8' Bes \qquad (7)$$

[26] We should note at this point the effect of ignoring ability in the estimating equation. If the equation

$$Y = C_1 + C_2 A + C_3 S + C_4 AS \qquad (8)$$

is estimated by ordinary least squares while equation (6) is the correct

(continued on next page)

Now let us consider the problem of measurement error in the monotonic transformation from the production efficiency index to measured ability. The appropriate relationship is then

$$\beta^c = m_1 + m_2 B + e.$$

Let us assume that

$$e_i \sim IIDN(0, \sigma^2)$$

and that the measurement error is uncorrelated with A, S, B and the error of measuring Y (call it ϵ). Equation (7) now becomes

$$Y = D_1' + D_2' A + D_3' S + D_4' AS + D_5' B + D_6' BA + D_7' BS + D_8' BSA + (D_1 + D_2 A + D_3 S + D_4 SA)e \quad (9)$$

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26 (concluded)
specification and has the usual desired properties of the error term then the error term associated with (8) will not.

Since the omitted terms are obviously positively correlated with the included variables, to the extent that the variance in B is small, the estimated coefficients C_i are biased upward. Also ignoring the interaction of ability with the included terms will make the variance of the calculated error term positively correlated with predicted income. For example, measured ability has the effect of increasing the rate of increase of earnings over the life cycle at any schooling level, thus causing the predicted variance of earnings to increase with the age class. If the effect of ability in this respect is not accounted for the larger variance will show up in the error variance.

Clearly the error term associated with estimating equation (7) when equation (9) is the true relationship does not satisfy the usual assumptions for least squares estimates. The combined error term, $\eta = (D_1 + D_2 A + D_3 S + D_4 SA) e + \varepsilon$, is normally distributed with mean zero but the covariance structure is heteroschedastic. The error variance increases with the square of $(D_1 + D_2 A + D_3 S + D_4 SA)$.[27] Estimates using this weighting factor are considered later.

Results of Empirical Estimation

Two samples of cohort data are examined to estimate the empirical earnings function developed in the previous section. The samples differ substantially in size, populations from which they are drawn, number of points observed over the life cycle and supplementary data available. Each sample is described briefly before the results of estimation are presented. The wider range of ages observed over the life cycle in the NBER-Thorndike sample allow more detailed analysis of earnings late in the life cycle and present value calculations. Inferences from the IRSS-Eckland sample are restricted to early life cycle earnings behavior.

Results Based on the NBER-Thorndike Data

The NBER-Thorndike sample is based on a group of males volunteering for Air Force pilot, navigator, and bombadier programs in the last half

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[27] Identical statements apply irrespective of degree of the polynomial transformation used if there is also an error of measurement. Estimates are unbiased.

of 1943. These volunteers were given initial screening tests and a set of seventeen tests to measure various abilities, to be described later, in 1943. Thorndike and Hagen sent a questionnaire to a sample of 17,000 of these men in 1955 which included a question on 1955 earnings. The NBER sent to a subset of these a subsequent questionnaire in 1969 which included additional questions on earnings in later years and questions on schooling and initial job earnings.

The data includes five separate approximately equally spaced points[28] on the age-income profile as well as the year of initial job, year of last full-time schooling, years of schooling and seventeen separate measures of ability. The age-income points are approximately initial job, 1955, 1960, 1964, and 1968. The individuals in the Thorndike sample differ from the U.S. male population as a whole in several ways.[29] First the sample includes a high ability group. All of the men completed high school or high school equivalency examinations, and passed the initial screening for the Air Force flight program. Their general health was better than the general population[30] in 1969. They were more homogenous in height

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[28]Any observation which might cause special problems is omitted. These include those individuals disabled, unemployed, in the military, or who is a pilot at his major occupation. Particular year observations for an individual are omitted if for example the year of initial job was questionable.

[29]Many of these comments originated with T. F. Juster who directed the data collection for the NBER.

[30]The model response was excellent with 57 per cent, 38 per cent were good, 3 per cent fair, and less than 1 per cent each were poor or non-response.

and weight due to military qualifications. They seem to have a high degree of self confidence, self reliance and risk preference. They tend to be entrepreneurs, an unusual 20 per cent work longer hours. Some of these factors may however be related to the high ability.

The age distribution of the individuals in the sample in 1969 is presented in Table A1. An annual earnings and year of earnings are recorded for the five-year intervals 1945-52 (first post-World War II job), 1953-57, 1958-62, 1962-66, and 1967-70. The 1955 and 1969 earnings figures are current while the others are recall responses. There is then considerable age variation within these year intervals.

All individuals have at least a high school degree. The distribution of adjusted schooling for the sample is presented in Table A2.

Seventeen different measures of ability, or indicators of likelihood of success in completing the Air Force training programs, were collected at the time of application in 1943. Scores include tests of reading comprehension, mechanical principles, dial and table reading, spacial orientation, numerical operations, speed of identification, mathematics, rotorary pursuit, two-hand coordination, complex coordination, aiming stress, discrimination reaction time, and finger dexterity. These scores have been combined into an aggregate measure of ability designed to approximate an I.Q. type measure.[31] The distribution of the sample with respect

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[31] The aggregate measure was constructed by Al Beaton.

TABLE A1

Age Distribution of the NBER-Thorndike Sample in 1969 Mean Age is 46.7 and the Standard Deviation is 3.1 Years. Non-response is .22 Per Cent

Age	Relative Frequency (per cent)
≤ 42	.04
43	.22
44	10.32
45	21.73
46	22.42
47	15.54
48	10.00
49	5.73
50	4.63
51	3.93
52	3.60
53	1.48
54	.10
≥ 55	.04

TABLE A2

Equivalent Years of Schooling Distribution for the NBER-Thorndike Sample Mean Years is 5.02 and the Standard Deviation is 2.4 Years

"Equivalent" Years of Schooling	Adjusted Years	Relative Frequency (Per cent)
12	2	24.24
13	3	11.46
14	4	7.67
15	5	6.56
16	6	28.32
17	7	5.35
18	8	7.69
19	9	1.17
20 or more	10 or more	7.53

Note: "Equivalent" years of education

High school graduate	=	12
Some college training, no degree	=	13-15
College graduate	=	16
Some post-graduate work, no M.A. degree	=	17
Masters degree or equivalent	=	18
Some post-masters work, no Ph.D.	=	19
Professional degree or Ph.D.	=	20

to this aggregate ability measure[32] and the distribution of ability within schooling classes are presented in Table A3. The overall distribution is scaled to have mean one and standard deviation one-fourth so that results are comparable between samples. The mean and standard deviations for the various schooling groups are in Table A4. Table A5 presents the means and standard deviations of earnings over all points observed over the life cycle for various schooling classes and ability classes.

The earnings function is estimated from the NBER-Thorndike data using all of the age-earnings points observed for each individual. The degree of polynomial approximation for age, schooling, and measured ability are determined by the data on the basis of signifigant reduction of error variance. The estimated equations are presented in Table A6. Equation (1) illustrates that a strict application of the theoretical form, with linear approximation, does result in coefficients with the predicted sign. Quadratic terms and interactions of schooling and ability and cubic terms in age and their interaction are the resulting "best" equation, equation (7). Both cubic and quadratic equations in age will be considered and the age-earnings profiles for each are presented in Table A7. The observed age range of the sample as seen in Table A1 is adjusted ages from 3 to 41, but with less than one percent outside the range 3 to 39. Even though the equation cubic in age is a significantly better predictor inside the observed interval, it is quite poor beyond adjusted age 39. Therefore the equation quadratic in age[33/] is

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[32] All earnings are in 1957-59 dollars.

[33/] The cubic but not quadratic term in ability is included in the quadratic age equation since it is a slightly better predictor.

TABLE A3

The Distribution of Measured Ability for the NBER-Thorndike Sample and for Schooling Classes. The Overall Mean is 1.00 and the Standard Deviation is .25.

Ability Class	Overall Relative Frequency	Relative Frequencies for Adjusted Schooling Classes							
		2	3	4	5	6	7	8	> 8
0.0 - 0.124	0.000	.001							
0.125 - 0.374	.003	.005	.002	.005	.003	.001	.004		
0.375 - 0.624	.052	.084	.085	.039	.055	.036	.030	.026	.023
0.625 - 0.874	.279	.371	.345	.332	.277	.208	.234	.220	.200
0.875 - 1.124	.382	.387	.382	.389	.385	.378	.358	.365	.401
1.125 - 1.374	.208	.132	.150	.182	.228	.257	.264	.265	.258
1.375 - 1.624	.065	.020	.030	.047	.049	.104	.087	.102	.093
1.625 - 1.874	.011	.001	.007	.005	.000	.016	.023	.018	.026
1.875 - 2.00	.000				.003			.003	
Sample size	4954	1201	568	380	325	1403	265	381	431

Note: The simple correlation between ability and schooling is .2451.

TABLE A4

Means and Standard Deviations of Ability for the Various Schooling Classes of the NBER-Thorndike Sample

Adjusted Schooling	Mean	Standard Deviation	Sample Size
2	.906	.217	1201
3	.930	.230	568
4	.967	.228	380
5	.989	.234	325
6	1.059	.256	1403
7	1.047	.263	265
8	1.069	.254	381
$\geq$ 8	1.063	.248	431

TABLE A5

Mean and Standard Deviations of Earnings Over All Life Cycle Points for Various Schooling Classes and Ability Classes for the NBER-Thorndike Sample

	Mean	Standard Deviation	Sample Size
Overall	9434.56	7093.44	15578
S = 2	8025.80	5024.03	3459
S = 3	9216.74	7464.38	1694
S = 4	9511.62	7321.70	1166
S = 5	9819.96	7502.72	1041
S = 6	9933.13	7349.03	4693
S = 7	9331.96	6905.64	882
S = 78	9163.44	6496.23	1302
	11619.42	9371.92	1341
0 < B < .25	9243.39	6112.84	20
.25 < B < .50	8166.55	7407.76	173
.50 < B < .75	8681.53	6403.80	2112
.75 < B < 1.0	9016.06	6748.65	5761
1.0 < B < 1.25	9744.36	7345.09	4894
1.25 < B < 1.50	10210.08	7428.57	2082
1.50 < B <	11568.95	8793.34	536

TABLE A6

Empirical Estimates of the Earnings Function from the NBER-Thorndike Sample Based on 15,578 Age-Earnings Points from 4,956 Individuals

	Coefficient						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
		7291.40	7338.47	2535.49	6671.05	5211.24	21108.50
A			-98.84	-141.70	-408.75	-636.00	-3921.20
S			-1413.70	-1428.70	-2692.30	-596.62	877.25
SA			158.13	432.84	482.88	432.56	148.02
A^2			1.80	7.62	12.53	21.16	206.09
S^2			83.85	164.11	255.24	104.08	-794.20
SA^2			-2.62	-8.93	-10.03	-14.26	6.87
S^2A			-15.45	-47.46	-49.70	-54.85	116.42
S^2A^2			.37	.96	1.04	1.94	-7.82
B	4608.60	-3043.90	-1294.80	10073.00	406.47	5934.30	-45197.00
BA	170.01	165.07	47.43	-27.59	430.96	1060.30	11015.00
BS	-719.87	-549.69	226.89	-248.14	1859.20	-5226.9	4721.40
BSA	42.39	42.42	-37.21	-538.15	-497.07	-266.00	-1820.80
BA^2			2.79	-7.79	-12.93	-31.86	-594.93
BS^2			-36.04	-163.27	-272.32	280.74	1065.0
BSA^2			1.00	13.33	11.96	21.11	83.505
BS^2A			8.51	69.28	57.90	61.99	-122.05
BS^2A^2			-.21	1.39	-1.21	-3.52	8.56
B^2				-6307.40		-5757.90	28134.00
B^2A				121.37		-570.03	-6738.40
B^2S				490.95		7401.90	-5035.2
B^2SA				210.94		-286.40	1435.20
B^2A^2				4.36		11.61	371.38
B^2S^2				42.60		-613.47	-240.65
B^2SA^2				-5.65		-5.42	-72.59
B^2S^2A				-26.99		6.29	5.86
$B^2S^2A^2$				.55		1.81	.99

(concluded on next page)

TABLE A6 (concluded)

	Coefficient						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
B^3					-1134.20	761.58	
B^3A					-53.97	115.29	
B^3S					-277.67	-2697.80	
B^3SA					112.40	217.46	
B^3A^2					4.13	2.53	
B^3S^2					53.78	264.68	
B^3SA^2					-2.99	-2.34	
B^3S^2A					-12.71	-17.62	
$B^3S^2A^2$					.28	-.15	
A^3							-2.99
A^3S							-.31
A^3S^2							.15
BA^3							9.09
BA^3S							-1.04
BS^3S^2							-.17
B^2A^3							-5.74
B^2A^3S							1.04
$B^2A^3S^2$							.03
R^2 =	.2063	.2642	.2693	.2704	.2705	.2710	.2759

TABLE A7.a

Quadratic Age-Earnings Profiles Based on the NBER-Thorndike Sample. $Y = C_0 + C_1 A + C_2 A^2$.

Ability (B)	Adjusted Schooling (S)	C_0	C_1	C_2
.50	0	6831.50	-200.02	6.58
	2	3752.38	207.41	-0.38
	6	613.26	486.71	-3.04
	10	1499.65	49.81	9.32
.75	0	6645.91	-108.30	4.58
	2	4152.73	160.14	0.94
	6	934.71	417.68	-0.37
	10	74.45	302.79	6.30
1.0	0	6141.31	-31.76	3.73
	2	4166.52	146.62	2.05
	6	1097.47	395.11	1.25
	10	-797.57	499.24	3.88
1.25	0	5211.40	24.54	4.44
	2	3664.91	178.09	2.89
	6	1048.66	433.60	1.46
	10	-932.00	620.34	2.25
1.50	0	3749.83	55.54	7.00
	2	2519.04	265.81	3.39
	6	735.37	548.45	-.10
	10	-144.45	647.23	1.60

Note: These estimates include a cubic but not a quadratic ability term.

TABLE A7.b

Cubic Age-Earnings Profiles Based on the NBER-Thorndike Sample.

$$Y = C_0 + C_1 A + C_2 A^2 + C_3 A^3.$$

Ability (B)	Adjusted Schooling (S)	C_0	C_1	C_2	C_3
.50	0	5543.50	-98.30	1.47	0.12
	2	8214.34	-678.00	47.29	-0.75
	6	5831.35	-472.66	48.16	-0.82
	10	-6851.19	1552.35	-71.99	1.34
.75	0	3036.13	549.70	-31.21	0.60
	2	5684.86	-158.12	18.33	-0.28
	6	7842.77	-897.36	70.59	-1.14
	10	5814.63	-734.71	60.44	-.84
1.00	0	4045.50	355.40	-17.46	.36
	2	5293.00	-118.82	17.14	-.24
	6	8511.60	-1061.58	80.55	-1.28
	10	12695.01	-1996.80	136.23	-2.10
1.25	0	8571.63	-681.20	42.71	-.60
	2	7038.76	-560.07	43.73	-.66
	6	7837.82	-965.33	78.04	-1.25
	10	13790.13	-2233.91	155.37	-2.43
1.50	0	16614.50	-2560.10	149.30	-2.27
	2	10922.15	-1481.90	98.11	-1.51
	6	5821.54	-608.59	63.00	-1.00
	10	9099.82	-1446.04	117.86	-1.85

used for prediction in this interval. Comparisons of Cubic and quadratic estimates are made later.[34] Consider the effect on earnings of age, schooling, ability and their interaction. The effect of schooling on the age-earnings profile is illustrated in Figure 5 for the average ability individual. The profile for schooling level S=0 is included for completeness, but it is a pure extrapolation since all persons in the sample were at least high school graduates. The initial earnings upon entering the labor force are scarcly affected by schooling while the rate of increase in earnings is enhanced. Individuals with more schooling initially have lower earnings than individuals who have less schooling but have been in the labor force longer. The earning of the more schooled person rise faster to overtake the less schooled

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[34] The weighted estimates were attempted for both the Cubic and Quadratic age equations, but were not obtained because the weighted cross product matrix was to close to singular to be inverted accurately.

Even the Unweighted equations are highly collinear due to the correlations between the polynomial age and schooling terms, for example CORR(A,A^2) = .983, CORR(A,A^3) = .945, CORR(A^2,A^3) =.989, CORR(S,S^2) = .974, COORR(B,B^2) =.986 , CORR(B,B^3) =.951 , and CORR(B^2,B^3) =.989 . Weighted estimated equations were obtained for lower degree polynomials in ability using the weight implied by the cubic age equation. Both weighted equations linear in ability and omitting ability entirely had larger calculated R^2, .47 and .46 respectively, than the R^2 for the unweighted equations cubic in age with R^2 = .276 or quadratic in age with R^2 = .270. However, these are weighted R^2's. Comparisons based on the squared simple correlation between predicted and observed values, unweighted R^2, indicates that the weighted estimates are clearly poorer. $\rho^2_{Y,\hat{Y}}$ for the weighted equations linear in ability and omitting ability ability are .240 and .247 respectively.

Similar comparisons can be made for alternative estimating equations. Two alternatives are compared here, both involve simple transformation of the dependent variable. One is the widely used log-linear earnings function and the other is one suggested by Heckman and Polachek (1972). Heckman and Polacheck estimate γ such the Y^γ is the appropriate dependent variable. They find that γ is approximately .33. Equations with both lnY and $Y^{.33}$ as the dependent variables were estimated as linear functions of age, schooling, ability and their interactions resulting in calculated R^2 of .44 and .39 respectively. Since R^2 in these cases represents the squared simple correlation between predicted and observed lnY and $Y^{.33}$ values respectively, they are not strictly comparable with the estimate based on linear Y. The resulting squared simple correlation between predicted and observed earnins are .260 and .256 respectively as compared with .269 for the corresponding estimate linear in Y.

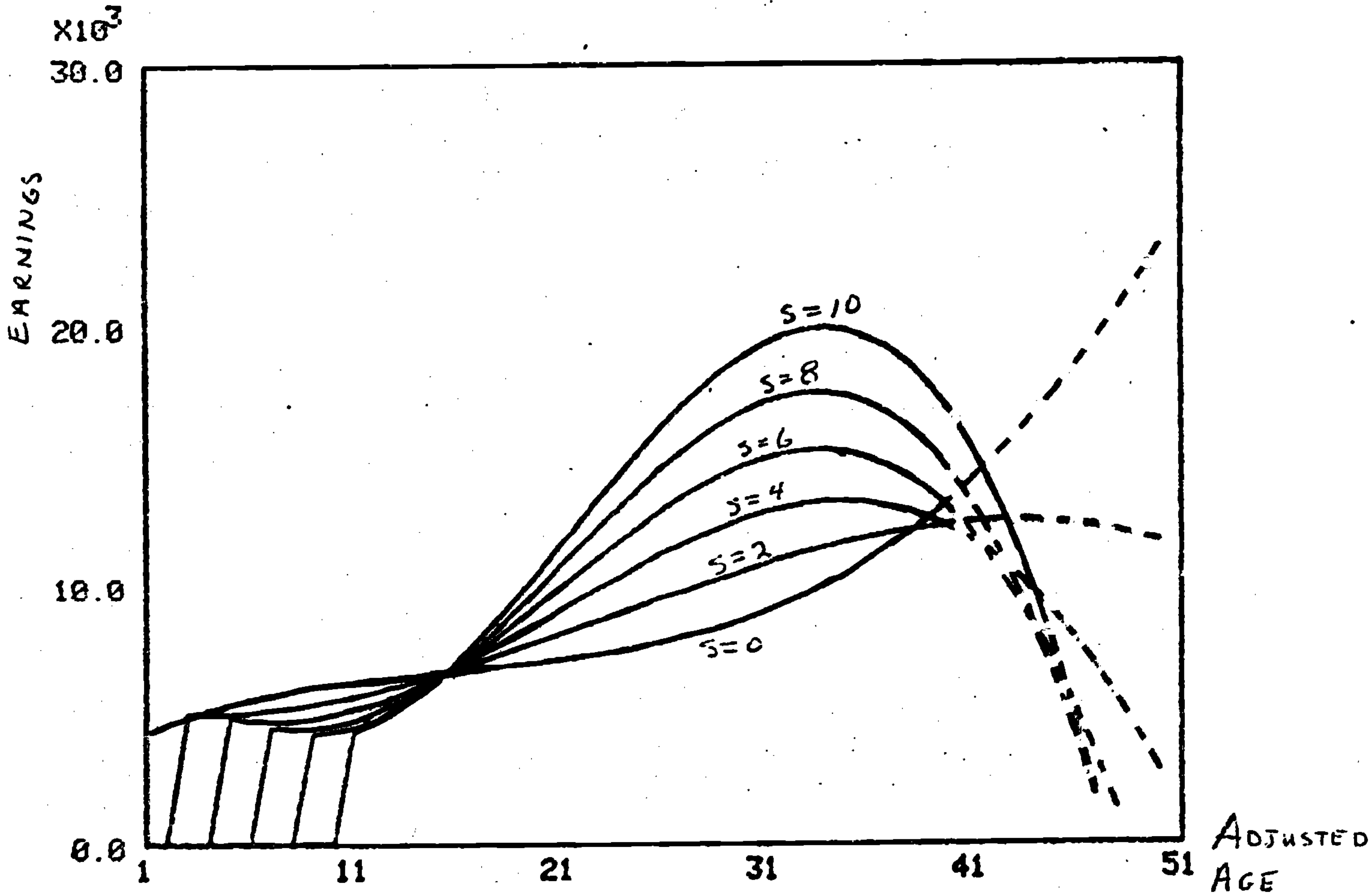

FIGURE 5.a. CUBIC ESTIMATED AGE-EARNINGS PROFILES BASED ON THE NBER-THORNDIKE SAMPLE FOR SEVERAL SCHOOLING LEVELS AT THE AVERAGE ABILITY LEVEL.

NOTE: ALL EARNINGS ARE IN 1957-59 DOLLARS.

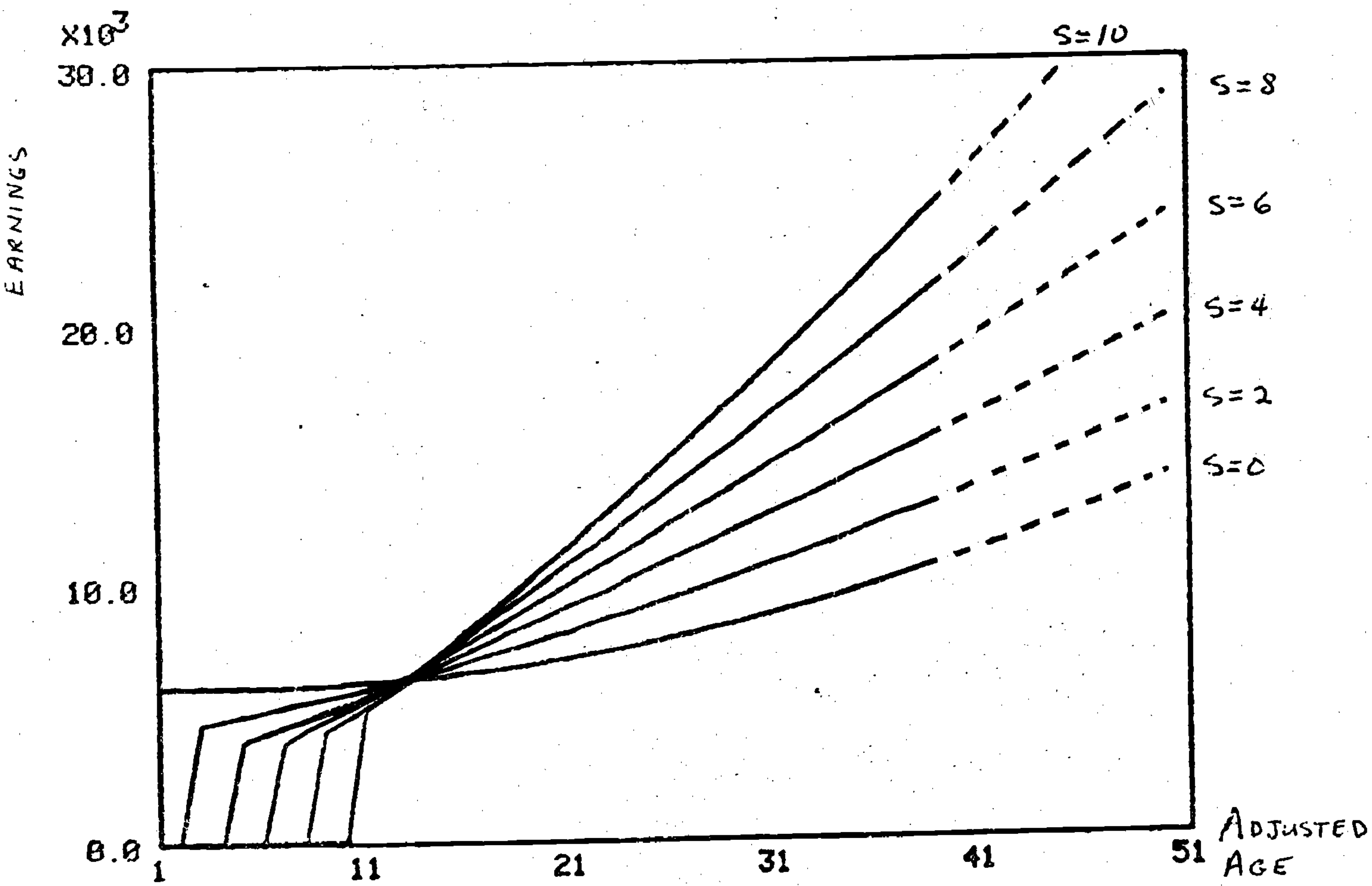

FIGURE 5.6. QUADRATIC AGE-EARNINGS PROFILES BASED ON THE NBER-THORNDIKE SAMPLE FOR SEVERAL SCHOOLING LEVELS AT THE AVERAGE ABILITY LEVEL.

person's earnings at about adjusted age sixteen and remain higher throughout the observed life cycle. The cubic estimates indicate that earnings begin to level off toward the end of the observed life cycle and decline absolutely after approximately adjusted age thirty-four, exactly when depending on the level of schooling. The absolute decline in earnings is not a clear prediction since less than one-fifth of one percent of the sample were observed beyond age thirty=nine and only nine percent were observed beyond age thirty-six and since the leveling-off of earnings later in the life cycle was not strong enough to dominate the shape of the quadratic age-earnings profiles, the quadratic estimates do not turn down at all.[35/]

These results can be couched in terms of experience rather than age where experience is defined simply as years of work beyond schooling.[36/] Experience-earnings profiles for individuals of average ability are presented in Figures 5c and d. Clearly increased schooling enhances the effect of experience even for a given ability level. Differences in earnings are even more pronounced within experience groups than within age groups.

The effect of an additional year of schooling obviously depends on the individuals position in the life cycle. Whether expressed in terms of age or experience. This is illustrated in terms of age for average individuals in Figure 6.a. Early in the life cycle, for example, schooling has little or even a small negative effect on earnings.[37/] The earnings schooling relationship becomes progressively stronger over the life cycle increasing in both level and rate of increase.

[35/] It should be noted that the effect of additional schooling is constrained to be a smooth quadratic by the functional form. Previous empirical work indicates that highschool, college and graduate degree, graduates may earn slightly more and the corresponding dropouts may earn slightly less than the smooth prediction.

[36/] This simple definition ignores the problem of schooling completed before military service and assumes that military service counts fully toward experience. A much more complete consideration of experience rather than age for working women is attempted by Mincer and Polachek (1973).

[37/] Again the relationship for A=40 is a prediction and is ambiguous for the Cubic age estimates.

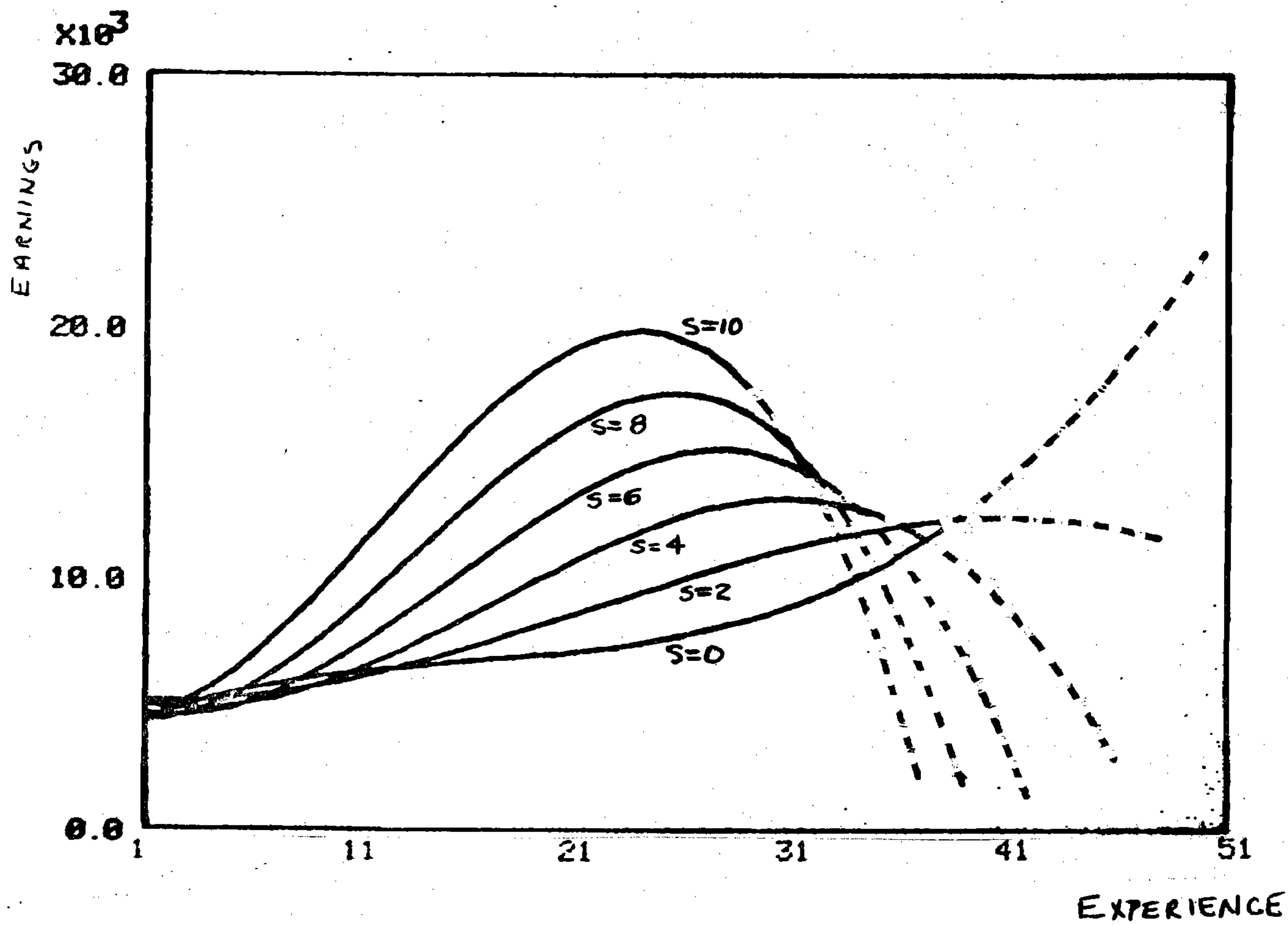

FIGURE 5.C. CUBIC EXPERIENCE-EARNINGS PROFILES BASED ON THE NBER-THORNDIKE SAMPLE FOR SEVERAL SCHOOLING LEVELS AT THE AVERAGE ABILITY LEVEL.

NOTE: EXPERIENCE IS DEFINED SIMPLY AS AGE LESS YEARS OF SCHOOLING.

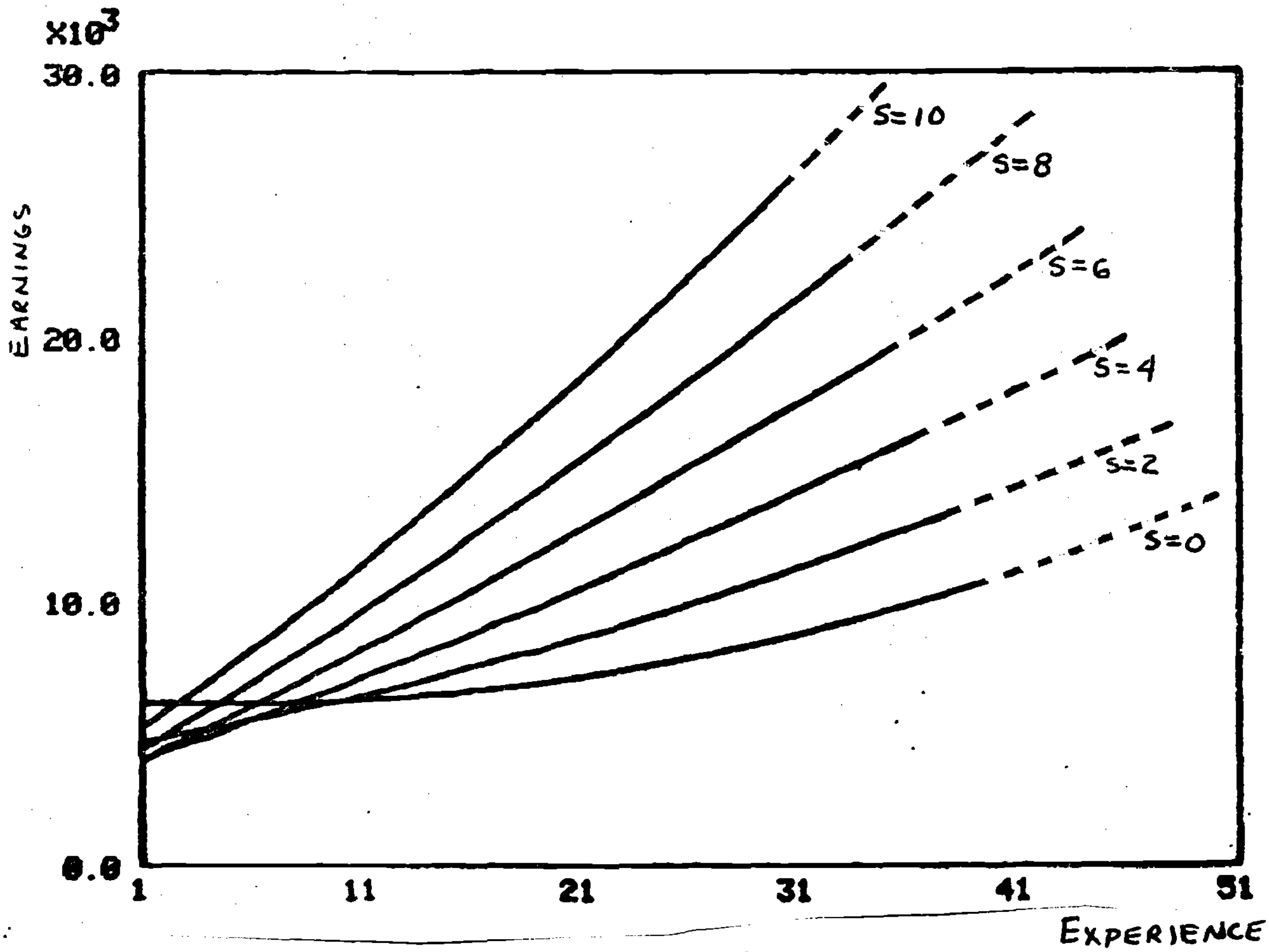

FIGURE 5.d. QUADRATIC EXPERIENCE-EARNINGS PROFILES BASED ON THE NBER-THORNDIKE SAMPLE FOR SEVERAL SCHOOLING LEVELS AT THE AVERAGE ABILITY LEVEL.

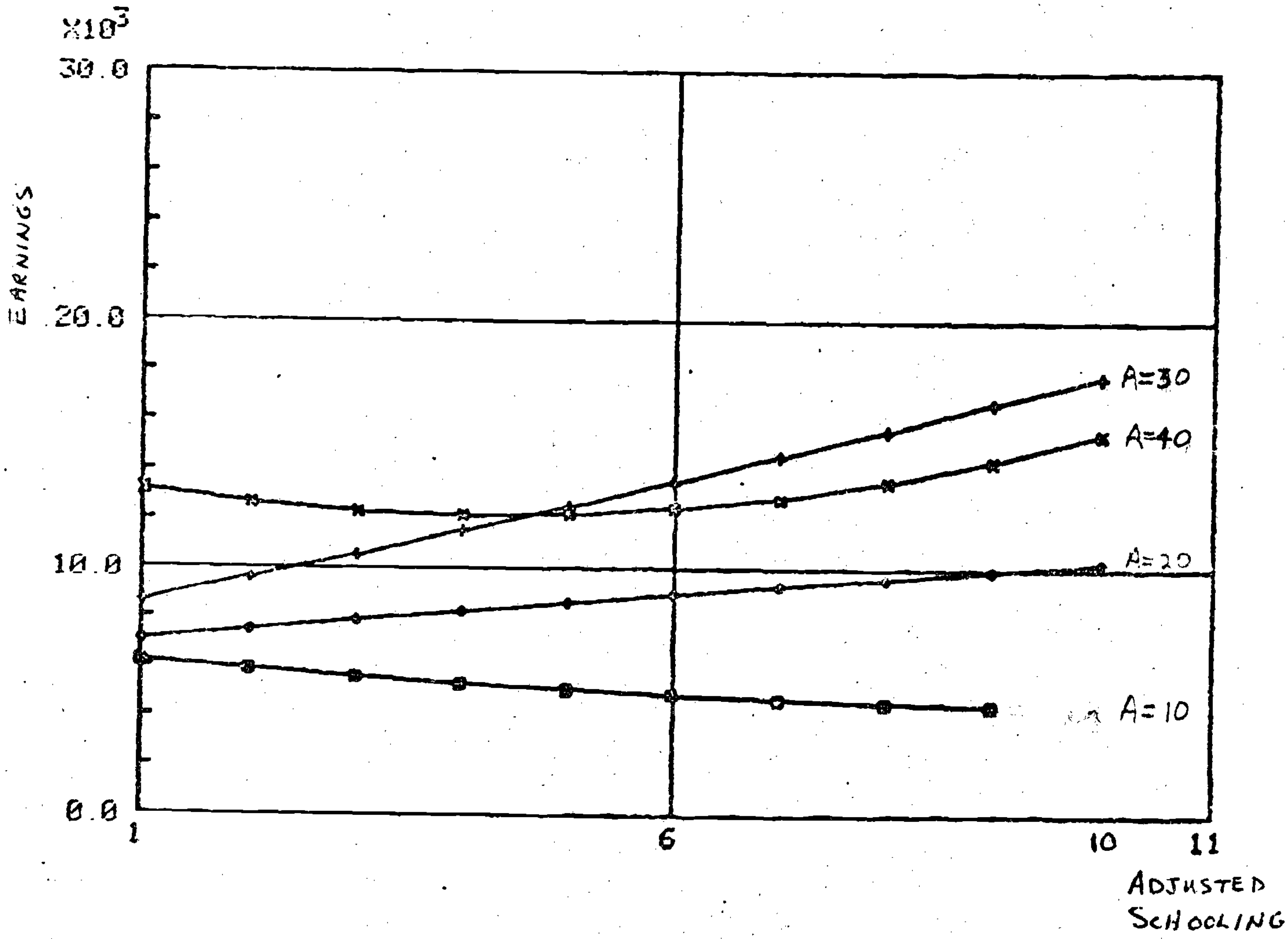

FIGURE 6.a EARNINGS AS A FUNCTION OF SCHOOLING LEVEL FOR SEVERAL AGES (ADJUSTED) AND ABILITY INDEX B = 1.0.

The level of ability as well as age affects the relationship between earnings and schooling. Figures 6.b,c, and d illustrate that the positive interaction between ability and schooling increase with age. Ability has little affect on the earnings schooling relationship at early ages, but increasingly raises the level of earnings in the relationship over the life cycle. This indicates a positive three way interaction.

Secondly, consider the relationship between earnings and ability. The effect of ability is less pronounced over the life cycle than that of schooling and depends on the level of schooling itself.

Figure 7 illustrates the effect of ability on the age-earnings profile for high school and college graduates. At high levels of schooling, college graduates and above, ability increases earnings at every point in the observed life cycle, and its effect is to increase the rate of growth of earnings, but not the initial level of earnings. However, for levels of schooling below the college graduate, there appears an initial period in which earnings are actually lower for higher ability persons. The length and magnitude of this period diminishes with increased schooling. The period seems to lower earnings most for high ability people. After this initial period, ability increases the level of earnings and the rate of growth of earnings. This finding is consistent with the predictions of the model, since for a given schooling level, higher ability persons are predicted to invest more at all points in the life cycle while the absolute difference in investment diminishes, with time resulting in lower earnings initially. The positive relationship between ability and the simple definition of experience is obvious since schooling is held constant.

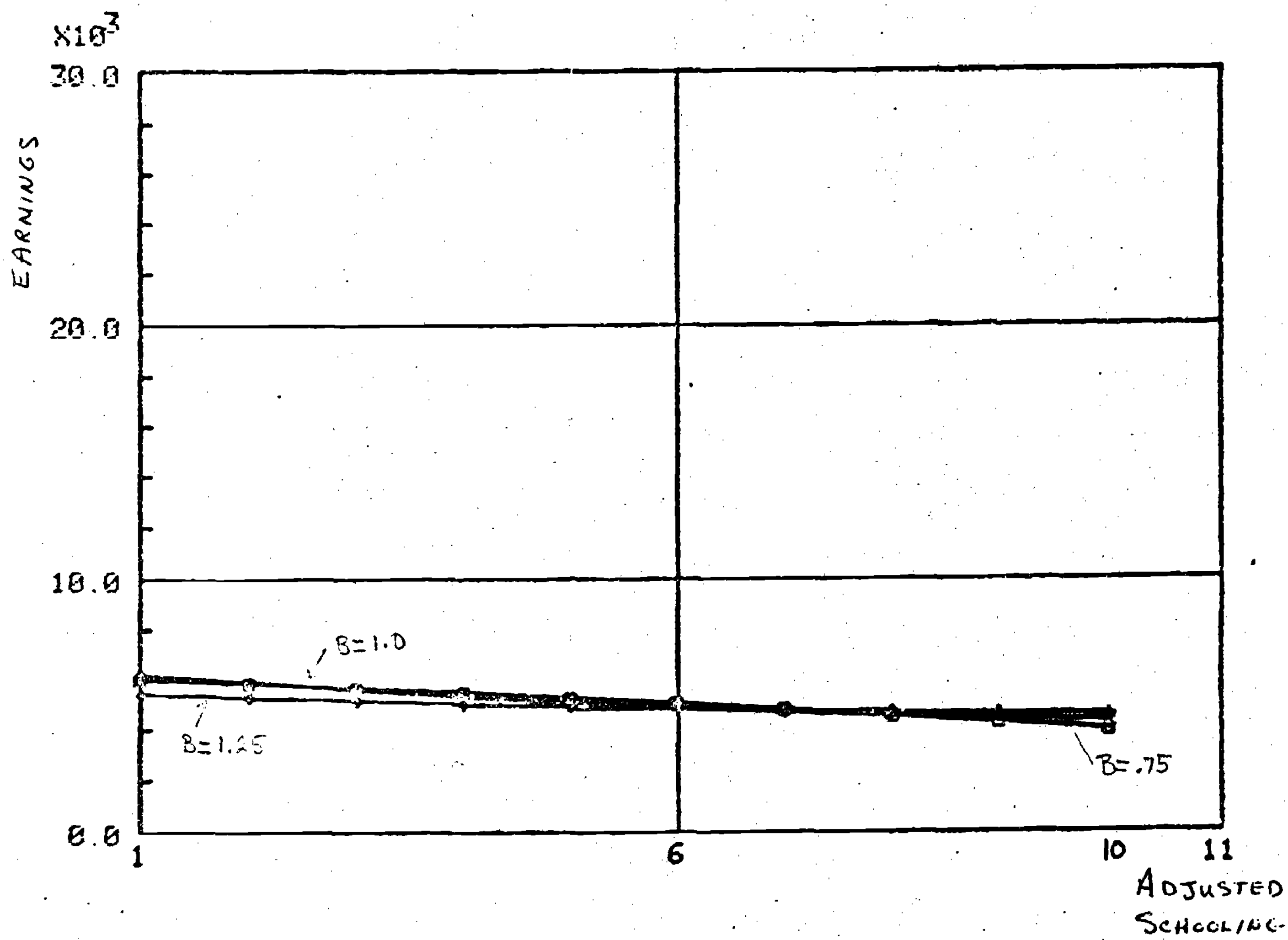

FIGURE 6.b. EARNINGS AS A FUNCTION OF SCHOOLING LEVEL FOR AVERAGE ABILITY AND ONE STANDARD DEVIATION (.25) ABOVE AND BELOW, AT ADJUSTED AGES (b) A=10, (c) A=20, AND (d) A=30. THESE ARE BASED ON THE CUBIC ESTIMATE.

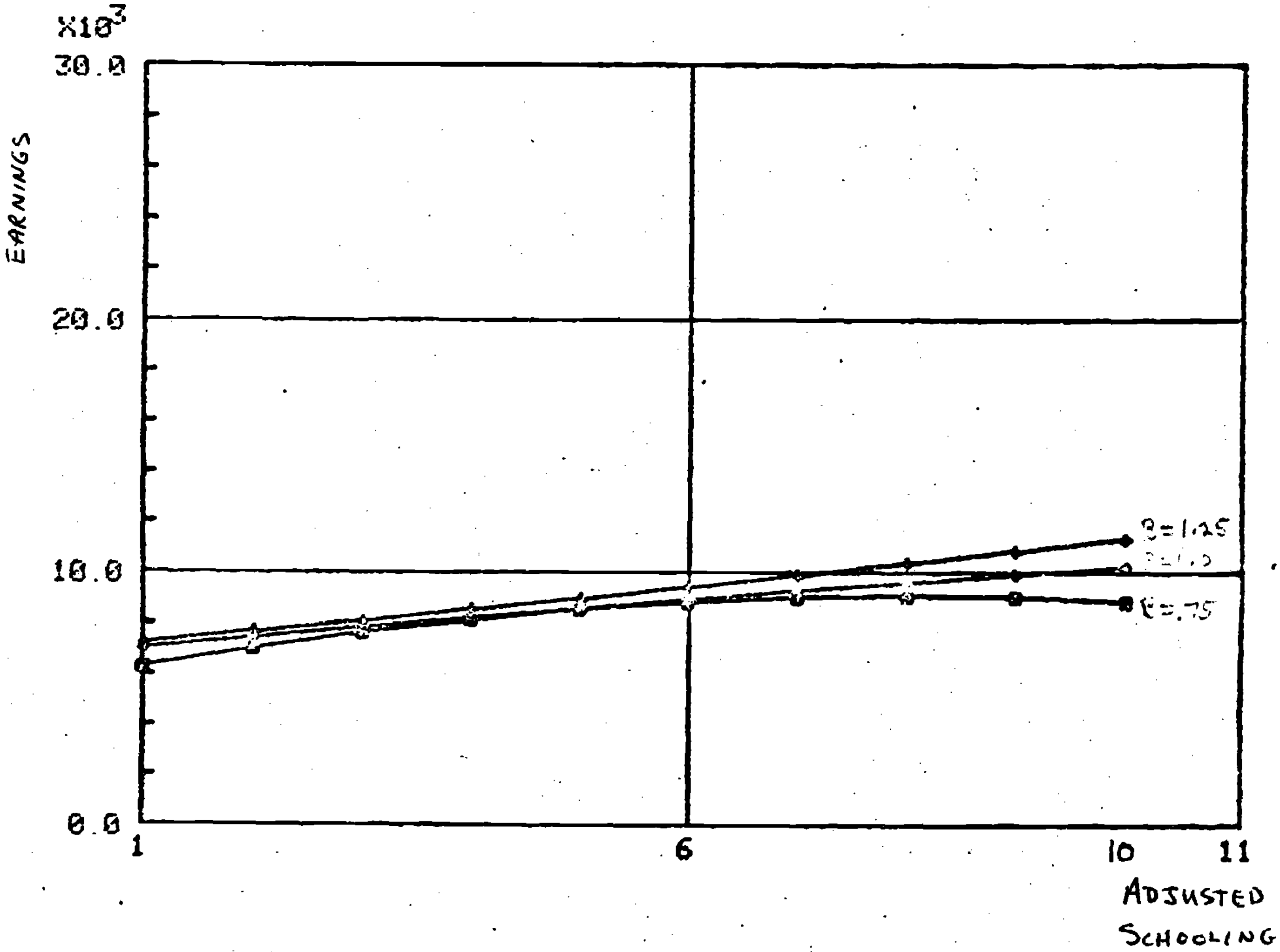

FIGURE 6.C. A=20.

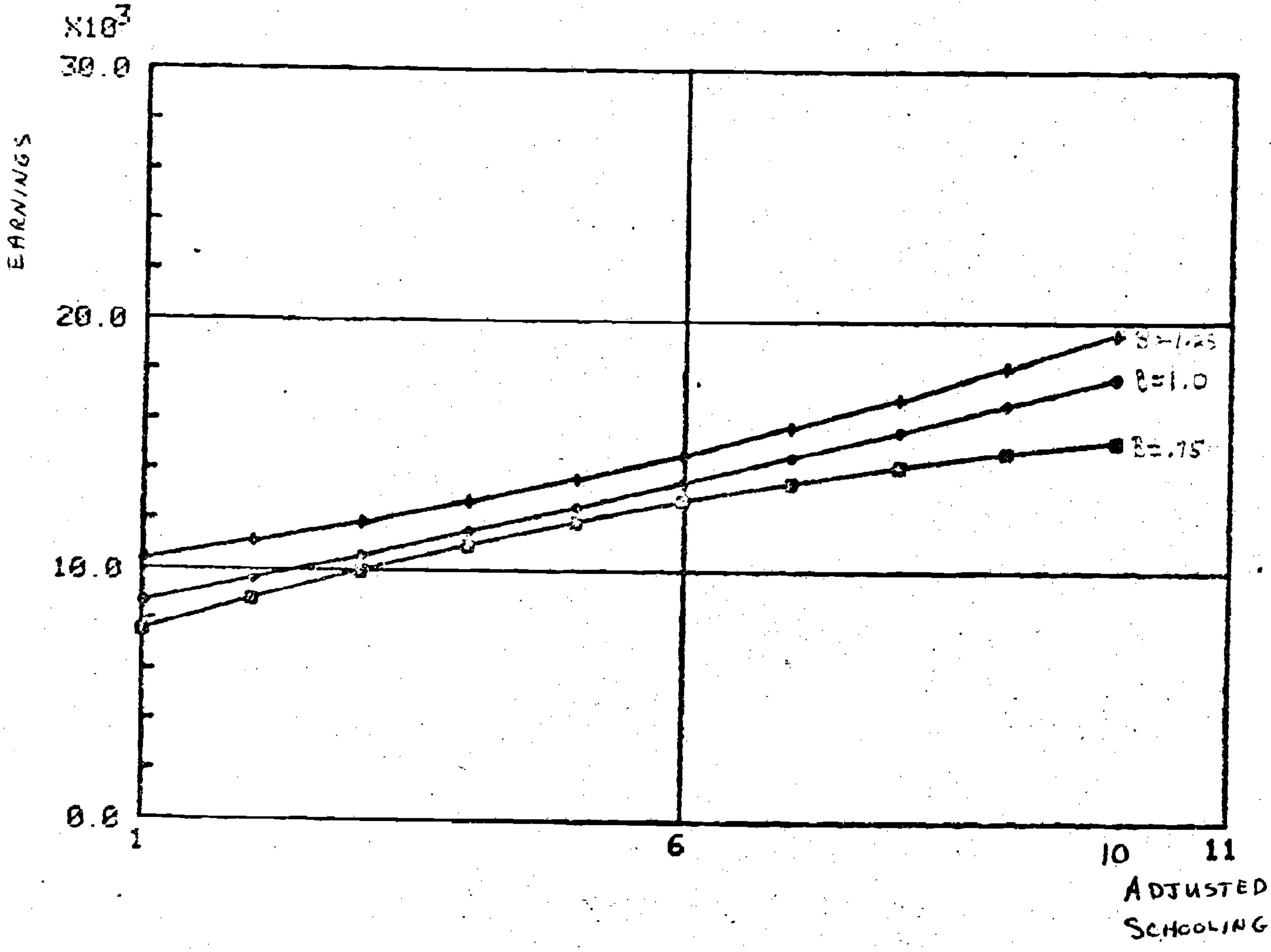

FIGURE 6.d. A=30

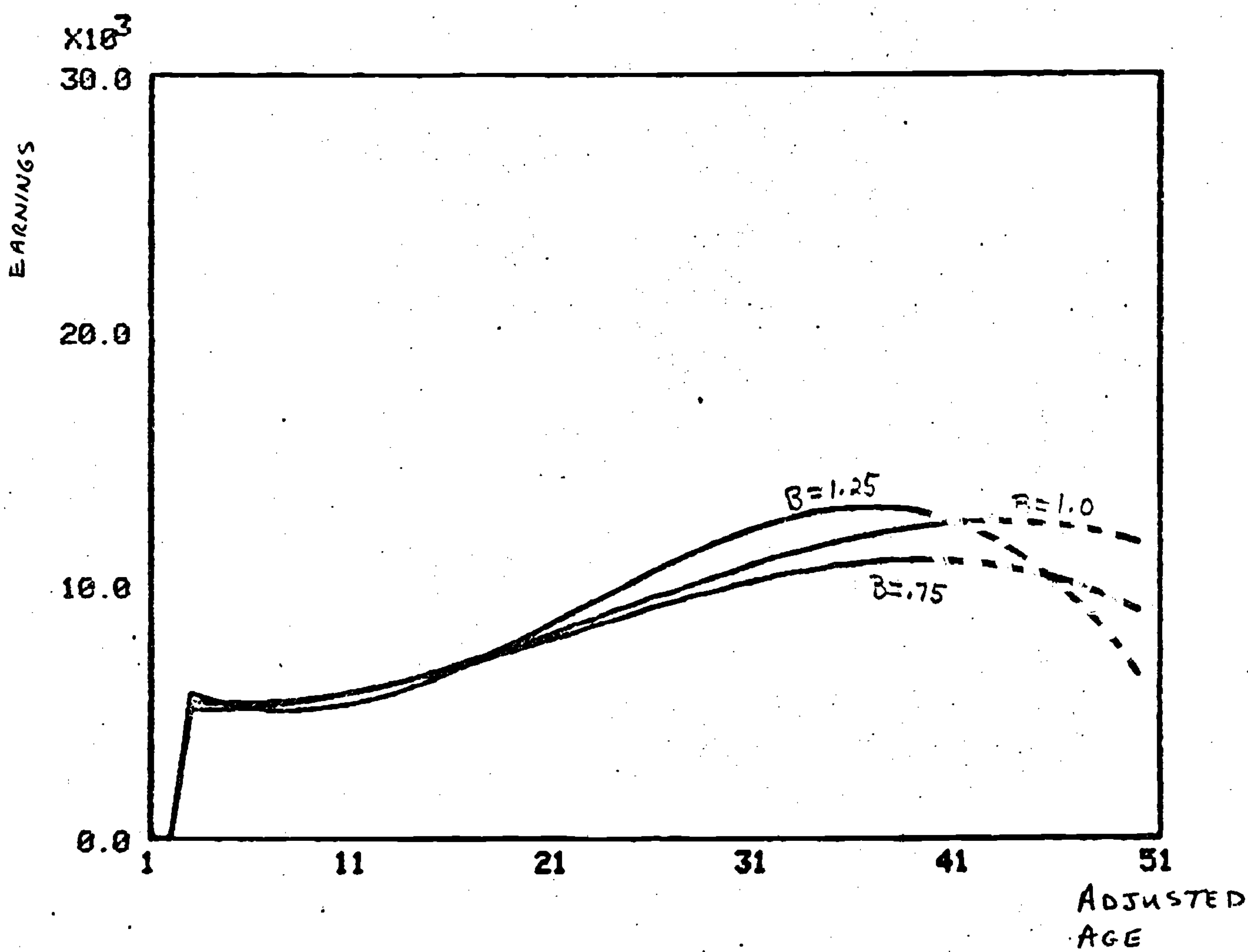

FIGURE 7.a. CUBIC (a,c) AND QUADRATIC (b,d) ESTIMATED AGE-EARNINGS PROFILES BASED ON THE NBER-THORNDIKE SAMPLE FOR AVERAGE ABILITY AND ONE STANDARD DEVIATION (.25) ABOVE AND BELOW, FOR HIGH SCHOOL GRADUATES (S=2) AND COLLEGE GRADUATES (S=6). S=2, CUBIC.

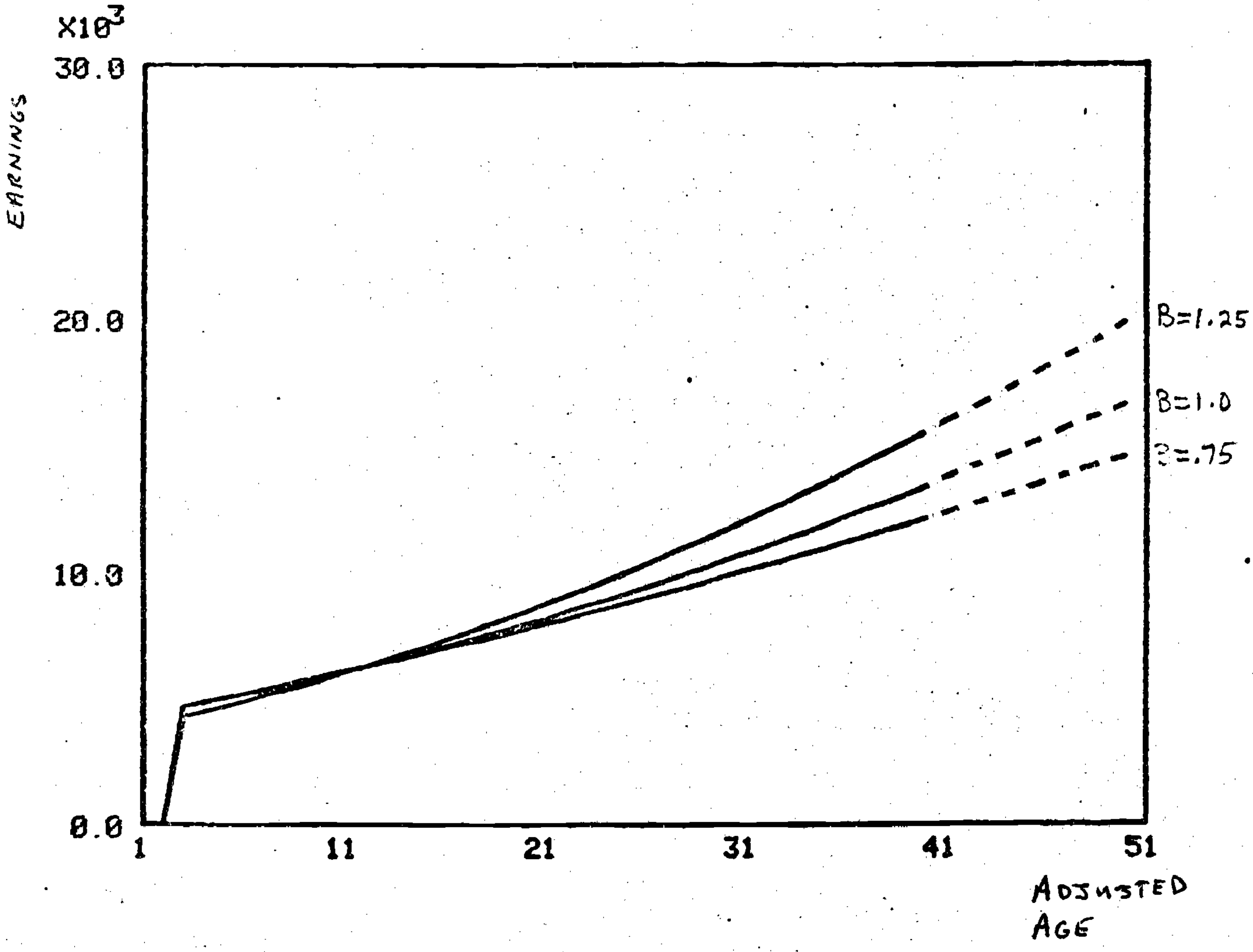

FIGURE 7.b, QUADRATIC, S=2.

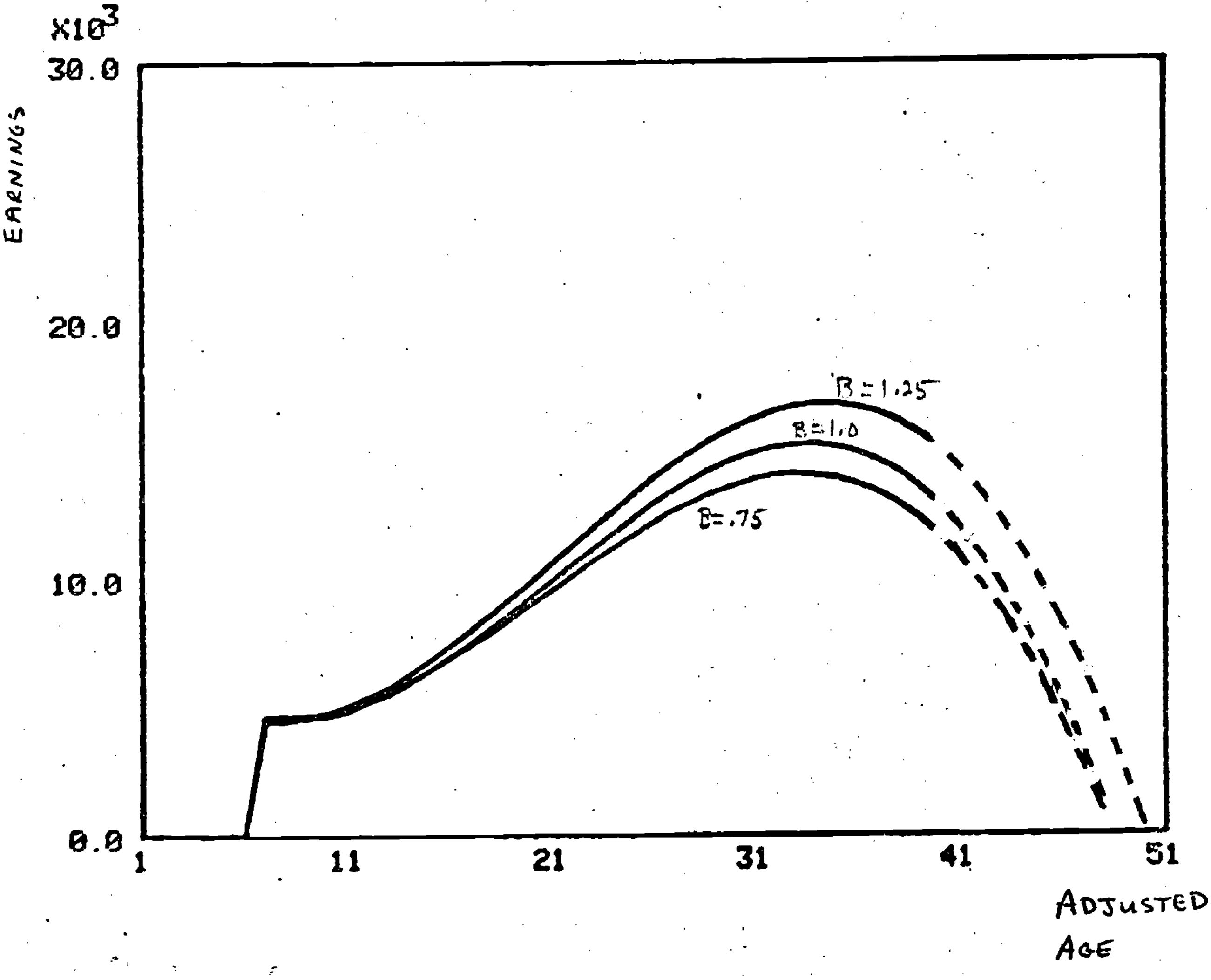

FIGURE 7.C. CUBIC, S=6.

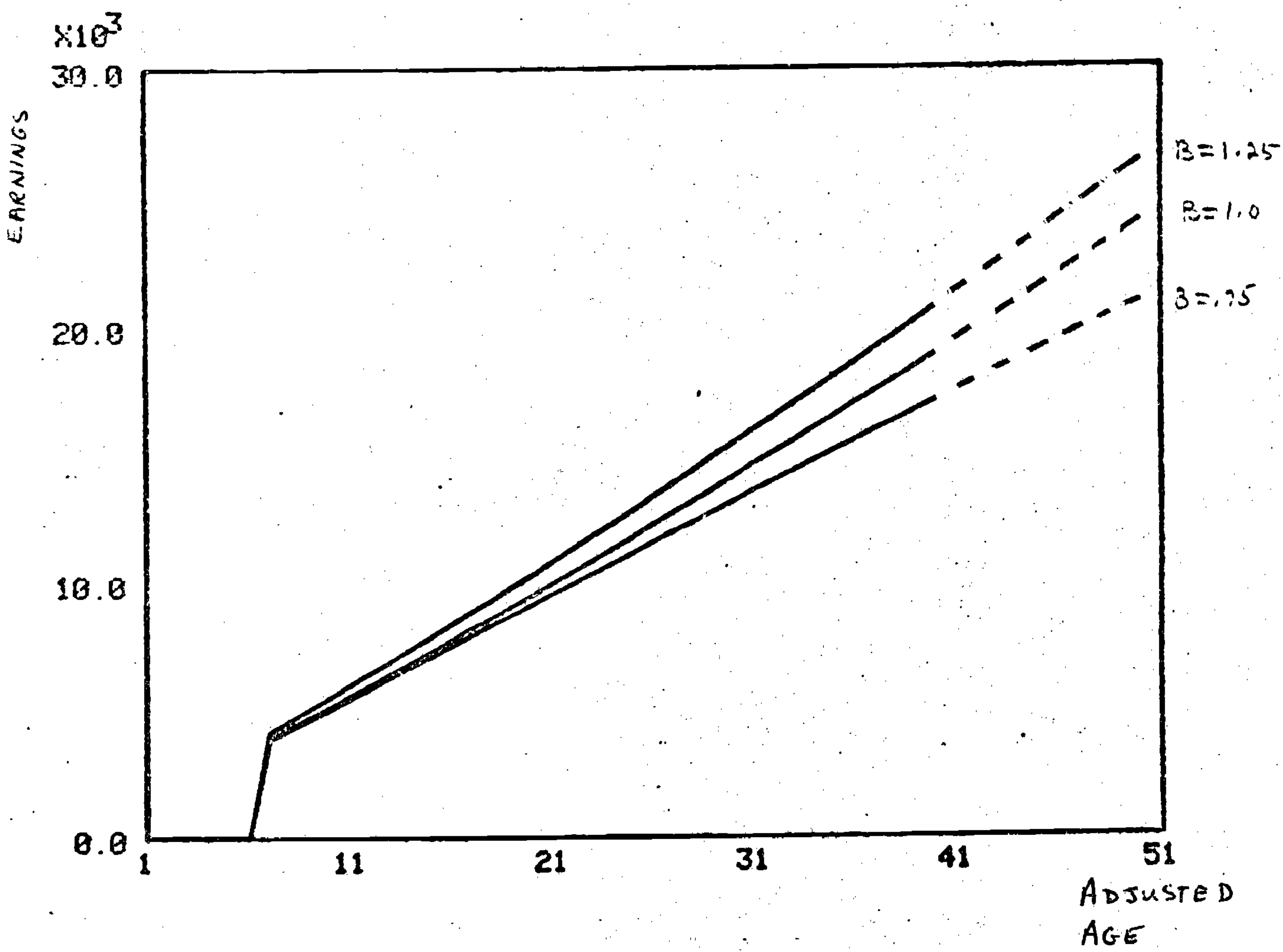

FIGURE 7.d. QUADRATIC, S=6.

The earnings ability relationship is very much dependent on both the level of schooling and the level of experience or age as illustrated in Figures 8. The earnings-ability relationship for high school graduates is negative at A = 10, constant at A = 20, and positive in the range B > .7 at age A = 30. The relationship is negative even at A = 30 for very low ability people as illustrated in Figure 8.a. For college graduates in Figure 3.b, the earnings-ability relationship begin approximately constant at A = 10 and becomes increaseingly positive with age. By the Ph.D. and professional level, S = 10 illustrated,in Figure 8.c.,the relationship is strongly and increasingly positive at all ages.[37/] These relationships are illustrated in a slightly different way in Figures 8.d, e, and f with the earnings ability relationship for different levels of schooling in the same figure.

Figures 9 illustratesthe comparison of age-earnings profiles when ability and schooling are positively and then negatively correlated over individuals. Since the two interact positively,all effects are exagerated when the two are positively correlated,and are dampened when they are netatively correlated. A point to note is that the age at which the earnings of the more schooled individual overtake the earnings of the less schooled individual is negatively related to the level of schooling,if schooling and ability are positively correlated,and is positively related to schooling,if schooling and ability are negatively correlated. Since ability and schooling are usually observed to be positively correlated (.24 in this sample),the age of overtaking is less important and the interaction of ability and schooling is accentuated.

[37/] Again, the prediction at A = 40 is unclear.

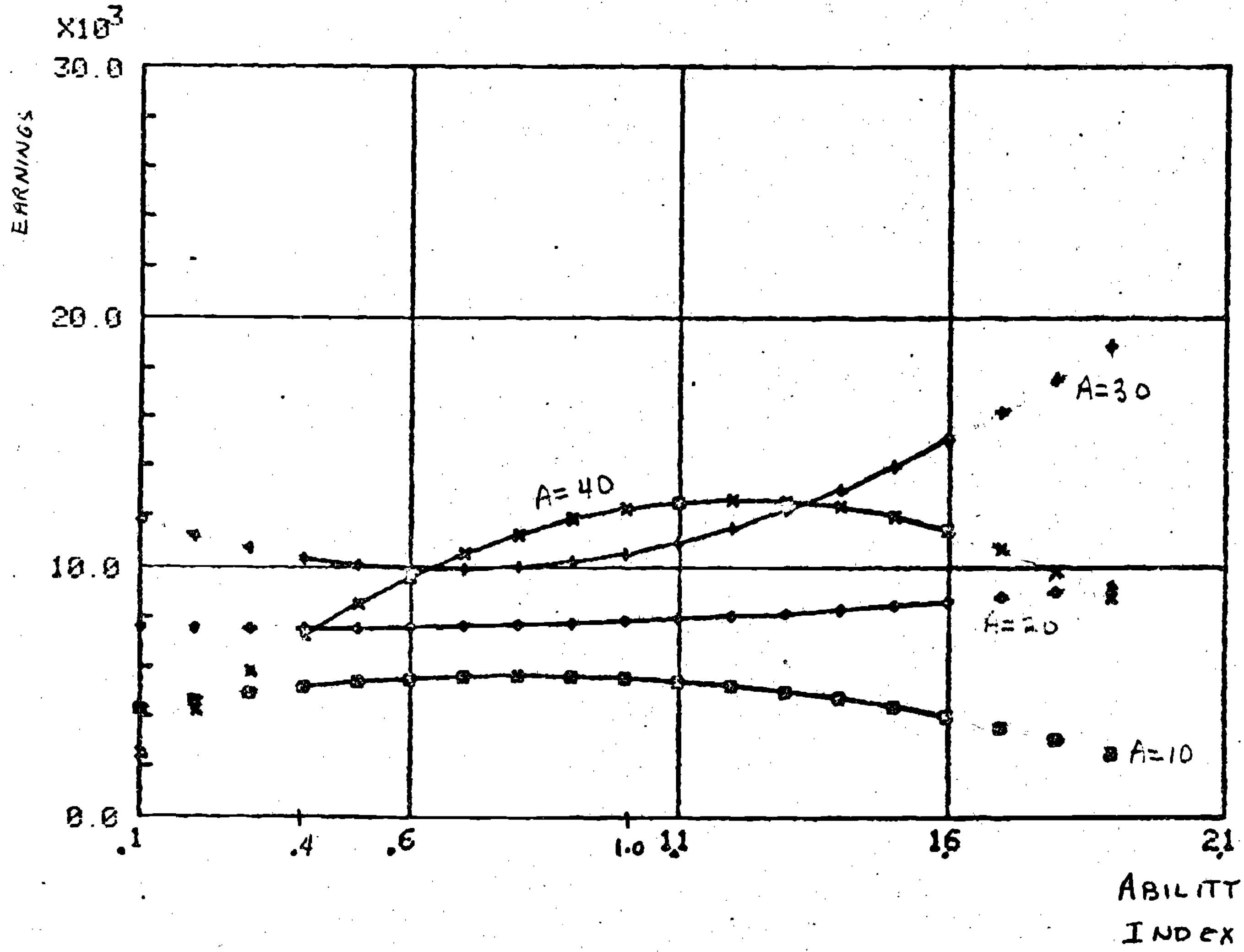

FIGURE 8, a. EARNINGS AS A FUNCTION OF ABILITY FOR SEVERAL AGE (ADJUSTED) AND SCHOOLING (ADJUSTED) COMBINATIONS. THESE ARE BASED ON THE CUBIC ESTIMATE, S=2.

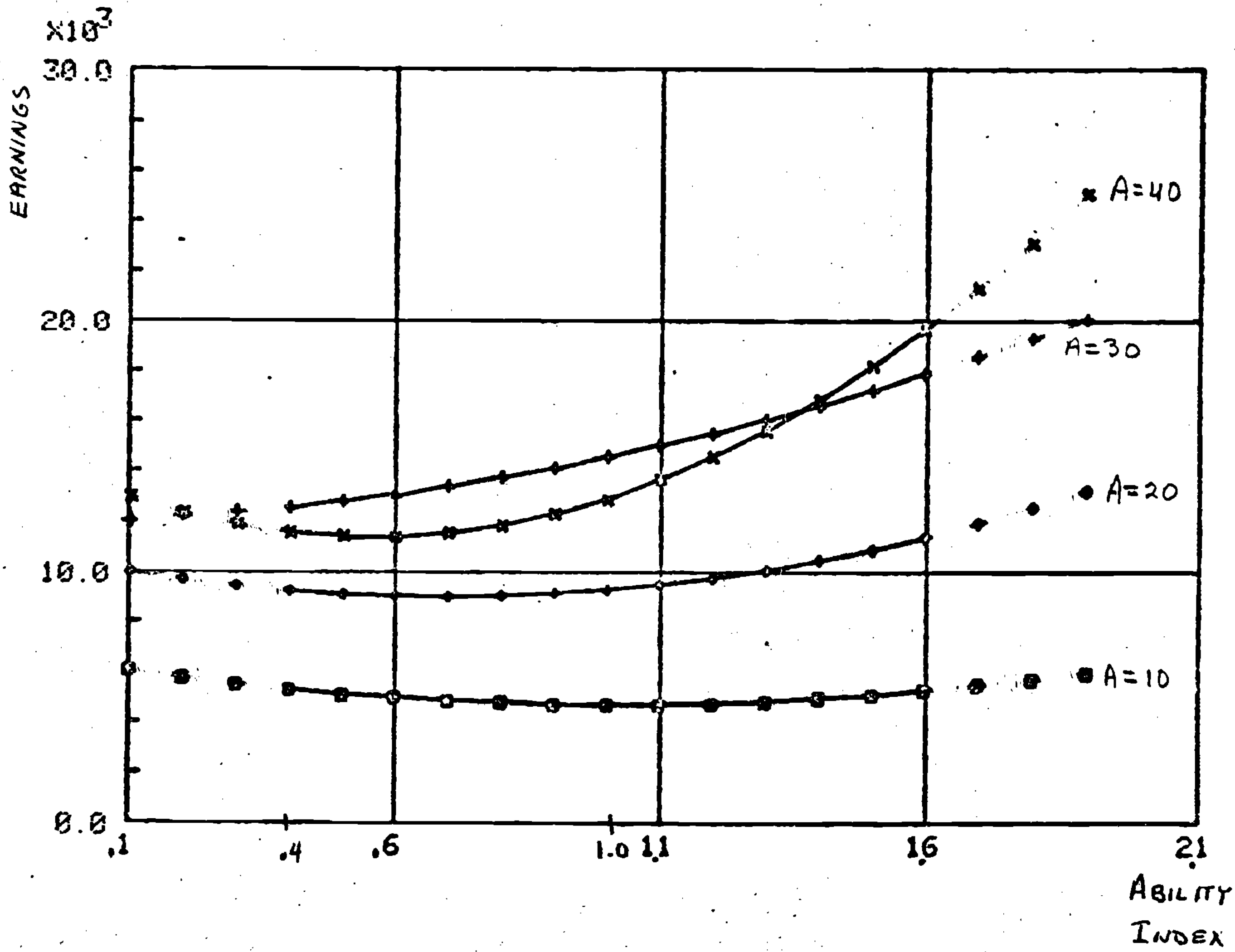
X10^3
EARNINGS
30.0
20.0
10.0
0.0
.1
.4
.6
1.0
1.1
16
21
ABILITY INDEX
A=40
A=30
A=20
A=10

FIGURE 8.b. S=6.

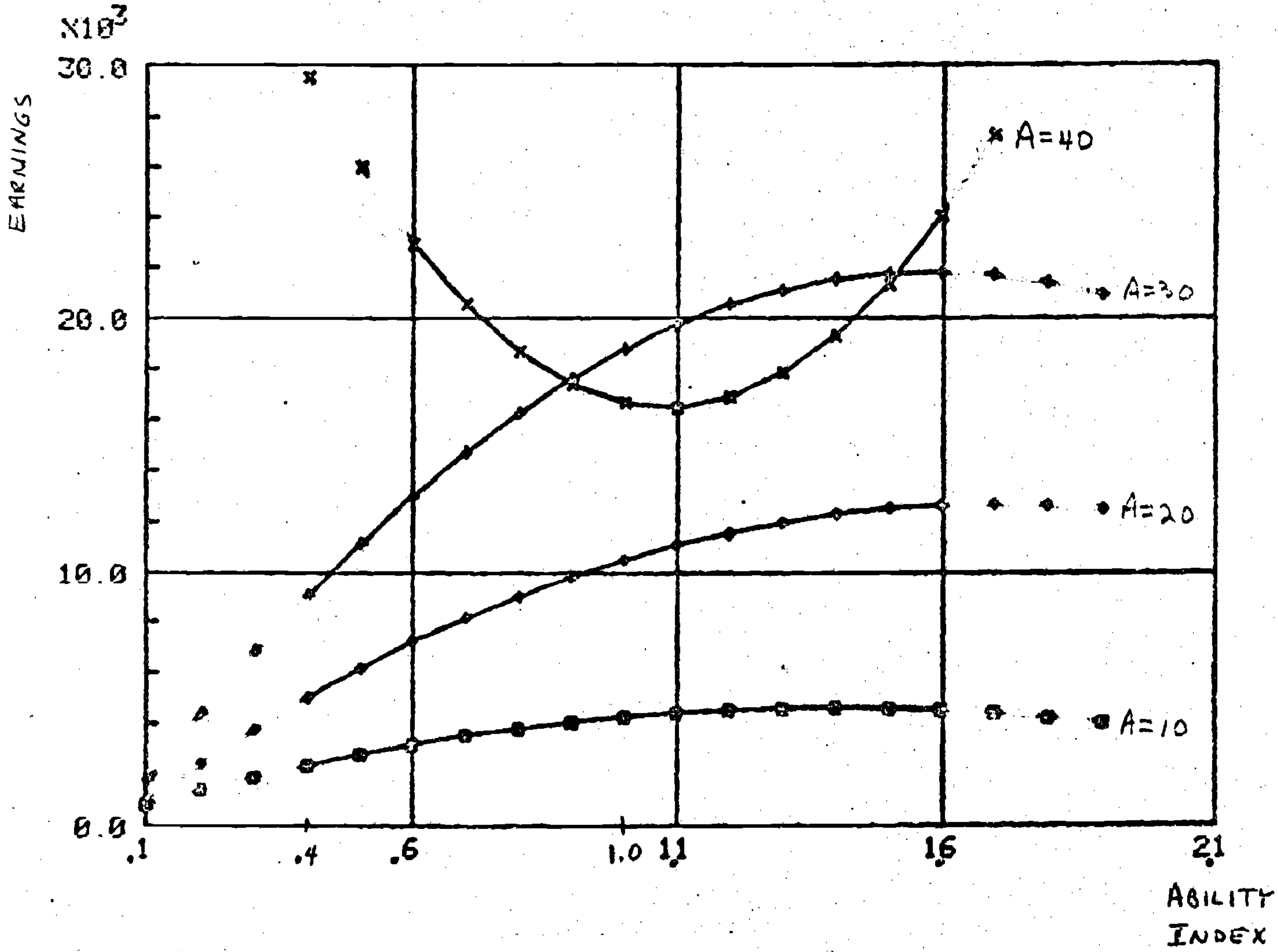

FIGURE 8.c. S=10.

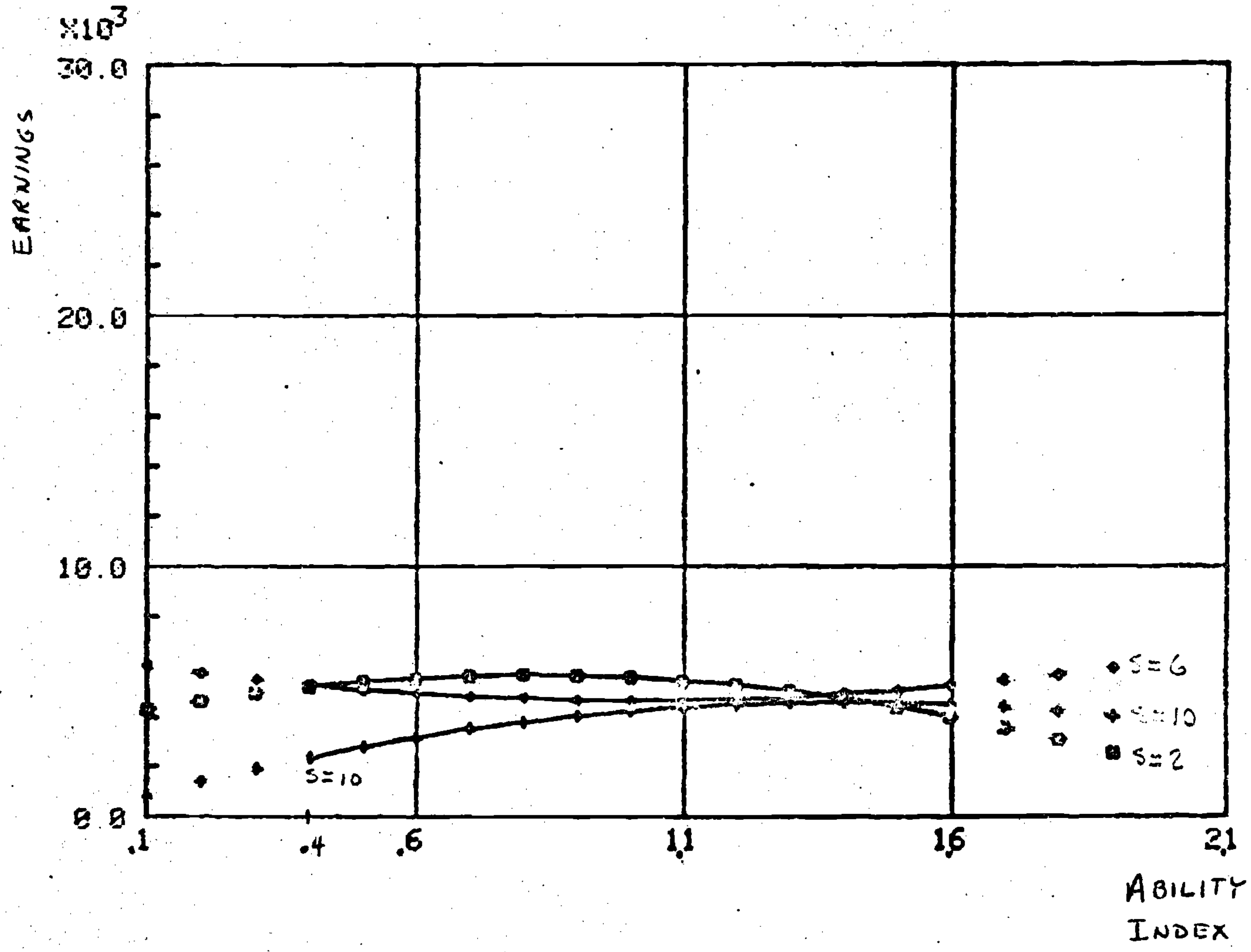

FIGURE 8.d. $A=10$

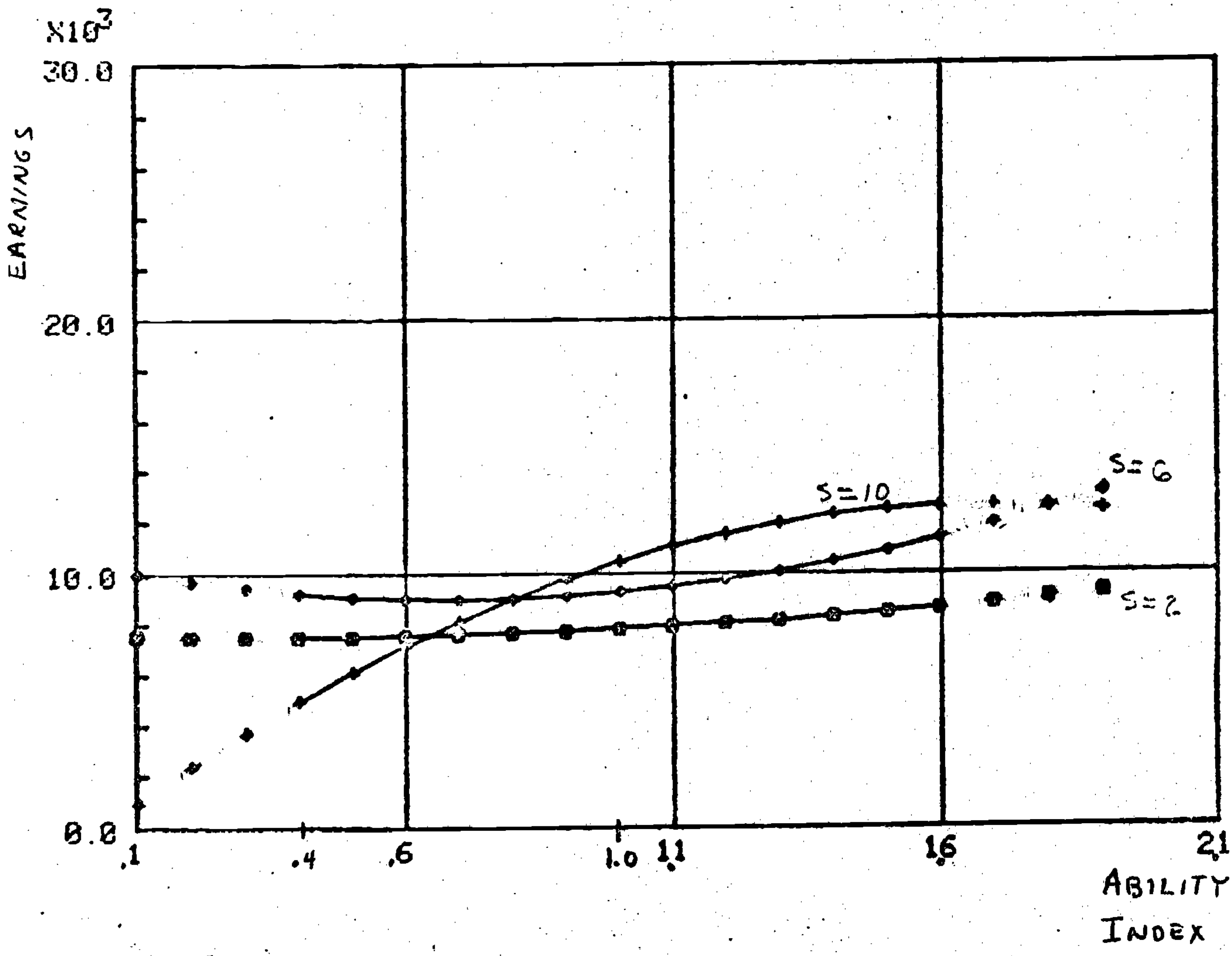

FIGURE 8.e. A=20.

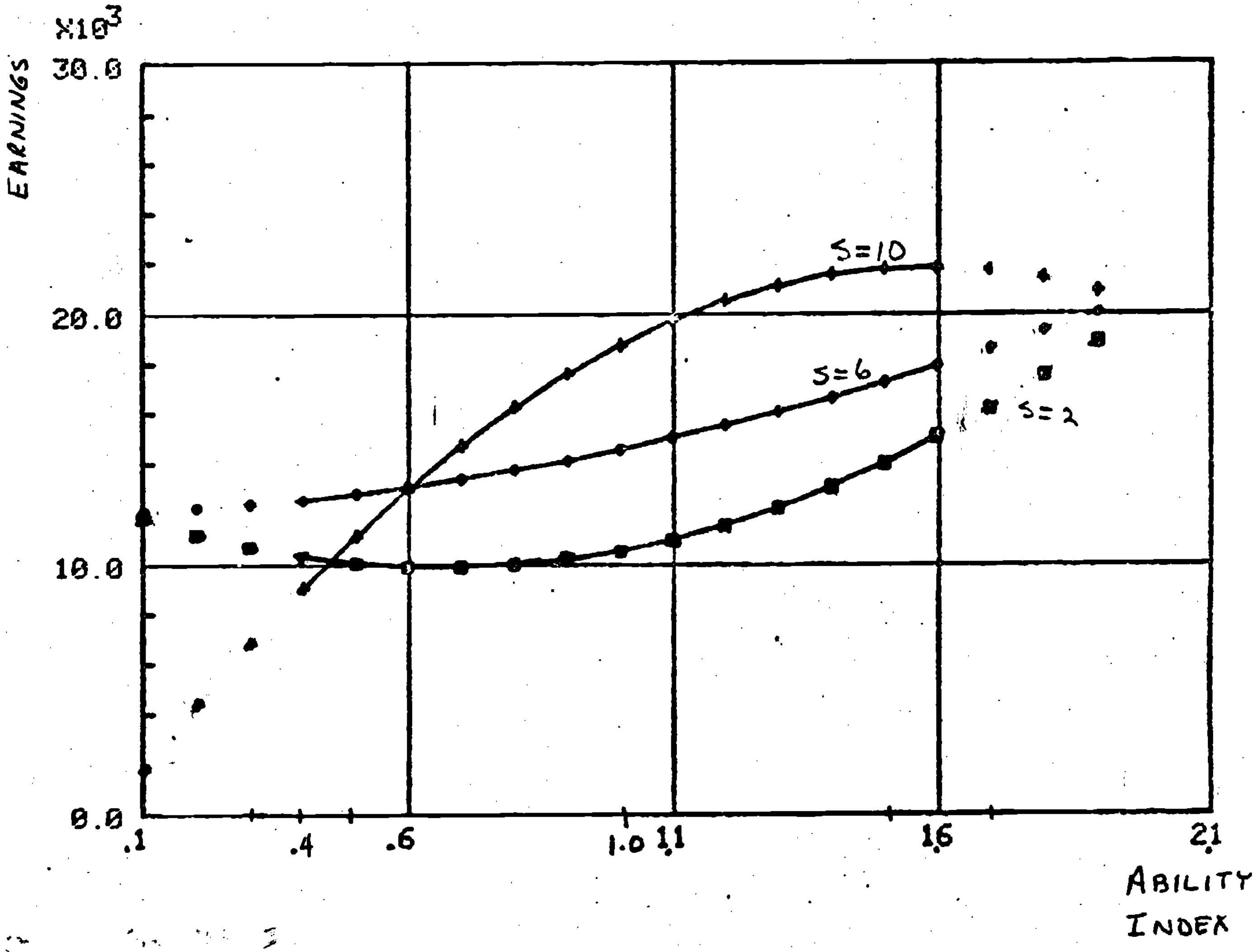

FIGURE 8.f. A=30.

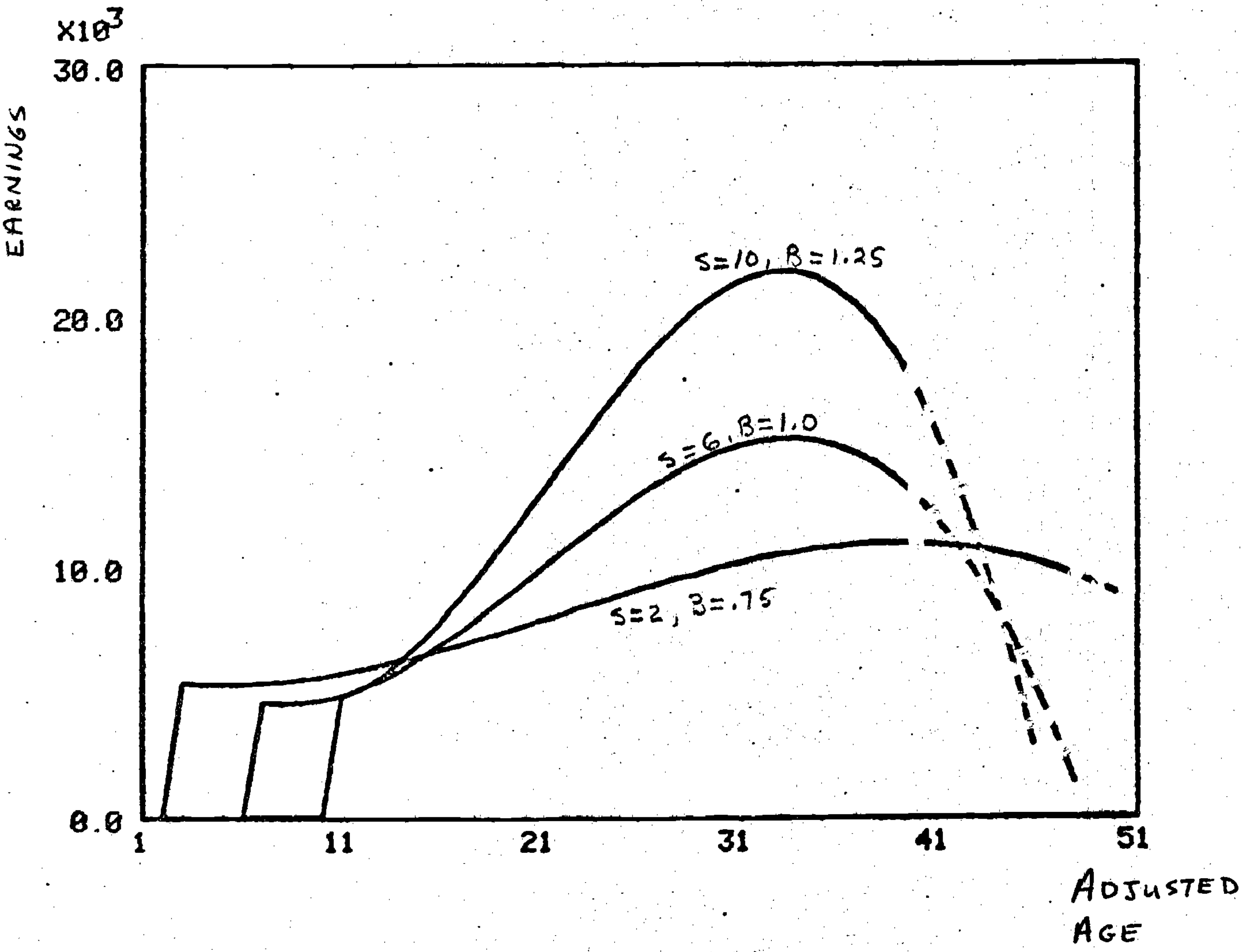

FIGURE 9.a. ESTIMATED AGE-EARNINGS PROFILES RESULTING WHEN ABILITY AND SCHOOLING ARE POSITIVELY CORRELATED.

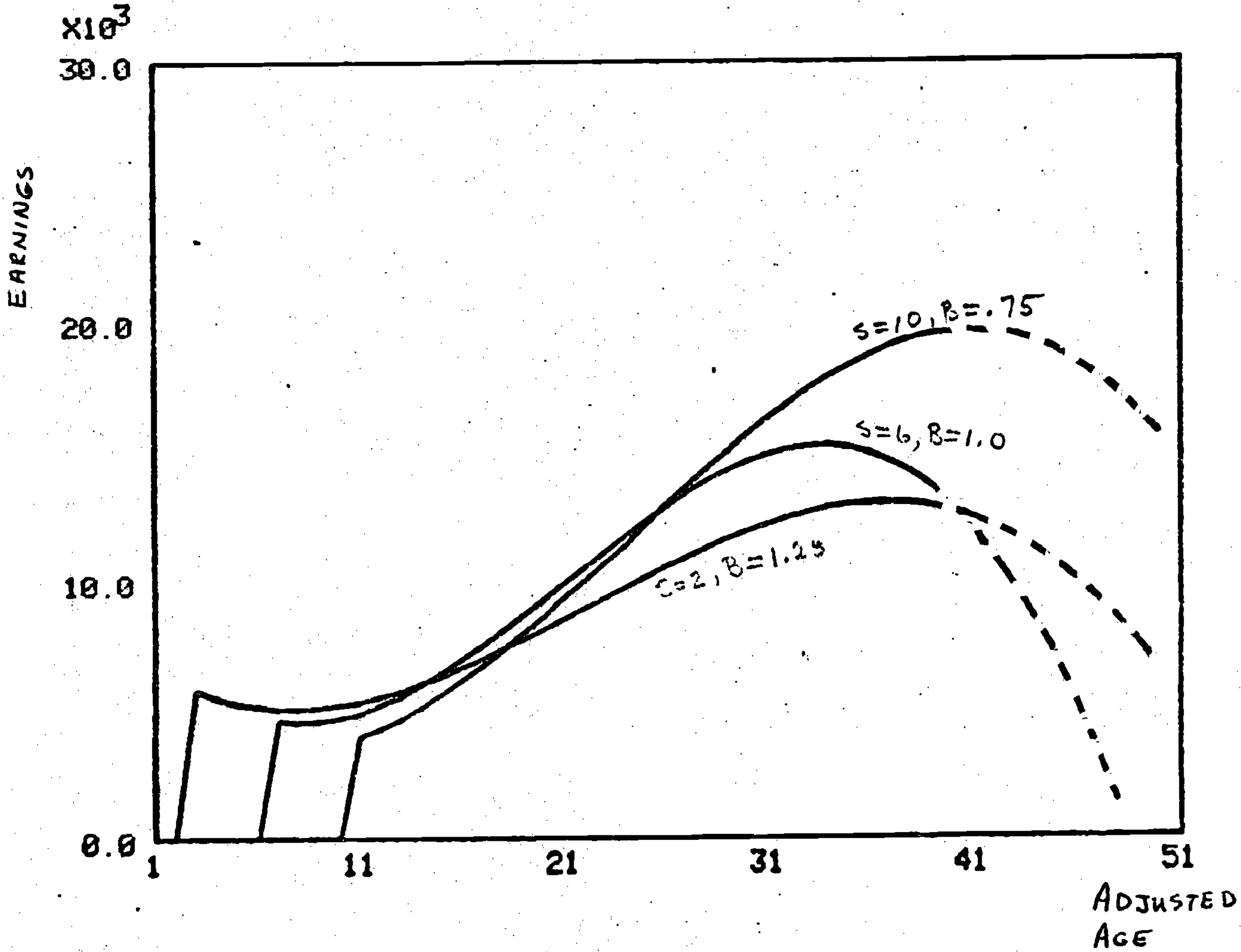

FIGURE 9.b, ESTIMATED AGE-EARNINGS PROFILES RESULTING WHEN ABILITY AND SCHOOLING ARE NEGATIVELY CORRELATED.

A comparison of the cubic and quadratic age-earnings profile estimates are presented in Figure 10.

The life cycle of earnings model as we have developed it, assumes that the individual behaves in such a way as to maximize the present value of net earnings, then maximizes an intertemporal utility function subject to that human wealth constraint. Clearly we need to consider the human wealth or present value of the observed earnings function we have estimaed. Clearly the present value of observed earnings represents an upper bound on the present value of earnings net of direct investment expenditures. The present value and internal rate of return calculations presented here are based on the quadratic rather than the cubic estimates of the earnings function, since the cubic estimates yield unreasonable predictions beyond adjusted age 36.[38/] The end of working life is taken to be N = 50 or chronologic age sixty-six. The resulting internal rates of return which equate the present value of earnings for a given schooling level with the present value of earnings for one year less schooling for various ability levels is presented in Table 8. The internal rate tends to rise to a peak then fall off slowly as the schooling level increases at all levels of ability, except average ability where it rises continuously. The internal rate of return appears to be a minimum at the average ability level at low levels of schooling, below S = 4, to rise monotonically with ability in the middle schooling range, S = 4 to S = 6, and to peak at the average ability level at high levels of schooling. The internal rate of return to a college degree over a highschool degree is 5.40 percent for those persons one-standard deviation below mean ability, 5.89 % at the mean, and 6.83

[38/] An example of an unreasonable prediction is to predict negative earnings at the level of minus ten thousand dollars or more for all high schooling levels. The resulting rates of return are all quite negative. Estimates of rates of return and present values using the cubic equation and assuming earnings are constant after the maximum at the maximum are presented in Appendix B.

TABLE 8

Internal Rate of Return which Equates the Present Value of Earnings for a Given Schooling Level with the Present Value of Earnings for One Year Less Schooling for Various Ability Levels

B \ S	1	2	3	4	5	6	7	8	9	10
.25	6.66	7.63	7.78	6.31	2.89					
.50	4.99	5.70	5.94	5.63	4.64	3.27	1.89			
.75	3.84	4.54	5.06	5.47	5.61	5.52	5.23	4.85	4.47	4.03
1.00	3.34	4.10	4.90	5.70	6.41	6.98	7.53	7.86	8.07	8.21
1.25	3.71	4.66	5.61	6.79	7.40	8.14	8.77	9.17	9.52	9.67
1.50	6.61	7.76	8.64	9.12	9.27	9.09	8.78	8.40	7.97	7.68
1.75	18.75	22.72	22.97	19.57	14.77	10.13	6.60	4.40	3.10	2.43

Note: These calculations are based on the quadratic age estimating equation based on the NBER-Thorndike sample.

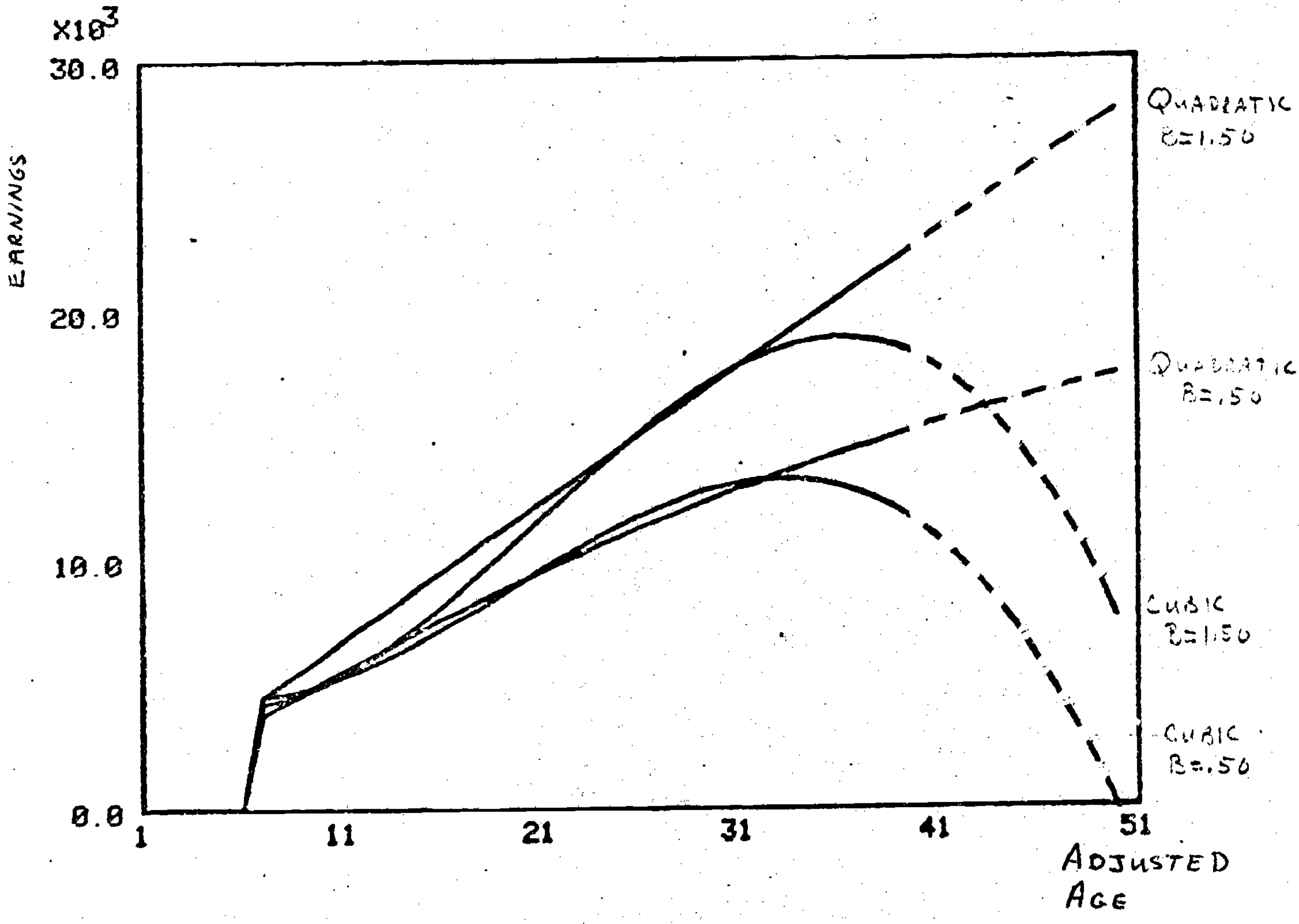

FIGURE 10. COMPARISON OF CUBIC AND QUADRATIC ESTIMATES FOR COLLEGE GRADUATES (S=6).

for those one-standard deviation above mean ability. The internal rate of return to a professional degree, S = 10, over a college degree, S = 6, is 4.67% for persons one-standard deviation below mean ability, 7.89% at mean ability, and 9.27% for those one-standard deviation above the mean.

The estimates of human wealth or the present value of predicted observed earnings are presented in Tables 9 and in Figures 11. The estimated present value is very much dependent upon the rate of discount. Table 9 presents estimates for rates of discount of four through seven percent,while Figures 11 illustrate the effect of schooling and ability on the present value of rates of discount four and six percent. The present value of predicted earnings increases with schooling at four percent, but not at six percent. The present value will increase with schooling only when the discount rate is less than the internal rate of return. The interaction of schooling and ability is illustrated by the fact that increased ability increases both the level of the present value-schooling relationship and the rate of change. For example, the present value increases with schooling at six percent for very high ability levels but decreases at low ability levels. The affect of ability and schooling are accentuated by a low discount rate,since higher earnings occur late in the life cycle.

The relationship between ability and the present value of predicted earnings is positive at all levels of schooling,and at both four and six percent as illustrated in Figure 11.b. The effect of ability is accentuated at lower interest rates but is positive even at seven percent. The positive effect of ability is greater at higher levels of schooling, even if the level of the present value is not higher as at low levels of ability at the six percent rate.

TABLE 9.a. PRESENT VALUE OF PREDICTED OBSERVED EARNINGS STREAM FROM THE ESTIMATED QUADRATIC AGE-EARNINGS PROFILES BASED ON THE NBER-THORNDIKE SAMPLE, (N=50).

Discount Rate	Adjusted Schooling Years	Ability Index .50	.75	1.00	1.25	1.50
.04	0	139918.63	149703.06	158701.19	166651.06	173290.56
	2	146637.00	150389.94	156926.06	167172.88	182058.19
	4	152637.88	155253.63	162050.69	174423.69	193765.38
	6	152387.94	160190.00	171066.00	186041.25	206140.69
	8	144101.19	163239.56	182025.75	200343.19	218072.25
	10	128468.63	163897.38	193739.94	216133.25	229215.31
.05	0	116433.25	124541.88	130951.25	135095.13	136406.69
	2	117689.75	120517.00	125047.00	131847.13	141485.06
	4	119861.88	121237.75	125858.81	134807.38	149164.50
	6	117418.31	122566.88	130332.31	141587.44	157204.81
	8	108597.81	122570.75	136553.56	150550.06	164562.19
	10	93996.44	120741.94	143393.25	160584.38	170950.88
.06	0	98845.69	105603.94	110141.69	111718.88	109595.06
	2	96065.19	98265.94	101425.38	105862.94	111898.13
	4	95544.00	96168.56	99329.63	105873.25	116644.19
	6	91750.63	95140.31	100753.75	109332.75	121618.88
	8	82945.44	93270.13	103812.13	114644.44	125838.44
	10	69636.19	90047.44	107429.50	120771.63	129064.38
.07	0	85436.81	91101.81	94273.50	94121.19	89813.63
	2	79665.31	81436.56	83653.13	86463.44	90016.06
	4	77253.19	77434.25	79611.00	84448.75	92612.50
	6	72666.94	74887.94	78998.63	85629.25	95409.94
	8	64185.80	71907.06	79956.38	88443.44	97476.63
	10	52235.40	67977.06	81456.56	91919.44	98611.75

NOTE: ALL VALUES ARE IN 1957-59 DOLLARS.

TABLE 9.b. PRESENT VALUE OF PREDICTED OBSERVED EARNINGS STREAM FROM THE ESTIMATED QUADRATIC AGE-EARNINGS PROFILES BASED ON THE NBER-THORNDIKE SAMPLE. (N=50).

Discount Rate	Ability Index	Adjusted Years of Schooling: 0	1	4	6	8	10
.04	.50	139918.63	146637.00	152637.88	152387.94	144101.19	128468.63
	.75	149703.06	150389.94	155253.63	160190.00	163239.56	163897.38
	1.00	158701.19	156926.06	162050.69	171066.00	182025.75	193739.94
	1.25	166651.06	167172.88	174423.69	186041.25	200343.19	216133.25
	1.50	173290.56	182058.19	193765.38	206140.69	218072.25	229215.31
.05	.50	116433.25	117689.75	119861.88	117418.31	108597.81	93996.44
	.75	124541.88	120517.00	121237.75	122566.88	122570.75	120741.94
	1.00	130951.25	125047.00	125858.81	130332.31	136553.56	143393.25
	1.25	135095.13	131847.13	134807.38	141587.44	150550.06	160584.38
	1.50	136406.69	141485.06	149164.50	157204.81	164562.19	170950.88
.06	.50	98845.69	96065.19	95544.00	91750.63	82945.44	69636.19
	.75	105603.94	98265.94	96168.56	95140.31	93270.13	90047.44
	1.00	110141.69	101425.38	99329.63	100753.75	103812.13	107429.50
	1.25	111718.88	105862.94	105873.25	109332.75	114644.44	120771.63
	1.50	109595.06	111898.13	116644.19	121618.88	125838.44	129064.38
.07	.50	85436.81	79665.31	77253.19	72666.94	64185.80	52235.40
	.75	91101.81	81436.56	77434.25	74887.94	71907.06	67977.06
	1.00	94273.50	83653.13	79611.00	78998.63	79956.38	81456.56
	1.25	94121.19	86463.44	84448.75	85629.25	88443.44	91919.44
	1.50	89813.63	90016.06	92612.50	95409.94	97476.63	98611.75

NOTE: ALL VALUES ARE IN 1957-59 DOLLARS.

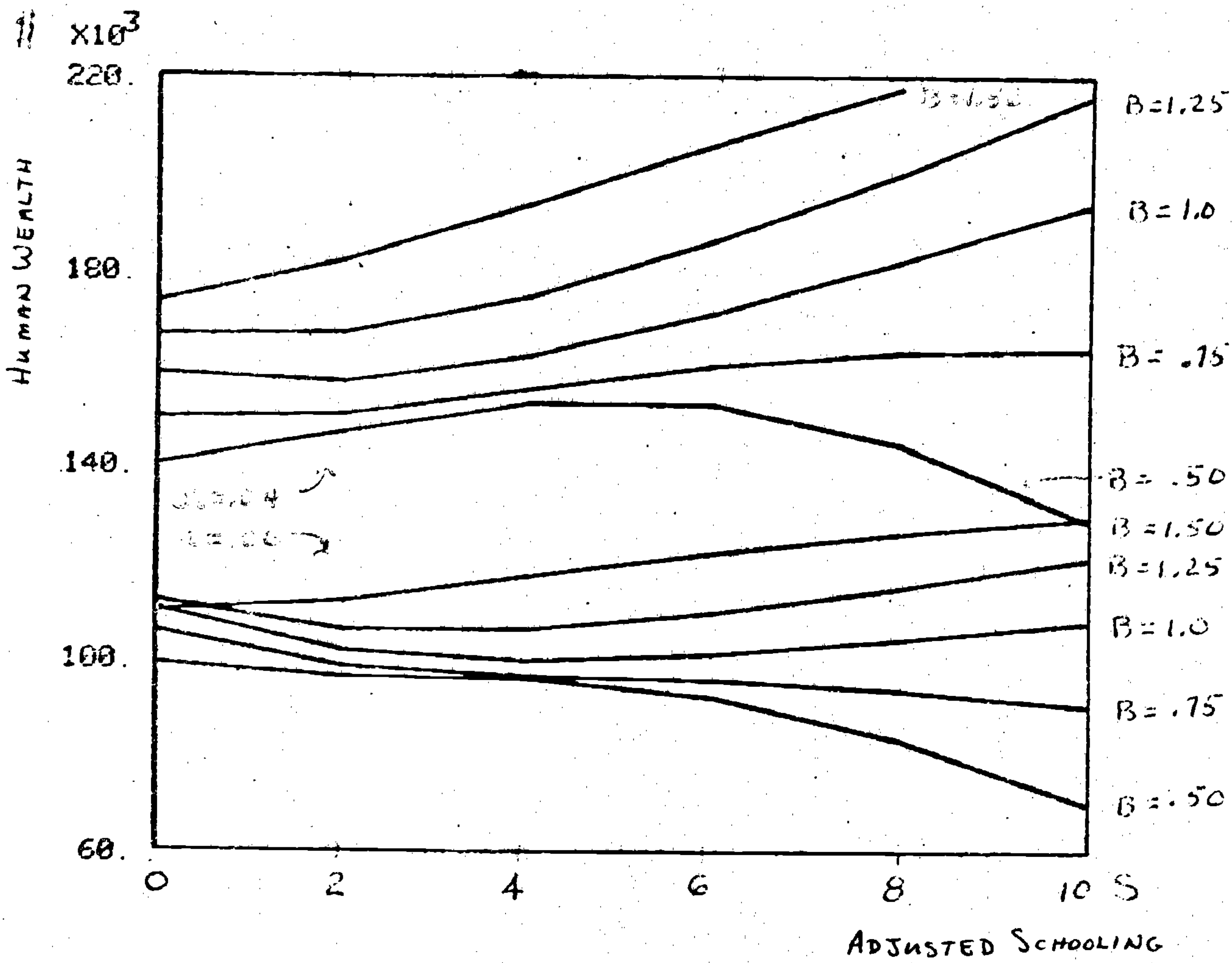

FIGURE 11.a. PRESENT VALUE OF PREDICTED OBSERVED EARNINGS FROM THE ESTIMATED QUADRATIC AGE-EARNINGS PROFILES BASED ON THE NBER-THORNDIKE SAMPLE AS A FUNCTION OF SCHOOLING, (N=50), DISCOUNTED AT BOTH 4 PERCENT AND 6 PERCENT.

Even though ability appears to have a small effect on earnings relative to the effect of schooling on earnings, the effect of ability on the present value of observed earnings is greater and positive over a wider range of discount rates. This is largely due to the period of foregone earnings with increased schooling.

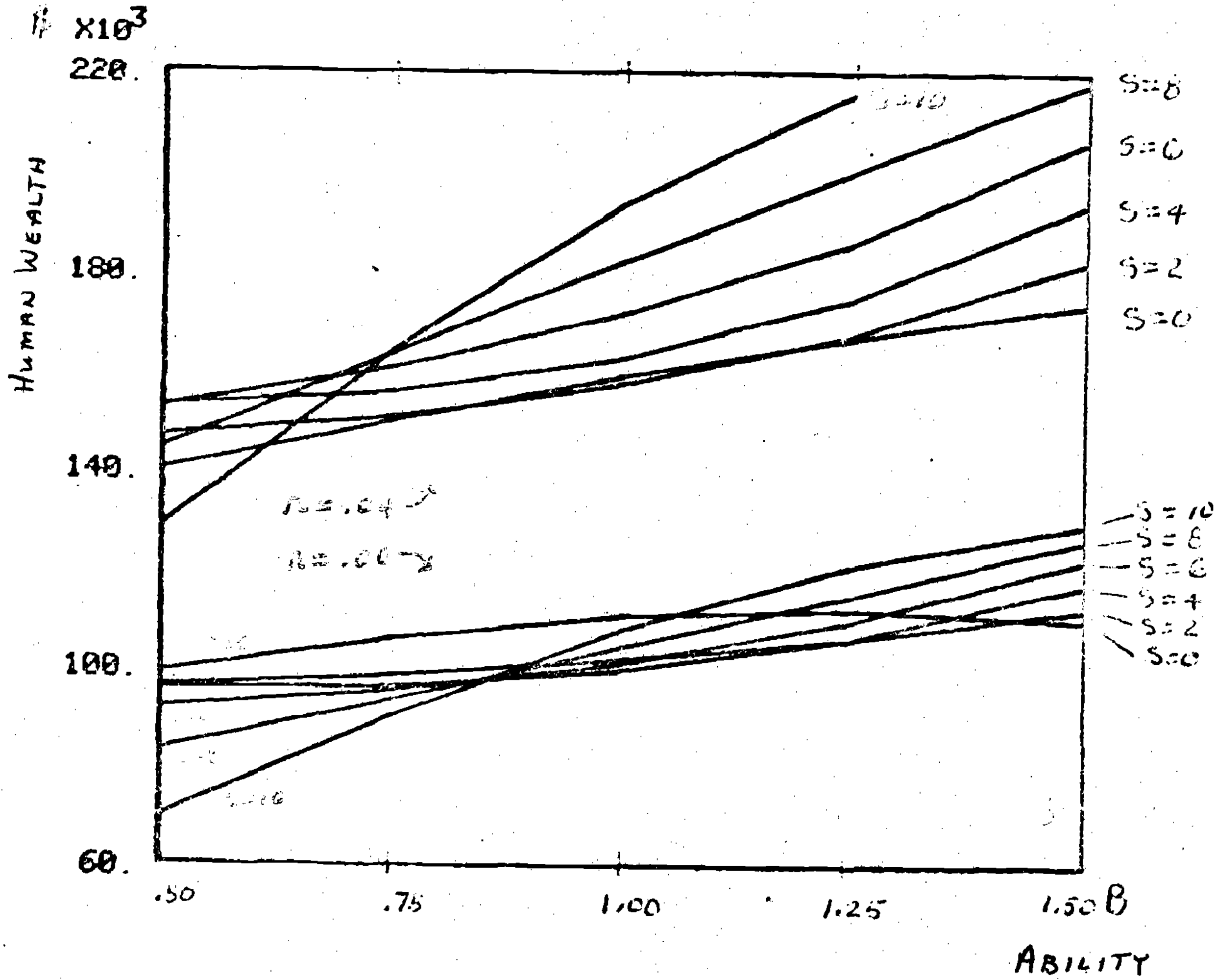

FIGURE 11.6. PRESENT VALUE OF PREDICTED OBSERVED EARNINGS AS A FUNCTION OF ABILITY FROM THE ESTIMATED QUADRATIC AGE-EARNINGS PROFILES BASED ON THE NBER-THORNDIKE SAMPLE. (N=50). DISCOUNTED AT BOTH 4% AND 6%.

RESULTS BASED ON THE IRRSS-ECKLAND DATA

In Spring1955, the Educational Testing Service conducted a survey of some 40,000 students in the nation's public schools called "A National Study of High School Students and Their Plans." Sponsored by the National Science Foundation, the main purpose of the survey was to access certain aspects of the American Educational System related to the encouragement and development of human resources in the sciences. Aptitude tests and questionnaires were administered to a national representative sample of two cohorts of students, comprising all the seniors in 516 schools and the sophomores in 97 schools, the latter representing roughly 0.5 percent of the tenth graders in public high schools listed in the U.S. Office of Education's 1951-52 Biennial Survey.[39/]

The aptitude test was constructed to give equal weight to vocabulary and arithmetic reasoning. The test consisted of 20 items chosen for proven validity and reliability in measuring these attributes.

In Spring 1970, the sophomores were sent a questionnaire entitled "A Fifteen Year Follow-up Survey" by Bruce K. Eckland through the Institute for Research in Social Sciences,[40/] supported by the National Science Foundation. Rather than trying to locate the entire group included in the 1955 study, a stratified sample of 42 schools was selected in order to provide a proportionate representation of schools from all regions of the country, with variations in school size, social class composition, mean test scores, and school dropout rates. About 84 percent of the students

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[39/] The data from the original survey were reported in two ETS monographs (Stice et al., 1956 and Educational Testing Service, 1957).

[40/] Institute for Research in Social Science, University of North Carolina, Chapek Hill, North Carolina.

in the sample eventually were found. Four mailings were administered between Spring and Fall of 1970. Members of the final sample received a 36-page questionnaire asking about their families, educational and career histories, and the events that had most affected their lives.

The basic sample used for this analysis contained 951 males who were sophomores in 1955 of which 689 were included for analysis.[41/] Sophomores were selected as the primary population because (according to Eckland) (a) due to attrition between the 10th and 12th grades, they were more representative of their total age cohort than were the seniors, and (b) after an inventory of the 1955 ETS records, the original answer sheets for the sophomores were found to be more complete; and the data for them were immediately available. All respondents were approximately fifteen years old in 1955 and thus thirty years old in 1970. The survey questionnaire included information on 1969 earnings,[42/] earnings for the first job after schooling (before 1969), and an estimate or guess of what earnings will be in 1979. Therefore, either two or three points on the individual earning profile are observed. Estimates are made with the 1979 estimated earnings included. This income value is the response to "What would you guess you personally will be earning ten years from now?"[43/] The corresponding age is a = 23.

41/ An observation was omitted if the respondent failed to answer questions about his schooling or if the answer about his schooling was unreasonable. Unreasonable is interpreted as leaving school before 1955 when the first survey was taken or after 1969 when the questionnaire itself was completed. A specific year observation for an individual was omitted if the individual failed to report that year's income or had zero income.

42/ All income data are mid-points of income classes used on the questionnaire. Class intervals are length ($1,000 up to $20,000 and are $4,000 between $20,000 and $36,000). For analysis these income figures are deflated by the published United States Consumer Price Index with base period 1957-59. Estimated income in 1979 is deflated by assuming a constant 4 percent rate of increase in the index from 1969 to 1979.

43/ The non-response rate for this question is 17.7 percent, double the 8.6 and 9.5 percent non-response rate of the other two income variables.

Ability is defined as the aptitude test score obtained in 1955 transformed to have mean one and standard deviation one-fourth. This test score, based on twenty items divided equally between vocabulary and arithmetic reasoning, is a fairly rough and general ability measure, and is much less likely to represent the theoretical notion of production efficiency. The overall distribution of ability and the distribution of ability by schooling class are presented in Table A.10. The mean ability by schooling class is presented in Table A.11. There does not appear to be any relation between mean ability and adjusted schooling as there was in the NBER-Thorndike sample.

Adjusted schooling is defined as the number of years the individual remained in school beyond 1956, when he would have been age sixteen. This definition is a little different from the "equivalent" years of schooling used with the NBER-Thorndike sample. The distribution of adjusted years of schooling is presented in Table A.12.

The earnings function is estimated from the IRSS-Eckland data using all three of the age-earnings points reported in the survey for each individual. The degree of the polynomial approximation for age, schooling and measured ability are determined by the data on the basis of signifigant reduction of error variance. The estimated equations including the weighted regression suggested earlier are presented in Table A.13. Again, the signs of the coefficients of the equation (1) estimated with no intercept are correctly predicted by the model. The "best" equation includes quadratic age and schooling but linear ability terms and their interactions. Both weighted and unweighted estimated quadratic age-earnings profiles are presented in Tables 14. The observed adjusted age range in the sample is from zero to twenty-three with no observations between thirteen and twenty-three. Earnings predictions beyond adjusted age twenty-three will not be very good.

TABLE 10

The Distribution of Measured Ability for the IRSS-Eckland Sample and for Schooling Classes
The Overall Mean is 1.00 and the Standard Deviation is .25

Ability Class	Overall Relative Frequency	Relative Frequencies for Adjusted Schooling Classes								
		1	2	3	4	5	6	7	8	> 8
0.625-0.674	0.210	0.163	0.200	0.182	0.200	0.284	0.175	0.109	0.394	0.308
0.875-0.924	0.263	0.260	0.275	0.242	0.300	0.284	0.300	0.304	0.242	0.221
0.925-0.974	0.148	0.152	0.225	0.061	0.133	0.054	0.250	0.217	0.091	0.154
1.125-1.174	0.103	0.131	0.150	0.152	0.100	0.027	0.050	0.065	0.030	0.106
1.175-1.224	0.155	0.135	0.075	0.212	0.167	0.270	0.175	0.239	0.091	0.115
1.425-1.474	0.029	0.038	0.000	0.030	0.033	0.014	0.000	0.965	0.030	0.019
1.475-1.524	0.091	0.121	0.075	0.121	0.066	0.068	0.050	0.000	0.121	0.077
Sample Size	689	289	40	33	30	74	40	46	33	104

TABLE 11

Means and Standard Deviations of ABility for the Various Schooling Classes of the IRSS-Eckland Sample

Adjusted Schooling	Mean	Standard Deviation	Sample Size
1	1.03	.255	289
2	.969	.223	40
3	1.052	.260	33
4	.995	.240	30
5	.972	.252	74
6	.974	.205	40
7	1.009	.202	46
8	.935	.285	33
8	.954	.254	104

TABLE 12

Adjusted Years of Schooling Distribution for the IRSS-Eckland Sample
Mean Years is 4.05 and the Standard Deviation is 3.4 Years

Adjusted Years	Relative Frequency (Per cent)
0	4.93
1	37.01
2	5.81
3	5.60
4	4.35
5	10.74
6	5.81
7	6.68
8	5.60
9	5.66
10	3.77
11	5.66

TABLE A.13

Empirical Estimates of the Earnings Function from the IRSS-Eckland Sample Based on 1825 Age-Earnings Points from 689 Individuals

	Coefficient					
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept		10256.00	3019.97	9066.27	2665.54	2449.45
A			429.77		408.36	454.85
S			513.77		815.65	1021.20
SA			76.14		165.45	129.72
A^2			-10.12		-13.34	-14.23
S^2			-324.66		-524.28	-545.46
SA^2			-1.56		-2.60	-1.88
S^2A			25.75		38.91	42.37
S^2A^2			-.69		-1.10	-1.17
B	4100.40	-5536.40		-5357.90	344.78	640.43
BA	198.20	210.99		373.83	25.81	-39.51
BS	-232.74	-324.13		329.40	-308.31	-604.57
BSA	28.64	30.85		83.51	-90.67	-32.00
BA^2				-8.52	2.96	4.65
BS^2				-288.35	203.79	234.38
BSA^2				-1.90	1.12	-.51
BS^2A				22.56	-13.50	-19.42
BS^2A^2				-.59	.41	.58
R^2 =		.2633	.3309	.2957	.3416	.3632

Note: Column (6) is coefficients estimated by weighted regression using the weight suggested in the empirical estimating proceedure. The R^2 is correspondingly a weighted value.

TABLE A.14:a

Quadratic Age-Earnings Profiles Based on the IRSS-Eckland Sample $Y = C_0 + C_1 A + C_2 A^2$.

Ability (B)	Adjusted Schooling (S)	C_0	C_1	C_2
.75	0	2924.12	427.72	-11.12
	2	2607.23	737.75	-17.81
	6	-6940.89	2048.66	-50.21
	10	-28374.83	4280.68	-107.97
1.0	0	3010.32	434.17	-10.34
	2	2743.07	685.37	-16.10
	6	-5482.98	1797.61	-44.10
	10	-23964.46	3722.96	-94.18
1.25	0	3096.51	440.62	-9.64
	2	2878.91	632.99	14.39
	6	-4025.07	1546.56	-37.99
	10	-19-54.09	3165.24	-80.39

Note: These estimates include a linear but not a quadratic ability term.

TABLE A.14.b.

Quadratic Age-Earnings Profiles Based on the IRSS-Eckland Weighted Regressions $Y = C_0 + C_1 A + C_2 A^2$.

Ability (B)	Adjusted Schooling (S)	C_0	C_1	C_2
.75	0	2929.77	425.22	-10.74
	2	2586.61	747.88	-18.72
	6	-6971.89	2060.52	-55.37
	10	-28360.00	4262.91	-119.62
1.0	0	3089.88	415.34	-9.58
	2	2678.82	702.58	-17.40
	6	-5609.22	1827.86	-51.28
	10	-23851.00	3686.52	-109.48
1.25	0	3249.99	405.46	-8.42
	2	2771.02	657.28	-16.08
	6	-4246.55	1595.20	-47.19
	10	-19343.64	3112.15	-99.34

Note: These estimates include a linear but not a quadratic ability term.

Earnings are predicted to peak before adjusted age twenty-three and to decline quadratically to large negative earnings toward the end of the life cycle. The prediction that earnings peak between ages thirteen and twenty-three is ill founded, since no observations are available in that interval.[44] The most reasonable implication is that earnings rise more sharply early in the life cycle and taper off before age twenty three.

The effect of schooling on the age earnings profile is illustrated in Figure 12 for the average ability indivdidual. Initial earnings upon entering the labor force are fairly constant, while more schooled individuals have earnings which rise more rapidly. The effect of schooling is to increase earnings after some age of overtaking which increases with the level of schooling. The level of earnings increases with schooling after the age of overtaking, but at a decreasing rate so that adjacent schooling profiles are progressively closer together at higher schooling levels.

Earnings-experience profiles are presented in Figure 13. The positive effect of experience on earnings increases with the level of schooling.

Measured ability in this sample has a negative effect on earnings. This is illustrated in Figures 14 for individuals with adjusted years of schooling zero, two, and six. Ability has no effect on initial earnings but then appears to retard the rate of growth of earnings.

The predicted relationships and profiles resulting from the weighted regression are almost identical. The weighted and unweighted profiles are not distinguishable when plotted on the same figure.

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[44] Since there are no observations in the interval $13 < A < 23$ no contribution to error variance is accumulated there. The same basic negative effect of ability is found when the predicted earnings at age $A = 23$ are omitted and a linear age-earnings profile is obtained.