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# Human Face Profile Recognition 

 by
## Vincent Wong

A Thesis Submitted<br>in<br>Partial Fulfillment of the<br>Requirements for the Degree of<br>MASTER OF SCIENCE<br>in<br>Computer Engineering

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July, 1994

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## Abstract

The purpose of this thesis is to implement an automatic person identification system based on face profiles. Each person's face profile can be quite unique within a small sample population and therefore it can be used as the basis of an automatic person identification system. To quantify human face profiles for use in the recognition system, Fourier descriptors are used to describe the open curve extracted from a face profile. Fourier descriptors in the low-frequency range are shown to be useful for human face profile recognition. By using 16 Fourier coefficients, a correct recognition rate of $92 \%$ for 60 subjects was achieved.

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## Human Face Profile Recognition

## Chapter 1. Introduction

Using computers to identify human faces has always been an attractive field to scientists and engineers. There are other person identification systems based on fingerprinting, iris-scanning, or retina-scanning available nowadays; however, none of them is more natural than identifying a person with his/her own face. This type of person identification system using the human face as the basis can be used for identification of criminals. It can also be used for authentication in secure systems. For example, it can add additional security on top of the required personal identity number (PIN) at the automatic teller machines (ATM), or it can automate the personnel check-in/check-out at the entrances of office buildings. Probably one of the most unique features to this kind of person identification systems is that the person being examined may not even be aware of the examination taking place. The person's face can be zoomed in with a video camera hidden in a place where nobody can see. This type of non-contact and non-interactive automatic person identification system is most valuable in areas of surveillance and security.

There are two types of human face images that can be used in a person identification system. ${ }^{(12)}$ Of course, the most natural way to identify a person is from the frontal image (i.e the camera is focused on the front of the face.) Many studies have been devoted to this area. ${ }^{(13,14,15,16)}$ However, the frontal image of a person can be very difficult to analyze because of its complexity and variability. In addition, the amount of
information that one can extract from a frontal image can be overwhelming, which means it can take quite an amount of computing time to perform a single identification.

On the other hand, it is possible to identify a person from the face profile (i.e. the camera points towards one side of the face.) Work in this area dates back to the last century when Francis Galton proposed algorithmic techniques for quantifying normalized profile traces with characteristic lengths and angles. ${ }^{(9,10)}$ Each person has a unique face profile. We often can recognize a person from the face profile with very casual inspection. We do that in many social occasions. The profile image of a person's face is easier to analyze than the frontal image since the only information that we need to process is the shape of the profile.

Researches on face profile recognition using fiducial points are commonly found ${ }^{\text {(6, 7, 8. }}$ ${ }^{11)}$ Systems with this approach use fiducial marks such as chin, nose, forehead, bridge, mouth and so forth. The distances between the fiducial points, angles between them, and areas of some triangles formed by the fudicial points are used as the features.

Aibara et al. used a different approach to identify face profiles. ${ }^{(4,5)}$ Fourier descriptors in the low-frequency range were shown to be useful for human face profile recognition. A correct recognition rate of $93.1 \%$ for 130 subjects had been reported.

In this thesis, a person identification system based on the face profile is implemented. Fourier descriptors are used to characterize face profile curves. The input to the system is assumed to be an image containing predominantly a human face profile. Otherwise, detection of face profiles becomes an additional problem. It is important to distinguish between face profile detection and face profile identification. In face profile detection, an algorithm is devised to search for the presence of one or more face profiles in an image. If a face is present, its size and location in the image must also be determined. This problem is fairly complex and computational intensive. On the other hand, face profile identification assumes that the profile curve of a face has been perceived, and the next step is to associate a name to the face.

This thesis concerns face profile identification only. Therefore there are some strict rules on how a person may pose in front of the camera. First of all, it is required that the size of the face profile be at least half the vertical size of the image. Secondly, the face should be upright although a little tilt is tolerable. Moreover, there should be no occlusion (i.e. glasses are not permitted.) Lastly, mouths should also be closed naturally to ensure consistency in the face profiles. To simplify the problem, it is also required that the left face profile be captured.

## Chapter 2. System Overview

Like most of the recognition systems, the human face profile recognition system has two modes of operations:

1) collect and store data in a database, and
2) match input data against stored data in a database.


Figure 2.1 Block diagram of the human face profile recognition system.

Figure 2.1 shows the block diagram of the human face recognition system. The input data to the database are the feature vectors extracted out of the profile images captured with a CCD camera. Input data are stored in a database for future reference in the training mode, or matched against the stored data in a database in the recognition mode. The human face profile recognition is divided into five major functional units, each of which will be briefly discussed in this section.

## A. Capture profile image



Figure 2.2 Setup of the human face profile recognition system.

The domain of our recognition system is the human face profile. Therefore, the input to the recognition system are profile images of faces. The input device for capturing images is a CCD camera. In Figure 2.2, the setup of the experimental human face profile recognition system is shown. The person under examination sits beside a white background. In front of the person is a video monitor which is connected to a CCD camera and a frame-grabber board resides in a computer. The CCD camera captures the profile image of the person's face. The live video image of the person's face profile provides a feedback to the person so that adjustment of the posture can be made accordingly. A set of photographic spotlights is used to brighten the white background so that high contrast images of face profiles can be obtained. Binary images of the profiles are then obtained with a simple threshold operation with the frame-grabber board for the profile curve extraction.

## B. Profile-curve extraction

Once we have a binary image of a person's face profile, we have to obtain a characteristic vector from the image so that all subsequent operations will be based on it. In order to do that, we have to extract the outline of the face profile. From the outline of the face profile, the characteristic vector is computed.

In order to extract the outlines of face profiles consistently, we utilize some feature points on a face profile. The feature points help to mark the position of the face profile and give clues to the size of the face profile in an image. There are six feature points
that we are interested in. Among them, two are derived from the others. These feature points are:

1) the tip of nose,
2) the top of nose,
3) the bottom of nose,
4) the chin position,
5) the upper terminating point (derived), and
6) the lower terminating point (derived).

The feature point extraction relies on the general shape of human face profiles. For instance, there must be a nose protrusion and a chin protrusion in a face profile, regardless of who the person is. Once we obtain the upper and the lower terminating points, the outline of the face profile is simply the boundary curve running from one terminating point to the other. The details of the curve extraction process will be discussed in Chapter 3. Face Profile Curve Extraction.

## C. Compute characteristic vector

Once the processor has determined the face profile curve of a person, it is not far from being able to compare it with others. The comparison should concern only the shape difference between two profile curves. Therefore we need to get a characteristic vector from the profile curve which can represent the curve independent of its size, position, and orientation in the image plane. We have chosen Fourier descriptors to
represent face profile curves since Fourier descriptors are invariant of size, position, and orientation of any closed boundary. In our case, a face profile curve can be viewed as a line object with a closed boundary. The mathematics of Fourier descriptors and how we use them to describe open curves such as face profile curves will be discussed in detail in Chapter 4. Fourier Descriptors.

## D. Update database

The characteristic vector, in which we use Fourier descriptors to represent face profile curves, is invariant of size, position, and orientation. Therefore it is useful for comparison purpose. As we have seen in Figure 2.1, there are two operations that we can perform after we obtain a characteristic vector from a face profile. We can either Store the characteristic vector in a database for future reference or we can match it with the stored vectors in a database.

A database is simply a collection of characteristic vectors which represent face profiles of people. The vectors in a database are identified by names. To learn a new face profile is to merely put a new entry in the database with the person's name associated with it. Usually more than one characteristic vector are obtained from the same person in various poses. So, we take the average and store the averaged values in the database.

## D. Matching

The human face profile recognition system identifies an unknown person by comparing the characteristic vector obtained from the person's face profile to the set of known vectors in a database. Distance measurement is made between the vector of the unknown person and each vector in the database. The unknown face profile is identified to be the person whose characteristic vector yields the shortest distance in the distance measurement.

The performance of the human face profile recognition system greatly depends on how the well the Fourier descriptors can resolve the differences in the face profiles of different persons. A complete analysis of the system and the test results will be presented in Chapter 5. Analysis of the System Performance.

## Chapter 3. Face Profile Curve Extraction

Before we can perform any kind of comparisons on human faces, we have to extract the face profile curve from the image. To automate the extraction process, we have to successfully locate the upper and the lower terminating positions on the face profile. The face profile curve is then defined as the curve running from the upper to the lower terminating positions along the face-to-background boundary.

To help locate the upper and the lower terminating positions, a few feature points on the face profile must be identified. These feature points mainly identify the positions of the nose and the chin.


Figure 3.1 Tip of nose.

The first feature point that we identify is the tip of nose (Point $\mathbf{A}$ in Figure 3.1). Since the tip of nose is a protrusion that is most highlighted in a human face profile,
we choose that as the point of reference in our face profile curve extraction. To locate the tip of nose, we search for the extreme left point in the face silhouette in a window that is $1 / 3$ down from the top of the image and $1 / 4$ up from the bottom of the image. The upper and the lower positions of the window are determined by examining a large number of images posed by a large number of people.

In order to successfully extract the face profiles from the face images of various sizes, some kind of distance measurement on the face profile is necessary. The nose distance, defined as the distance between the top and the bottom of nose, is used as a reference. The upper and the lower terminating positions will be determined based on this reference distance.


Figure 3.2 Bottom of nose.

The second feature point that we identify is the bottom of nose (Point B in Figure 3.2). First, a small portion of the face profile curve is extracted out from the tip of nose and downward. The extracted curve is encoded in chain code. The details of the
chain code and the curve extraction will be discussed in Chapter 3.1 Chain Code and Curve Extraction. For now, we have a small portion of the face profile curve. The bottom of nose is defined as the position of the first clockwise turn on the curve, starting from the tip of nose. The details of the turning point detection algorithm will be discussed in Chapter 3.2 Turning Point Detection Algorithm.


Figure 3.3 Top of nose.

The third feature point that we identify is the top of nose (Point $\mathbf{C}$ in Figure 3.3). Unlike the bottom of nose, it is not easy to use the turning point detection algorithm to locate the top of nose because of the large variation that is found in the shapes of noses and eyes of people. In the turning point detection algorithm, only the chain code of the curve is examined. The actual shape of the face profile is not taken into consideration. Therefore, a different approach is used to locate the top of nose.

Since the position of the top of nose is a point on the face profile curve above the tip of nose, we extract a portion of the face profile curve as shown in Figure 3.3. Then,
the line of sight from the bottom of nose to each point on the curve is examined, starting from the tip of nose. If the immediate extension of the line of sight from the bottom of nose does not belong to the background, the top of nose is said to be found. The immediate extension is a line of a few pixel units in length, an extension that is sufficient to distinguish the nose-to-background boundary and the peak at the top of nose seen from the inside of the face silhouette.

Now that we have clearly defined the position of the nose with three feature points, we can compute the distance between the top and the bottom of nose. We will use this distance, called a "reference distance", $d$, in our chin position definition.


Figure 3.4 Chin position.

The next feature point that we identify is the chin position (Point D in Figure 3.4). For the chin position, the turning point algorithm is used along with a distance constraint. First, the curve of the face profile below the bottom of nose is extracted. Then the chin position is defined as the first counterclockwise turn on the curve that is
at least a reference distance away from the bottom of nose. This distance constraint provides the discrimination against the feature positions at the lips that may be mistaken by the turning point detection algorithm.


Figure 3.5 Terminating positions.

The last two points that we have to locate are the upper and the lower terminating positions (Points $\mathbf{E}$ and $\mathbf{F}$ in Figure 3.5). These two points are defined by the distances from the other feature points on the face profile. These distances are defined in terms of the reference distance $d$ that we computed from positions of the top and the bottom of nose. The upper terminating position is defined as the point that is $0.75 d$ above the top of nose and the lower terminating position is defined as the point that is $0.5 d$ below the chin position. The ratios defined here also come from the observation of a large number of human face samples.


Figure 3.6 Complete face profile curve.

Finally, the complete face profile curve is extracted from the upper to the lower terminating positions as shown in Figure 3.6.

## Chapter 3.1 Chain Code and Curve Extraction

To get the face profile curve is to get the contour of the face silhouette. The curve is encoded in chain code. Eight-connectedness is used to represent the positions of the neighboring pixels. The 8 directions are numbered in a clockwise modulo- 8 fashion as shown in Figure 3.1.1. To minimize the noise from the rough edges that may be found in the face silhouette during the contour-tracing, a large contour-tracing mask is used. The contour-tracing mask is analogous to a ball rolling on a hill surface. The larger the ball is, the less bumpy is the resulting locus of the center of the ball. This technique is similar to the erosion operation in morphological image processing except that a constraint is applied to the movement of the mask in our case. Using a mask in contour-tracing is essentially applying a low-pass filter to the curve. All the high-frequency components such as the noisy edges are eliminated in the resulting curve. However, if we use too large a mask, the critical curvature information may also be removed. With a working image size of $512 \times 480$ pixels, the size of the contour-tracing mask is chosen to be 7 pixels in diameter. The digitized version of the contour-tracing mask is shown in Figure 3.1.2. During the contour-tracing, the contour-tracing mask is moved along the inside edge of the face silhouette. The locus of the center of the contour-tracing mask is then encoded in chain code.


Figure 3.1.1 Eight-connectedness.


Figure 3.1.2 Contour-tracing mask.

One constraint we impose on the movement of the curve is that the maximum directional change from pixel to pixel is limited to 1 unit. In other words, the angular change from one pixel to the next pixel is limited to 45 degrees. The reason for imposing such constraint on contour-tracing is to eliminate the ambiguity found in the curve length calculation.


Curve A


Curve B

Figure 3.1.3 Ambiguity in curve length calculation.

Consider the two situations in Figure 3.1.3. Let the displacement from any pixel to its horizontal or vertical neighbors be 1 unit and 1.414 units (root 2) for its diagonal neighbors. The curve length for curve $A$ and curve $B$ are 1.414 units and 2 units respectively. Curve $B$ has a directional change of 2 units from pixel $b$ to pixel $c$. As we can see, pixel $b$ and pixel $c$ are both neighbors of pixel $a$. Therefore curve $B$ could have been 1.414 units in length if it went from pixel a to pixel $c$ directly. Since the curve length directly affects how we sample the curve, we simply impose a constraint to limit the movement from pixel to pixel. The constraint allows the contour-tracing mask to be moved in 3 directions only. They are:

1) 45 degrees clockwise,
2) no change in direction since the last movement, and
3) 45 degrees counterclockwise.

The decision on the movement from pixel to pixel is made by examining the 3 possible moves in the order listed above. The order of the examination is important because it
ensures that the contour-tracing mask always leans towards the edge of the face silhouette. The path of a move is considered clear when none of the edge pixels of the contour-tracing mask overlaps with the background. When the path of a move being examined is clear, the position of the current pixel is advanced with that move. If none of the 3 possible moves is legal, the position of the current pixel is advanced with move \#3 (45 degree away from the edge.) This condition may occur with a right-angled turn in a face silhouette. An example is shown in Figure 3.1.4. The right-angled turn in a face silhouette does not cause any problem in contour-tracing. However, it should be handled properly. It should also be mentioned that it is a rare case since a human face profile is usually smooth. A sharp angle on a face profile like this is very unlikely to exist.


Figure 3.1.4 Right-angled silhouette. All 3 possible moves are illegal.

## Chapter 3.2 Turning Point Detection Algorithm

The bottom of nose and the chin position are located with the turning point detection algorithm. The algorithm is to apply a series of filters to the chain code of a curve and find the points on the curve which represent the points of inflections. The filtering process is shown in the block diagram of Figure 3.2.1.


Figure 3.2.1 Turning point detection algorithm block diagram.

There are 6 steps in the process. The chain code is encoded with 8 allowable directions as discussed in Chapter 3.1 Chain Code and Curve Extraction. The directions are numbered in a modulo- 8 fashion so that the incremental change in direction from pixel to pixel can be computed. To simplify the chain encoding and eliminate the ambiguity
found in the curve length calculation, the maximum directional displacement cannot be greater than 1 unit between neighboring pixels.

In Figure 3.2.1, the first filter converts the chain code to its incremental values. The output is then passed to an averaging filter. The averaging filter performs a convolution on the input data with a kernel of all 1's. Only the portion of the curve with a large net directional change would yield a large magnitude. The choice of the kernel size is dependent upon the feature size on the curve. A kernel size of 7 is sufficient for our case. The output of the averaging filter is passed to an accumulation filter which is essentially the same filter as the averaging filter but with a larger kernel size. The accumulation filter generates large magnitudes at positions in which the curve has apparent directional changes. The next two filters, the median and the clipping filters, are used to process the data before the peak detection.

In the accumulation filter, a small kernel is used when the target is a sharp turning position. In the case of detecting the bottom of nose, a kernel size of 11 is used. When the filter is used to detect a slow turn on the curve, a larger kernel is used. In the case of detecting the chin position, a kernel size of 31 is used. The reason for using a larger kernel for a slower turn is obvious. If a small kernel was used on a slow turn, each segment of the curve as seen by the kernel would become nearly a straight line and it would be impossible to detect the turn.

In Figure 3.2.2, we are trying to detect and locate the bottom of nose. The intermediate output for each stage of the filtering process is shown in Figure 3.2.3.


Figure 3.2.2 Bottom of nose.

(a) Input chain code.

(b) Incremental values.

(c) Average. (kernel size $=7$ )

(d) Accumulation. (kernel size $=11$ )

(e) Median. (filter size $=11$ )

(f) Clipping. (threshold value $=3$ )

Figure 3.2.3 Using the turning point detection algorithm to locate the bottom of nose.

## Chapter 4. Fourier Descriptors

After we obtain the outline of the face profile of a person in the form of chain code, we compute a characteristic vector of the profile curve using Fourier descriptors.

There are many ways to define Fourier descriptors that represent closed curve contour functions. Two of them were reviewed by Persoon and $\mathrm{Fu} .{ }^{(1)}$ The Fourier descriptors presented by Zahn and Roskies are based on the function of arc length by the accumulated change in direction of the curve since the starting point. ${ }^{(2)}$ Granlund defined the Fourier descriptors in the complex space that is immediately related to the Cartesian image plane. ${ }^{(3)}$ The Fourier descriptors used in the human face profile recognition system is based on it. The following represents the mathematical considerations of the technique used by Granlund.


Figure 4.1 A contour function in a complex space.

A contour function, a closed-curve $C$, is included in a complex space as shown in
Figure 4.1. A complex-valued function $u$ is generated by moving a point around the contour. Assume that the point is moving at a constant speed along C. Let the parameter of length covered by the movement of the point at every time $t$ be $l$. Then the complex function is represented by

$$
u(l)=x(l)+j y(l)
$$

Let the total arc length of the closed-curve $C$ be $L$ and let the complex function $u$ be periodic with period $L$. Now the complex function $u$ can be expressed as a Fourier series. The Fourier coefficients become

$$
a_{n}=\frac{1}{L} \int_{0}^{L} u(l) e^{-j n 2 \pi / L} d l
$$

and

$$
u(l)=\sum_{n=-\infty}^{\infty} a_{n} e^{j n 2 \pi / L}
$$

For simplicity, we let $L=2 \pi$. Then the formulas become

$$
a_{n}=\frac{1}{2 \pi} \int_{0}^{2 \pi} u(l) e^{-j n l} d l
$$

and

$$
u(l)=\sum_{n=-\infty}^{\infty} a_{n} e^{j n l}
$$

The Fourier coefficients here are not unique for a specific contour. They are dependent upon the starting position. We are also interested in the effect on the Fourier coefficients when the contour undergoes translation, rotation, and dilation.

## A. Starting position

There is a set of Fourier coefficients for each starting position of the contour-tracking. That means that there is a set of Fourier coefficients for each $\delta$ of the function

$$
u=u(l+\delta)
$$

We now assume that there exists a function

$$
u(l)=u^{(0)}(l)
$$

and let $a^{(0)}$ be the set of Fourier coefficients of this specific contour function. All the other functions are given by

$$
u(l)=u^{(0)}(l+\delta)
$$

The resulting Fourier coefficients become

$$
\begin{aligned}
a_{n} & =\frac{1}{2 \pi} \int_{0}^{2 \pi} u^{(0)}(l+\delta) e^{-j n l} d l \\
& =\frac{1}{2 \pi} \int_{0}^{2 \pi} u^{(0)}(l) e^{-j n(l-\delta)} d l \\
& =\frac{1}{2 \pi} \int_{0}^{2 \pi} u^{(0)}(l) e^{-j n l} e^{j n \delta} d l \\
& =e^{j n \delta} a_{n}^{(0)}
\end{aligned}
$$

Therefore, the Fourier coefficients differ from that of the specific contour function by a factor of $e^{j n \delta}$.

## B. Translation

When the specific contour function is translated with a complex vector $Z$, it becomes

$$
u(l)=u^{(0)}(l)+Z
$$

The Fourier coefficients then become

$$
\begin{aligned}
a_{n} & =\frac{1}{2 \pi} \int_{0}^{2 \pi}\left[u^{(0)}(l)+Z\right] e^{-j n l} d l \\
& =\frac{1}{2 \pi} \int_{0}^{2 \pi} u^{(0)}(l) e^{-j n l} d l+\frac{1}{2 \pi} \int_{0}^{2 \pi} Z e^{-j n l} d l \\
& =a_{n}^{(0)} \quad \text { for } n \neq 0, \text { or } \\
& =a_{n}^{(0)}+Z \quad \text { for } n=0
\end{aligned}
$$

Therefore, all coefficients except $a_{0}$ are invariant of translation.

## C. Rotation



Figure 4.2 A rotated coordinate system.

In Figure 4.2, the original coordinate system is rotated counterclockwise by $\theta$. From the elementary trigonometry, the new and old axes are related by the equations

$$
\begin{aligned}
& x^{\prime}=x \cos \theta+y \sin \theta \\
& y^{\prime}=-x \sin \theta+y \cos \theta
\end{aligned}
$$

or in matrix format,

$$
\left[\begin{array}{l}
x^{\prime} \\
y^{\prime}
\end{array}\right]=\left[\begin{array}{cc}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{array}\right]\left[\begin{array}{l}
x \\
y
\end{array}\right] .
$$

The $2 \times 2$ matrix that transforms the original coordinate system to the new system can be written as a complex vector $e^{-j \theta}$ if the image plane is viewed as a complex space. Therefore, when the specific contour function is rotated counterclockwise by $\theta$ in the complex image plane, the contour function become

$$
u(l)=e^{-j \theta} u^{(0)}(l)
$$

and hence the Fourier coefficients become

$$
a_{n}=e^{-j \theta} a_{n}^{(0)}
$$

## D. Dilation (Scaling)

The size of the specific contour can be scaled with a factor $R$. Similarly, it can be shown that the Fourier descriptors are simply multiplied with $R$.

$$
a_{n}=R a_{n}^{(0)}
$$

## General form of Fourier coefficients

As a result, the general form of the Fourier coefficients generated by translation, rotation, dilation, and changes in the starting position can be expressed as

$$
a_{n}=e^{-j n \delta} \cdot e^{-j \theta} \cdot R \cdot a_{n}^{(0)}+Z \delta(n),
$$

where $\delta(n)$ is a delta function.

Table 4.1 summarizes the Fourier coefficients for a contour function that undergoes all possible geometric transformations and changes in starting position.

| Transformation | Contour function | Fourier coefficients |
| :--- | :--- | :--- |
| Translation | $u(l)=u^{(0)}(l)+Z$ | $a_{n}=a_{n}^{(0)}+Z \delta(n)$ |
| Rotation | $u(l)=e^{-j \theta} u^{(0)}(l)$ | $a_{n}=e^{-j \theta} a_{n}^{(0)}$ |
| Dilation | $u(l)=R u^{(0)}(l)$ | $a_{n}=R a_{n}^{(0)}$ |
| Starting position | $u(l)=u^{(0)}(l+\delta)$ | $a_{n}=e^{j n \delta} a_{n}^{(0)}$ |

Table 4.1 Some basic properties of Fourier coefficients.

In order to make the Fourier coefficients useful in shape discrimination, the coefficients of a contour function must be independent of translation, rotation, dilation, and the starting position. Consider the following set of coefficients, derived from the Fourier coefficients in their general form $a_{n}$ :

$$
\begin{aligned}
b_{n} & =\left(a_{1+n} a_{1-n}\right) / a_{1}^{2} \\
& =\left[a_{1+n}^{(0)} e^{j(1+n) \delta} R e^{-j \theta} \cdot a_{1-n}^{(0)} e^{j(1-n) \delta} R e^{-j \theta}\right] /\left[a_{1}^{(0)} e^{j \delta} R e^{-j \theta}\right]^{2} \\
& =\left[a_{1+n}^{(0)} a_{1-n}^{(0)}\right] /\left[a_{1}^{(0)}\right]^{2} \quad \text { for } n \neq 1 .
\end{aligned}
$$

Both sets of coefficients are complex numbers. The difference in these two sets of coefficients is that the new coefficients of the contour function are invariant of the
starting position, rotation, and dilation. Because the new coefficients do not contain $a_{0}$, they are also independent of translation.

## Fourier descriptors for open curves

In order to use the Fourier descriptors discussed above on open curves, we trace the line pattern once and then retrace it so that a closed boundary is obtained. An example is shown in Figure 4.3.


Figure 4.3 Trace the line pattern once and retrace it.

Since the curve is an open curve, the positions of the two terminating points are known. We can define the Fourier descriptors based on the magnitudes of the general Fourier coefficients. We have

$$
\begin{aligned}
c_{n} & =\left|a_{n+1}\right| /\left|a_{1}\right| \\
& =\left[\left|a_{n+1}^{(0)}\right| \cdot\left|e^{j(n+1) \delta}\right| \cdot R \cdot\left|e^{-\jmath \theta}\right|\right] /\left[\left|a_{1}^{(0)}\right| \cdot\left|e^{\prime \delta}\right| \cdot R \cdot\left|e^{-j \theta}\right|\right] \\
& =\left[\left|a_{n+1}^{(0)}\right| \cdot\left|e^{j(n+1) \delta}\right|\right]\left[\left[\left|a_{1}^{(0)}\right| \cdot\left|e^{e^{\delta}}\right|\right] \quad \text { for } n \geq 0 .\right.
\end{aligned}
$$

The resulting coefficients are independent of translation, rotation, and scaling. Although the coefficients are still sensitive to the starting position, we can let the starting position be one of the terminating points of the open curve.

## Implementation

The mathematics of the Fourier descriptors we discussed so far assumed the contour function was continuous in space. The Fourier transform we applied was the Fourier Series. In reality, the curve is represented by a discrete number of points in the complex image plane. Since we trace the curve and retrace it back to the starting point during the curve sampling, the sample values represent a periodic discrete-time signal. Therefore, we can express the signal in a discrete-time Fourier series. Obviously, the sample points of a boundary curve should be evenly spaced in curve length. The details of the curve sampling will be discussed in Chapter 4.1 Curve Sampling.

Let $u(k)$ be a complex-valued function that represents the coordinates of the points on the boundary curve of a line object sampled at a fixed arc length. Let $N$ be the number of data points in $u(k)$. The discrete-time Fourier series becomes

$$
a(k)=\frac{1}{N} \sum_{n=0}^{N-1} u(n) e^{-j 2 \pi k n N} \quad \text { for } 0 \leq k<N,
$$

and

$$
u(k)=\sum_{n=0}^{N-1} a(n) e^{2 \pi k n N} \quad \text { for } 0 \leq k<N .
$$

The normalized Fourier coefficients are then given by

$$
c_{k}=\left|a_{k+1}\right| /\left|a_{1}\right|
$$

Due to the symmetry in the Fourier coefficients, that is

$$
\left|a_{k}\right|=\left|a_{N-k}\right|
$$

there are $N / 2$ normalized Fourier coefficients for an $N$-point transformation. So, the Fourier descriptors for an open curve are given by $c_{0}$ to $c_{N / 2-1}$.

## Another way of looking at Fourier descriptors for open curves



Figure 4.4 An example of a way of open curve sampling.

Consider the open curve as shown in Figure 4.4. Let $u(k)$ be a complex-valued function that represents the coordinates of the points on the boundary curve of a line object sampled at a fixed arc length. Let $N$ be the number of samples in $u(k)$ and let $N$ be an odd number as well. Moreover, let the starting point be one of the open curve terminals. Then, we have

$$
u(k)=u(N-k) \quad \text { for } 0<\mathrm{k}<\mathrm{N}
$$

The discrete-time Fourier series becomes

$$
\begin{aligned}
a(k)= & \frac{1}{N} \sum_{n=0}^{N-1} u(n) e^{-j 2 \pi k n / N} \\
= & {\left[u(0)+u(1) e^{-j 2 \pi k / N}+u(2) e^{-j 2 \pi 2 k / N}+\ldots+u\left(\frac{N-1}{2}\right) e^{-j 2 \pi\left(\frac{N-1}{2}\right) k / N}\right.} \\
& \left.+u(N-1) e^{-j 2 \pi(N-1) k / N}+u(N-2) e^{-j 2 \pi(N-2) k / N}+\ldots+u\left(\frac{N-1}{2}+1\right) e^{-j 2 \pi\left(\frac{N-1}{2}+1\right) k / N}\right] / N \\
= & {\left[u(0)+u(1) e^{-j 2 \pi k / N}+u(2) e^{-j 2 \pi 2 k / N}+\ldots+u\left(\frac{N-1}{2}\right) e^{-j 2 \pi\left(\frac{N-1}{2}\right) k / N}\right.} \\
& \left.+u(1) e^{+j 2 \pi k / N}+u(2) e^{+j 2 \pi 2 k / N}+\ldots+u\left(\frac{N-1}{2}\right) e^{+\rho 2 \pi\left(\frac{N-1}{2}\right) k / N}\right] / N \\
= & {[u(0)+2 u(1) \cos (2 \pi k / N)+2 u(2) \cos (2 \pi 2 k / N)+\ldots} \\
& \left.+2 u\left(\frac{N-1}{2}\right) \cos \left(2 \pi\left(\frac{N-1}{2}\right) k / N\right)\right] / N \\
= & \left\{2 \sum_{n=0}^{(N-1) / 2}\left[u(n) \operatorname{Real}\left(e^{-j 2 \pi k n / N}\right)\right]-u(0)\right\} / N .
\end{aligned}
$$

The formula implies that the discrete-time Fourier series of the closed boundary can be computed without explicitly closing the open curve. If we separate the real part and the imaginary part of the complex-valued function, $u(k)$, the Fourier transformation changes from a discrete-time Fourier series to a discrete Fourier transform (DFT). The Fourier coefficients become

$$
\begin{aligned}
& \operatorname{Real}[a(k)]=\left\{2 \operatorname{Real}\left\{\sum_{n=0}^{(N-1) / 2} \operatorname{Real}[u(n)] e^{-j 2 \pi k n / N}\right\}-\operatorname{Real}[u(0)]\right\} / N, \\
& \operatorname{Imag}[a(k)]=\left\{2 \operatorname{Real}\left\{\sum_{n=0}^{(N-1) / 2} \operatorname{Imag}[u(n)] e^{-j 2 \pi k n / N}\right\}-\operatorname{Imag}[u(0)]\right\} / N .
\end{aligned}
$$

The equations show that the Fourier coefficients can be computed with the sample points of an open curve. In order to use discrete Fourier transform in our computation, the input signal to the transformation must be in real numbers. Therefore, it is necessary to separate the complex-valued function, $u(k)$, into its $x$ and $y$ components.

We have seen two ways of computing Fourier descriptors for open curves. In the human face profile recognition system, we choose the first method in our implementation. By explicitly closing an open curve, the Fourier transformation is straight forward since it is rather natural to represent the sample points in complex values.

## Chapter 4.1 Curve Sampling

After the face profile curve extraction, the face profile curve is represented by a chain code. The characteristic vector of the face profile curve is then obtained using Fourier descriptors. The input signal to the discrete-time Fourier series transformation is a complex-valued function. It represents the periodic sequence of the locations of the sample points of a face profile curve. The open curve is viewed as a line object which has a boundary curve that can be traced and retraced back to the starting position. Therefore, the boundary curve is a function of $(x, y)$ coordinates and its independent variable is the arc length since the starting position. To obtain a uniform sampling of the boundary curve, we have to sample the curve at a fixed arc length interval. The arc length interval is equal to the perimeter of the boundary curve divided by the number of sample points. Obviously, the perimeter of the boundary curve is 2 times the arc length of the open curve. Since the open curve that we obtained is represented by a sequence of pixels, the arc length of the open curve is approximated by summing the incremental distances from one pixel to the next pixel, from one end to the other end of the open curve. The incremental distance of the horizontal moves and the vertical moves are counted as 1 pixel unit while the diagonal moves are counted as 1.414 pixel units (root 2 ). To retain the fidelity of the original curve, all sample points are linearly interpolated from the original pixel coordinates of the open curve.

The positions of the sample points closest to the terminals of the open curve can affect the overall layout of the samples. Consider the three different sets of samples of the same curve in Figure 4.1.1. In all three cases, the sampling arc length interval is $\ell$. In cases (a) and (b), the sample points of the retrace overlap with those of the first trace. In case (c), the starting point is neither at the terminal of the open curve nor half way of the sampling interval from the terminal of the curve. As a result, the sampling points in the first trace do not overlap with those of the retrace.


Figure 4.1.1 Different ways of sampling the boundary curve of a line object.

Obviously, it is easier to sample the boundary curve with cases (a) and (b) as shown in Figure 4.1.1. Sample points of the retrace of the curve can be duplicated from those obtained in the first trace.

In the human face profile recognition system, the open curves of face profiles are sampled as in case (a) of Figure 4.1.1. The total number of points that we obtain from the boundary curve is a power of 2 . In that case, we can use the fast Fourier transform
to calculate our Fourier coefficients. The arc length interval for the curve sampling is therefore the total length of the open curve divided by $2^{n-1}$ where $2^{n}$ is the number of sample points on the boundary curve.

## Chapter 5. Matching

In the human face profile recognition system, matching is the final process that determines the identity of an unknown person. Preceding the matching process, a Fourier descriptors vector is computed from the face profile of the person under examination. This vector is called a test vector, from which a distance measurement is made to each vector (template vector) in a database. A match is said to be found when the shortest distance falls below a certain threshold value. The threshold value is a maximum allowable distance for the system to consider a match. It is used to discriminate against people that are not registered in the database.

To quantify the difference between the Fourier descriptors vectors of the face profiles of two persons, we use Euclidean distance measurement. The $m$-value Euclidean distance between two $n$-dimensional vectors is given by

$$
d_{m}=\sum_{j=0}^{m-1}\left[v_{1}(j)-v_{2}(j)\right]^{2}
$$

where $m \leq n$ and $n$ is the size of vectors $v_{1}$ and $v_{2}$. The smaller the Euclidean distance is, the closer the two vectors are in the $n$-dimensional space. To ensure that the Fourier descriptors can truly represent a person's face profile, the Euclidean distance between the Fourier descriptors vectors of two persons should be large. In other words, there should be a significant difference between the two Fourier descriptors vectors.

In the following, we will examine the differences in the Fourier descriptors vectors of people's face profiles as well as the consistency in the Fourier descriptors vectors of the face profiles of the same person. The size of the Fourier descriptors vector, of course, affects the effectiveness of Fourier descriptors in describing human face profiles. For now, we will use a vector size of 64 coefficients for the following tests. Formal evaluations on how the vector size affects the performance of the system will be presented in Chapter 6. Analysis of the System Performance.

## A. Differences in the Fourier descriptors vectors of people's face profiles

Figure 5.1 shows 4 face profiles and their Fourier descriptors vectors. The size of the Fourier descriptors vector is 64 . (i.e. 65 points are sampled from the open curve.) The Euclidean distances between the vectors are shown in Figure 5.2. There are 4 tables in Figure 5.2. Different numbers of values in the Fourier descriptors vectors are used in the Euclidean distance calculation. In an $m$-value Euclidean distance calculation, only the first $m$ values of the Fourier descriptors vectors are used. Since the profile of the Fourier descriptors vector diminishes as the index increases, the $m$-value Euclidean distance between two vectors converges as $m$ increases. For this reason, it is not necessary to use all the values in the Fourier descriptors vector for our Euclidean distance calculation. We can see that the Euclidean distances increase by a fair amount when 16 values are used in the calculation instead of 8 . However, the distances do not increase as much when the calculation is switched from 16 to 32
values and from 32 to 64 values. Using only a small fraction of the Fourier descriptors vector for the Euclidean distance calculation also reduces the computation time in the matching process. The effect will become more noticeable if a large database is used.

In Table (b) of Figure 5.2, a typical value of the Euclidean distance between the Fourier descriptors vectors of two persons is about $1000 \times 10^{-6}$. We can choose our threshold value based on this number.


Figure 5.1 Fourier descriptors vectors of size 64 of 4 face profiles. ( $c_{0}$ is not shown, which is equal to 1.0.)

| Vectors | $v 1$ | $v 2$ | $v 3$ | $v 4$ |
| :---: | :---: | :---: | :---: | :---: |
| $v 1$ |  | 469 | 1,857 | 1,238 |
| $v 2$ | 469 |  | 1,447 | 2,659 |
| $v 3$ | 1,857 | 1,447 |  | 5,223 |
| $v 4$ | 1,238 | 2,659 | 5,223 |  |

(a) 8 values are used in the calculations.

| Vectors | $v 1$ | $v 2$ | $v 3$ | $v 4$ |
| :---: | :---: | :---: | :---: | :---: |
| $v I$ |  | 648 | 2,013 | 1,532 |
| $v 2$ | 648 |  | 1,506 | 2,889 |
| $v 3$ | 2,013 | 1,506 |  | 5,531 |
| $v 4$ | 1,532 | 2,889 | 5,531 |  |

(b) 16 values are used in the calculations.

| Vectors | $v 1$ | $v 2$ | $v 3$ | $v 4$ |
| :---: | :---: | :---: | :---: | :---: |
| $v 1$ |  | 688 | 2,035 | 1,576 |
| $v 2$ | 688 |  | 1,554 | 2,965 |
| $v 3$ | 2,035 | 1,554 |  | 5,570 |
| $v 4$ | 1,576 | 2,965 | 5,570 |  |

(c) 32 values are used in the calculations.

| Vectors | $v 1$ | $v 2$ | $v 3$ | $v 4$ |
| :---: | :---: | :---: | :---: | :---: |
| v1 |  | 693 | 2,041 | 1,584 |
| $v 2$ | 693 |  | 1,561 | 2,973 |
| v3 | 2,041 | 1,561 |  | 5,577 |
| $v 4$ | 1,584 | 2,973 | 5,577 |  |

(d) 64 values are used in calculations.

Figure 5.2 Euclidean distances of 4 Fourier descriptors vectors of size 64. Various numbers of coefficients are used in the calculations. (Values are in $10^{-6}$.)

## B. Consistency in the Fourier descriptors vectors of the face profiles of the same person

In order to show that Fourier descriptors can be used as the basis for identifying human face profiles, consistent results must be obtained from different images of the same person. In other words, small Euclidean distances must be obtained among different images of the same person.

Figure 5.3 shows 7 face profiles of the same person and their corresponding Fourier descriptors vectors. In images $1,2,3$, and 4, the person was asked to keep his mouth closed naturally. In images 5,6 , and 7 , the person was asked to open his mouth gradually. The Euclidean distances between the vectors are shown in Figure 5.4. The size of the Fourier descriptors vector is 64. In Table (a) of Figure 5.4, 16 values are used in the Euclidean distance calculations, while in Table (b), 64 values are used. Once again, we can see that the Euclidean distances do not increase as much when the number of values used in the distance calculation switched from 16 values to 64 values. Further investigation on this respect will be detailed in Chapter 6. Analysis of System Performance. Now, recall from the distance comparison of the Fourier descriptors vectors of different people, the typical value of the Euclidean distance of a 16 -value comparison using Fourier descriptors of size 64 is $1000 \times 10^{-6}$. If we choose $300 \times 10^{-6}$ as our threshold value, images 1-4 will all fall below this value when comparing with one another. The Euclidean distances from the vectors of images 5, 6, and 7 to the vectors of images 1 to 4 are larger since there are subtle changes in the face profiles when the person's mouth is allowed to open widely. With the open
mouth, the curve length of the face profile becomes longer. Since a fixed number of points are sampled from the face profile curve for the vector calculation, the positions of the sampled points are spread out more on the profile curve, causing the Fourier descriptors vector to vary slightly.

In Figure 5.4, we can see that there is certainly a consistency in the Fourier descriptors vectors of the face profiles of the same person with the mouth closed naturally. The open mouth images have some impact on the resulting Fourier descriptors, depending on the degree of openness. Therefore, the Euclidean distances from these vectors to the vectors of the naturally posed images are slightly larger.




Figure 5.3 Fourier descriptors of size 64 of the face profiles of the same person. ( $c_{0}$ is not shown, which is equal to 1.0.) The person's mouth is open in pictures 5,6, and 7. Picture 5 is the least open and picture 7 is the most open.

| Vectors | $v 1$ | $v 2$ | $v 3$ | $v 4$ | $v 5$ | $v 6$ | $v 7$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $v 1$ |  | 101 | 178 | 31 | 306 | 224 | 464 |
| $v 2$ | 101 |  | 44 | 109 | 147 | 112 | 341 |
| $v 3$ | 178 | 44 |  | 189 | 59 | 141 | 386 |
| $v 4$ | 31 | 109 | 189 |  | 352 | 199 | 415 |
| $v 5$ | 306 | 147 | 59 | 352 |  | 206 | 424 |
| $v 6$ | 224 | 112 | 141 | 199 | 206 |  | 184 |
| $v 7$ | 464 | 341 | 386 | 415 | 424 | 184 |  |

(a) 16 values are used in the calculations.

| Vectors | $v 1$ | $v 2$ | v3 | $v 4$ | $v 5$ | $v 6$ | $v 7$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $v 1$ |  | 122 | 192 | 39 | 317 | 243 | 508 |
| $v 2$ | 122 | , | 52 | 131 | 163 | 132 | 376 |
| $v 3$ | 192 | 52 | N | 206 | 71 | 159 | 428 |
| $v 4$ | 39 | 131 | 206 |  | 370 | 218 | 461 |
| $v 5$ | 317 | 163 | 71 | 370 |  | 217 | 455 |
| $v 6$ | 243 | 132 | 370 | 218 | 217 |  | 209 |
| $v 7$ | 508 | 376 | 218 | 461 | 455 | 209 |  |

(b) 64 values are used in the calculations.

Figure 5.4 Euclidean distances between the Fourier descriptors vectors of the face profiles of the same person. Vectors $\mathbf{v 5}, \mathbf{v} \mathbf{6}$, and $\mathbf{v 7} 7$ are computed from images with the person's mouth open. 16 and 64 values are used in the calculations. (Values are in $10^{-6}$.)

## Chapter 6. Analysis of the System Performance

The correct recognition rate of the human face profile recognition system decreases as the sample population increases. Besides that, there are other parameters that can affect the performance of the system as well. The two most important ones are the size of the Fourier descriptors vector and the number of coefficients in the vector that are used in the matching process.

## A. Sample population and vector size

We begin our investigation by seeing how the sample population affects the recognition rate. At the same time, we investigate the effect of the size of the Fourier descriptors vector on the system performance. First, 24 databases are constructed based on the images obtained from 60 people. Each database is characterized by the number of samples it contains and the size of the Fourier descriptors vector representing each sample. Each vector in the databases is constructed using 3 different images of the same person. To test the system, 4 different images of the same person, which differ from those used to construct the databases, are presented to the system for each corresponding entry in each database. The results of the tests are plotted in Figure 6.1.


Figure 6.1 Recognition rate versus sample population with various vector sizes.

As expected, the correct recognition rate decreases as the sample population increases. Moreover, the larger the vector size, the better the system performance with large sample populations. The recognition rate also seems to converge as the vector size increases. As we can see, there is not a big difference in the system performance between vector sizes 64 and 128 .

Now, let us take a close examination of the above tests. Specifically, we choose the test with the vector size of 64 coefficients and the sample population of 60 people. In Figure 6.2, the confusion matrix of the test results is shown.


Figure 6.2 Confusion matrix of the test results for the vector size of 64 coefficients and the sample population of 60 people. (Output 0 represents the unidentifiables.)

The confusion matrix shows the output of the system for the presented input. The numbers along the diagonal of the matrix represent the numbers of correct recognitions for the given input. Let us take a look at person \#12. The confusion matrix shows that the system has mistaken him as another person for 2 times, one as person \#40 and another one as person \#59. Person \#33 has also been mistaken by the system as person \#40. Similarly, person \#9 has also been mistaken by the system as person \#59. We shall examine the face profile curves of these people and see what
similarities they possess. In Figure 6.3, the face profile curves of these people are shown.


Figure 6.3 Look-alike face profiles. Person \#I2 was mistaken as persons \#40 and \#59. Person \#33 was mistaken as person \#40. Person \#9 was mistaken as person \#59.

At the first glance, these face profile curves may not look alike. However, they all have similar features that make the computer mistake one as the other. First of all, while we are looking at these images, we should remember that the system makes comparisons based on face profile curves only. The face silhouettes of these images obviously do not come close to one another. So, when we compare these images, we should pay more attention to the flow of the profile curves. One of the similarities that may be found among these people is the proportions of the main features on the face profile.

These may include the ratio of the distance between the eye and the bottom of nose to the distance between the bottom of nose and the chin position. On the other hand, the errors made by the system can also be coming from the angular position of the face. Therefore, it is difficult to conclude what causes the faults.

In Figure 6.4, the relative distances of all the vectors to that of person \#12 are shown. Four people with their vectors which are farthest away in the Euclidean space to that of person \#12 are chosen for close examination. The face profile curves for these people are shown in Figure 6.5.


Figure 6.4 Relative distances of the vectors to that of person \#12.


Figure 6.5 Vectors of the face profiles \#5, \#19, \#22, and \#48 are far away from that of the face profile \#12 in the Euclidean space. (Euclidean distances are calculated from the database of sample population of 60 people and vector size of 64 coefficients.)

Now, let us compare these face profile curves. The face profile curve of person number \#12 is quite rounded compared with the others. The area covered by the curve and the straight line joining the two terminating points is obviously larger than those of the other face profile curves. Also, the eyebrow area of person \#12 is not as distinctive as the rest. Person \#5 has a sharp chin and person \#19 has a big round chin. The chin of person \#12 looks like those of persons \#22 and \#48. However, the noses of person \#22 and \#48 are flatter than that of person \#12. With all these differences, there is no doubt these 4 face profile curves yield quite different Fourier descriptors vectors compared to that of person \#12.

## B. Number of coefficients used in the matching process

Another system parameter which directly affects the recognition rate is the number of coefficients in the Fourier descriptors vector that are used in the matching process. Since the profile of the Fourier descriptors vector diminishes as the index of the vector increases, the Euclidean distance between two vectors converges to a stable value as the number of coefficients used in the calculation increases. Therefore, we can find out the number of coefficients that is adequate for computing the Euclidean distance between two vectors. In Figure 6.6, the correct recognition rate is plotted against the number of coefficients used in the matching process. We have chosen the vector size of 64 coefficients for the test since it is most appropriate for a large sample population size as determined earlier.


Figure 6.6 Recognition rate versus the number of coefficients used in the matching process.

As we can see, the recognition rate increases as the number of coefficients used in the matching process increases and it reaches its maximum at a quarter of the total number of coefficients in the vector. In other words, the recognition rate of the system with a vector size of 64 coefficients does not get any better with more coefficients used in the matching process than that obtained with 16 values.

## C. Face direction

In general, the human face profile recognition system requires that the person under examination be facing absolutely perpendicular to the view of the camera. Any deviation from the perpendicular position causes subtle changes or sometimes big changes in the shape of the face profile curve, depending on person to person. In Table 6.7, the Euclidean distances between the deviated face profile curves and the straight face profile curves for a few people are shown.

| Angle (deg.) | $\mathbf{- 1 0}$ | $\mathbf{- 7 . 5}$ | $\mathbf{- 5}$ | $\mathbf{- 2 . 5}$ | $\mathbf{0}$ | $\mathbf{2 . 5}$ | $\mathbf{5}$ | $\mathbf{7 . 5}$ | $\mathbf{1 0}$ |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Person\#1 | 2,170 | 1,186 | 619 | 827 | 0 | 128 | 129 | 71 | 356 |
| Person\#2 | 3,178 | 1,345 | 345 | 231 | 0 | 78 | 110 | 164 | 403 |
| Person\#3 | 2,048 | 809 | 509 | 574 | 0 | 109 | 124 | 289 | 558 |
| Person\#4 | 3,809 | 1,348 | 467 | 157 | 0 | 89 | 78 | 147 | 438 |

Table 6.7 The Euclidean distances between the deviated face profile curves and the straight face profile curve. (The vector size is 64 and 16 values are used in the distance calculation. Values shown are in $10^{-6}$.)

The angular position of the straight face profiles is at zero degrees. Faces that turn towards the camera have positive angular positions and negative for the opposite direction. Notice that there is a difference between the two directions of angular deviation. The face profile curves of people that turn towards the camera are much more tolerable than those of the opposite direction. Intuitively, we think that the face profiles captured at two different directions with the same magnitude of angular deviation should be very comparable. However, as the person's head turns away from the camera, part of the face begins to block or distort some of the essential features found in the straight face profile. These include the nose and the lips. On the other hand, the general shape of the straight face profile suffers minimum distortion as the person's head turns towards the camera at a small angle since both sides of the face hardly obstruct those essential features.

## Conclusion

The human face profile recognition system presented in this thesis has clearly shown that it is possible to construct a system to identify human individuals through their face profiles. The use of Fourier descriptors to represent face profile curves has demonstrated its practical application in this domain. Although considerable effort is still required to construct a more robust and convenient system, this thesis has confirmed the ideas and feasibility of building such systems that researchers have developed over the past two decades.

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## Appendix <br> Program Listings

This appendix contains the program listings of the human face profile recognition system. The two main modules are matching.c and training.c for the identification process and the registration process respectively. The rest of the supporting modules are listed below with brief descriptions provided.

## Module name

classify.c classify.h
complex.h
costable.c
curvelib.c curvelib.h
database.c database.h extract.c extract.h
feature.c feature.h
fft.c
fft.h
fgrabber.c fgrabber.h
filters.c
filters.h
global.c
global.h
humanf.h
image.c
image.h
sampling.c
sampling.h
vector.c
vector.h

## Description

Matching process.
Macros for working with complex numbers.
Table to use for cosine and sine table lookups.
Chain code arithematics.
Database manipulations.
Curve extraction with chain code
Feature detection and location.
Fast Fourier transform.
Frame-grabber board operations.
Filters for the Turning point detection algorithm.
Global variables.
Useful macros and definitions for the system.
Image file operations for non-realtime processing.
Curve sampling and display.
Highest in the program hierarchy besides the main programs.

File: classify.c

*include - . . includelhumant $\mathrm{h}^{-}$-
Ainclude - -. \databaseldatabase. $\mathrm{h}^{*}$

```
    Function: classify the closest vector in the database to the
    Arguaents: input vector
    atabase
            vector
            *vize
```

 vector size the distance troa sach vector in the
database to the test vector.

```
    Return value: the vector number (index) of the closest notch.
int
classify(database, vector, vaize, distance)
```



```
lint
        vaizo:
    static double e_distance(double *, double *, int ):
    double dist: /% distance between 2 vectors */
    double sin_dist = NUGE_vaL: M, minimum distance coaputed so tor*
    tor (1 = 0:, < database.nvectors: (+*)
        distancell| = dist
```



```
        if (dist < sindist) !
        ain_index = &is:
```

    returntain_index 1:
    
static double
edistancel $x, y, n)$
double $-x$, " y :

${ }_{i}$
double dift:
double dist -0.0 difterence between 2 elements */
$\begin{array}{ll}\begin{array}{l}\text { double } \\ \text { diuble } \\ \text { int }\end{array} & \text { dirt } \\ \text { dist }\end{array}=0.0$ :
for ( $1-0 ; 1<\mathrm{n} ; 1+\mathrm{l}$ )

retuen (dist):

File: classify.h


Hitndet
Udetine Classify_H
classify_h
extern int classify( DATARASE, double P, int, double - ) if

File: complex.h

```
*inelude cmath,h>
#itndet COMPLEX_H
typedet struct complex COMPLEx:
detine COMPLEX(c,a,b) &\
    (c). }x=(a):
```


$0.242980179903264,0.248927605745720,0.254865659604514,0.260794117915276$ $0.242980179903264,0.246927605745720,0.254865659604514,0.260794117915276$
$0.26671275741469,0.272621355449949,0.278519689385053,0.284407537211212$, $0.290284677254462,0.296150088243623,0,302005949319228,0,307649640041535$ $0.313681740398891,0.319502030816015,0.325310292162263,0.331106305759876$ $0.336689853392220,0.342660717311995,0.344418680249435,0.354161525420490$
$0.359995036534989,0.365612997804774,0.371317193951837,0,377007410216416$ $0.339895036534988,0,365612997804774,0,371317193951637,0,377007410216416$
$0.362663432365090,0.388345046698926,0.393992040061048,0.399624199845646$ O
$0.405241314004990,0.410843171057904,0.416429560097632,0.422000270799799$ $0.427555093430282,0.433093018653152,0.438616236538527,0,444122144570429$ $.449611329654607,0.455083587126344,0.460538710958240,0.465976495767966$ $.471396736825998,0,476799230063322,0,482183712079122,0,487550100146436$,
$.492998192229784,0.498227666972782,0.501539393725718,0,508830142541107$, $0.514102744193222,0.519355990165589,0.52458969267 e 469,0.529803624686295$ $0.534997619867097,0.540171472729892,0.545324988422047,0.550457972936605$ $0.355570233019602,0.560661376197336,0.565731810783613,0.570780745886967$ $0.575008191417845,0.580813958095765,0.585797857456439,0.590759701858874$,
$0.595699304492433,0.600616479383869,0.605511041404325,0.610362606276309$ $0.615231590580627,0.620057211763289,0.624859489142366,0.629638238914927$ $0.614393284163646,0.619124444861776,0.643831542889791,0.648514401022112$ $0.653172842953777,0.657806693297079,0.662415777590172,0.666999922303618$, $0.671559954847018,0.676092703575316,0.660660999795425,0.685093667772701$
$0.699549544777067,0693971460999654,069876249409572,0,702754744457225$ $0.707106781196547,0.711432195745216,0.715730025283819,0.720002507961292$, $.724247082951467,0,728464390448225,0,732654271672413,0,736816568877370$ $0.740951125354559,0.745057795441466,0.749136394523459,0.753186799043612$ $0.757208846506484,0.761202385444266,0,765167265622459,0.769103337645580$
$0.77301045316273,0,77688465673233,0.740772228572094,0.79455697715575$ $0.786346427626606,0,792106577300212,0.795836904608883,0.799537269107903$ $0.803207531480645,0.806847553543799,0,810457196252595,0.814036329705946$ $0.017594813151584,0,821102514991105,0.224599302785025,0.82604504525775$ $0.031469612302545,0.834862874986380,0.836224705554838,0.8415 S 4977436899$
$0 . \theta 44853565249707,0.848120344803297,0.85135193105265,0.05455798 \theta 365401$ $0.857728610000272,0,860866938637767,0,063972956121596,0.867046245515693$ $0.870096991109711,0,873094979418290,0.876070094195407,0,879012226426633$ $0.881921264348355,0.894797098430938,0,887639620402854,0,890448723244758$ $0.893224301195515,0.895966249756185,0.998674465693954,0.901348947046022$
$0.903969293123443,0.906595704514915,0.909167963090522,0.911706032005430$ $0.914209755703531,0.916679059921043,0.919113851690050,0.921514019342042$ $0.923879532511297,0.926210242138311,0.928506000473216,0.930766961079984$ $0.932992798834739,0.935183509933948,0.937339011912575,0.939459223602190$ $0.941544065183021,0.943593458161960,0.945607325380521,0.947585591017741$,
$0.949528180393017,0.951415020969000,0.953106040354194,0.955141108105171$ . $956940335732209,0.958703474695 \mathrm{e} 72,0.960430519415566,0,962121404269042$ $0.963776065795440,0.965394441697689,0.966976471044852,0.968522094274417$ $0.970031253194544,0.971503990986252,0.972939952205560,0.9743393827 e 5576$ $.975702130038529,0.977020142657754,0.976317370719620,0.979569765685441$
$.980785280403230,0,98196386910955 S, 0.963105487431216,0.984210092386929$ $.985217642388941,0.986100097244599,0,987301418157858,0.988257567730750$ $0.989176509964781,0.990058210262297,0.990902635427780,0.991709753669099$ $0.992479534599710,0.993211949234795,0.993906970002356,0.99456457073425 \mathrm{~s}$ $0.995184726672197,0.995767414467660,0.996312612182778,0.996820299291166$,
$0.997290456678690,0.997723066644192,0.998119112900149,0.998475500573295$ $0.998795456205172,0.999071727752645,0.999322384589349,0.999529417501093$ 0.999698818696204 .0 .999830581795823 .0 .999924701839145 .0 .999961175202601

File: curvelib.c

-include *-. includelhuzant .h*


| Punction: | position |  |
| :---: | :---: | :---: |
| Intent: | Return the | index in an array of run-length. The index |
|  | represents length | the closest position of the given curve |
| Axguanents: | runien | the array of run-length. |
|  | clen | the curve length. |
|  | index | the index in the array. |

```
position! runien, cien, index)
double runien:
louble clen:
int
ine 1-0:
while (runlentif s cien)
\prime` Deteraine which point is closer the desired curve length.
if & runlenti) - cien < {runlen|fi-runienfi-1)//2.)
Mndex - .l:
```


## File: curvelib.h



| - | File: | database.c |  |
| :---: | :---: | :---: | :---: |
| * | Intent: | Databa | nanipulation tunctions. |
| - | Routines: | void | create_database ( Datailase - int ) |
| . |  | int | Load_databage ( DATMAASE * chat - ) |
| ** |  | void | save database ( datalise, char * |
| * |  | void | add_vector ( Database *, double *, char *) |
| * |  | int | blend_vector ( database ${ }^{\circ}$, double $*$, int ) |
| $\because$ |  | int | find_vector_1dt DATABASE, char *) |
| */ |  |  |  |

-include ". . Include\humant.h"


fscant ( fp. '1d\n', tdatabaso-*vsize ): / vector size */
database--vector -
(double ") malloc ( sizeot (double ")*database-~nvectors if $\rightarrow$ vector_14
(char $\cdots$ ) walloc( sizeot (chat -3*database->nvectors ) :

tot $(1-0.1 \times$. database->nvectors: i*)
telose ( ip i:
retuent 41 :


```
static vold
```




```
char but 1801 :
int
len:
int int
```



```
database-svector_idivnua) - (char *lasilioc (sizeot (char)*1en ) is
stecpy( database->vector_idivnum), but if
tscant ( \(f_{p} \cdot \cdot \mid d \backslash n^{*}\), sdatabase-svector_ctrivnuas) if
database->vector (vnum) \(=\)
(double *) mallioc ( sizeot (double)*database->vsite )
for \(t i=0\) : i < database->valze: it+
```






```
*detine ccu 1 /* counter-clockrise */
extern int Ghere_is_boundaryt int, int if
extern void get_chain_codel COORD, int, int, int, double, COORD,
```


## dit

File: feature.c

| * | File: | teature.c |  |
| :---: | :---: | :---: | :---: |
| * | Intent : |  |  |
| $\because$ |  | betect and locate taature points of a tace protile troa chain code. The pecson is assuased to be |  |
| $\because$ |  | facin | lett in the tange. |
| * | Routines: | vold | nose_tip ( CCORC - ) |
| $\because$ |  | void | nose_bottcal COCRD, COORD * |
| $\because$ |  | void | nose_top ( COCRD, COCRD, COORD - ) |
| $\because$ |  | void | uppeí_tersination ( COORD, COORD, COORD - ) |
| $\because$ |  | void | Chin_position( COORD, COORD, COORD - ) |
| $\because$ |  | void | lover__teraination (COORD, COCRD, CCORD. COCRD. - |
| */ |  |  |  |

Hinclude * . $\backslash$ includelhuasant. $\mathrm{h}^{*}$.

-include ...lcurveliblcurvelib.h



 ratio for the lover teraination *
static void sark_teature( int ', int, int •, int *);

noid


X = vhere_t9_boundary! DAGE_DX/2, (NOSE_ZIN_TOP+NOSE_wIN_BOTTCAI/2 )
while $\binom{x>0}{$ not yet } 1
I. adjust window top position so that the vertical acan starts froe a backpround area. The lowest window top position is the center of the isage.

$\ddot{\prime \prime} \quad$ wook tor the tace silhouette.


(f (not_yet)
else $\begin{aligned} & \mathrm{x}-\mathrm{F} \text {; } \\ & \text { break; }\end{aligned}$
/- just pass the nose tip */
x++/ /P sove back to the nose tip */
$\because \quad$ rind out the center position of the nose tip.
y - NOEE WIN BOTTCM:
Whilel RĖAD PDEEL $x, y$, to 011
$y 2=y:$
While
$y_{2}-7$

pos->x $=\frac{x}{2}:$
pos->y $=(y+y 2) / 2:$



* last pixel position in tha chaln */
/* curve length of the chaln */
* choin codo w/o the starting coord */
is run-length of the curve */
/* run-length of the curve */
/* coordinates of eoch pixel
* feoture points, indexes in datal/, */
/* number of fearure polncs found */

Out_pos.x $=$ Mage_ox;
Out_pos $y=$ Mage

get_cheln_codel start_pos, west, ccu, start_pos.y+phagedpy/e,
out pos, choin, 256 , ©0polnt, send pos, acien )
dato - (int *, molloc (sizeof (int) *npoints ):
locerion (coon )
** Prepore data for processing. data() contalng all tha volues In chaln"1 without the starting coordinates. dataid 19 made equal to the tirst direction code in the inpur chaln code.
data\{0 $=$ choint2 ;
icopy \{ chain+2, data+1, npoints-1):
** Get the run-lengths and the locations of each pixel on
the curve.
chain_intol chain, npoints, funlen, location);
** Apply a series of ifiters to locate the feature points on the curve.
ince (doto, npoints ):
sum $\left\{\begin{array}{l}\text { date, npolnts, } \\ \text { sum } \\ \text { date, npolnts, } \\ 11\end{array}\right\}$ : $: ~$
sum (data, npolnts, 11 )
clippling ${ }^{\text {daca, npoints, } 3 \text { ); }}$
merk_teoture (doto, npoints, fpoint, sntp)
** The tirgt positive turn is considered the nose bottom.
for $\left(\mathrm{i}=0 ; \mathrm{i}\right.$ e $\left.\mathrm{nf}_{\mathrm{p}} ; \mathbf{1 + +}\right)$
 pos->y
brear:
free (choin):
ree
ree ( dato
cunlen
l
free (locacion):

*/
done

- 1;

(double) (noseborton_pos.x-locotion $(\mathrm{i} \mid \mathrm{x}) * \mathrm{~d} /$ dist) :


If (READPDEEL ( $x, y$ ) |= 0) \{ done a $0 ;$
breok:
1
i (done $=1$ )

pos-sy a locotion(i)-y;
break:
,
free (chain):
ree (runlen ):
ree (location):

pos->x $=$ end_pos.x;
pos-;y $=$ end_pos. $y$
free (chain ):

out_pos.x = TRAGE_OX:
out_pos.y $=$ TRAGE_OY:



data $=(\text { int })^{2}$ malloc (sizeol (int)*npointa $)$
runcen = (double *, malloc ( sizeot (double) *npoints)

*) sade equal to the tirst direction cose in the input chain code.
datal01 = chain 121
icopy ( chain+2, datat1, npoints-1 ):
I. Get the run-lengths and the locations of each pixel on
$\because$ the curve.
chain_intof chain, npoints, runten, location )
$\because$
$\because$
$\because$
Apply a series of fllters to locate the teature points
on the curve.
ince ( data, npoints ):
$\operatorname{sua}(d a t a$,
sua (data, npoints, 11$)$,
sua ( data, npoints, 31 if
gediant data, npoints, if
clipping( data, npoints, 10):
sark_feature ( data, apoints, tpoint, infp):
". The ticst positive tuen is considered the nose bottoo.
tor (it = 0: it <ntp: $1+1$ )
dist $=$ orstance ( nose botton_pos. $x$, nose_botton_pos.y.
locationitpointifil|.x, locationitpoint |i) |i.y |:
It I datalfpointifl| < 064 dist $>$ ret_len ) |
poss>x = lecationtfpoint ifilix:
pos->y
break: break:

```
tree(chain ):
    tree( data):
    free( location):
```

```
#*)
```


chain - (int *)alloci sizeot (int) * 256 )
dist $=$ DISTAKCE ( nose_top_pos. x , nose_top_pos. y ,
nose_botion_pos. x , nose_botton_pos. y ).

pos->x $=$ end_pos. $\cdot x$
pos->y $=$ end_pos. $y$
treet chain 3


## sign **-1 th datalif ? max 1 | /- -asax

 (point intp) - 1-sax_oce/2 - is nfpt+:found

| $\because$ |
| :--- |
| $\because$ |
| $\because$ |

It is possible the max is tound above.
it i tound) !

$$
\begin{array}{ll}
\text { if }(\text { datalif } \geq 0) \\
\text { elge } \quad 3 \mathrm{gn}=12
\end{array} \text { /. +ve -) }
$$

$$
\text { else } \operatorname{sign}=-1 ; \quad \text { |- -ve - }
$$

$$
\begin{aligned}
& \operatorname{sax}=\text { dataliti / recocd lse aax -/ } \\
& \text { sax occ }
\end{aligned}
$$

$$
\begin{aligned}
& \text { sax_ooc - } \\
& \text { tound - } 0 \text { : }
\end{aligned}
$$

$$
\begin{aligned}
& \text { f }(\text { mul < 0) } 1 \text { /: exit region and enter * } \\
& \text { sign - -sign; }
\end{aligned}
$$

$$
\begin{aligned}
& 3 \text { lyn esign: } \\
& \text { sax datalili } \\
& \text { max occ - 1: }
\end{aligned} \quad / \text { recora ist sax }+/
$$

$$
\begin{aligned}
& \max \text { ooc }=\text { if } \\
& \text { tound }=0 \text { : }
\end{aligned}
$$

```
ntpoints - ntp
```


## File: feature.h


*include * ., includelhumant .h*


```
#: Function declarations.
extern void nose_tip( coond -)
extern void nose-bottcol cocke, COORD.1.1.
extern void nosettopl cOORD, COORD, CONRD.', COND - I 
extern void upper_tersination( coNMD, COORD, COCRD,
extern void lover_tersination! cOORD, COOPD, COORD, COORD ? '.
```

File: fft.c

| * | File: | fte, e |  |
| :---: | :---: | :---: | :---: |
| * | Intent: | Fast four | rourier transtocs. |
| * |  | void | titi complex - |
| $\because$ |  | void | iftti complex |
| * |  | void | loupass ( COMPL |
| * |  |  |  |
| */ |  |  |  |
| *include * . , includelinumant - $\mathrm{h}^{*}$ |  |  |  |
| tinclude 'fit, $\mathrm{h}^{\text {\% }}$ ) |  |  |  |
| *include *coaplex.th* |  |  |  |
| static |  | void |  | -, int, int ). |
| static | void | csel | complex *, int |
| extern | double | lel1. |  |


| * | Function: | ttt |  |
| :---: | :---: | :---: | :---: |
| ** | Intent: | Porward | rit. $l$ is the size of the array. |
| * | Arguasents | $\times$ | input data in coaplex values. |
| $\because$ |  | $N$ | size of the input array. |
| void |  |  |  |
|  |  |  |  |
| void |  |  |  |
|  |  |  |  |
| int | N. |  |  |
| 1 | ftt21x, $n$ | SE: |  |



```
```

    CHPLEX ctesp:
    ```
```

    CHPLEX ctesp:
    \
    \
    tor (1-0: + < N-1: 1++) 1
    tor (1-0: + < N-1: 1++) 1
        if ()>>1)
        if ()>>1)
        /- Interchange x(1) and x13) -
        /- Interchange x(1) and x13) -
        CCMPLEX_ASSIGNT CTEap, XIF) |
        CCMPLEX_ASSIGNT CTEap, XIF) |
        COMPLEX_ASSIGN( x/1), cresp i, 
        COMPLEX_ASSIGN( x/1), cresp i, 
    ,
    ,
    k=n2:
    k=n2:
    +
    ```
```

    +
    ```
```




```
```

File: fft.h

```
```

```
```

File: fft.h

```
```

| ** | File: Intent: | ftt,h |
| :---: | :---: | :---: |
| $\because$ |  | FrT tunction declatations. |
| $*$ |  |  |
| Ainclude | *complex.h* |  |
| Aitndet | Frta |  |
| *detine | - $\mathrm{FET}_{\sim} \mathrm{H}$ |  |
| -itndet | true |  |
| *define | true 1 |  |
| sitndet | false |  |
| sdetine fat | false | 0 |
| sendit | falie |  |
| /* |  |  |
| $\because$ | Macros to be with the array defined in costabie.c. |  |
| *idine |  |  |
| *define | $\cos (x, n)$ | ) Cos_tablel ( (inc) ( $1024 . / \mathrm{n}$ ) *x) $) 110241$ |
| *detine | $\sin (x, n)$ |  |
| /* | Function deciarationg. |  |
| $\because$ |  |  |
| exteen |  | ftt\| Complex - int ) . |
| extern | void | Iftt ( COMPLEX * int ) |
| extern | int | lowpass ( Complex a, int, int ) |
| extern | void | aagnitudel COMPLEX * double * int $/$ \% |

File: fgrabber.c







tinclude "..रincludelhumant . $\mathrm{h}^{*}$.
-include - ...curveliblcurvelib, h -
meigreor neighborll -


File: global.h






| $\because$ | Macro: |
| :--- | :--- | :--- |
| Intent: | FABS |
| $\because /$ | Floating point absolute tunction |

$$
\text { sdetine fabs }(x) \quad\{(x)>0.0>(x):-(x))
$$

| . | Macro: | MNX, MIM |
| :---: | :---: | :---: |
| * | Intent: | Maximun and minisun of two numbecs. |
| */ |  |  |
| *define | $\max (\mathrm{a}, \mathrm{b})$ | ( (a) > (b) ? (a) : (b) ) |
| * detine | MIN(a,b) | ( (a) - (b) ? (a) - (b) ) |
| *detine | true | 1 |
| sdetine | FALSE | 0 |
| *setine | trace_ox | S12 /- horizontal size of the isage */ |
| Adefine | nage py | 480 / vertical size of the inage */ |

File: image.c


```
%old
get_imagel vold I
        init_itexpClI:
        Printfl *Press any hlt to treaze the langa* I;
        frcczell:
lyelse
**----------------------------------------------------------------------------
get_imagell
    int loadim_pCl char * , lnt *, int * 1;
    Char atringle0l;
    printt। "pleage enter input tile namc: * 1;
    gcanfl "!s*, string l
    if I !loadimpe| string, sdx, bdy | | |
        #printt! stderr, *Unable to open input file.\n* I;
    | exitl11:
l
```




```
        close1tod
    <a, (1)
    lead td, sc:ize, 2 1: /* gize fo comment area */
    lol
    Mrad to, x, 2 1: 
    readi td, y, 2 1; 2 1; % * upper left-hand corner y */'
    Feadl to, tormat, 2 1; ;; /* reserved by ITE: */
    read to, coment, csize 1: /* comment */
    loccltd l;
    return\ 54tcgize 1: /* oftget to image data */
#* Function: Iondim_PC L
** Intent: Load an uncompressed ITEX
    Arguments: filemame the image filename.
    width
    Return values: length % length of image in pixe le
int
loadin_pCl file_name, width, length
Char *filc namei mon:
    int offset: 
    char X, commenti2s61;
    int ta;
    ima len, tenp, i:
    it | lottget = itex_image_neader I tile_name,
```



```
    fd = openl tile_name, o_rdomly I O_binary l:
    if | tornat == 1; ; % unable to procegs compregged image */
    read to, coment, offget 1: /* move to the beginning of the data */
    ** Now ccad the inage data
    -yetvideomodel vVEsi6color i;
    bp = IUCHAR *lmalloci EuF size
    x=y*0;
```


tor 1 i O; i< len: $1+1$ l

if $1+x=0 \begin{array}{ll}d x \\ x=0\end{array} 1$
$x=0 ;$
$y^{++} ;$
close 1 td $1:$
trec| bp 1:
width $=d x$;
return 1 ;

fwitel bp, 1, ctr. ff 1;
fclosel fp i:


While $\{$ read td, bp, dx*reduction ) ) t
for $1+=0 ; 1<\langle\mathrm{x}:+1+$ reduction )
,
i

$$
\begin{gathered}
=\pi>\text { th }) \text { ( } 25 s): \\
\text { setcolor }
\end{gathered}
$$

$$
\begin{aligned}
& \text { setcolort } 253 \text { in } \\
& \text { setpixel x, y in }
\end{aligned}
$$

$$
\text { else } 1
$$

setcolor ( 0 ):

close ( td ):
-setcolori 255 )
return 1
File: image.h


```
\begin{tabular}{|c|c|c|c|}
\hline * & File: & \multicolumn{2}{|l|}{\multirow[t]{2}{*}{astching.c}} \\
\hline * & Intent: & & \\
\hline * & & \multicolumn{2}{|l|}{This is the main progran tor the recognition systen. It compares the test vector of an input profile to} \\
\hline - & & \multicolumn{2}{|l|}{the vectors in the database and outputs the result.} \\
\hline \multicolumn{4}{|l|}{-.} \\
\hline \multicolumn{4}{|l|}{\(\cdot /\)} \\
\hline \multicolumn{4}{|l|}{} \\
\hline \multicolumn{4}{|l|}{*include ". .ddatabaseldatabase . \(\mathrm{h}^{*}\) *} \\
\hline \multicolumn{4}{|l|}{} \\
\hline \multicolumn{4}{|l|}{\#include *..ivectorlvector. \(\mathrm{h}^{*}\) *} \\
\hline * define & e theesholdi6 & 0.000400 & /* threshold for 16-point coaparison */ \\
\hline \multicolumn{4}{|l|}{void} \\
\hline \multicolumn{4}{|l|}{main( arge, argv)} \\
\hline \multicolumn{4}{|l|}{\multirow[t]{2}{*}{int arge:
chat
Oargy:}} \\
\hline & & & \\
\hline \multicolumn{2}{|l|}{1} & \multicolumn{2}{|l|}{\multirow[t]{2}{*}{fill_page ( char *):}} \\
\hline & void & & \\
\hline & char & but l801: & \\
\hline & double & -vector : & /. test vector */ \\
\hline & оптдвдse & database: & 1 \% databage being used */ \\
\hline & int & neoapare: & \\
\hline & int & vnua & \(\%\) vector * selected for the satch */ \\
\hline & double & -distance: & /* distances froo the test vector -/ \\
\hline & int & vector_ok: & /- get_vector () successfui \% \\
\hline & int & batch \(=0\) : & \%. Itor non-batch processing \%/ \\
\hline & int & b_ctr \(=0\) : & /- nuaber of taces processed */ \\
\hline & file & \({ }^{-1 \mathrm{Tp}}\) \% & /* batch output tile pointer */ \\
\hline
\end{tabular}
```

it ( arge, 1 th istrcap $\left.(\text { argulit })^{*-b^{*}}\right)$ ) 1
$t_{\text {tp }}=$ fopen $\left(\right.$ "batch.out $^{*}, w^{*}$ ):
printt © - pleage enter the nane of the database: - 1
scant ( $-15^{\circ}$, but ):
It ( Itoad_database ( ddatabase, but ) )
fprint f( stderr, *Unable to open the database file. $\ln { }^{*}$ ) exit $(-1)$ :
1
print ( ) Database loaded. $\mathrm{In}^{*}$ ):
printf( -vector size: $1 d \backslash \mathrm{n}^{*}$, database.vsize)

vector - (double *) malloc( sizeof (double)*database.vsize) ) distance - (double *) malloc (sizeot (double) *database, nvectocs ).
whlle (1) 1
printti Hou many coefficients do you wish to use in
scant t 'id " conparison? ${ }^{-}$
if ( nccopare i i| ncoapare > database.vsize)
if incoapare continue:
m
( (1) if ' 'get_vector d database.vsize, vector ) 1 i
it is iget vector
vector_or
else
vector_ok $=1$ :
vnun - classifyt database, vector, ncompare, distance ).
It indet _ FRAME_GrabBer
it ( itatch) $\begin{aligned} & \text { getch (1): }\end{aligned}$
endif _retvideomodel _oEFAULTMSOE i:
D ctret.
(vector_ok at distancelvnua) < TMRESHOLD16)
fili_page ( database.vector_1divnual) ]
if i-batch)

-190 1
If i vector_ox I \& $\begin{aligned} & \text { fili_pagei *Unable to identify muan subject* ) : }\end{aligned}$

else 1 till paget *Unable to extract tace profile* 1 : it 'tbatch )
-Idtunable to extract tace profile. in ${ }^{*}$. D_ctr ).
1
if ( batch )
If ( vector_ok )
printt ( - \nindo you wigh to continue) (y/n) - 1

1
if ( batch )
princt( "Goodbye.\n"):



```
printt \({ }^{-1} \mathrm{n}^{*}\) )
```



## File: sampling.c


tor ( 1 - $1: 1$ ( C nsasples: $1++$ ) curr_x - ROUND (curve sample fil. x). cure $y$ - Round icurve_sample lil.-y URITE_PRXEL (frame_x+Curt $x_{\text {, }}$ frase_ $y+$ curr_y i)


```
*/
(clisplay2t curve, trane_x, trase_y, reduction)
    curve:
    trase x, frase_y:
    lown
    $ouble curr_x, curr_y: % % current position */ 
    # The header contains the starting position
    curr_x = (double) curve (01/reductionz
    frase-x = ROUND (idoublel frase_x/reduction)
    (rame_y = ROUND (ddouble) frame_y/reduction)
    WRITE_PDCEL( trase_x+ROMND (Cure_x), trame_y+ROUND (curr_y) )=
    Ghile (curvelsi it cham, tebaination i
        ext move-curvelo
        urr - to (double) nelahborinext movel . x/reduction
        curr y +* (doub1e) neighbor (next oovel, y/reduction:
        lol
```

    File: sampling.h
    

```
    rintfi -Pleage enter the name of the database: - I
    scont! - 19*), database_name):
    If (1load_database( sdatabase, database_naer )) 1
    rint+1**)Unoble to open the database tile,in**)
    *rintf(*)
    if & "but ** 'y' I I I'P create a new database:
        printt! -What is the vectoc slze for the:
        scant( "Id", tvelze):
        create_database(suatabase, v9ize )
    elge I
        printe!(Goodbye.in*)
    1
    it I Inew databare)
    -1se
    print!t "Database loaded.\n* )
    print!( *Database sreatedi\n*)
    print:( "Vector alme: Wan", amta
    printf, "muber of vectors in the database: id\n*, database,nvectors),
    vector - (double *)mallocl sizeot (double) *database.vgize )
    O}\mathrm{ Get tace protile vector hore
    while (1),
    It { get_vector { database.vsize, vector ) )
    else
        vector_ok - 0
*itndet _rRAME_GRABBER
    If (tbatch)
        getch 0:
*endit
    _getvideceodel _DEFANLTMODE 1:
    it ( Ivector or,
        prinṫ{! "Unable to extract vector.\n* ),
    if ( isase_person) (
        printt (:What is the name of the perzon) -
        M, (first nase last_nasel *).
        streat(but, *)';
    if ( (vnum = tind vector id( database, but )) =- -1) (
        printf({-Unable to find the nase, '15
        printf( "Do you vant to odd this person -
        grant ( -119", but2) (yatase? (y/n) -)
        If I vector_ot &f but2 * ' ' ' | |
            add_vectori sdatabase, vector, but ).
                ave= database( database, database_nase)
        else
            print:( "Vector discarded.\n*)
    =15e।
        It ( Isane_person
            print:( -Person', name found.in*)
        printf1 *Nuaber of vectors blended together: \d\n*,
```



```
        print:( Do you vant blend this vector-*
        scant( "119*, but2 1:
        If ( vector_ok && "but2 =.
            blend_vector( slatabase, vector, vnua ):
        else
            printr( vector discarded.un- I%
    printf! *Do you wish to continue? (y/n) - ),
    ocant( -115*, but2):
    it ( 'buf2 =* 'n')
    else break:
        print:1-Will that be the sase pecson7 (y/n) - )
        3cant! *115**, butz)
        sase_person - 1.
        elye
        gase_pecsoo = 0
    1
    save_database! database, database_nase l.
    printi( cosbye.lo- !
    exit(0)!
```

File: vector.c


nose_tipt tnose_tip_pos i.
hose-topt nose tip posp nose bottoonpos i)
upper_tecaination ' nose_top_pos, nose, botton pos, Aupper tern pos it
tovet_tersinationt nose top, nos, nose botionton, thin, chin pos pos, sHower_ters_pos ):-
asa - (double ") salloct sizeot (doublel - fte size )
get_chain_codel uppertere_pos, vest, CCW, pear_ pr 1, $9999 .$. tover_tecs_pos, chain, MAX_cMaIM_sIzE, snpoints. send pos. Geten I
oflset = olsplay_cffiet.
displayi chain. " of tret.
traw_square_narker ioftret +nose_top_pos, $x_{1}$ nose_top_pos-y ).
draw_square_zarkerl oftret those_botion_pos, $x$, nose_botton pos. $y$ i
draw-square_sarker 1 oftset+upper_tece_pos.x. upper_tecr_pos-y if

drav_ditine of tset+no9e_top_po9. . , nose top pos.y.
oftset+nose_bottoo_pos. $x$. nöse_botcoa_pos. y.
REF nPoints


draw_dimel ot tset+chin pos. $x$, chin_Dos. $y$,

I. saaple the curvo and obtain the Fourier descriptors vector.
csample (chain, nsasples, elen, scurve_saaple $1 /$
oftset *- DISPLAY_OFFEET-3/2:
(ti ncurve, fit bive bit
fiti nacurve, fttisize ),
$\because$ Nocsalize the vector such that vector $101=1.0$.
tor (1-1: + < vaize: i*


```
    tree(chain)
    freel surve_sanple )
    treel ncurvél:
    return 1: /* operation successtu1 *)
#
atatic vord
erace back{ curve_sanple, nsasples. closed_curve)
int curve-sanple
CcMPLEX **closed_eurve:
    complex *curve:
    * closed curve = curve 
    (COMPLEX *)=alloc( sizeot (COMPLEX)*(nsaaples-1)*2 1
    tor (i = 0; A < nsanples: it+)
        COMPLEX_ASSIGN( curvelil. Curve_samplell) ,
    # Trace back. Note that the two ending points are not included
        n the retrace.
    1. nsasples:
        COMPLEX_ASSIGNT curvel1), curve_sampleinsamples-1-11),
            j++:
```




```
    Arguments: x1, y1, x2, y2 two points,
```

    Arguments: x1, y1, x2, y2 two points,
    lol
lol
static void
draw_diine( x1, y1, x2, y2, npoints)
int x1, y1, x2, y2
npoints:
double slopet
doutle dx, dy
double x, y
Clindet _rpame_graboer
*endit
sotcoior! 25s ):
If ( abs(x1-x2) > abs(y1-y2) )
*)
tor (1.0: \& < npoints: i+c) 1
y= (double) y1 -slope* (double) x1-x)
WRITE_PDXEL( ROUND (x), ROUND (y) ):
Blsel
slope - (double) (x1-x2)/(double) (y1-y2);
(double) (yz-yl)/(double) (npoints-1)
for { 1 = 0: \& < mpoints; i+ ) {
y= (double) y1 * dy*(double) I:
WRITE_PIXEL( ROMND (x) , RONND (y) If

```

```

*/
draw_cross_macker ( }x,
int x, y
*itndet _FRAME_grasber
sendit
MRITE_PTXEL( x, y ):
WRITE_PINEL!
WRITEPPDEL, (
WRITEPPDELI (
MRIT-PDEL, }x+1\mathrm{ PNEL,
WRITE_PDELS( }x+3,y
WRITE PDXEL|
GRITEPPDEL( x, y-1
MRITE_PDXEL( x, y-2 )
WRITE-PDKEL( x, y*1),
RITE PIELL(x,y*2)
MRITEPDELL( ( }x,$$
\begin{array}{l}{(,y+3}\\{(}\end{array}
$$
lum,
static vold

```
```

*ifndet vector - H
*detine _vector_"\#
exteen ine get_vectori int, double * 1/
*enalt

```

\section*{File: vector.h}

```

