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# HUMAN WEALTH AND HUMAN CAPITAL

by

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The past few years have witnessed an outpouring of research directed toward discovering the relationship between human wealth and human capital. Many of the papers have been theoretical [e.g., Becker (1964), Ben-Porath (1967), Rosen (1973), Haley (1973)] while others, more data-oriented pieces, have sought empirical description of the relationship [see, among others, Hanoch (1967), Griliches and Mason (1972), T. Johnson (1970), Mincer (1974) and Welch (1973)]. The theoretical underpinnings for much of what has transpired are derived from Schultz (1962), Becker (1967) and Mincer (1970). Controversial from the start, most of the criticism has consisted of questioning the relevance of the analysis rather than the theoretical validity of it. Two notable exceptions are Becker (1967) and Rosen (1973). In this paper, a few theoretical issues will be raised regarding the relationship between the distribution of human capital and that of human wealth. Special attention will be paid to the empirical implications of the analysis.

## I. Inequality of Income v. Inequality of Wealth

This section will make the following rather simple point: There is no simple combination of discounted flows of wage and property incomes that reflects an individual's true wealth position. Even in the absence of property income, the present value of the stream of earnings is likely to overstate an individual's wealth and overstate it to a larger extent for wealthy individuals.[1]

To see this, consider a two-period world in which individual A is endowed with $B_0$ (inheritance) of physical capital in period zero, $L_0$ of raw labor in period zero and $L_1$ in period one the rental price of which is R per unit. All individuals are not alike except to the extent that there are securities that all can purchase yielding rate r. Suppose that A also has the option of investing some $kB_0$ in a machine which will return $(1 + r^*)kB_0$ in period one. In addition, he may use some given proportion h of raw labor which, when combined with $cB_0$ of physical capital $(c + k < 1)$, produces human capital sold in period one for $(1+r')(hRL_0 + cB_0)$ dollars. The individual has four possibilities:

(1) No investment

a. Consume $B_0 + RL_0$ in period zero and $RL_0$ in period one $(L_1=L_0)$.

b. Consume $n(B_0 + RL_0)$ in period zero and purchase (or sell) $(1-n)(B_0 + RL_0)$ of securities to consume $(1-n)(B_0 + RL_0)(1+r) + RL_0$ in period one.

(2) Invest in the machine, but not in human capital. Then consumption in period zero is $n[(1-k)B_0 + RL_0]$ while that in period one is $(1+r^*)kB_0 + RL_0 + (1-n)[(1-k)B_0 + RL_0](1+r)$.

(3) Invest in human capital, but not in the machine. Then consumption in period zero is $n[(1-h)RL_0 + (1-c)B_0]$ while that in period one is $(1+r')(hRL_0 + cB_0) + (1+r)(1-n)[(1-h)RL_0 + (1-c)B_0]$.

(4) Invest in both human capital and the machine. Then consumption in period zero is $n[(1-h)RL_0 + (1-k-c)B_0]$ while that in period one is $(1+r')(hRL_0 + cB_0) + (1+r^*)kB_0 + (1+r)(1-n)[(1-h)RL_0 + (1-k-c)B_0] + RL_0$.

Wealth levels in (1)a. and (1)b. are necessarily identical when future consumption is discounted at rate r. If $r^*>r$ then (2) dominates (1). The difference between (2) and (1)b. is that consumption in period zero is $nkB_0$ smaller for (2) and consumption in period one is $kB_0(1+r)(1-n) - (1+r^*)kB_0$ larger for (2). The present value of the difference between (2) and (1)b. is then $-kB_0 + \frac{1+r^*}{1+r} kB_0$. If $r^*>r$ then (2) yields higher wealth than (1)b. and thereby dominates it. But the difference between wealth levels is overstated by $r^*kB_0$ since the opportunity cost of funds is $rkB_0$. I.e., $rkB_0$ is the normal flow return on previous income and only $(r^*-r)kB_0$ is profit which should be counted when looking at wealth differences across individuals. The point is especially relevant in the context of wage v. property income. An individual who earns \$100 in wages in each period and who purchases a \$50 security during period zero will have a larger observed income flow in period one than the same individual who consumed the entire \$100 in period zero. Yet both clearly have the same wealth. The "profit" on the purchase of security is zero since $r^*=r$. Although observed income is higher in period one when the security is purchased, the cost is reduced consumption in period zero, the market value of which is identical to the period one return. If, on the other hand, the \$50 were invested in an asset that yielded, on average, $r^*>r$, $50(r^*-r)$ is "profit" and should be counted as actual income in period one.[2]

Treatment of investment in human capital is perfectly analogous. The only important difference is that a major part of investment costs here take the form of foregone earnings. Since wage income in period zero is $hRL_0$ lower as the result of the investment in human capital, $(hRL_0)(1+r)$ of income in period one should be counted as actual income in the wealth calculation. To this again, should be added the inframarginal return to the investment, $(r'-r)(RL_0 + cB_0)$. Another way to say this is that wage wealth is the present value of the permanent income flow $RL_0$ plus profits from investment in human capital.

Thus, there is no simple way to add property income to wage income to obtain a measure of true income. Part of the property income simply reflects the normal return on previously earned (and counted) wages and property income. return above "normal" return can be computed once r is known to obtain an accurate measure of wealth. Similarly, current wage income includes the return to previous investment in human capital, part of which is "normal" and part of which $(r'- r)$ is inframarginal and super-normal. It is only the latter component that should be included in the computation of actual income. Thus, the discounted flow of observed earnings overstates true wage wealth by

$$\frac{rD_0}{(1+r)} + \frac{r(D_0+ D_1)}{(1+r)^2} + \frac{r(D_0+ D_1+ D_2)}{(1+r)^3} +\ldots+ \frac{r(D_0+ D_1+\ldots+ D_{T-1})}{(1+r)^T}$$

where $D_\tau$ is the amount of direct outlay for human capital acquisition in period $\tau$. Since higher wealth individuals tend to invest in more human capital, $D_\tau$ tends to be larger for high wealth individuals and the overstatement therefore larger.

## II. Human Capital and Wealth-Augmentation

Can the acquisition of human capital augment an individual's wealth? In a trivial sense, the answer is no. The situation is analogous to a firm that is granted a government-enforced monopoly. The entire value of the monopoly profits can be capitalized and attributed as rent to the franchise right. A similar treatment of human capital profits can be made. Birth is essentially the granting of a franchise which allows a maximum-wealth level when optimal (profit-maximizing) investment behavior is followed. Wealth can be destroyed if the individual does not act to maximize it; but in a trivial sense it can never be created by human capital.[3]

Yet this point is a semantic one. A more substantive one still underlies most of the empirical work on human capital. Becker (1967) and more recently Rosen (1973), have pointed out that the "rate of return" as obtained from cross-sectional analyses tells nothing about the marginal return to additional acquisition of human capital (usually schooling). Rosen is more specific. He argues that the Mincer (1970) semi-log human capital function is inconsistent with optimization on the part of the individual. The point that Rosen makes relates to the assumption in Becker (1964) of a constant average rate of return over the life cycle. Rosen suggests that if this rate were constant, one would observe bang-bang dynamics in the human capital accumulation process. The analysis has implications for the relationship between wealth and human capital across individuals. It is also best discussed in the terms of Mincer's (1970) derivation of the human capital function, which although equivalent to the earlier Becker (1964) specifications, lends itself more readily to analysis of wealth.

Mincer's procedure begins by defining an individual's labor wealth as

(1) $$R_S = \int_S^N W_S e^{-rt} dt$$

Where S is the highest level of schooling completed, r is the discount rate, N is the retirement age and $W_S$ is the (per period) market wage of an individual with S years of schooling. (Define period zero to be the year that school attendance begins and assume that the total cost of attending school consists of foregoing all wages during the period of attendance.) Wealth must be equalized across individuals, it is argued, or individuals would alter their school attendance until the market adjusted wages in such a way as to equalize wealth. Thus, an individual who acquired no schooling would have wealth:

$$R_0 = \int_0^N W_0 e^{-rt} dt \tag{2}$$

Since $R_0$ must equal $R_S$,

$$\int_0^N W_0 e^{-rt} dt = \int_S^N W_S e^{-rt} dt \tag{3}$$

or upon integration,

$$W_S = W_0 e^{rS} \tag{4}$$

so that

$$\ln W_S = \ln W_0 + rS \tag{5}$$

Note that up to this point, nothing has been said about rates of return. So far r is simply the discount rate. This becomes a rate of return only when it is added that if the marginal rate of return were to exceed r, individuals would continue to invest and would necessarily acquire additional schooling until the marginal rate of return were equal to r.[4]

Thus the inference that r is a rate of return requires that equation (5) be an equilibrium condition.[5] The model, by construction, assumes that individuals specialize either in work or in investment in human capital so that the observed wage is the actual wage which equals the workers's VMP. (When it is acknowledged that some compensation takes the form of on-the-job training, the observed wage is no longer the appropriate unit of analysis. This is the case for the reason that the observed payment understates true payment and that the true wage rate increases over the lifetime so that $W_S$ represents a weighted average of discounted wages.)

The assumption that $R_S$ equals $R_0$ for all individuals was defended by claiming that the result is the natural outcome of competition between workers. This implies one of the two things. On the one hand, we may assume that all workers are alike. If so, the only situation under which one would observe different levels of schooling attainment by different individuals is one where the marginal rate of return was equal to the discount rate at all levels of schooling. This implies that the marginal rate of return is equal to the average rate of return so that investment in human capital does not alter an individual's wealth. If this were not the case, all individuals would choose the same amount of schooling. This can be more easily seen by looking at the following diagrams:

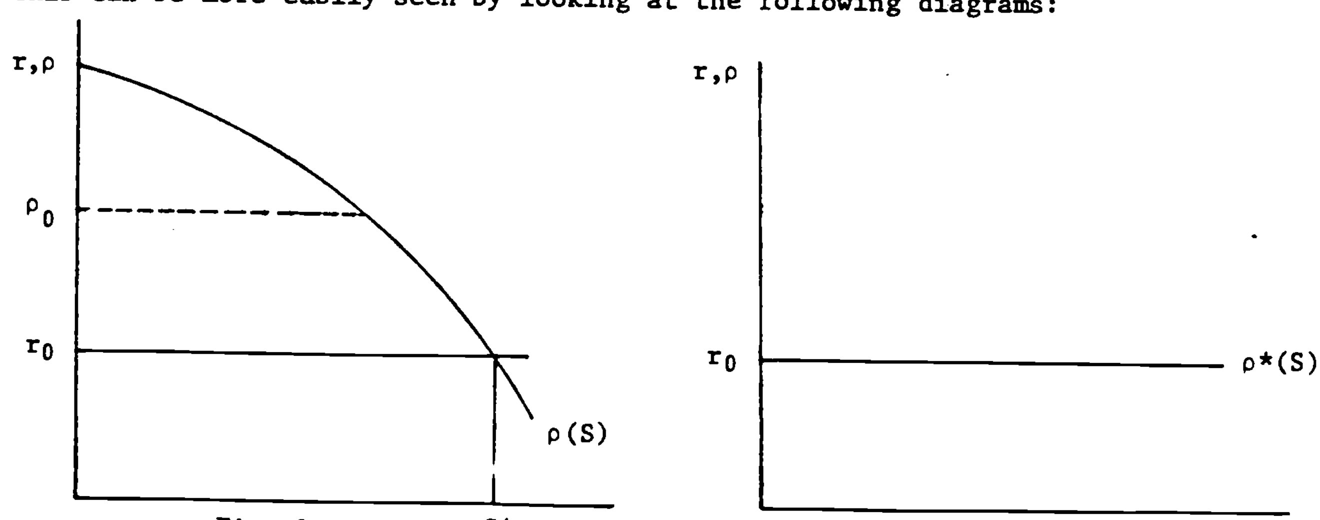


Fig. 1

Fig. 2

If the rate of return function were of the shape illustrated in Fig. 1, all individuals would invest in S* of schooling, the marginal rate of return would equal $r_0$ and would lie below the average rate of return, $\rho_0$. Only when the rate of return function is like $\rho^*(S)$ in Fig. 2 are individuals indifferent between different levels of schooling. Here, the marginal rate of return equals the average rate of return which equals $r_0$. (If $\rho^*(S)$ were horizontal above $r_0$, an infinite amount of schooling would be acquired driving down the rate of return to schooling until it equaled $r_0$. If $\rho^*(S)$ were horizontal below $r_0$, no schooling would be acquired.)[6] Investment in human capital under these circumstances is simply a way to finance different consumption time preferences. Individuals who preferred to consume more later (say, because their family size was expected to increase) could invest in human capital as a form of savings which returns present income compounded at the market rate of interest at some date in the future. However, the individual could just as easily work now at the wage $W_0$ and put his current income in the bank. Starting in period S he would then be able to withdraw an amount equal to the difference between $W_S$ and $W_0$ until the period N so that his received "income" would equal $W_S$. Thus, under the assumption of equality across individuals, investment in human capital is identical to saving. There is no need to analyze it any differently than one would analyze the purchase of a security. "Investment in human capital is not investment in any wealth increasing sense, but rather a way to save. The r that is estimated in (5) is not only the rate at which one borrows and lends in the securities market, but is also the internal borrowing and lending rate. If the assumption of worker equality is maintained, human capital becomes impotent in the usual sense. It has no effect on wealth and merely offers another way to express time preference. There are no infrmarginal profits to be taken by investing in human capital.

On the other hand, one could explain different levels of schooling acquisition by arguing that individuals have different abilities. One can imagine a situation in which some workers are more adept at manual labor jobs requiring no schooling while others have a comparative advantage in the performance of tasks which require schooling. Both groups might have equal wealth, but this does not imply that investment in human capital does not produce wealth. Since individuals in both groups maximize their wealth levels, by investing in fewer years of schooling, the academic group would suffer a wealth decrease. Similarly, an investment in additional schooling by a blue-collar individual results in a wealth decrease on his part. In this situation, investment in human capital yields inframarginal rents. Although the individual earns only the normal return r for a final unit of human capital, he captures $r' > r$ for previous units and thereby produces wealth through his investment. Implicit here is the fact that the unschooled wage rate obtainable by the academic population is less than the unschooled wage rate of the manual labor group. Similarly, $W_S$ for the school-oriented individuals exceeds the potential $W_S$ for blue-collar workers. It is clear, however, that $W_S$ earned by the academics exceeds $W_0$ earned by the laborers since the former spend less time in the labor force and both groups by assumption have the same wealth. An obvious implication of this kind of reasoning is that by requiring that an individual attend more school than he would otherwise choose, his wealth necessarily decreases. (Nor can one argue that even though his wealth decreases, his child's increases through externalities which cause the child to move from the blue-collar to academic group. Even if this were accomplished, nothing is gained since, by assumption, both groups have equal wealth.)

At this point, however, it is necessary to consider the original assumption. Wealth was equalized across individuals by competition in the labor and occupation market. The above reasoning, however, was based on the notion of inequality between individuals, i.e., of non-competing groups. But if that is the case, there is no reason to assume that wealth levels are equal across the two groups. Once one allows wealth to differ between groups, the Mincer derivation of the semi-log wage function disintegrates.

Let us summarize the discussion to this point: One is able to derive the semi-log wage function by assuming that wealth is equalized across individuals. One observes, however, that different individuals acquire differing amounts of schooling over their lifetimes. If all individuals have equal ability and cpportunity, differences in schooling simply reflect different consumption time preferences. No wealth is produced by school attendance and "investment" in human capital is not investment at all, but rather a way to save. If, on the other hand, individuals are not equal in ability, schooling levels may differ. However, then the justification for the assumption of equal wealth across individuals breaks down and so, then, does the derivation of the semi-log wage function. Nor is the point specific to the semi-log wage function. If cross-sectional analysis is appropriate, it must be the case that individuals have the same opportunities or it is inappropriate to compare two values of a particular variable. If individuals differ in values of that variable, either they face different opportunities, in which case they are non-competing groups, or they are indifferent between the two values, in which case wealth cannot differ by values of that variable. In the former case, as Rosen points out, the estimates of rate of return (or any schooling coefficient) is a not very informative average across different individuals with different opportunities. In the latter case, it simply is an estimate of the discount rate.

An alternative rationale:

Longitudinal data sets (as the National Longitudinal Survey and Michigan-Income Dynamics Study) allow an alternative derivation of the semi-log wage function. Let us start with the fact that observed wages grow over an individual's lifetime. Approximate this growth by a general form of growth equation of the form

$$(6) \qquad W_\tau = AW_t e^{[\gamma(t,\ldots)](\tau-t)}$$

where t is age at the beginning of the period and $\tau$ is age at the end. A is a shift parameter which relates to the chronological period and is therefore invariant across individuals. The fact that the growth rate itself, $\gamma$, is a function of life-cycle time means that long-term wage growth may assume any shape and is not restricted to exponential growth. When an individual acquires additional human capital through schooling his marginal product rises. This tends to cause wages to grow. It is then reasonable to postulate that in its simplest form

(7) $\gamma = \alpha_0 + \alpha_1(t) + \alpha_2(S_\tau - S_t)$

where S is the level of schooling in period $\tau$ and $S_t$ is the level of schooling in period t. $\alpha_1(t)$ represents the effect of age itself on wage growth, i.e., it is the term responsible for shifting the wage growth function over the lifetime (see Lazear (1975), for a more complete discussion of the rationale). Upon substitution and after taking the log of both sides of (6), one obtains

(8) $\ln W_\tau = \ln A + \ln W_t + \alpha_0 + \alpha_1(t) + \alpha_2(S - S_t)$,

which is essentially the same equation as the Mincer semi-log wage function.

If the discrete investment construct of the Mincer model is maintained, $\alpha_2$ must exceed r in order for individuals to be indifferent between investment in human capital and the purchase of a security. This can be seen by considering Figure 3.

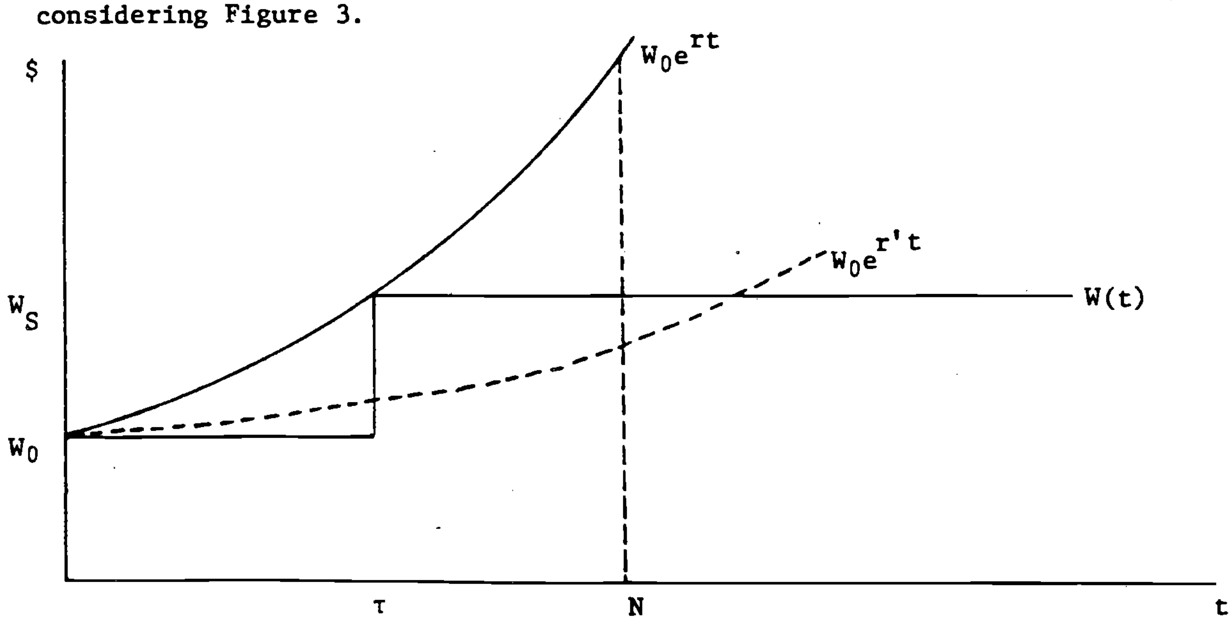


Fig. 3

The present value of the stream $W_0e^{rt} = {}_0\int^N (W_0e^{rt})e^{-rt}dt = NW_0$ necessarily exceeds the present value of the stream

$$W(t) = {}_0\int^\tau W_0e^{-rt}dt + {}_\tau\int^N W_Se^{-rt}dt \tag{9}$$

There exists, however, a discount rate $r' < r$ such that

$$ {}_0\int^\tau W_0e^{-r't}dt + {}_0\int^N W_Se^{-r't}dt = {}_0\int^N (W_0e^{r't})e^{-r't}dt \tag{10}$$

since the right hand side equals the constant $NW_0$ and the left hand side, which is less than $NW_0$ for $r' = r$ is equal to $\tau W_0 + (N-\tau)W_S > NW_0$ for $r'=0$. Continuity of the left-hand side guarantees that for some $r'$, (10) holds with equality. Thus if $\alpha_2 = r$ so that wage growth between 0 and $\tau$ equals $r$, the discount rate must not exceed $r' < \alpha_2$ in order for the individual to undertake the investment.

The interpretation of $\alpha_2$ is important and differs from the usual one of a marginal rate of return. The $\alpha_2$ coefficient is an average rate of growth which must exceed the marginal rate of return. This interpretation, unfortunately, says less than did the marginal rate of return concept. The latter tells us the value of an additional year of investment in schooling; the former does not. The average growth rate tells us about inframarginal returns as well as marginal ones. Unless we are willing to specify a rate of returns function which gives us the relationship between the marginal and average rates (essentially the approach employed by Rosen (1973) and by Brown (1974)), one cannot infer the value of an additional year of schooling from the estimates.

The tale is not quite as grim as it at first appears. Equation (8) allows us to mitigate the effects of this problem by estimating the coefficients in a growth context rather than by considering wage levels. Rewrite (8) as

$$\ln W_\tau - \ln W_t = \ln A + \alpha_0 + \alpha_1(t) + \alpha_2(S_\tau - S_t) \tag{11}$$

As $\tau - t$ becomes small, the difference between the marginal and average rates of growth shrink. Thus, by using longitudinal data across individuals one may obtain estimates which are fairly close approximations for the marginal effect of schooling on wages.

There is still the difficulty, however, that if the individual is currently investing in on-the-job training, the observed wage does not reflect the true wage. However, as long as the proportion of compensation received in money wages does not change greatly between t and $\tau$, the growth form of wage function which considers only observed wage differences will eliminate most of this bias.

In the same way that lumping years of schooling and wage growth together reduces the information content of the model, combining school attendance years with non-school attendance years confounds effects as well. Consider Figure 4:

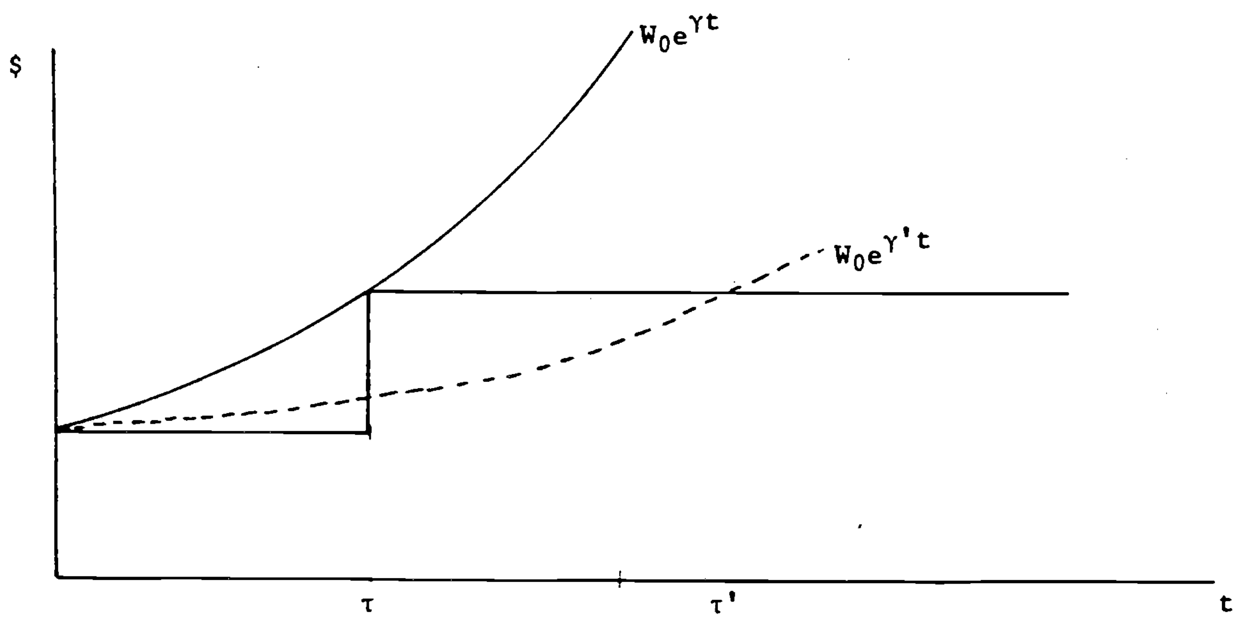


Fig. 4

It is much more accurate to say that the average growth rate between 0 and $\tau$ is $\gamma$ and between $\tau$ and $\tau'$ is zero than to say that the average growth rate between 0 and $\tau'$ is $\gamma'$, which tends to understate the effects of schooling.

There is an additional justification for using the longitudinal wage growth specification. If "ability" affects wage rates, it enters the wage level function as an omitted variable. However, as long as the effect of ability on wages is invariant over time (chronological and life-cycle), looking at wage growth differences out the ability affect so that the estimates do not suffer from omitted variable bias.[7] Finally, as shown in the appendix, the wage growth specification is not theoretically inconsistent with optimization although it may be empirically. I.e., the actual estimates may violate stability conditions.

The important difference between the Becker-Mincer derivation and wage growth derivation of the semi-log wage function is that the latter permits wealth to be augmented by investment in human capital. In addition, it allows for externalities that result in the increased wealth of other persons - the individual's children for example.[8] If this were the case, children would be willing to compensate parents for obtaining additional units of education. If it were difficult to arrange the intergenerational transfer, parents would underinvest in their own schooling, thereby providing justification for government subsidization of education.[9]

## Summary and Conclusion

This paper considers two related theoretical issues concerning the relationship between wealth and human capital. The first part of the paper is devoted to a discussion of measures of wealth inequality. It is argued that there is no simple way to add discounted property income flows to discounted wage flows to obtain a measure of wealth because a part of the former may simply reflect the normal returns on the latter. In fact, even if property income were zero throughout, the discounted stream of earnings would tend to overstate the individual's wealth since part of the earnings are the normal return to direct outlays made in order to produce the earnings stream.

The second part of the paper discusses the role of human capital in wealth-augmentation. It is argued that traditional human capital theories either imply that human capital is a non-wealth increasing security, simply yielding the market rate of interest, or that the derivation of the usual empirical specification is inconsistent. The former is true if all individuals have the same ability and opportunities; the latter is true when abilities differ so that individuals become members of non-competing groups.

Finally, an alternative rationale for the semi-log wage function is offered. This specification is appropriate only when the analysis is conducted with longitudinal data so that the estimates result from observing the same individual during different points in the life-cycle.

FOOTNOTES

[1]A related point is made by Smith (1975). There it is argued that as the result of optimal life-cycle savings, current assets cannot be used as a proxy for wealth in labor supply functions.

[2]Note that $r^* > r$ should not merely reflect a risk premium so that the corrected riskless market rate of return is equal to r. Instead, it must reflect a true return to disequilibrium; it is inframarginal return that individual A can capture, but that others cannot. This can be the result of specialized information or differential ability. The "profit" on the investment is simply a rent to these specialized factors. See Friedman (1955) and Levhari and Weiss (1974) for related discussion of this point.

[3]See Pesek and Saving (1967) for a related discussion with respect to creation of wealth by the monetary system.

[4]Assume that age-earnings profiles cross only once so that the rate of return is well-defined. See J. Hirschliefer (1970), pp. 51-57, for a complete discussion of the validity of the rate of return rule.

[5]Equilibrium also requires that the wage rate $W_S$ be a "permanent" wage.

[6]An implicit assumption is that S is the appropriate unit of analysis. In this model and in those generally used empirically, no variation in the rate of schooling acquisition is permitted. Thus, the condition that an individual acquires either one or zero units of schooling per year (or corresponding fractions) is imposed.

It should also be mentioned that the curves drawn in Figs. 1 and 2 do not exhaust the entire possibility of functional shapes. Consider a negatively sloped marginal rate of return function and an inverted U-shaped average return function. If the peak of the inverted U were at S=r, investment in S* would have no effect on wealth <u>and</u> all individuals would have the same level of schooling. What we seek to explain here, however is how one might observe levels of schooling differing across individuals.

[7]If ability affects wage growth per se, i.e., is part of $\gamma$, obviously nothing is gained by the wage growth formulation.

[8]It was argued that under the non-wealth increasing formulation, there could be no spill over effects of education on other individuals because their wealth levels were always equalized.

[9]This question is considered in greater detail in Lazear (1975a).

APPENDIX

In this section the conditions, under which the wage growth specifications is consistent with optimization, are derived. Start with

(A1) $$W_{t+1} = AW_t e^{\alpha_0 + \alpha_1 t + \alpha_2 \Delta S_t}$$

then

(A2) $$\frac{\partial W_{t+1}}{\partial \Delta S_t} = \alpha_2 W_{t+1}$$

Assume also that

(A3) $$\frac{\partial W_{t+i}}{\partial \Delta S_t} = \alpha_2 W_{t+1} \qquad \text{for } i = 2,\ldots,N-t$$

Then the marginal return to $S_t$ is

(A4) $$\int_1^{N-t} (\alpha_2 W_{t+1}) e^{-r\tau} d\tau = MR$$

or

(A5) $$\frac{\alpha_2 W_{t+1}}{r} [ e^{-r} - e^{-r(N-t)} ]$$

If the marginal cost of an additional year of schooling consists only of foregone earnings, the condition that marginal cost equals marginal returns implies that

(A6) $$W_t = \frac{\alpha_2 W_{t+1}}{r} [ e^{-r} - e^{-r(N-t)} ]$$

or substituting from (A1)

(A7) $$\frac{1}{A} e^{-(\alpha_0 + \alpha_1 t + \alpha_2 \Delta S_t)} = \frac{\alpha_2}{r} [ e^{-r} - e^{-r(N-t)} ]$$

Taking logs and rearranging terms yields

(A8) $$\Delta S_t = \frac{1}{\alpha_2}\left[-\ln \alpha_2 - \ln (e^{-r} - e^{-r(N-t)}) + \ln r - (\ln A + \alpha_0) - \alpha_1 t\right]$$

Three conditions must be met by (A8) in order for (A1) to be consistent. First, as the result of the discussion on page 12, $\alpha_2$ must exceed r. Second, $\Delta S_t$ must be non-negative. Finally, $\frac{\partial \Delta S_t}{\partial t}$ must become negative eventually, i.e., school attendance reaches zero at death.

Nothing in (A8) violates the first requirement. Whether or not $\alpha_2$ exceeds r is an empirical matter. In previous work, with longitudinal data, (Lazear (1975a)), I estimated $\alpha_2$ to be $\underset{(.0363)}{.1467}$ for men 14 - 24 years old in 1966. This estimate is unlikely to violate the condition that $\alpha_2 > r$.

The second condition, that $S_t$ be non-negative depends upon the sign of the term bracketed in (A8). Again, using estimates from Lazear (1975a), one obtains $\Delta S_t < 0$ for the mean individual in the sample used (National Longitudinal Survey - Young Men, 1966-69) if r = .10. This casts doubt on the acceptability of the wage growth formulation as a justification for the semi-log wage function.

Finally, upon differentiating (A8) with respect to t we obtain

(A9) $$\frac{\partial \Delta S_t}{\partial t} = \frac{1}{\alpha_2}\left[\frac{-re^{-r(N-t)}}{e^{-r} - e^{-r(N-t)}} - \alpha_1 - t\,\frac{\partial \alpha_1}{\partial t}\right]$$

The first term is clearly negative. $\alpha_1$, however, is also negative (wage growth slows down over the life cycle) but $\frac{\partial \alpha_1}{\partial t}$ is positive (the difference between being 16 and 17 years old is more significant for wage growth than is the difference between being 40 and 41). Thus, the sign of $\frac{\partial \Delta S_t}{\partial t}$ is indeterminate at any point. However, as t increases, $\alpha_1$ becomes increasingly close to zero (since $\frac{\partial \alpha_1}{\partial t} > 0$ and $\alpha_1 < 0$) so that one expects that eventually, $\frac{\partial \Delta S_t}{\partial t} < 0$ as required.

REFERENCES

Becker, G. Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education. (New York: NBER, 1964).

_____. Human Capital and the Personal Distribution of Income. W.S. Woytinski Lecture #1, University of Michigan, 1967.

Ben-Porath, Y. "The Production of Human Capital and the Life Cycle of Earnings," JPE (August, 1967) 352-365.

Brown, C. "Optimal Self-Investment and the Earnings of Young Workers," unpublished doctoral dissertation, Harvard University, 1974.

Friedman, M. "Comment on Survey of Empirical Evidence on Economies of Scale, Caleb Smith", Business Concentration and Price Policy (Princeton: NBER, 1955).

Griliches, Z. and W. Mason. "Education, Income and Ability," JPE (May-June, 1972), S74-S103.

Haley, W. "Human Capital: The Choice Between Investment and Income," AER, (December, 1973) 929-944.

Hanoch, G. "An Economic Analysis of Earning and Schooling," Journal of Human Resources (Summer, 1967, 2) 310-329.

Hirschleifer, J. Investment, Interest, and Capital (Englewood Cliffs, N.J.: Prentice-Hall, 1970).

Johnson, T. "Returns from Investment in Human Capital," AER (1970) 546-560.

Lazear, E. "Age, Experience and Wage Growth," University of Chicago, Center for Mathematical Studies in Business and Economics, Report #7502 (January, 1975).

_____. "Intergenerational Externalities: The Case of Schooling," Unpublished manuscript. (July, 1975. [1975a]).

Levhari, D. and Y. Weiss. "The Effect of Risk and Investment in Human Capital," AER, LXIV, 6 (December, 1974) 950-963.

Mincer, J. "The Distribution of Labor Incomes: A Survey," J.E. Lit., 8 (March, 1970) 1-26.

_____. Schooling, Experience and Earnings (New York: NBER, 1974).

Pesek, Boris and Thomas R. Saving. Money, Wealth and Economic Theory. (New York: Macmillan, 1967).

Rosen, S. "Income Generating Functions and Capital Accumulation," HIER Discussion Paper Number 306, Harvard University, June, 1973.

Schultz, T.W. The Economic Value of Education. (Princeton: Princeton University Press, 1962).

Welch, F. "Black-White Differences in Returns to Schooling," AER (December, 1973) 893-907.