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Hybrid Bit-to-Symbol Mapping for Spatial Modulation

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***Abstract*—In spatial modulation (SM), the information bit stream is divided into two different sets: the transmit antenna index bits (TA-bits) and the amplitude and phase modulation bits (APM-bits). However, the conventional bit-to-symbol mapping (BTS-MAP) scheme maps the APM-bits and the TA-bits independently. For exploiting their joint benefits, we propose a new BTS-MAP rule based on the traditional 2-D Gray mapping rule, which increases the Hamming distance (HD) between the symbol pairs detected from the same transmit antenna (TA) and simultaneously reduces the average HD between the symbol pairs gleaned from different TAs. Based on the analysis of the distribution of minimum Euclidean distance (MED) of SM constellations, we propose a criterion for the construction of a meritorious BTS-MAP for a specific SM setup, with no need for additional feedback links or extra computational complexity. Finally, Monte Carlo simulations are conducted for confirming the accuracy of our analysis.**

***Index Terms*—Gray mapping, hamming distance (HD), multiple-input–multiple-output (MIMO), spatial modulation (SM).**

I. Introduction

Spatial modulation (SM) is a new 3-D hybrid modulation scheme conceived for multiple-input–multiple-output (MIMO) transmission, which exploits the indexes of the transmit antennas (TAs) as an additional dimension invoked for transmitting information, apart from the classic 2-D amplitude and phase modulation (APM) [1]–[5]. In SM, the information bit stream is divided into two different sets: the bits transmitted through the TA indexes and the APM constellations [6]. For simplicity, we refer to these two sets of bits as TA-bits and APM-bits.

In conventional 2-D APM constellations, the choice of the bit-to-symbol mapping (BTS-MAP) rule plays an important role in determining the achievable bit error ratio (BER) performance [7]. For example, an optimized BTS-MAP is capable of providing a low error floor in both bit-interleaved coded modulation and in its iterative decoding and demodulation aided counterpart (BICM-ID) [8]. It is widely known that, for equally likely and statistically independent 2-D APM constellations, Gray mapping is optimal in terms of minimizing the BER [7]. In general, the optimal BTS-MAP depends on the specific geometry of the signal constellation, particularly on the location of the phasors separated by the minimum Euclidean distance (MED) [9]. Compared with APM schemes, SM has a higher constellation dimension and a different distribution of MED due to its hybrid modulation principle. Hence, the classic Gray mapping proposed for 2-D constellations will no longer achieve the optimal BER in fading MIMO channels.

Recently, the wide-ranging studies disseminated in [1], [4], [6], and [10]–[14] have characterized some of the fundamental properties of SM, such as its energy efficiency [12], [13] and the effects of power imbalance [14]. In these contributions, a general framework was established for the BTS-MAP rule of SM [1], where the APM-bits are mapped according to the classic Gray mapping rule, whereas the TA-bits are mapped to the active TA index. Due to its intuitive nature and low complexity, this rule has been considered in diverse SM-based systems [15]–[18]. For example, in [15] and [16], this general framework was developed for an arbitrary number of TAs and hence strikes a flexible tradeoff in terms of the attainable BER performance and capacity. In [17] and [18], this philosophy has been extended to trellis coded modulation aided SM systems for the sake of achieving reliable digital transmission. However, this classic BTS-MAP scheme maps the APM-bits and the TA-bits to symbols independently and hence may sacrifice the classic Gray-coded benefits. Moreover, the MED distribution of SM was not considered in the design process of the conventional BTS-MAP scheme, which is a measure of separation between two constellation points, and as a result, it has a dominant influence on the BER.

Against this background, the novel contributions of this treatise are as follows.

- We propose a new BTS-MAP rule based on the classic Gray-coded principles for exploiting the interaction of the TA-bits and the APM-bits in fading channels, which differs from the conventional BTS-MAP, since it reduces the average Hamming distance (HD) between the SM symbol pair of the different TAs, and simultaneously, it increases the HD between the symbol pair of the same TA.
- Based on the analysis of the distribution of the MED, we propose a criterion for the construction of a beneficial BTS-MAP rule for a specific SM-MIMO setup, which is achieved without the need for an additional feedback link and with no extra computational complexity. Finally, performance comparisons with the conventional BTS-MAP scheme of [1] are provided for different constellation sizes and signal-to-noise ratios (SNRs).

Notation: $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^{\mathrm{H}}$ denote conjugate, transpose, and Hermitian transpose, respectively. The probability of an event is represented by $P(\cdot)$. Furthermore, $\|\cdot\|$ and $|\cdot|$ denote the Euclidean norm and magnitude operators, respectively; whereas $\varepsilon(\mathbf{x},\mathbf{y})$ is the HD between the binary strings $\mathbf{x}$ and $\mathbf{y}$. N_t is the number of TAs, and M is the size of the APM constellation adopted. Let $\mathbf{b}_i^m$ be the transmit bit vector mapped to the SM symbol $\mathbf{x}_i$ and the APM symbol s_i^m, which corresponds to TA i, whereas $\mathbf{b}_j^k$ is the transmit bit vector mapped to the SM symbol $\mathbf{x}_j$ related to TA j and the APM

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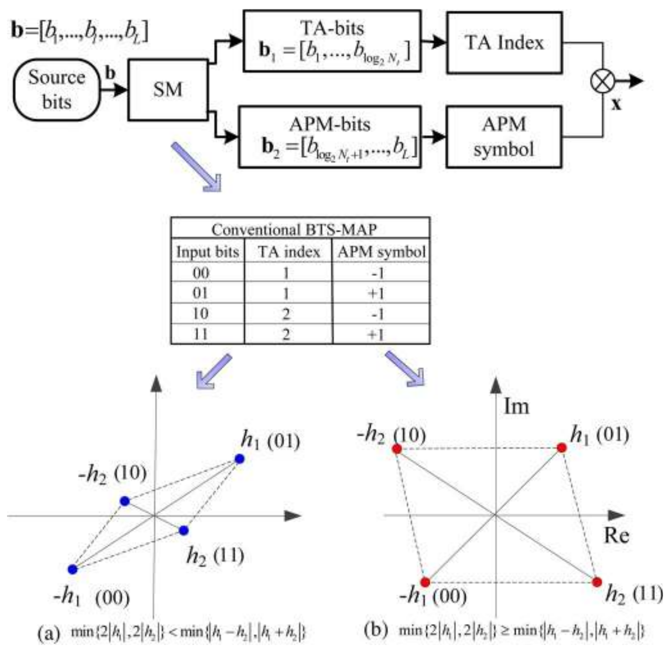


Conventional BTS-MAP		
Input bits	TA index	APM symbol
00	1	-1
01	1	+1
10	2	-1
11	2	+1

Fig. 1. Conventional BTS-MAP of SM: an example for the BPSK-modulated (2×1)-element SM. (a) MED encountered on the same TA, where the conventional BTS-MAP is optimal. (b) MED encountered on different TAs, where the conventional BTS-MAP is suboptimal.

symbol s_j^k. The average HD between the symbol pair gleaned from different TAs, termed as HDD, is defined as $\text{HDD} = (1/(N_t(N_t - 1)M^2)) \sum_{i\neq j, i,j\in\{1,\ldots,N_t\}} \varepsilon(\mathbf{b}_i^m, \mathbf{b}_j^k)$. Moreover, the average HD between the symbol pair detected from the same TA, termed as HDS, is defined as $\text{HDS} = (1/(N_t M(M-1))) \sum_{i\in\{1,\ldots,N_t\}} \varepsilon(\mathbf{b}_i^m, \mathbf{b}_i^k)$. For a fixed MIMO channel $\mathbf{H}$, $d_{\min}^{\text{Same}}(\mathbf{H})$ is the MED between the symbol pair of the same TA, $d_{\min}^{\text{Diff}}(\mathbf{H})$ is the MED between the symbol pair of different TAs, and the overall MED of a specific SM is $d_{\min} = \min\{d_{\min}^{\text{Same}}(\mathbf{H}), d_{\min}^{\text{Diff}}(\mathbf{H})\}$.

II. Conventional Bit-To-Symbol Mapping Rule for Spatial Modulation

Consider a MIMO system having N_t transmit and N_r receive antennas. The $(N_r \times N_t)$-element channel matrix $\mathbf{H}$ is used for modeling a flat-fading channel with elements having complex Gaussian distributions with unit variance. We not only focus our attention on the independent and identically distributed Rayleigh case but discuss Nakagami-m channels as well. Let $\mathbf{b} = [b_1, \ldots, b_L]$ be the transmit bit vector in each time slot, which contains $L = \log_2(N_t M)$ bits. As shown in Fig. 1, the input vector $\mathbf{b}$ is divided into two subvectors of $\log_2(N_t)$ and $\log_2(M)$ bits, which are denoted by $\mathbf{b}_1$ and $\mathbf{b}_2$, respectively. The bits in the subvector $\mathbf{b}_1$ are used for selecting a unique TA index q for activation, whereas the bits in the subvector $\mathbf{b}_2$ are mapped to a Gray-coded APM symbol s_l^q. Hence, the resultant SM symbol $\mathbf{x} \in \mathbb{C}^{N_t\times 1}$ is formulated as [6]

$$\mathbf{x} = s_l^q \mathbf{e}_q \tag{1}$$

where $\mathbf{e}_q (1 \le q \le N_t)$ is selected from the N_t-dimensional standard basis vectors.

To expound a little further, we exemplify the binary phase-shift keying (BPSK)-modulated (2×1)-element SM in Fig. 1. As shown in Fig. 1, $L = 2$ input bits are divided into two single-bit streams, and then, the first bit determines the activated TA (1 or 2), whereas the second single bit generates the classic BPSK symbol ($+1$ or -1). The aforementioned BTS-MAP method considers the APM-bits and the TA-bits independently and hence facilitates a simple implementation of SM.

However, this BTS-MAP method may result in a Gray-coding penalty [9], which degrades the BER. Specifically, for the example of BPSK-modulated (2×1)-element SM in Fig. 1, assuming that the channel matrix is $\mathbf{H} = [h_1, h_2]$, there are four received constellation points denoted by h_1, h_2, $-h_1$, and $-h_2$; and we investigate two scenarios: 1) the MED is $d_{\min} = 2|h_2|$; and 2) the MED is $d_{\min} = \min\{|h_1 - h_2|, |h_1 + h_2|\}$. Note that scenario (1) corresponds to the case when the MED is encountered on the same TA (TA 2), whereas scenario (2) corresponds to the case when the MED is encountered on different TAs (between TAs 1 and 2). For scenario (1), the most likely erroneously detected pattern is given by the nearest constellation points $(-h_2, h_2)$. If the conventional BTS-MAP is adopted, these points differ only in a single bit, i.e., by the difference between the bits "10" and "11" in Fig. 1(a), whereas other adjacent constellation points may differ in more than one bits. This mapping rule obeys the concept of Gray mapping, where the probability of having a bit error is minimized; hence, it has an optimal performance in this specific scenario. However, for scenario (2), it is suboptimal, because the nearest constellation points become $(-h_1, h_2)$, which differ in two bits. This implies that more than one bit errors are associated with the most likely error pattern of $(-h_1, h_2)$; hence, a performance penalty will occur. This indicates that the MED distribution should be considered in the design of BTS-MAP.

III. Proposed Bit-To-Symbol Mapping for Spatial Modulation

A. Principle of the Proposed BTS-MAP

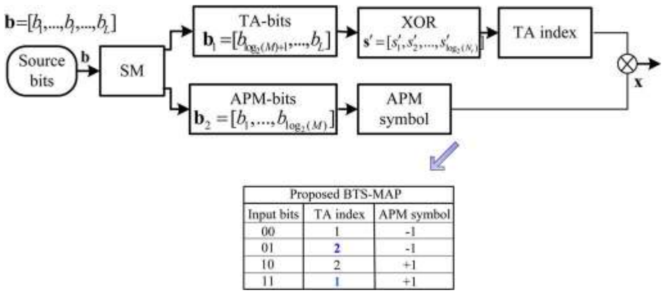
We found in Section II that the design of the BTS-MAP scheme depends on the MED, which may be achieved between the symbol pair gleaned from the same TA or between the symbol pair from different TAs. The conventional BTS-MAP rule does not consider this distribution of the MED, and hence, it becomes suboptimal for some channel scenarios, as illustrated in Fig. 1(b).

For exploiting the mapping gain of SM, we propose a new BTS-MAP scheme based on traditional Gray-coded modulation. To be specific, similar to the conventional BTS-MAP of SM, the L-bit input vector $\mathbf{b} = [b_1, \ldots, b_L]$ is divided into a pair of subvectors $\mathbf{d} = [d_1, \ldots, d_{\log_2(M)}] = [b_1, \ldots, b_{\log_2(M)}]$ and $\mathbf{s} = [s_1, \ldots, s_{\log_2(N_t)}] = [b_{\log_2(M)+1}, \ldots, b_L]$. As a result, the transmit bit vector can be represented as $\mathbf{b} = [\mathbf{d}, \mathbf{s}]$. Then, the first subvector $\mathbf{d}$ is mapped to an APM symbol s_l^q, rather than the TA index in conventional BTS-MAP. Then, the subvector $\mathbf{s}$ is transformed to a new input vector by using the bit-by-bit XOR operation with jointly considering the last APM-bit, which is represented as

$$\begin{aligned}\mathbf{s}' &= \left[s_1', s_2', \ldots, s_{\log_2(N_t)}'\right] \\ &= \left[d_{\log 2(M)} \oplus s_1, s_1 \oplus s_2, \ldots, s_{\log_2(N_t)-1} \oplus s_{\log_2(N_t)}\right] \\ &= \left[b_{\log_2(M)} \oplus b_{\log_2(M)+1}, \ldots, b_{\log_2(L)-1} \oplus b_{\log_2(L)}\right]\end{aligned} \tag{2}$$

where "$\oplus$" denotes the XOR operation. Then, the bit vector $\mathbf{s}'$ is mapped to a specific TA index of $q = (\sum_{k=1}^{\log_2(N_t)} 2^{\log_2(N_t)-k} s_k') + 1$, which is used for transmitting the APM symbol.

The rationale of introducing the XOR operation in (2) is to increase the HDS, while decreasing the HDD. To be specific, the last APM-bit $b_{\log_2(M)}$ of $\mathbf{d}$ and the subvector $\mathbf{s} = [b_{\log_2(M)+1}, \ldots, b_L]$ form a new vector $\tilde{\mathbf{s}} = [b_{\log_2(M)}, \mathbf{s}]$ for generating the TA-bits in (2), where we have $b_{\log_2(M)} = 0$ or $b_{\log_2(M)} = 1$. Assuming that there are two subvectors $\mathbf{s}_a$ and $\mathbf{s}_b$ and then provided that two different subvectors



Proposed BTS-MAP		
Input bits	TA index	APM symbol
00	1	-1
01	2	-1
10	2	+1
11	1	+1

Fig. 2. Proposed BTS-MAP for SM.

$\tilde{\mathbf{s}}_a = [0, \mathbf{s}_a]$ and $\tilde{\mathbf{s}}_b = [1, \mathbf{s}_b]$ map to the same TA index, the HD $\varepsilon(\tilde{\mathbf{s}}_a, \tilde{\mathbf{s}}_b)$ is maximized to $\log_2(N_t) + 1$. For example, the vectors $\tilde{\mathbf{s}}_a = [0, 0, 0]$ and $\tilde{\mathbf{s}}_b = [1, 1, 1]$ are mapped to the same TA index for SM using $N_t = 4$ TAs. This result implies that the XOR operation maps the subvector pair $\tilde{\mathbf{s}}_a$ and $\tilde{\mathbf{s}}_b$ having the highest HD to the same TA, and hence, it is capable of increasing the HDS, while decreasing the HDD.

B. *Example and Selection Criterion*

Compared with Fig. 1, we present the new BTS-MAP table in Fig. 2 for the simple BPSK-modulated (2×1)-element SM example mentioned in Section II, where the HDD is reduced to 1 and the HDS is increased to 2. This BTS-MAP may be more suitable for the channel scenario (2) in Fig. 1, because it performs a Gray mapping when considering the adjacent constellation points $-h_1$ and h_2 associated with the MED.

As indicated in Section II, the design of BTS-MAP depends on the distribution of MED, which can be encountered either on the same TA or on different TAs. To be specific, for a given channel matrix $\mathbf{H}$, the MED between the SM symbol pair $(\mathbf{x}_i, \mathbf{x}_j)$ of different TAs is given by

$$d_{\min}^{\text{Diff}}(\mathbf{H}) = \min_{\substack{\mathbf{x}_i, \mathbf{x}_j \in \mathbb{X}, \\ \mathbf{x}_i \neq \mathbf{x}_j, i \neq j}} \|\mathbf{H}(\mathbf{x}_i - \mathbf{x}_j)\|_F = \min_{s_i^m, s_j^k \in \mathbb{Q}, i \neq j} \left\| \left(\mathbf{h}_i s_i^m - \mathbf{h}_j s_j^k \right) \right\| \tag{3}$$

where $\mathbb{X}$ is the set of all legitimate transmit symbols, $\mathbf{h}_i$ is the ith column of $\mathbf{H}$, and s_i^m and s_j^k represent the classic APM constellation points from the set $\mathbb{Q}$. Moreover, the MED between the symbol pair of the same TA is defined as

$$d_{\min}^{\text{Same}}(\mathbf{H}) = \min_{\substack{s_i^m, x_i^k \in \mathbb{Q}, \\ s_i^m \neq s_i^k}} \left\| \mathbf{h}_i \left(s_i^m - s_i^k \right) \right\|. \tag{4}$$

Based on (3) and (4), the probability of the MED $d_{\min}$ encountered on different TAs is defined as $P_{\text{Diff}} = P(d_{\min}^{\text{Diff}}(\mathbf{H}) < d_{\min}^{\text{Same}}(\mathbf{H}))$, whereas the probability of the MED $d_{\min}$ on the same TA is defined as $P_{\text{Same}} = P(d_{\min}^{\text{Diff}}(\mathbf{H}) > d_{\min}^{\text{Same}}(\mathbf{H}))$. To minimize the probability of having a bit error, the HD of the more likely adjacent constellation points having the MED should be lower than that of other points located at a higher distance than the MED. This is also the basic concept of Gray mapping for a 2-D APM constellation. In the proposed BTS-MAP, the HDD is lower than the HDS, and hence, it is more suitable for the specific scenario, when the MED occurs more often in the context of different TAs, which can be expressed as $P_{\text{Diff}} > P_{\text{Same}}$. In other words, when we have $P_{\text{Diff}} > P_{\text{Same}}$ for an SM setup, most of the error events are typically imposed by the SM symbols of different TAs; hence, the minimization of the HD between these nearest points (i.e., the HDD) leads to directly minimizing the probability of bit errors. Based on this observation, the BTS-MAP selection criterion conceived for a specific SM scheme is formulated as follows.

Selection Criterion: If $P_{\text{Diff}} > P_{\text{Same}}$ (or $P_{\text{Diff}} \geq 1/2$) is satisfied by a specific SM, then the proposed BTS-MAP is superior to the conventional one in terms of reducing the BER. Otherwise, the conventional BTS-MAP is preferred.

IV. Theoretical Analysis and Mapping Optimization

Here, the probabilities of P_{Diff} and P_{Same} are derived, which are used as an evaluation criterion for selecting a meritorious BTS-MAP for a specific SM setup. As shown in [4] and [10], the phase-shift keying (PSK) modulation schemes are preferred in SM; hence, PSK is adopted for our theoretical analysis.

A. *M-PSK-Modulated (2×1)-Element SM*

For the M-PSK-modulated (2×1)-element SM, the associated channel matrix can be expressed as $\mathbf{H} = [h_1, h_2]$, where h_1 and h_2 are the fading coefficients of the first and second TAs, respectively, which have zero mean and unit variance. The corresponding MED $d_{\min}^{\text{Diff}}(\mathbf{H})$ of (3) and the MED $d_{\min}^{\text{Same}}(\mathbf{H})$ of (4) are given by

$$\begin{cases} d_{\min}^{\text{Same}}(\mathbf{H}) = \min \left\{ 2\sin\left(\frac{\pi}{M}\right)|h_1|, 2\sin\left(\frac{\pi}{M}\right)|h_2| \right\} \\ d_{\min}^{\text{Diff}}(\mathbf{H}) = \min \left\{ \left| h_1 e^{j\frac{2k\pi}{M}} - h_2 \right|, k = 0, \ldots, M-1 \right\}. \end{cases} \tag{5}$$

Now, we can derive the distribution functions of $d_{\min}^{\text{Diff}}(\mathbf{H})$ and $d_{\min}^{\text{Same}}(\mathbf{H})$. Since the amplitudes of $h_i (i = 1, 2)$ obey the Rayleigh distribution having probability density functions (PDFs) of $f_{|h_i|}(x) = xe^{-(x^2/2)}$, $i = 1, 2$, the cumulative distribution functions (CDFs) of $\eta_i = 2\sin(\pi/M)|h_i|$, $i = 1, 2$ are formulated as

$$F_{\eta_i}(x) = \int_0^{\frac{x}{2\sin\left(\frac{\pi}{M}\right)}} xe^{-\frac{x^2}{2}} dx = 1 - e^{-\frac{x^2}{8\sin^2\left(\frac{\pi}{M}\right)}}, \quad x \geq 0; i = 1, 2. \tag{6}$$

Based on the distribution function of η_i in (6), the CDF and the PDF of the random variable $d_{\min}^{\text{Same}}(\mathbf{H}) = \min\{\eta_i, i = 1, 2\}$ in (5) are given by

$$F_{d_{\min}^{\text{Same}}(\mathbf{H})}(x) = 1 - e^{-\frac{x^2}{4\sin^2\left(\frac{\pi}{M}\right)}}, \quad x \geq 0 \tag{7}$$

$$f_{d_{\min}^{\text{Same}}(\mathbf{H})}(x) = \frac{d\left[F_{d_{\min}^{\text{Same}}(\mathbf{H})}(x)\right]}{dx} = \frac{x}{2\sin^2\left(\frac{\pi}{M}\right)} e^{-\frac{x^2}{4\sin^2\left(\frac{\pi}{M}\right)}}, \quad x \geq 0. \tag{8}$$

Let us now derive the PDF of the variable $d_{\min}^{\text{Diff}}(\mathbf{H})$. Since the amplitudes of $\beta_k = |h_1 e^{j(2k\pi/M)} - h_2|$, $k = 1, \ldots, M-1$, obey the Rayleigh distribution having PDFs of $f_{\beta_k}(x) = (x/2)e^{-(x^2/4)}$, $k = 1, \ldots, M-1$, the associated CDFs are

$$F_{\beta_k}(x) = \int_0^x f_{\beta_k}(x) dx = 1 - e^{-\frac{x^2}{4}}, \quad x \geq 0; k = 1, \ldots, M-1. \tag{9}$$

Based on the theory of order statistics, the CDF and the PDF of $d_{\min}^{\text{Diff}}(\mathbf{H}) = \min\{\beta_k, k = 1, \ldots, M-1\}$ are

$$F_{d_{\min}^{\text{Diff}}(\mathbf{H})}(x) = 1 - \left(1 - \left(1 - e^{-\frac{x^2}{4}}\right)\right)^M = 1 - e^{-\frac{Mx^2}{4}}, \quad x \geq 0 \tag{10}$$

$$f_{d_{\min}^{\text{Diff}}(\mathbf{H})}(x) = \frac{d\left[F_{d_{\min}^{\text{Diff}}(\mathbf{H})}(x)\right]}{dx} = \frac{M}{2} x e^{-\frac{Mx^2}{4}}, \quad x \geq 0. \tag{11}$$

As illustrated in Section III, for a fixed channel matrix $\mathbf{H}$—provided that the proposed BTS-MAP performs better than the conventional BTS-MAP—the inequality $P_{\text{Diff}} = P\{d_{\min}^{\text{Same}}(\mathbf{H}) > d_{\min}^{\text{Diff}}(\mathbf{H})\} > 1/2$ should be satisfied, which is equivalent to $P\{z \leq 0\} > 1/2$, where $z = d_{\min}^{\text{Diff}}(\mathbf{H}) - d_{\min}^{\text{Same}}(\mathbf{H})$. Based on (8) and (11), the probability $P_{\text{Same}} = P(z > 0)$ for the M-PSK-modulated (2×1)-element SM is given by

$$\begin{aligned} P_{\text{Same}} &= P(z > 0) \\ &= \int_0^{\infty} \left[\int_0^{+\infty} f_{d_{\min}^{\text{Diff}}(\mathbf{H})}(z+y) f_{d_{\min}^{\text{Same}}(\mathbf{H})}(y) dy \right] dz \\ &= \int_0^{\infty} \int_0^{+\infty} \left(\frac{M(z+y)}{2} e^{-\frac{M(z+y)^2}{4}} \right) \\ &\quad \cdot \left(\frac{y}{2\sin^2\left(\frac{\pi}{M}\right)} e^{-\frac{y^2}{4\sin^2\left(\frac{\pi}{M}\right)}} \right) dy dz \\ &= \frac{1}{\left(M \sin^2\left(\frac{\pi}{M}\right) + 1\right)}. \end{aligned} \tag{12}$$

Due to the constraint of $P_{\text{Diff}} + P_{\text{Same}} = 1$, the probability $P_{\text{Diff}} = P(z \leq 0)$ is calculated as

$$P_{\text{Diff}} = P(z \leq 0) = 1 - P(z > 0) = 1 - \frac{1}{M \sin^2\left(\frac{\pi}{M}\right) + 1}. \tag{13}$$

According to (13), the values of P_{Diff} for BPSK, QPSK, 8-PSK, and 16-PSK are 0.67, 0.67, 0.54, and 0.39, respectively. The result in (13) indicates that $P_{\text{Diff}} > P_{\text{Same}}$ is satisfied for $M \leq 8$. In this case, the proposed BTS-MAP performs better than the conventional scheme.

B. BPSK-Modulated (4×1)-Element SM

Next, consider the case of $N_t > 2$. Here, we investigate the (4×1)-element SM using BPSK. Let us denote the channel coefficients by $\mathbf{H} = [h_1, h_2, h_3, h_4]$. In this system, the MEDs of (3) and (4) can be represented as

$$d_{\min}^{\text{Same}}(\mathbf{H}) = \min\{2|h_1|, 2|h_2|, 2|h_3|, 2|h_4|\} \tag{14}$$

$$\begin{aligned} d_{\min}^{\text{Diff}}(\mathbf{H}) = \min\{&|h_1 \pm h_2|, |h_1 \pm h_3|, |h_1 \pm h_4|, \ldots \\ &|h_2 \pm h_3|, |h_2 \pm h_4|, |h_3 \pm h_4|\}. \end{aligned} \tag{15}$$

TABLE I
METRICS OF HDS AND HDD OF THE CONVENTIONAL BTS-MAP AND THE PROPOSED BTS-MAP OF SM FOR DIFFERENT MIMO SETUPS. MOREOVER, THE CORRESPONDING METRICS P_{Same} AND P_{Diff} ARE ALSO PROVIDED

N_t/N_r	APM scheme	HDS/HDD (Conventional)	HDS/HDD (Proposed)	P_{Same} (%)	P_{Diff} (%)
2/1	BPSK	1.00/1.50	2.00/1.00	28.8	71.2
2/1	QPSK	1.33/2.00	2.00/1.50	33.3	66.7
4/1	BPSK	1.00/1.83	3.00/1.50	13.4	86.6
4/1	QPSK	1.33/2.33	2.67/2.00	14.8	85.2
2/2	BPSK	1.00/1.50	2.00/1.00	16.8	83.2
2/2	QPSK	1.33/2.00	2.00/1.50	33.2	66.8
4/2	BPSK	1.00/1.83	3.00/1.50	6.6	93.4
4/2	QPSK	1.33/2.33	2.67/2.00	13.3	86.7

Similar to Section IV-A, if the proposed BTS-MAP outperforms the conventional BTS-MAP for the fading channel, the inequality $P\{d_{\min}^{\text{Same}}(\mathbf{H}) > d_{\min}^{\text{Diff}}(\mathbf{H})\}$ should be satisfied. To simplify the analysis, we shall assume that the Euclidean distances in the receive constellation are statistically independent. Strictly speaking, this is not true, since the constellation points created by each channel are indeed interdependent through the transmit symbols. However, based on regression analysis [20], we can state that the correlation between $d_{\min}^{\text{Same}}(\mathbf{H})$ and $d_{\min}^{\text{Diff}}(\mathbf{H})$ is low. Moreover, we will demonstrate using Monte Carlo simulations in Table I that this assumption does not impose high inaccuracy.

First, for a normalized transmit constellation, the received vectors $2|h_i|$ $(i = 1, 2, 3, 4)$ obey the Rayleigh distribution of

$$f_{2|h_i|}(x) = \frac{x}{4} e^{-\frac{x^2}{8}} \quad (i = 1, 2, 3, 4). \tag{16}$$

Based on the theory of order statistics [20] and on the four distances $2|h_i|$ $(i = 1, 2, 3, 4)$ in the receive SM constellation, the CDF and the PDF of the random variable $d_{\min}^{\text{Same}}(\mathbf{H})$ are

$$F_{d_{\min}^{\text{Same}}}(\mathbf{H})(x) = 1 - \left[1 - F_{2|h_i|}(x)\right]^4 = 1 - e^{-\frac{x^2}{8} \times 4} = 1 - e^{-\frac{x^2}{2}} \tag{17}$$

$$f_{d_{\min}^{\text{Same}}}(\mathbf{H})(x) = \frac{d\left[F_{d_{\min}^{\text{Same}}(\mathbf{H})}(x)\right]}{dx} = x e^{-\frac{x^2}{2}}, \quad x > 0. \tag{18}$$

Let us now derive the PDF of the MED $d_{\min}^{\text{Diff}}(\mathbf{H})$. Since h_1 and h_2 are Gaussian random variables, the PDF of $|h_i \pm h_j| (i \neq j)$ formulated in (15) obeys the Rayleigh distribution, which can be expressed as

$$f_{|h_i - h_j|}(x) = \frac{x}{2} e^{-\frac{x^2}{4}}, \; x \geq 0. \tag{19}$$

Then, the CDF and the PDF of the random variable $d_{\min}^{\text{Diff}}(\mathbf{H})$ are given by

$$F_{d_{\min}^{\text{Diff}}}(\mathbf{H})(x) = 1 - e^{-3x^2}, \quad x > 0 \tag{20}$$

$$f_{d_{\min}^{\text{Diff}}}(\mathbf{H})(x) = \frac{d\left(F_{d_{\min}^{\text{Diff}}(\mathbf{H})}(x)\right)}{dx} = 6x e^{-3x^2}, \quad x > 0. \tag{21}$$

Similar to the M-PSK-modulated (2×1)-element SM, we have the following probability:

$$\begin{aligned} P\left(z = d_{\min}^{\mathrm{Diff}}(\mathbf{H}) - d_{\min}^{\mathrm{Same}}(\mathbf{H}) > 0\right) &= \int_0^\infty \int_0^\infty f_{d_{\min}^{\mathrm{Diff}}(\mathbf{H})}(y+z) f_{d_{\min}^{\mathrm{Same}}(\mathbf{H})}(y) dy dz \\ &= \int_0^\infty \int_0^\infty 6(y+z)e^{-3(y+z)^2} \cdot y e^{-\frac{y^2}{2}} dy dz \\ &= \int_0^\infty y e^{-\frac{7}{2}y^2} dy = \frac{1}{7}. \end{aligned} \tag{22}$$

From (22), we have $P_{\mathrm{Same}} = P(z > 0) = 1/7$ and $P_{\mathrm{Diff}} = P(z \le 0) = 1 - P(z > 0) = 6/7$, which satisfies the condition $P_{\mathrm{Diff}} > P_{\mathrm{Same}}$. Hence, for the BPSK-modulated 4×1)-element SM, the proposed BTS-MAP is preferred.

C. Other MIMO Setups

In case of a high modulation order M and a large number of TAs N_t, there exist too many received distances associated with different values. In this case, it may be a challenge to theoretically evaluate the probability $P\{d_{\min}^{\mathrm{Same}}(\mathbf{H}) > d_{\min}^{\mathrm{Diff}}(\mathbf{H})\}$, because the exact distribution of the random variable $d_{\min}^{\mathrm{Diff}}(\mathbf{H})$ depends on both the channel matrix and on the symbol alphabet.

To deal with these challenging scenarios, the statistical P_{Diff} and P_{Same} results based on Monte Carlo simulations can be invoked for selecting the appropriate 3-D mapping schemes. To be specific, we can create a parameter lookup table for the SM schemes associated
AQ1 with the MIMO setups considered, similar to Table II. For a specific SM transmission, we assume that the relevant statistical information, concerning the fading type, the MIMO antenna setup, and the PSK scheme adopted, is available for the transmitter. Then, we can use this information to select the appropriate BTS-MAP scheme according to the lookup table designed offline. Moreover, if we consider the adaptive SM schemes of [22] and [23], we can use a feedback link for appropriately selecting the BTS-MAP directly by using the information $d_{\min}^{\mathrm{Diff}}(\mathbf{H})$ and $d_{\min}^{\mathrm{Same}}(\mathbf{H})$. If the constraint of $P_{\mathrm{Diff}} > P_{\mathrm{Same}}$ ($d_{\min}^{\mathrm{Diff}}(\mathbf{H}) < d_{\min}^{\mathrm{Same}}(\mathbf{H})$ for adaptive SM) is satisfied for a specific MIMO setup, the proposed BTS-MAP is adopted. Otherwise, the conventional BTS-MAP scheme is utilized.

V. Performance Results

A. HDD and HDS Metrics for Different BTS-MAP Schemes

Here, the HDs HDD and HDS of the proposed BTS-MAP and of the conventional BTS-MAP are compared under different MIMO setups. The simulation setup is based on 2–4 bits/symbol transmissions over independent flat Rayleigh block-fading channels. Furthermore, the probabilities P_{Diff} and P_{Same} of the occurrence of the MED $d_{\min}$ are also investigated.

As shown in Table I, the XOR operation of (2) allows the proposed BTS-MAP scheme to achieve higher HDD and lower HDS values compared with those of the conventional BTS-MAP. Moreover, the inequality $P_{\mathrm{Diff}} > P_{\mathrm{Same}}$ is satisfied in diverse MIMO setups in Table I. It means that the MED $d_{\min}$ is encountered between different TAs with a high probability, and hence, the proposed BTS-MAP, which has a lower HDD, is preferred. For example, P_{Diff} of the SM system associated with $N_t = 4$, $N_r = 1$, and BPSK modulation is higher than 86.6%, whereas the HDD is reduced from 1.83 to 1.5 by using the proposed scheme. The minimization of this HD between these nearest points leads to a BER performance gain.

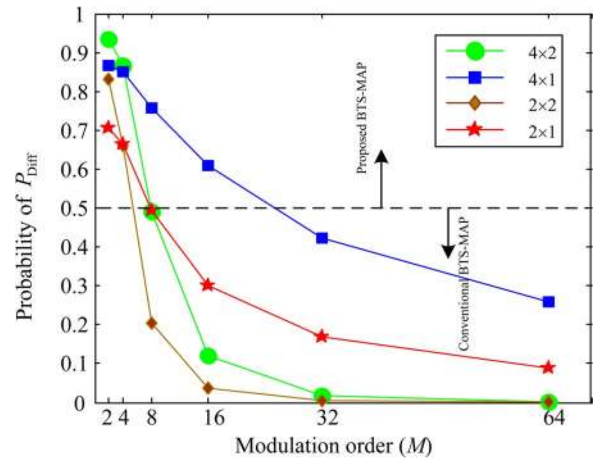


Fig. 3. Probability P_{Diff} for SM under various modulation orders and different antenna configurations $N_t \times N_r$.

Moreover, Table I shows that the simulation results of P_{Diff} match the theoretical results for the BPSK-modulated (2×1)- and (4×1)-element SM systems in Section IV. Note that the modest difference observed between the theoretical and simulation results is due to the approximation process invoked for the evaluation of P_{Diff} in Section IV.

Furthermore, observe in Table I that, as the modulation order increases, the corresponding P_{Diff} is reduced. To expound a little further, we investigate the effect of the modulation order and the number of TAs on the probability P_{Diff} in Fig. 3. Explicitly, observe in Fig. 3 that a higher modulation order may achieve a lower P_{Diff} value for a fixed $(N_t \times N_r)$-element MIMO. This is due to the fact that, if M is significantly higher than N_t, the APM symbol errors dominate the performance of SM. By contrast, if the number of TAs N_t is increased while maintaining a fixed value of M, we have an increased value of P_{Diff} due to the fact that the TA decision errors dominate the performance of SM. Moreover, since the increase of N_r can reduce both the TA and APM decision errors in SM, the specific effect of this parameter depends on the particular SM setup considered.

As shown in Fig. 3, our BTS-MAP rule is that, if we have $P_{\mathrm{Diff}} > 0.5$, then the proposed BTS-MAP may achieve a better BER performance. Otherwise, the conventional BTS-MAP can be utilized. Note that, even if the statistics of P_{Diff} are available for an SM-based MIMO system (such as the adaptive SM of [22] and [23]), our BTS-MAP selection rule still remains appropriate. Moreover, the proposed scheme can be also readily extended to other types of fading channel distributions, such as Rician and Nakagami fading [19].

B. BER Performance

Here, we characterize the BER performance of the proposed BTS-MAP compared with the conventional BTS-MAP in MIMO Rayleigh and Nakagami-m fading channels. Moreover, the optimal maximum- AQ2
likelihood detector is adopted. Here, the notation "Pro." represents the proposed BTS-MAP scheme, whereas "Con." denotes the conventional BTS-MAP.

Fig. 4 shows the BER performance of the (2×1)-element SM systems associated with different PSK schemes. As expected, in Fig. 4, the proposed BTS-MAP provides SNR gains of about 0.9 dB for $M = 2$ and 0.6 dB for $M = 4$ at BER $= 10^{-2}$ over the conventional BTS-MAP scheme. More important, similar to the result achieved by

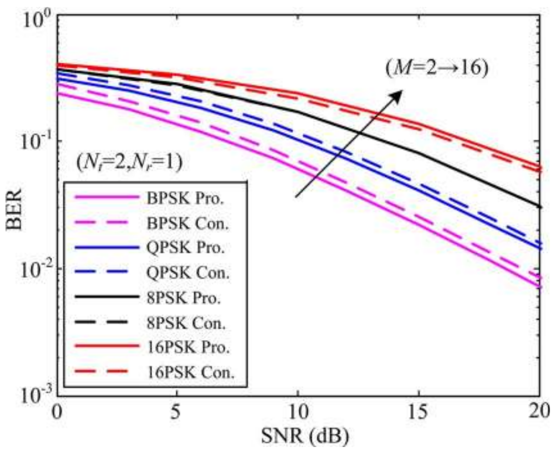


Fig. 4. BER performance of the proposed BTS-MAP and the conventional BTS-MAP schemes having $N_t = 2$, $N_r = 1$ and employing M-PSK signal sets.

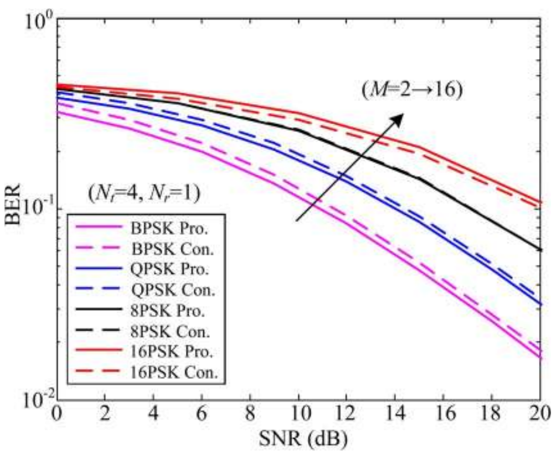


Fig. 5. BER performance of the proposed BTS-MAP and the conventional BTS-MAP schemes associated with $N_t = 4$, $N_r = 1$ and M-PSK schemes.

conventional Gray mapping for classic 2-D constellations, the specific SNR value only has a modest effect on the mapping gain of the proposed scheme [9]. Observe in Fig. 4 that, for the case of $M > 8$, the conventional BTS-MAP outperforms the proposed BTS-MAP. This result is consistent with the findings in Fig. 3, where the constraint of $P_{\mathrm{Diff}} > P_{\mathrm{Same}}$ is no longer met. Additionally, for the case of $M = 8$, it is found that the proposed BTS-MAP and the conventional BTS-MAP achieve almost the same BER performance. This is due to the fact that, for this scheme, we have $P_{\mathrm{Diff}} \approx 0.5$. The aforementioned trends of these BTS-MAP schemes recorded for SM are also visible in Fig. 5, where (4×1)-element SM systems are considered. Moreover, in Fig. 6, the performance of the proposed BTS-MAP is investigated in Nakagami-m fading channels. As shown in Fig. 6, the proposed scheme outperforms the conventional one in (2×1)-element MIMO channels having $m = 1.5$ and $m = 0.8$. Since we have a higher P_{Diff} for the case of $m = 1.5$, the corresponding BER gain is more attractive than that of $m = 0.8$.[1]

[1]In our simulations, the value of P_{Diff} for $m = 1.5$ is approximately 0.7, whereas this value for $m = 0.8$ is about 0.55. Moreover, our proposed BTS-MAP can be also directly extended to the SM in conjunction with M-QAM modulation. Due to space limitations, the related simulation results are not included here.

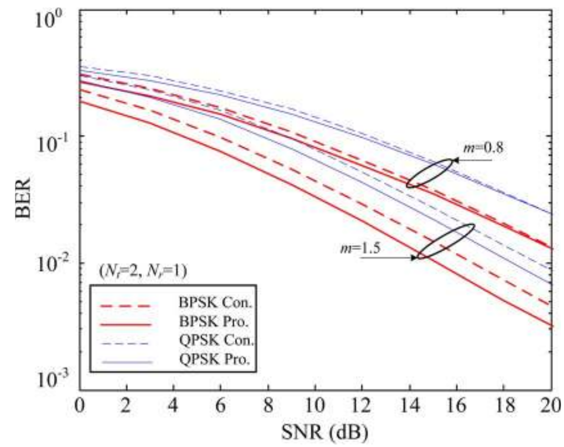


Fig. 6. BER performance of the proposed BTS-MAP and the conventional BTS-MAP schemes for (2×1)-element Nakagami-m channels.

VI. Conclusion

A novel BTS-MAP scheme has been proposed for SM systems with the objective of increasing the HDS and simultaneously reducing the average HDD. Based on the theoretical analysis of the MED distribution of SM constellations, a criterion was proposed for the construction of a beneficial BTS-MAP scheme for a specific MIMO setup. The proposed mapping rule exhibited is attractive for employment in SM systems. For achieving a further improved BER performance, our further work will be focused on the integration of adaptive SM and channel coding with the proposed scheme. AQ3

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AUTHOR QUERIES

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AQ1 = A citation for Table II was provided. The document, however, contains only one table. Please check.

AQ2 = ML was expanded as “maximum likelihood”. Please check if appropriate. Otherwise, please provide the corresponding expanded form.

AQ3 = The word “exhibit” in the sentence “The proposed mapping rule exhibits is attractive for employment in SM systems” was changed to “exhibited”. Please check if appropriate. Otherwise, please make the necessary changes.

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Correspondence

Hybrid Bit-to-Symbol Mapping for Spatial Modulation

Ping Yang, Yue Xiao, Lu Yin, Qian Tang, Shaoqian Li, *Senior Member, IEEE*, and Lajos Hanzo, *Fellow, IEEE*

***Abstract*—In spatial modulation (SM), the information bit stream is divided into two different sets: the transmit antenna index bits (TA-bits) and the amplitude and phase modulation bits (APM-bits). However, the conventional bit-to-symbol mapping (BTS-MAP) scheme maps the APM-bits and the TA-bits independently. For exploiting their joint benefits, we propose a new BTS-MAP rule based on the traditional 2-D Gray mapping rule, which increases the Hamming distance (HD) between the symbol pairs detected from the same transmit antenna (TA) and simultaneously reduces the average HD between the symbol pairs gleaned from different TAs. Based on the analysis of the distribution of minimum Euclidean distance (MED) of SM constellations, we propose a criterion for the construction of a meritorious BTS-MAP for a specific SM setup, with no need for additional feedback links or extra computational complexity. Finally, Monte Carlo simulations are conducted for confirming the accuracy of our analysis.**

***Index Terms*—Gray mapping, hamming distance (HD), multiple-input–multiple-output (MIMO), spatial modulation (SM).**

I. Introduction

Spatial modulation (SM) is a new 3-D hybrid modulation scheme conceived for multiple-input–multiple-output (MIMO) transmission, which exploits the indexes of the transmit antennas (TAs) as an additional dimension invoked for transmitting information, apart from the classic 2-D amplitude and phase modulation (APM) [1]–[5]. In SM, the information bit stream is divided into two different sets: the bits transmitted through the TA indexes and the APM constellations [6]. For simplicity, we refer to these two sets of bits as TA-bits and APM-bits.

In conventional 2-D APM constellations, the choice of the bit-to-symbol mapping (BTS-MAP) rule plays an important role in determining the achievable bit error ratio (BER) performance [7]. For example, an optimized BTS-MAP is capable of providing a low error floor in both bit-interleaved coded modulation and in its iterative decoding and demodulation aided counterpart (BICM-ID) [8]. It is widely known that, for equally likely and statistically independent 2-D APM constellations, Gray mapping is optimal in terms of minimizing the BER [7]. In general, the optimal BTS-MAP depends on the specific geometry of the signal constellation, particularly on the location of the phasors separated by the minimum Euclidean distance (MED) [9]. Compared with APM schemes, SM has a higher constellation dimension and a different distribution of MED due to its hybrid modulation principle. Hence, the classic Gray mapping proposed for 2-D constellations will no longer achieve the optimal BER in fading MIMO channels.

Recently, the wide-ranging studies disseminated in [1], [4], [6], and [10]–[14] have characterized some of the fundamental properties of SM, such as its energy efficiency [12], [13] and the effects of power imbalance [14]. In these contributions, a general framework was established for the BTS-MAP rule of SM [1], where the APM-bits are mapped according to the classic Gray mapping rule, whereas the TA-bits are mapped to the active TA index. Due to its intuitive nature and low complexity, this rule has been considered in diverse SM-based systems [15]–[18]. For example, in [15] and [16], this general framework was developed for an arbitrary number of TAs and hence strikes a flexible tradeoff in terms of the attainable BER performance and capacity. In [17] and [18], this philosophy has been extended to trellis coded modulation aided SM systems for the sake of achieving reliable digital transmission. However, this classic BTS-MAP scheme maps the APM-bits and the TA-bits to symbols independently and hence may sacrifice the classic Gray-coded benefits. Moreover, the MED distribution of SM was not considered in the design process of the conventional BTS-MAP scheme, which is a measure of separation between two constellation points, and as a result, it has a dominant influence on the BER.

Against this background, the novel contributions of this treatise are as follows.

- We propose a new BTS-MAP rule based on the classic Gray-coded principles for exploiting the interaction of the TA-bits and the APM-bits in fading channels, which differs from the conventional BTS-MAP, since it reduces the average Hamming distance (HD) between the SM symbol pair of the different TAs, and simultaneously, it increases the HD between the symbol pair of the same TA.
- Based on the analysis of the distribution of the MED, we propose a criterion for the construction of a beneficial BTS-MAP rule for a specific SM-MIMO setup, which is achieved without the need for an additional feedback link and with no extra computational complexity. Finally, performance comparisons with the conventional BTS-MAP scheme of [1] are provided for different constellation sizes and signal-to-noise ratios (SNRs).

Notation: $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^{\mathrm{H}}$ denote conjugate, transpose, and Hermitian transpose, respectively. The probability of an event is represented by $P(\cdot)$. Furthermore, $\|\cdot\|$ and $|\cdot|$ denote the Euclidean norm and magnitude operators, respectively; whereas $\varepsilon(\mathbf{x}, \mathbf{y})$ is the HD between the binary strings $\mathbf{x}$ and $\mathbf{y}$. N_t is the number of TAs, and M is the size of the APM constellation adopted. Let $\mathbf{b}_i^m$ be the transmit bit vector mapped to the SM symbol $\mathbf{x}_i$ and the APM symbol s_i^m, which corresponds to TA i, whereas $\mathbf{b}_j^k$ is the transmit bit vector mapped to the SM symbol $\mathbf{x}_j$ related to TA j and the APM

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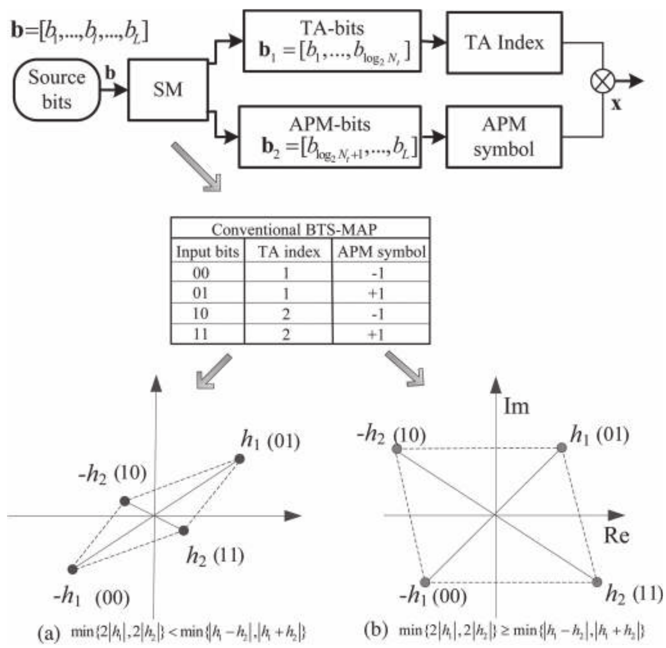


Fig. 1. Conventional BTS-MAP of SM: an example for the BPSK-modulated (2×1)-element SM. (a) MED encountered on the same TA, where the conventional BTS-MAP is optimal. (b) MED encountered on different TAs, where the conventional BTS-MAP is suboptimal.

symbol s_j^k. The average HD between the symbol pair gleaned from different TAs, termed as HDD, is defined as $\text{HDD} = (1/(N_t(N_t - 1)M^2)) \sum_{i \neq j, i,j \in \{1,\ldots,N_t\}} \varepsilon(\mathbf{b}_i^m, \mathbf{b}_j^k)$. Moreover, the average HD between the symbol pair detected from the same TA, termed as HDS, is defined as $\text{HDS} = (1/(N_t M(M-1))) \sum_{i \in \{1,\ldots,N_t\}} \varepsilon(\mathbf{b}_i^m, \mathbf{b}_i^k)$. For a fixed MIMO channel $\mathbf{H}$, $d_{\min}^{\text{Same}}(\mathbf{H})$ is the MED between the symbol pair of the same TA, $d_{\min}^{\text{Diff}}(\mathbf{H})$ is the MED between the symbol pair of different TAs, and the overall MED of a specific SM is $d_{\min} = \min\{d_{\min}^{\text{Same}}(\mathbf{H}), d_{\min}^{\text{Diff}}(\mathbf{H})\}$.

II. Conventional Bit-To-Symbol Mapping Rule for Spatial Modulation

Consider a MIMO system having N_t transmit and N_r receive antennas. The $(N_r \times N_t)$-element channel matrix $\mathbf{H}$ is used for modeling a flat-fading channel with elements having complex Gaussian distributions with unit variance. We not only focus our attention on the independent and identically distributed Rayleigh case but discuss Nakagami-m channels as well. Let $\mathbf{b} = [b_1, \ldots, b_L]$ be the transmit bit vector in each time slot, which contains $L = \log_2(N_t M)$ bits. As shown in Fig. 1, the input vector $\mathbf{b}$ is divided into two subvectors of $\log_2(N_t)$ and $\log_2(M)$ bits, which are denoted by $\mathbf{b}_1$ and $\mathbf{b}_2$, respectively. The bits in the subvector $\mathbf{b}_1$ are used for selecting a unique TA index q for activation, whereas the bits in the subvector $\mathbf{b}_2$ are mapped to a Gray-coded APM symbol s_l^q. Hence, the resultant SM symbol $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$ is formulated as [6]

$$\mathbf{x} = s_l^q \mathbf{e}_q \tag{1}$$

where $\mathbf{e}_q (1 \leq q \leq N_t)$ is selected from the N_t-dimensional standard basis vectors.

To expound a little further, we exemplify the binary phase-shift keying (BPSK)-modulated (2×1)-element SM in Fig. 1. As shown in Fig. 1, $L = 2$ input bits are divided into two single-bit streams, and then, the first bit determines the activated TA (1 or 2), whereas the second single bit generates the classic BPSK symbol ($+1$ or -1). The aforementioned BTS-MAP method considers the APM-bits and the TA-bits independently and hence facilitates a simple implementation of SM.

However, this BTS-MAP method may result in a Gray-coding penalty [9], which degrades the BER. Specifically, for the example of BPSK-modulated (2×1)-element SM in Fig. 1, assuming that the channel matrix is $\mathbf{H} = [h_1, h_2]$, there are four received constellation points denoted by h_1, h_2, $-h_1$, and $-h_2$; and we investigate two scenarios: 1) the MED is $d_{\min} = 2|h_2|$; and 2) the MED is $d_{\min} = \min\{|h_1 - h_2|, |h_1 + h_2|\}$. Note that scenario (1) corresponds to the case when the MED is encountered on the same TA (TA 2), whereas scenario (2) corresponds to the case when the MED is encountered on different TAs (between TAs 1 and 2). For scenario (1), the most likely erroneously detected pattern is given by the nearest constellation points $(-h_2, h_2)$. If the conventional BTS-MAP is adopted, these points differ only in a single bit, i.e., by the difference between the bits "10" and "11" in Fig. 1(a), whereas other adjacent constellation points may differ in more than one bits. This mapping rule obeys the concept of Gray mapping, where the probability of having a bit error is minimized; hence, it has an optimal performance in this specific scenario. However, for scenario (2), it is suboptimal, because the nearest constellation points become $(-h_1, h_2)$, which differ in two bits. This implies that more than one bit errors are associated with the most likely error pattern of $(-h_1, h_2)$; hence, a performance penalty will occur. This indicates that the MED distribution should be considered in the design of BTS-MAP.

III. Proposed Bit-To-Symbol Mapping for Spatial Modulation

A. Principle of the Proposed BTS-MAP

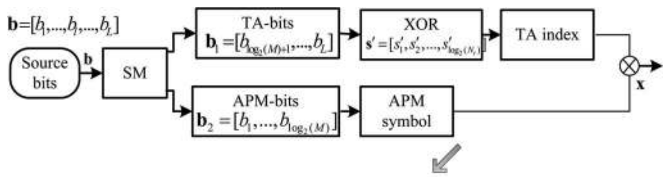
We found in Section II that the design of the BTS-MAP scheme depends on the MED, which may be achieved between the symbol pair gleaned from the same TA or between the symbol pair from different TAs. The conventional BTS-MAP rule does not consider this distribution of the MED, and hence, it becomes suboptimal for some channel scenarios, as illustrated in Fig. 1(b).

For exploiting the mapping gain of SM, we propose a new BTS-MAP scheme based on traditional Gray-coded modulation. To be specific, similar to the conventional BTS-MAP of SM, the L-bit input vector $\mathbf{b} = [b_1, \ldots, b_L]$ is divided into a pair of subvectors $\mathbf{d} = [d_1, \ldots, d_{\log_2(M)}] = [b_1, \ldots, b_{\log_2(M)}]$ and $\mathbf{s} = [s_1, \ldots, s_{\log_2(N_t)}] = [b_{\log_2(M)+1}, \ldots, b_L]$. As a result, the transmit bit vector can be represented as $\mathbf{b} = [\mathbf{d}, \mathbf{s}]$. Then, the first subvector $\mathbf{d}$ is mapped to an APM symbol s_l^q, rather than the TA index in conventional BTS-MAP. Then, the subvector $\mathbf{s}$ is transformed to a new input vector by using the bit-by-bit XOR operation with jointly considering the last APM-bit, which is represented as

$$\begin{aligned}\mathbf{s}' &= \left[s_1', s_2', \ldots, s_{\log_2(N_t)}'\right] \\ &= \left[d_{\log 2(M)} \oplus s_1, s_1 \oplus s_2, \ldots, s_{\log_2(N_t)-1} \oplus s_{\log_2(N_t)}\right] \\ &= \left[b_{\log_2(M)} \oplus b_{\log_2(M)+1}, \ldots, b_{\log_2(L)-1} \oplus b_{\log_2(L)}\right]\end{aligned} \tag{2}$$

where "$\oplus$" denotes the XOR operation. Then, the bit vector $\mathbf{s}'$ is mapped to a specific TA index of $q = (\sum_{k=1}^{\log_2(N_t)} 2^{\log_2(N_t)-k} s_k') + 1$, which is used for transmitting the APM symbol.

The rationale of introducing the XOR operation in (2) is to increase the HDS, while decreasing the HDD. To be specific, the last APM-bit $b_{\log_2(M)}$ of $\mathbf{d}$ and the subvector $\mathbf{s} = [b_{\log_2(M)+1}, \ldots, b_L]$ form a new vector $\tilde{\mathbf{s}} = [b_{\log_2(M)}, \mathbf{s}]$ for generating the TA-bits in (2), where we have $b_{\log_2(M)} = 0$ or $b_{\log_2(M)} = 1$. Assuming that there are two subvectors $\mathbf{s}_a$ and $\mathbf{s}_b$ and then provided that two different subvectors



Proposed BTS-MAP		
Input bits	TA index	APM symbol
00	1	-1
01	2	-1
10	2	+1
11	1	+1

Fig. 2. Proposed BTS-MAP for SM.

$\tilde{\mathbf{s}}_a = [0, \mathbf{s}_a]$ and $\tilde{\mathbf{s}}_b = [1, \mathbf{s}_b]$ map to the same TA index, the HD $\varepsilon(\tilde{\mathbf{s}}_a, \tilde{\mathbf{s}}_b)$ is maximized to $\log_2(N_t) + 1$. For example, the vectors $\tilde{\mathbf{s}}_a = [0, 0, 0]$ and $\tilde{\mathbf{s}}_b = [1, 1, 1]$ are mapped to the same TA index for SM using $N_t = 4$ TAs. This result implies that the XOR operation maps the subvector pair $\tilde{\mathbf{s}}_a$ and $\tilde{\mathbf{s}}_b$ having the highest HD to the same TA, and hence, it is capable of increasing the HDS, while decreasing the HDD.

B. Example and Selection Criterion

Compared with Fig. 1, we present the new BTS-MAP table in Fig. 2 for the simple BPSK-modulated (2×1)-element SM example mentioned in Section II, where the HDD is reduced to 1 and the HDS is increased to 2. This BTS-MAP may be more suitable for the channel scenario (2) in Fig. 1, because it performs a Gray mapping when considering the adjacent constellation points $-h_1$ and h_2 associated with the MED.

As indicated in Section II, the design of BTS-MAP depends on the distribution of MED, which can be encountered either on the same TA or on different TAs. To be specific, for a given channel matrix $\mathbf{H}$, the MED between the SM symbol pair $(\mathbf{x}_i, \mathbf{x}_j)$ of different TAs is given by

$$d_{\min}^{\text{Diff}}(\mathbf{H}) = \min_{\substack{\mathbf{x}_i, \mathbf{x}_j \in \mathbb{X}, \\ \mathbf{x}_i \neq \mathbf{x}_j, i \neq j}} \|\mathbf{H}(\mathbf{x}_i - \mathbf{x}_j)\|_F = \min_{s_i^m, s_j^k \in \mathbb{Q}, i \neq j} \left\| \left(\mathbf{h}_i s_i^m - \mathbf{h}_j s_j^k \right) \right\| \tag{3}$$

where $\mathbb{X}$ is the set of all legitimate transmit symbols, $\mathbf{h}_i$ is the ith column of $\mathbf{H}$, and s_i^m and s_j^k represent the classic APM constellation points from the set $\mathbb{Q}$. Moreover, the MED between the symbol pair of the same TA is defined as

$$d_{\min}^{\text{Same}}(\mathbf{H}) = \min_{\substack{s_i^m, x_i^k \in \mathbb{Q}, \\ s_i^m \neq s_i^k}} \left\| \mathbf{h}_i \left(s_i^m - s_i^k \right) \right\|. \tag{4}$$

Based on (3) and (4), the probability of the MED $d_{\min}$ encountered on different TAs is defined as $P_{\text{Diff}} = P(d_{\min}^{\text{Diff}}(\mathbf{H}) < d_{\min}^{\text{Same}}(\mathbf{H}))$, whereas the probability of the MED $d_{\min}$ on the same TA is defined as $P_{\text{Same}} = P(d_{\min}^{\text{Diff}}(\mathbf{H}) > d_{\min}^{\text{Same}}(\mathbf{H}))$. To minimize the probability of having a bit error, the HD of the more likely adjacent constellation points having the MED should be lower than that of other points located at a higher distance than the MED. This is also the basic concept of Gray mapping for a 2-D APM constellation. In the proposed BTS-MAP, the HDD is lower than the HDS, and hence, it is more suitable for the specific scenario, when the MED occurs more often in the context of different TAs, which can be expressed as $P_{\text{Diff}} > P_{\text{Same}}$. In other words, when we have $P_{\text{Diff}} > P_{\text{Same}}$ for an SM setup, most of the error events are typically imposed by the SM symbols of different TAs; hence, the minimization of the HD between these nearest points (i.e., the HDD) leads to directly minimizing the probability of bit errors. Based on this observation, the BTS-MAP selection criterion conceived for a specific SM scheme is formulated as follows.

Selection Criterion: If $P_{\text{Diff}} > P_{\text{Same}}$ (or $P_{\text{Diff}} \geq 1/2$) is satisfied by a specific SM, then the proposed BTS-MAP is superior to the conventional one in terms of reducing the BER. Otherwise, the conventional BTS-MAP is preferred.

IV. Theoretical Analysis and Mapping Optimization

Here, the probabilities of P_{Diff} and P_{Same} are derived, which are used as an evaluation criterion for selecting a meritorious BTS-MAP for a specific SM setup. As shown in [4] and [10], the phase-shift keying (PSK) modulation schemes are preferred in SM; hence, PSK is adopted for our theoretical analysis.

A. M-PSK-Modulated (2×1)-Element SM

For the M-PSK-modulated (2×1)-element SM, the associated channel matrix can be expressed as $\mathbf{H} = [h_1, h_2]$, where h_1 and h_2 are the fading coefficients of the first and second TAs, respectively, which have zero mean and unit variance. The corresponding MED $d_{\min}^{\text{Diff}}(\mathbf{H})$ of (3) and the MED $d_{\min}^{\text{Same}}(\mathbf{H})$ of (4) are given by

$$\begin{cases} d_{\min}^{\text{Same}}(\mathbf{H}) = \min\left\{2\sin\left(\frac{\pi}{M}\right)|h_1|, 2\sin\left(\frac{\pi}{M}\right)|h_2|\right\} \\ d_{\min}^{\text{Diff}}(\mathbf{H}) = \min\left\{\left|h_1 e^{j\frac{2k\pi}{M}} - h_2\right|, k = 0, \ldots, M-1\right\}. \end{cases} \tag{5}$$

Now, we can derive the distribution functions of $d_{\min}^{\text{Diff}}(\mathbf{H})$ and $d_{\min}^{\text{Same}}(\mathbf{H})$. Since the amplitudes of $h_i (i = 1, 2)$ obey the Rayleigh distribution having probability density functions (PDFs) of $f_{|h_i|}(x) = xe^{-(x^2/2)}$, $i = 1, 2$, the cumulative distribution functions (CDFs) of $\eta_i = 2\sin(\pi/M)|h_i|$, $i = 1, 2$ are formulated as

$$F_{\eta_i}(x) = \int_0^{\frac{x}{2\sin\left(\frac{\pi}{M}\right)}} xe^{-\frac{x^2}{2}} dx = 1 - e^{-\frac{x^2}{8\sin^2\left(\frac{\pi}{M}\right)}}, \quad x \geq 0; i = 1, 2. \tag{6}$$

Based on the distribution function of η_i in (6), the CDF and the PDF of the random variable $d_{\min}^{\text{Same}}(\mathbf{H}) = \min\{\eta_i, i = 1, 2\}$ in (5) are given by

$$F_{d_{\min}^{\text{Same}}(\mathbf{H})}(x) = 1 - e^{-\frac{x^2}{4\sin^2\left(\frac{\pi}{M}\right)}}, \quad x \geq 0 \tag{7}$$

$$f_{d_{\min}^{\text{Same}}(\mathbf{H})}(x) = \frac{d\left[F_{d_{\min}^{\text{Same}}(\mathbf{H})}(x)\right]}{dx} = \frac{x}{2\sin^2\left(\frac{\pi}{M}\right)} e^{-\frac{x^2}{4\sin^2\left(\frac{\pi}{M}\right)}}, \quad x \geq 0. \tag{8}$$

Let us now derive the PDF of the variable $d_{\min}^{\text{Diff}}(\mathbf{H})$. Since the amplitudes of $\beta_k = |h_1 e^{j(2k\pi/M)} - h_2|$, $k = 1, \ldots, M-1$, obey the Rayleigh distribution having PDFs of $f_{\beta_k}(x) = (x/2)e^{-(x^2/4)}$, $k = 1, \ldots, M-1$, the associated CDFs are

$$F_{\beta_k}(x) = \int_0^x f_{\beta_k}(x) dx = 1 - e^{-\frac{x^2}{4}}, \quad x \geq 0; k = 1, \ldots, M-1. \tag{9}$$

Based on the theory of order statistics, the CDF and the PDF of $d_{\min}^{\mathrm{Diff}}(\mathbf{H}) = \min\{\beta_k, k = 1, \ldots, M-1\}$ are

$$\begin{aligned} F_{d_{\min}^{\mathrm{Diff}}(\mathbf{H})}(x) &= 1 - \left(1 - \left(1 - e^{-\frac{x^2}{4}}\right)\right)^M \\ &= 1 - e^{-\frac{Mx^2}{4}}, \quad x \geq 0 \end{aligned} \tag{10}$$

$$\begin{aligned} f_{d_{\min}^{\mathrm{Diff}}(\mathbf{H})}(x) &= \frac{d\left[F_{d_{\min}^{\mathrm{Diff}}(\mathbf{H})}(x)\right]}{dx} \\ &= \frac{M}{2} x e^{-\frac{Mx^2}{4}}, \quad x \geq 0. \end{aligned} \tag{11}$$

As illustrated in Section III, for a fixed channel matrix $\mathbf{H}$—provided that the proposed BTS-MAP performs better than the conventional BTS-MAP—the inequality $P_{\mathrm{Diff}} = P\{d_{\min}^{\mathrm{Same}}(\mathbf{H}) > d_{\min}^{\mathrm{Diff}}(\mathbf{H})\} > 1/2$ should be satisfied, which is equivalent to $P\{z \leq 0\} > 1/2$, where $z = d_{\min}^{\mathrm{Diff}}(\mathbf{H}) - d_{\min}^{\mathrm{Same}}(\mathbf{H})$. Based on (8) and (11), the probability $P_{\mathrm{Same}} = P(z > 0)$ for the M-PSK-modulated (2×1)-element SM is given by

$$\begin{aligned} P_{\mathrm{Same}} &= P(z > 0) \\ &= \int_0^\infty \left[\int_0^{+\infty} f_{d_{\min}^{\mathrm{Diff}}(\mathbf{H})}(z+y) f_{d_{\min}^{\mathrm{Same}}(\mathbf{H})}(y) dy\right] dz \\ &= \int_0^\infty \int_0^{+\infty} \left(\frac{M(z+y)}{2} e^{-\frac{M(z+y)^2}{4}}\right) \\ &\quad \cdot \left(\frac{y}{2\sin^2\left(\frac{\pi}{M}\right)} e^{-\frac{y^2}{4\sin^2\left(\frac{\pi}{M}\right)}}\right) dy dz \\ &= \frac{1}{\left(M \sin^2\left(\frac{\pi}{M}\right) + 1\right)}. \end{aligned} \tag{12}$$

Due to the constraint of $P_{\mathrm{Diff}} + P_{\mathrm{Same}} = 1$, the probability $P_{\mathrm{Diff}} = P(z \leq 0)$ is calculated as

$$\begin{aligned} P_{\mathrm{Diff}} &= P(z \leq 0) = 1 - P(z > 0) \\ &= 1 - \frac{1}{M \sin^2\left(\frac{\pi}{M}\right) + 1}. \end{aligned} \tag{13}$$

According to (13), the values of P_{Diff} for BPSK, QPSK, 8-PSK, and 16-PSK are 0.67, 0.67, 0.54, and 0.39, respectively. The result in (13) indicates that $P_{\mathrm{Diff}} > P_{\mathrm{Same}}$ is satisfied for $M \leq 8$. In this case, the proposed BTS-MAP performs better than the conventional scheme.

B. BPSK-Modulated (4 × 1)-Element SM

Next, consider the case of $N_t > 2$. Here, we investigate the (4×1)-element SM using BPSK. Let us denote the channel coefficients by $\mathbf{H} = [h_1, h_2, h_3, h_4]$. In this system, the MEDs of (3) and (4) can be represented as

$$d_{\min}^{\mathrm{Same}}(\mathbf{H}) = \min\{2|h_1|, 2|h_2|, 2|h_3|, 2|h_4|\} \tag{14}$$

$$\begin{aligned} d_{\min}^{\mathrm{Diff}}(\mathbf{H}) = \min\{&|h_1 \pm h_2|, |h_1 \pm h_3|, |h_1 \pm h_4|, \ldots \\ &|h_2 \pm h_3|, |h_2 \pm h_4|, |h_3 \pm h_4|\}. \end{aligned} \tag{15}$$

TABLE I
METRICS OF HDS AND HDD OF THE CONVENTIONAL BTS-MAP AND THE PROPOSED BTS-MAP OF SM FOR DIFFERENT MIMO SETUPS. MOREOVER, THE CORRESPONDING METRICS P_{Same} AND P_{Diff} ARE ALSO PROVIDED

N_t/N_r	APM scheme	HDS/HDD (Conventional)	HDS/HDD (Proposed)	P_{Same} (%)	P_{Diff} (%)
2/1	BPSK	1.00/1.50	2.00/1.00	28.8	71.2
2/1	QPSK	1.33/2.00	2.00/1.50	33.3	66.7
4/1	BPSK	1.00/1.83	3.00/1.50	13.4	86.6
4/1	QPSK	1.33/2.33	2.67/2.00	14.8	85.2
2/2	BPSK	1.00/1.50	2.00/1.00	16.8	83.2
2/2	QPSK	1.33/2.00	2.00/1.50	33.2	66.8
4/2	BPSK	1.00/1.83	3.00/1.50	6.6	93.4
4/2	QPSK	1.33/2.33	2.67/2.00	13.3	86.7

Similar to Section IV-A, if the proposed BTS-MAP outperforms the conventional BTS-MAP for the fading channel, the inequality $P\{d_{\min}^{\mathrm{Same}}(\mathbf{H}) > d_{\min}^{\mathrm{Diff}}(\mathbf{H})\}$ should be satisfied. To simplify the analysis, we shall assume that the Euclidean distances in the receive constellation are statistically independent. Strictly speaking, this is not true, since the constellation points created by each channel are indeed interdependent through the transmit symbols. However, based on regression analysis [20], we can state that the correlation between $d_{\min}^{\mathrm{Same}}(\mathbf{H})$ and $d_{\min}^{\mathrm{Diff}}(\mathbf{H})$ is low. Moreover, we will demonstrate using Monte Carlo simulations in Table I that this assumption does not impose high inaccuracy.

First, for a normalized transmit constellation, the received vectors $2|h_i|$ $(i = 1, 2, 3, 4)$ obey the Rayleigh distribution of

$$f_{2|h_i|}(x) = \frac{x}{4} e^{-\frac{x^2}{8}} \quad (i = 1, 2, 3, 4). \tag{16}$$

Based on the theory of order statistics [20] and on the four distances $2|h_i|$ $(i = 1, 2, 3, 4)$ in the receive SM constellation, the CDF and the PDF of the random variable $d_{\min}^{\mathrm{Same}}(\mathbf{H})$ are

$$\begin{aligned} F_{d_{\min}^{\mathrm{Same}}}(\mathbf{H})(x) &= 1 - \left[1 - F_{2|h_i|}(x)\right]^4 \\ &= 1 - e^{-\frac{x^2}{8} \times 4} = 1 - e^{-\frac{x^2}{2}} \end{aligned} \tag{17}$$

$$\begin{aligned} f_{d_{\min}^{\mathrm{Same}}}(\mathbf{H})(x) &= \frac{d\left[F_{d_{\min}^{\mathrm{Same}}(\mathbf{H})}^{(x)}\right]}{dx} \\ &= x e^{-\frac{x^2}{2}}, \quad x > 0. \end{aligned} \tag{18}$$

Let us now derive the PDF of the MED $d_{\min}^{\mathrm{Diff}}(\mathbf{H})$. Since h_1 and h_2 are Gaussian random variables, the PDF of $|h_i \pm h_j| (i \neq j)$ formulated in (15) obeys the Rayleigh distribution, which can be expressed as

$$f_{|h_i - h_j|}(x) = \frac{x}{2} e^{-\frac{x^2}{4}}, \; x \geq 0. \tag{19}$$

Then, the CDF and the PDF of the random variable $d_{\min}^{\mathrm{Diff}}(\mathbf{H})$ are given by

$$F_{d_{\min}^{\mathrm{Diff}}}(\mathbf{H})(x) = 1 - e^{-3x^2}, \quad x > 0 \tag{20}$$

$$\begin{aligned} f_{d_{\min}^{\mathrm{Diff}}}(\mathbf{H})(x) &= \frac{d\left(F_{d_{\min}^{\mathrm{Diff}}(\mathbf{H})}^{(x)}\right)}{dx} \\ &= 6x e^{-3x^2}, \quad x > 0. \end{aligned} \tag{21}$$

Similar to the M-PSK-modulated (2×1)-element SM, we have the following probability:

$$\begin{aligned} P\left(z = d_{\min}^{\mathrm{Diff}}(\mathbf{H}) - d_{\min}^{\mathrm{Same}}(\mathbf{H}) > 0\right) &= \int_0^\infty \int_0^\infty f_{d_{\min}^{\mathrm{Diff}}(\mathbf{H})}(y+z) f_{d_{\min}^{\mathrm{Same}}(\mathbf{H})}(y)\, dy dz \\ &= \int_0^\infty \int_0^\infty 6(y+z)e^{-3(y+z)^2} \cdot y e^{-\frac{y^2}{2}} dy dz \\ &= \int_0^\infty y e^{-\frac{7}{2}y^2} dy = \frac{1}{7}. \end{aligned} \tag{22}$$

From (22), we have $P_{\mathrm{Same}} = P(z > 0) = 1/7$ and $P_{\mathrm{Diff}} = P(z \leq 0) = 1 - P(z > 0) = 6/7$, which satisfies the condition $P_{\mathrm{Diff}} > P_{\mathrm{Same}}$. Hence, for the BPSK-modulated 4×1)-element SM, the proposed BTS-MAP is preferred.

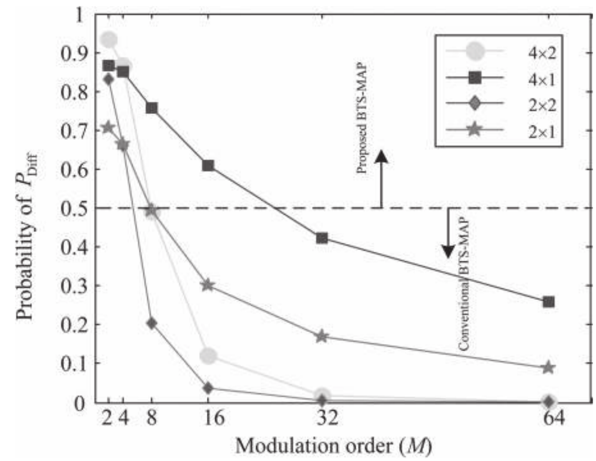


Fig. 3. Probability P_{Diff} for SM under various modulation orders and different antenna configurations $N_t \times N_r$.

C. Other MIMO Setups

In case of a high modulation order M and a large number of TAs N_t, there exist too many received distances associated with different values. In this case, it may be a challenge to theoretically evaluate the probability $P\{d_{\min}^{\mathrm{Same}}(\mathbf{H}) > d_{\min}^{\mathrm{Diff}}(\mathbf{H})\}$, because the exact distribution of the random variable $d_{\min}^{\mathrm{Diff}}(\mathbf{H})$ depends on both the channel matrix and on the symbol alphabet.

To deal with these challenging scenarios, the statistical P_{Diff} and P_{Same} results based on Monte Carlo simulations can be invoked for selecting the appropriate 3-D mapping schemes. To be specific, we can create a parameter lookup table for the SM schemes associated with the MIMO setups considered, similar to Table II. For a specific SM transmission, we assume that the relevant statistical information, concerning the fading type, the MIMO antenna setup, and the PSK scheme adopted, is available for the transmitter. Then, we can use this information to select the appropriate BTS-MAP scheme according to the lookup table designed offline. Moreover, if we consider the adaptive SM schemes of [22] and [23], we can use a feedback link for appropriately selecting the BTS-MAP directly by using the information $d_{\min}^{\mathrm{Diff}}(\mathbf{H})$ and $d_{\min}^{\mathrm{Same}}(\mathbf{H})$. If the constraint of $P_{\mathrm{Diff}} > P_{\mathrm{Same}}$ ($d_{\min}^{\mathrm{Diff}}(\mathbf{H}) < d_{\min}^{\mathrm{Same}}(\mathbf{H})$ for adaptive SM) is satisfied for a specific MIMO setup, the proposed BTS-MAP is adopted. Otherwise, the conventional BTS-MAP scheme is utilized.

V. PERFORMANCE RESULTS

A. HDD and HDS Metrics for Different BTS-MAP Schemes

Here, the HDs HDD and HDS of the proposed BTS-MAP and of the conventional BTS-MAP are compared under different MIMO setups. The simulation setup is based on 2–4 bits/symbol transmissions over independent flat Rayleigh block-fading channels. Furthermore, the probabilities P_{Diff} and P_{Same} of the occurrence of the MED $d_{\min}$ are also investigated.

As shown in Table I, the XOR operation of (2) allows the proposed BTS-MAP scheme to achieve higher HDD and lower HDS values compared with those of the conventional BTS-MAP. Moreover, the inequality $P_{\mathrm{Diff}} > P_{\mathrm{Same}}$ is satisfied in diverse MIMO setups in Table I. It means that the MED $d_{\min}$ is encountered between different TAs with a high probability, and hence, the proposed BTS-MAP, which has a lower HDD, is preferred. For example, P_{Diff} of the SM system associated with $N_t = 4$, $N_r = 1$, and BPSK modulation is higher than 86.6%, whereas the HDD is reduced from 1.83 to 1.5 by using the proposed scheme. The minimization of this HD between these nearest points leads to a BER performance gain.

Moreover, Table I shows that the simulation results of P_{Diff} match the theoretical results for the BPSK-modulated (2×1)- and (4×1)-element SM systems in Section IV. Note that the modest difference observed between the theoretical and simulation results is due to the approximation process invoked for the evaluation of P_{Diff} in Section IV.

Furthermore, observe in Table I that, as the modulation order increases, the corresponding P_{Diff} is reduced. To expound a little further, we investigate the effect of the modulation order and the number of TAs on the probability P_{Diff} in Fig. 3. Explicitly, observe in Fig. 3 that a higher modulation order may achieve a lower P_{Diff} value for a fixed $(N_t \times N_r)$-element MIMO. This is due to the fact that, if M is significantly higher than N_t, the APM symbol errors dominate the performance of SM. By contrast, if the number of TAs N_t is increased while maintaining a fixed value of M, we have an increased value of P_{Diff} due to the fact that the TA decision errors dominate the performance of SM. Moreover, since the increase of N_r can reduce both the TA and APM decision errors in SM, the specific effect of this parameter depends on the particular SM setup considered.

As shown in Fig. 3, our BTS-MAP rule is that, if we have $P_{\mathrm{Diff}} > 0.5$, then the proposed BTS-MAP may achieve a better BER performance. Otherwise, the conventional BTS-MAP can be utilized. Note that, even if the statistics of P_{Diff} are available for an SM-based MIMO system (such as the adaptive SM of [22] and [23]), our BTS-MAP selection rule still remains appropriate. Moreover, the proposed scheme can be also readily extended to other types of fading channel distributions, such as Rician and Nakagami fading [19].

B. BER Performance

Here, we characterize the BER performance of the proposed BTS-MAP compared with the conventional BTS-MAP in MIMO Rayleigh and Nakagami-m fading channels. Moreover, the optimal maximum-likelihood detector is adopted. Here, the notation "Pro." represents the proposed BTS-MAP scheme, whereas "Con." denotes the conventional BTS-MAP.

Fig. 4 shows the BER performance of the (2×1)-element SM systems associated with different PSK schemes. As expected, in Fig. 4, the proposed BTS-MAP provides SNR gains of about 0.9 dB for $M = 2$ and 0.6 dB for $M = 4$ at BER $= 10^{-2}$ over the conventional BTS-MAP scheme. More important, similar to the result achieved by

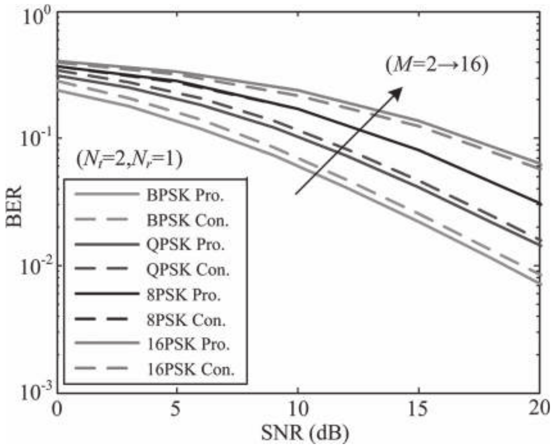


Fig. 4. BER performance of the proposed BTS-MAP and the conventional BTS-MAP schemes having $N_t = 2$, $N_r = 1$ and employing M-PSK signal sets.

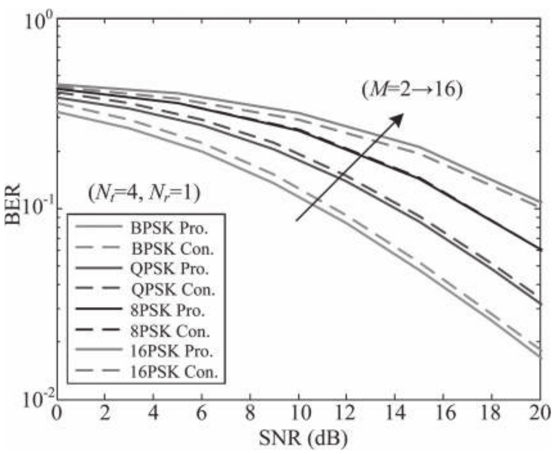


Fig. 5. BER performance of the proposed BTS-MAP and the conventional BTS-MAP schemes associated with $N_t = 4$, $N_r = 1$ and M-PSK schemes.

conventional Gray mapping for classic 2-D constellations, the specific SNR value only has a modest effect on the mapping gain of the proposed scheme [9]. Observe in Fig. 4 that, for the case of $M > 8$, the conventional BTS-MAP outperforms the proposed BTS-MAP. This result is consistent with the findings in Fig. 3, where the constraint of $P_{\mathrm{Diff}} > P_{\mathrm{Same}}$ is no longer met. Additionally, for the case of $M = 8$, it is found that the proposed BTS-MAP and the conventional BTS-MAP achieve almost the same BER performance. This is due to the fact that, for this scheme, we have $P_{\mathrm{Diff}} \approx 0.5$. The aforementioned trends of these BTS-MAP schemes recorded for SM are also visible in Fig. 5, where (4×1)-element SM systems are considered. Moreover, in Fig. 6, the performance of the proposed BTS-MAP is investigated in Nakagami-m fading channels. As shown in Fig. 6, the proposed scheme outperforms the conventional one in (2×1)-element MIMO channels having $m = 1.5$ and $m = 0.8$. Since we have a higher P_{Diff} for the case of $m = 1.5$, the corresponding BER gain is more attractive than that of $m = 0.8$.[1]

[1]In our simulations, the value of P_{Diff} for $m = 1.5$ is approximately 0.7, whereas this value for $m = 0.8$ is about 0.55. Moreover, our proposed BTS-MAP can be also directly extended to the SM in conjunction with M-QAM modulation. Due to space limitations, the related simulation results are not included here.

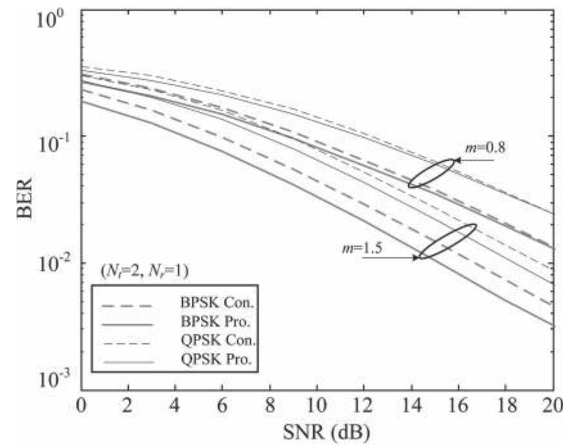


Fig. 6. BER performance of the proposed BTS-MAP and the conventional BTS-MAP schemes for (2×1)-element Nakagami-m channels.

VI. Conclusion

A novel BTS-MAP scheme has been proposed for SM systems with the objective of increasing the HDS and simultaneously reducing the average HDD. Based on the theoretical analysis of the MED distribution of SM constellations, a criterion was proposed for the construction of a beneficial BTS-MAP scheme for a specific MIMO setup. The proposed mapping rule exhibited is attractive for employment in SM systems. For achieving a further improved BER performance, our further work will be focused on the integration of adaptive SM and channel coding with the proposed scheme.

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AQ1 = A citation for Table II was provided. The document, however, contains only one table. Please check.

AQ2 = ML was expanded as “maximum likelihood”. Please check if appropriate. Otherwise, please provide the corresponding expanded form.

AQ3 = The word “exhibit” in the sentence “The proposed mapping rule exhibits is attractive for employment in SM systems” was changed to “exhibited”. Please check if appropriate. Otherwise, please make the necessary changes.

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