

# Hybrid Control of the Air-fuel ratio in Force Transients for Multi-point Injection Engines.<sup>1</sup>

A. Balluchi<sup>†</sup> L. Benvenuti<sup>†‡</sup> M. D. Di Benedetto<sup>‡</sup> S. Cardellino<sup>†</sup>  
C. Rossi<sup>§</sup> A. L. Sangiovanni-Vincentelli<sup>†¶</sup>

<sup>†</sup>PARADES, Via San Pantaleo, 66, 00186 Roma, Italy,  
balluchi, lucab, stefanoc, alberto@parades.rm.cnr.it

<sup>‡</sup>Dip. di Ingegneria Elettrica, Università degli Studi de L'Aquila,  
Poggio di Roio, 67040 L'Aquila, Italy, dibenede@giannutri.caspur.it

<sup>§</sup>Magneti Marelli S.p.A., Via del Timavo, 33, 40134 Bologna, Italy, Carlo.Rossi@bologna.marelli.it

<sup>¶</sup>Dep. of Electrical Engineering and Computer Science,  
University of California at Berkeley, CA 94720, alberto@eecs.berkeley.edu

## Abstract

The problem of delivering as quickly as possible a requested torque produced by a spark ignition engine equipped with a multi-point port injection manifold and with drive-by-wire electronics is addressed. The optimal control problem, subject to the constraint that the air-fuel ratio stays as close as possible to the stoichiometric ratio, is solved using a detailed, cycle-accurate hybrid model and hybrid control approaches. The quality of the control law, that is particularly simple to implement, has been analytically demonstrated and a set of simulations have been carried out.

## 1 Introduction

The ever increasing computational power of micro-controllers has made it possible to extend the performance and the functionality of electronic sub-systems controlling the motion of the car. This opportunity has exposed the need for control algorithms with guaranteed properties that can reduce substantially emission and gas consumption while maintaining the performance of the car.

In this paper, we deal with the problem of delivering as quickly as possible a requested torque produced by a spark ignition engine equipped with a multi-point port injection manifold and with drive-by-wire electronics to control the throttle valve position. The optimization problem is subject to the constraint that the air/fuel (A/F) ratio stays as close as possible to stoichiometric value of 14.64 (the ratio that guarantees minimum emission). A/F ratio evolution is subject to both air and fuel dynamics. After fuel is injected, partly va-

por turns into fluid that deposits onto the intake runner. Hence only part of the injected fuel is immediately available for combustion, while the fluid film contributes with a noticeable delay to the combustion process since it has first to evaporate again.

The most used solutions to this problem consist of feed-forward compensation of fuel dynamics [1, 6, 5], based on mean value engine models [4]. However, the mean values of the engine variables of interest may not be enough to guarantee small transient deviations from the optimal A/F ratio. In this paper, we propose an approach, based on a hybrid *detailed* model of the cyclic behavior of the engine, that is characterized by very small deviations over the optimal A/F ratio. The hybrid model describes accurately the detailed behavior of the drive-by-wire components, of the injections systems and of the torque-generation mechanism. The discrete abstraction for parts of the model allows the development of powerful closed-loop control laws.

The goal is to design a control law for the fuel injection durations and the throttle valve motor voltage so to drive the evolution of the system from an initial condition characterized by the delivery of a torque  $u^0$  to a final condition characterized by the delivery of a requested torque  $u^1$  in minimum time subject to constraints on emissions. The control problem solved here is new not only because a detailed model for the injection process is used but also because the entire control chain, from throttle motor to engine, is considered. Our approach to the control problem at hand is to first introduce and solve an auxiliary optimal control problem in the continuous time, and then to map the solution back in the hybrid domain with the appropriate inputs and models. By doing so, we are paying close attention to constraint satisfaction that is essential to obtain a control law that obeys emission limitations. The quality of the control law has been analytically demonstrated and extensive simulations have been carried out.

<sup>1</sup>This research has been partially sponsored by PARADES, a Cadence, Magneti-Marelli and SGS-Thomson GEIE, and by CNR.

## 2 Plant model and problem formulation

In this section, a hybrid model for vehicles with 4-stroke 4-cylinder gasoline engine equipped with a multi-point injection system and drive-by-wire is illustrated. The model (an expansion of the model in [2]) consists of four parts (see Fig. 1): two continuous-time systems, modeling the power-train dynamics and the air dynamics respectively, and two hybrid systems, one modeling the behavior of each cylinder and one modeling the behavior of each injection system.

**Air Dynamics.** The model of the quantity of air entering the cylinder during the intake run is obtained from the air flow balance equation of the manifold. The air mass  $m_a$ , loaded during an intake run, is subject to the manifold pressure ( $p$ ) dynamics which is controlled by the throttle valve actuated by a DC motor. The linearized model is

$$\dot{\alpha}(t) = a_\alpha \alpha(t) + b_\alpha v(t) \quad (1)$$

$$\dot{p}(t) = a_p p(t) + b_p \alpha(t) \quad (2)$$

$$m_a(t) = c_p p(t) \quad (3)$$

where  $\alpha$  is the throttle angle and  $v$  is the DC motor voltage.

**Powertrain Model.** Powertrain dynamics are modeled by the linear system

$$\dot{\zeta}(t) = A \zeta(t) + b u(t), \quad (4)$$

where  $\zeta = [\alpha_e, \omega_c, \omega_p]^T$  represents the axle torsion angle, the crankshaft revolution speed and the wheel revolution speed. The input signal  $u$  is the torque produced by the engine and acting on the crankshaft. Model parameters  $A$  and  $b$ , depend on the transmission gear which is assumed not to change. Integrating the crankshaft velocity  $\omega_c$ , a single-state hybrid system emits the event *dead\_point*, when pistons reach either the top or bottom dead centers.

**Torque-generation.** The behavior of each cylinder in the engine is abstractly represented by a Finite State Machine (FSM) and a Discrete Event System (DES) modeling torque generation. The FSM state  $S_i$  of the  $i$ -th cylinder assumes values in the set  $H_i, I_i, C_i, E_i$  which correspond to the Exhaust, Intake, Compression and Expansion strokes, respectively, in the 4-stroke engine cycle. When the event *dead\_point* is emitted, the piston reaches a dead center and a FSM transition occurs. The DES describing the torque generation process of the  $i$ -th cylinder increments its time counter  $k$  by one at each transition of the FSM. Its inputs are the masses  $m_{ai}$  and  $m_{vi}$  of air and fuel loaded during the intake phase; its output is the torque  $u_i(k)$  generated by the cylinder. At the transition  $I_i \rightarrow C_i$ , that is at time  $t_{i1}$ , the event *int\_end<sub>i</sub>* is generated and the DES reads its inputs, storing in  $q_{ai}$  and  $q_{vi}$  their values. The maximum amount of torque achievable during the next expansion phase, obtained by the

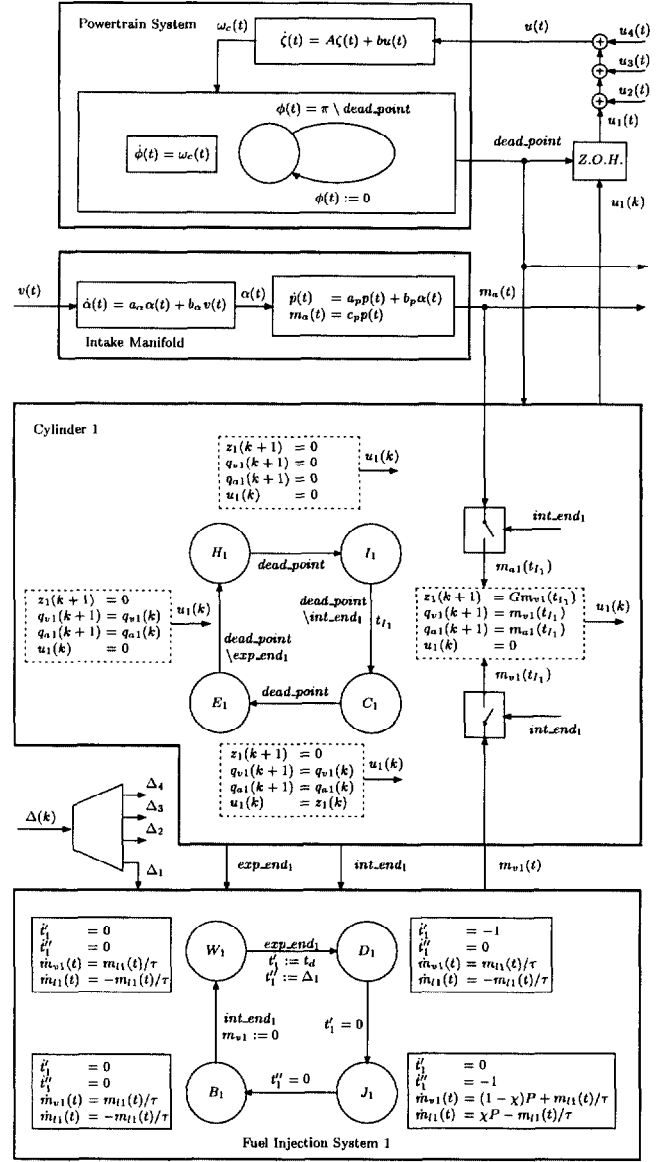


Figure 1: Engine hybrid model  $\mathcal{E}$ .

fuel-to-torque gain  $G$ , is stored in  $z_i$ . The DES output  $u_i(k)$  is always zero except at the  $C_i \rightarrow E_i$  transition when it is set to the value stored in  $z_i$ . Input  $u_i(t)$  to the powertrain dynamics is obtained from  $u_i(k)$  by a zero order hold block latched on the event *dead\_point*.

**Injection Process.** The  $i$ -th injection system is abstractly represented by a hybrid system, whose discrete state  $F_i$  assumes values  $J_i, B_i, W_i, D_i$  as follows:

- $J_i$ : the injector is open and delivers a constant flow  $P$  of vaporized fuel. A fraction  $\chi$  of it condenses in a fuel-puddle and increases the mass  $m_{li}$  of liquid fuel, fraction  $1 - \chi$  increases the mass  $m_{vi}$  of vaporized fuel in the intake runner. The mass of liquid fuel evaporates off with a time constant  $\tau$ . The injector remains open for  $\Delta_i$  seconds measured by timer  $t_i''$ .
- $B_i$ : the injector is closed and the evaporation process continues. When the next *dead\_point* event is emitted,

the intake valve opens, and the air-fuel mix begins to enter the cylinder. At the  $I_i \rightarrow C_i$  transition, the intake valve closes and the *int.end<sub>i</sub>* event is generated. The mass  $m_{vi}$  of vapor is reset to zero since all the vapor fuel has been loaded in the cylinder.

- $W_i$ : the injector is closed and evaporation proceeds.
- $D_i$ : the beginning of fuel injection is synchronized with respect to the the beginning of the exhaust phase with a time delay of  $t_d$  seconds measured by timer  $t'_i$ . Fuel dynamics is as in state  $W_i$ .

**Engine hybrid model.** The overall model of the engine  $\mathcal{E}$  is the combination of four hybrid systems representing the behavior of each cylinder and related injection system, and the power-train and intake manifold models which are shared among all cylinders. The pistons are connected to the crankshaft, so that dead-points are synchronous and the cycle of each one is delayed one step with respect to the cycle of the previous one. Then, the *dead.point* events and the time counter  $k$  are shared among all the cylinders and only one signal  $u_i(t)$  may be different from zero at any time. Input signals are: the input voltage  $v(t)$  to the DC motor actuating the throttle valve, a scalar continuous time signal in the class  $\mathcal{V}$  of functions  $\mathbb{R}_0^+ \rightarrow [-V, +V]$ ; the injection intervals  $\Delta(k)$ , a scalar discrete time signal in the class  $\mathcal{D}$  of functions  $\mathbb{Z}_0^+ \rightarrow [0, \Delta_{\max}]$ , which is sequentially distributed over the four injectors synchronously with the corresponding *exp.end<sub>i</sub>* event.

The state of the overall hybrid systems is a triple  $(\mathbf{q}, \mathbf{z}, \mathbf{x})$  where:  $\mathbf{q} = [S_1, F_1, S_2, F_2, S_3, F_3, S_4, F_4]$  is the state of the FSMs associated to each cylinder and each injection system,  $\mathbf{z} = [z_1, q_{v1}, q_{a1}, \dots, z_4, q_{v4}, q_{a4}]$  is the cylinder DES state and  $\mathbf{x} = [\zeta, \phi, \alpha, p, t'_1, t''_1, m_{v1}, m_{l1}, \dots, t'_4, t''_4, m_{v4}, m_{l4}]$  is the continuous state associated to the powertrain and air dynamics and to each injection system. The output of the overall system is the generated torque  $u$ .

## 2.1 Problem formulation

At the initial time the engine hybrid model is supposed to be at a steady state, where a fixed crankshaft velocity  $\omega_c^0$  is maintained by a torque  $u^0$ . The FMSs repeat a same sequence of configuration, which can be obtained from the initial state  $[C_1, W_1, E_2, W_2, H_3, D_3, I_4, B_4]$ . The powertrain equilibrium state is  $\zeta^0 = [\alpha_c^0, \omega_c^0, \omega_p^0] = -A^{-1}bu^0$  and the crank angle  $\phi$  evolves between 0 and  $\pi$  describing a triangular wave of frequency  $2\omega_c^0$ . The injection DESs steady state is given by  $z_i^0 = u^0$ ,  $q_{vi}^0 = u^0/G$ ,  $q_{ai}^0 = 14.64q_{vi}^0$ , when not 0. The manifold dynamics is at the equilibrium with  $p^0 = q_{ai}^0/c_p$ ,  $\alpha^0 = -a_p p^0/b_p$ . Finally, the fuel injection hybrid model exhibits a periodic steady state at frequency  $\omega_c^0$ , and the evolution of state variables  $t'_i$ ,  $t''_i$ ,  $m_{li}$  and  $m_{vi}$  can be readily obtained by integration of fuel dynamics.

Given a value  $u^0$  of torque produced by the engine, de-

fine in the hybrid state space the invariant set  $\mathcal{T}(u^0) \subset Q \times Z \times X$  given by the trajectories described by the hybrid model  $\mathcal{E}$  during its evolution in the steady state corresponding to the torque  $u^0$ , under inputs  $v = (a_\alpha a_p/b_\alpha b_p)(14.64/c_p G)u^0$  and  $\Delta_i = u^0/(GP)$ .

**Problem 1** Consider the engine hybrid model  $\mathcal{E}$ , depicted in Figure 1. Let  $u^0$  and  $u^1$  be the initial value and the desired value of the torque. Assume that, at the initial time  $t = 0$ , the hybrid state  $(\mathbf{q}^0, \mathbf{z}^0, \mathbf{x}^0)$  belongs to  $\mathcal{T}(u^0)$ , namely the steady state trajectory with torque  $u^0$ . Consider in the hybrid state space  $Q \times Z \times X$ , the target set  $\mathcal{T}(u^1)$ . Find  $\hat{\Delta}(t) \in \mathcal{D}$  and  $\hat{v}(t) \in \mathcal{V}$  such that

- the initial state  $(\mathbf{q}^0, \mathbf{z}^0, \mathbf{x}^0)$  is steered to  $\mathcal{T}(u^1)$ ;
- for all  $t \geq 0$  and for all  $k \geq 0$

$$0 \leq \alpha(t) \leq \pi/2 \quad (5)$$

$$L_{\min} q_{vi_C}(k) \leq q_{ai_C}(k) \leq L_{\max} q_{vi_C}(k) \quad (6)$$

where  $i_C$  denote the cylinder which enters the  $C$  state and  $L_{\min} < 14.64 < L_{\max}$ ;

- the time  $\bar{t}$  such that  $u(t) = u^1$  for all  $t \geq \bar{t}$  is minimized.

## 3 Auxiliary optimal control problem

In this section a continuous time model  $\mathcal{E}_{CT}$ , which approximates the behavior of the hybrid model  $\mathcal{E}$  is introduced and Problem 1 is reformulated for such model. In model  $\mathcal{E}_{CT}$  the intake manifold dynamics, given by (1-2), has as output the flow of air that, in average, enters the cylinders:  $f_a(t) = c_a p(t)$  with  $c_a = \frac{\pi}{\omega_c^0} c_p$ . Fuel dynamics is expressed by the average model  $\dot{m}_i(t) = a_i m_i(t) + b_i \Delta(t)$ , with fuel flow given by  $f_l(t) = c_l m_l(t) + d_l \Delta(t)$  where  $a_l = -\tau^{-1}$ ,  $b_l = X P \frac{\omega_c^0}{\pi}$ ,  $c_l = \tau^{-1}$ ,  $d_l = (1-X) P \frac{\omega_c^0}{\pi}$ . Since in  $\mathcal{E}_{CT}$  the crankshaft coupling is abstracted away, these two dynamics are decoupled. As standard in constrained optimal control, rewrite constraints in Problem 1 as  $g_i(\cdot) \leq 0$ . By (5),

$$g_1 = -\alpha \leq 0, \quad g_2 = \alpha - \pi/2 \leq 0. \quad (7)$$

Minimum time solutions for model  $\mathcal{E}_{CT}$  are obtained by the Maximum Principle. The Hamiltonian function should contain all the given constraints, in a formulation that explicitly shows the contribution of each control which affects the state components that appear in the constraint itself. This is achieved differentiating with respect to time the constraints, until the interesting controls appear (see [3]). Differentiating (7),

$$h_1 = \frac{dg_1}{dt} = -a_\alpha \alpha - b_\alpha v, \quad h_2 = \frac{dg_2}{dt} = a_\alpha \alpha + b_\alpha v, \quad (8)$$

where  $v$ , which controls  $\alpha$ , appears. In model  $\mathcal{E}_{CT}$  constraints (6) are imposed on  $f_a$  and  $f_l$ , that is

$$L_2 p + L_m m_l \leq \Delta(t) \leq L_1 p + L_m m_l \quad (9)$$

with  $L_1 = \frac{c_a}{L_{\min} d_l}$ ,  $L_2 = \frac{c_a}{L_{\max} d_l}$ ,  $L_m = -\frac{c_l}{d_l}$ . Constraints (9) introduce a coupling between the manifold dynamics and the fuel dynamics. To have both control variables  $v$  and  $\Delta$  appearing in the Hamiltonian, fuel dynamics is expanded as follows

$$\dot{x}_1(t) = \omega(t), \quad \dot{x}_2(t) = x_1(t) \quad (10)$$

$$\dot{m}_l(t) = a_l m_l(t) + b_l x_2(t) \quad (11)$$

where  $x_2$  stands for  $\Delta$  and  $\omega$  is the new input. By (9),

$$g_3 = -x_2 + L_2 p + L_m m_l \leq 0, g_4 = +x_2 - L_1 p - L_m m_l \leq 0. \quad (12)$$

Further, since  $\Delta$  is bounded to belong to  $[0, \Delta_{\max}]$ , then

$$g_5 = -x_2 \leq 0, \quad g_6 = +x_2 - \Delta_{\max} \leq 0. \quad (13)$$

Differentiating twice (12) and (13), one gets<sup>2</sup>

$$h_3 = +C_2 v - \omega + D_2 \alpha + E_2 p + F_1 x_1 + F_2 x_2 + F_0 m_l$$

$$h_4 = -C_1 v + \omega - D_1 \alpha - E_1 p - F_1 x_1 - F_2 x_2 - F_0 m_l$$

$$h_5 = -\omega \quad h_6 = +\omega.$$

Let  $p^1 = \frac{14.64}{c_p G} u^1$  and consider the new target set

$$\mathcal{M} = \left\{ (p, \alpha, x_1, x_2, m_l) \mid p = p^1, \alpha = -\frac{b_\alpha}{a_\alpha} p^1 \right\}. \quad (14)$$

Let  $\mathcal{O}$  denote the class of functions  $\mathbb{R}_0^+ \rightarrow [-\Omega, +\Omega]$  for some  $\Omega > 0$ . Solutions to Problem 1 are derived from the solutions of the following problem for  $\Omega \rightarrow \infty$ .

**Problem 2** Consider the dynamics (1-2), (10-11). Find  $\hat{v}(t) \in \mathcal{V}$  and  $\hat{\omega}(t) \in \mathcal{O}$  which steer in minimum time an initial state  $p^0 = 14.64/(c_p G) u^0$ ,  $\alpha^0 = -(a_p/b_p) p^0$  to the manifold  $\mathcal{M}$  as in (14), satisfying the constraints (7), (12) and (13) for all  $t \geq 0$ .

Necessary conditions on optimal solutions to Problem 2 are obtained from Pontryagin Maximum Principle. Introducing the adjoint variables  $\psi_\alpha, \psi_p, \psi_1, \psi_2, \psi_m$  and the Lagrange multipliers  $\gamma_i$ , the Hamiltonian is

$$H = 1 + \psi_\alpha \dot{\alpha} + \psi_p \dot{p} + \psi_1 \dot{x}_1 + \psi_2 \dot{x}_2 + \psi_m \dot{m}_l + \gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3 + \gamma_4 h_4 + \gamma_5 h_5 + \gamma_6 h_6 \quad (15)$$

where  $h_i$  are as above, if  $g_i(t) < 0$  then  $\gamma_i(t) = 0$ , and if  $g_i(t) = 0$  then  $\gamma_i(t) \geq 0$ . The adjoint variables dynamics ( $\dot{\psi} = -\frac{\partial H}{\partial x}$ ), are

$$\begin{aligned} \dot{\psi}_\alpha &= -a_\alpha \psi_\alpha - b_p \psi_p + a_\alpha (\gamma_1 - \gamma_2) - D_2 \gamma_3 + D_1 \gamma_4, \\ \dot{\psi}_p &= -a_p \psi_p - E_2 \gamma_3 + E_1 \gamma_4, \dot{\psi}_1 = -\psi_2 - F_1 (\gamma_3 - \gamma_4), \\ \dot{\psi}_2 &= -b_l \psi_m - F_2 (\gamma_3 - \gamma_4), \dot{\psi}_m = -a_l \psi_m - F_0 (\gamma_3 - \gamma_4). \end{aligned} \quad (16)$$

Along a trajectory  $\hat{\xi} = (\hat{\alpha}, \hat{p}, \hat{x}_1, \hat{x}_2, \hat{m}_l)$  solution to Problem 2 corresponding to the optimal controls  $\hat{v}, \hat{\omega}$ ,

<sup>2</sup>where  $h_i = \frac{dg_i}{dt}$ , for  $i = 3, \dots, 6$ , and  $C_1 = L_1 b_p b_\alpha$ ,  $C_2 = L_2 b_p b_\alpha$ ,  $D_1 = L_1 (a_\alpha + a_p) b_p$ ,  $D_2 = L_2 (a_\alpha + a_p) b_p$ ,  $E_1 = L_1 a_p^2$ ,  $E_2 = L_2 a_p^2$ ,  $F_1 = L_m b_l$ ,  $F_2 = L_m a_l b_l$ ,  $F_0 = L_m a_l^2$ .

there exist adjoint evolutions  $\psi = (\psi_\alpha, \psi_p, \psi_1, \psi_2, \psi_m)$ , which satisfy (16), and Lagrange multipliers  $\gamma_i$  s.t.

$$\min_{\substack{v = \mathcal{V} \\ \omega = \mathcal{O}}} H(v, \omega, \hat{\xi}, \psi) = H(\hat{v}, \hat{\omega}, \hat{\xi}, \psi) = 0 \quad (17)$$

is verified on (15) for almost any  $t \geq 0$ . Hence, optimal controls has to satisfy

$$\hat{v} = \begin{cases} -V & \text{if } \frac{\partial H}{\partial v} > 0 \\ [-V, V] & \text{if } \frac{\partial H}{\partial v} = 0 \\ V & \text{if } \frac{\partial H}{\partial v} < 0 \end{cases}, \hat{\omega} = \begin{cases} -\Omega & \text{if } \frac{\partial H}{\partial \omega} > 0 \\ [-\Omega, \Omega] & \text{if } \frac{\partial H}{\partial \omega} = 0 \\ \Omega & \text{if } \frac{\partial H}{\partial \omega} < 0 \end{cases} \quad (18)$$

where

$$\partial H / \partial v = b_\alpha \psi_\alpha + b_\alpha (\gamma_2 - \gamma_1) + C_2 \gamma_3 - C_1 \gamma_4 \quad (19)$$

$$\partial H / \partial \omega = \psi_1 - \gamma_3 + \gamma_4 - \gamma_5 + \gamma_6 \quad (20)$$

are required to meet the additional conditions

$$\text{if } g_1 = 0 \vee g_2 = 0, \quad \partial H / \partial v = 0, \quad (21)$$

$$\text{if } g_3 = 0 \vee g_4 = 0, \quad \partial H / \partial v = 0 \vee \partial H / \partial \omega = 0, \quad (22)$$

$$\text{if } g_5 = 0 \vee g_6 = 0, \quad \partial H / \partial \omega = 0. \quad (23)$$

The target set  $\mathcal{M}$  given in (14) is a 3-dimensional subspace parallel to the  $m_l$ ,  $x_1$  and  $x_2$  axes. Hence, at the final time  $T$ , the transversality condition gives

$$\psi_m(T) = \psi_1(T) = \psi_2(T) = 0. \quad (24)$$

Assuming that  $u^1$  is not on the boundary of feasible torques, (7) are not tight at the final time:  $g_1(T) < 0 \wedge g_2(T) < 0$ . Since, by (14),  $\alpha(T)$  is such that  $p(T) = p^1$  is an equilibrium point for (2), then  $\dot{p}(T) = 0$  and, by (15),  $H(T) = 1 + \psi_\alpha(T) (a_\alpha \alpha^1 + b_\alpha v(T)) = 0$ , i.e.

$$\psi_\alpha(T) = -(a_\alpha \alpha^1 + b_\alpha v(T))^{-1}. \quad (25)$$

### 3.1 Minimum time trajectories.

Due to space limitation, only the case  $u^1 < u^0$  is here illustrated and some proofs are not reported. We have already noticed that  $g_1(T) < 0 \wedge g_2(T) < 0$ . It can be verified

**Proposition 1** Solutions to Problem 2, with  $u^1 < u^0$ , can be found in the sufficient family given by trajectories that

- reach the target set by a constrained arc satisfying  $g_1 < 0 \wedge g_2 < 0 \wedge g_3 = 0 \wedge g_4 < 0 \wedge g_5 < 0 \wedge g_6 < 0$  (26)

- fulfill  $g_3 = 0$  along all the trajectory.

Hence, fuel injection is regulated in such a way that the evolution of  $m_l$  follows the evolution of the manifold pressure so to maintain  $\frac{f_a}{f_l} = L_{\max}$  (see (12)). The time to the target set depends only on manifold pressure evolution. Solutions obtained with  $g_4 = 0$  are

equivalent to (26), but the latter have been chosen since they minimize fuel consumption.

Optimal solutions to Problem 2 are obtained by backward integration of the extended dynamics given by (1–2), (10–11) and (16), from final conditions (24),(25),

$$p(T) = p^1 = \frac{q_a^1}{c_p}, \alpha(T) = -\frac{b_\alpha p^1}{a_\alpha}, m_{li}(T) \in [0, -\frac{L_2}{L_m} p^1] \quad (27)$$

on the target set (14), choosing  $\psi_p(T)$  such that  $p(t), \alpha(t), m_{li}(t)$  span the region in the space of interest.

Being  $g_3 = 0$  along all the optimal trajectory, by (12), the optimal injection control is  $\hat{\Delta} = L_2 p + L_m m_{li}$ .

The optimal  $\hat{v}$  at  $t = T$  is obtained for  $\psi_\alpha(T) < 0$ . By (19), we have  $\frac{\partial H}{\partial v}(T) = b_\alpha \psi_\alpha(T) < 0$ , and, by (18),  $\hat{v}(T) = V$ . Further, in a left neighborhood of  $t = T$ ,  $\gamma_3(t) = \psi_1(t) \geq 0$ ,  $\gamma_i(t) = 0$  for  $i \neq 3$ . From the backwards integration of (16), we verify that  $\frac{\partial H}{\partial v}(t) = b_\alpha \psi_\alpha(t) + C_2 \psi_1(t)$ , increases, while  $\alpha(t)$  decreases. Hence, at some time  $t = t_1 < T$  either  $\frac{\partial H}{\partial v}(t_1) = 0$  or  $\alpha(t_1) = 0$ , i.e.  $g_1(t_1) = 0$ . In the former case, by (18), the control  $v$  has to switch to  $\hat{v} = -V$ . Points  $(\alpha(t_1), p(t_1))$ , define the switching surface

$$\Sigma = \{(\alpha(t_1), p(t_1)) \mid b_\alpha \psi_\alpha(t_1) + C_2 \psi_1(t_1) = 0\}. \quad (28)$$

for  $\hat{v}$ . For  $t < t_1$ ,  $\alpha(t)$  increases in the backward integration and  $g_1$  never reaches 0. Otherwise, if  $\alpha(t_1) = 0$ , the singular control  $\hat{v}(t) = 0$  is applied until, at some time  $t_2 < t_1$ ,  $v$  switches to the nonsingular control  $-V$  and  $\alpha(t)$  starts to increase. Introduce the function  $\sigma(\alpha, p)$  such that

$$\begin{aligned} \sigma(\alpha, p) &= 0 & \text{if } (\alpha, p) \in \Sigma \\ \sigma(\alpha, p) &> 0 & \text{if } (\alpha, p) \notin \Sigma \text{ and } v = V \\ \sigma(\alpha, p) &< 0 & \text{if } (\alpha, p) \notin \Sigma \text{ and } v = -V \end{aligned} \quad (29)$$

see Figure 3. According to the construction above,

**Proposition 2** *The feedback laws*

$$\hat{\Delta}(p, m_{li}) = L_2 p + L_m m_{li} \quad (30)$$

$$\hat{v}(\alpha, p) = \begin{cases} -V & \text{if } \sigma(\alpha, p) < 0 \wedge \alpha > 0 \\ 0 & \text{if } \alpha = 0 \\ V & \text{if } \sigma(\alpha, p) > 0 \wedge \alpha > 0 \end{cases} \quad (31)$$

are optimal controls for Problem 2.

#### 4 Hybrid control and simulation results

The continuous control law described in Section 3, must be approximated to yield a feasible control law for the hybrid model introduced in Section 2. The main issues to address when we move from the continuous case to the hybrid case are the following ones.

In model  $\mathcal{E}$  there is a delay between the time at which the injection control  $\Delta_i$  is set (at the end of the expansion phase) and the time at which the vaporized fuel

$q_v$  is loaded (at the end of the intake phase).

Further, feasible control actions on  $\Delta_i$  are discrete time signals synchronized with the crank angle. This issue is the main cause of difficulty for devising a hybrid control strategy.

Finally, in model  $\mathcal{E}$ , there exist four independent fuel dynamics whose evolutions are constrained with respect to the same air flow evolution by A/F bounds.

The measures available for closing the control loop are: the pressure  $p$ , the angle  $\alpha$  and the crankshaft speed  $\omega_c$ . According to (30), fuel injection is regulated so to maintain in (6)  $q_a = L_{max} q_v$ , when the cylinder is in the compression stroke. The injection control  $\Delta_i$  for the  $i$ -th cylinder is set at the end of the expansion stroke (i.e., when the *exp\_end<sub>i</sub>* event is generated). The fuel mass  $q_v$  in the next compression phase can be expressed as a linear function of the mass of liquid fuel  $m_{li}(t_k)$  and the injection duration  $\Delta_i(t_k)$  at the end of the expansion stroke. This is achieved by integration of the fuel dynamics described in Figure 1, approximating  $t_{k+2} - t_k \approx 2\pi/\omega_c(t_k)$  and linearizing exponentials. Hence, since by (3)  $q_a = c_p p(t_{k+2})$ , the feedback is set as follows

$$\hat{\Delta} = \max(0, K_p \tilde{p}(k) + K_l \tilde{m}_{li}(k)) \quad (32)$$

where:  $\tilde{p}(k)$  is an estimate of the pressure  $p$  at the end of the intake phase, i.e.  $p(t_{k+2})$ , and  $\tilde{m}_{li}(k)$ , is an estimate of the current mass of liquid fuel  $m_{li}(t_k)$ . Since  $m_{li}$  is not measurable,  $\tilde{m}_{li}(k)$  is obtained by integration of the fuel dynamics from the value  $m_{li}(0)$ , known for the initial steady state. Control  $\hat{\Delta} = 0$  is applied only in the case where the initial condition cannot be controlled to the target set satisfying constraint (6). Due to the independent fuel mass evolutions a small chattering around the constraint  $q_a = L_{max} q_v$  appears in the hybrid model.

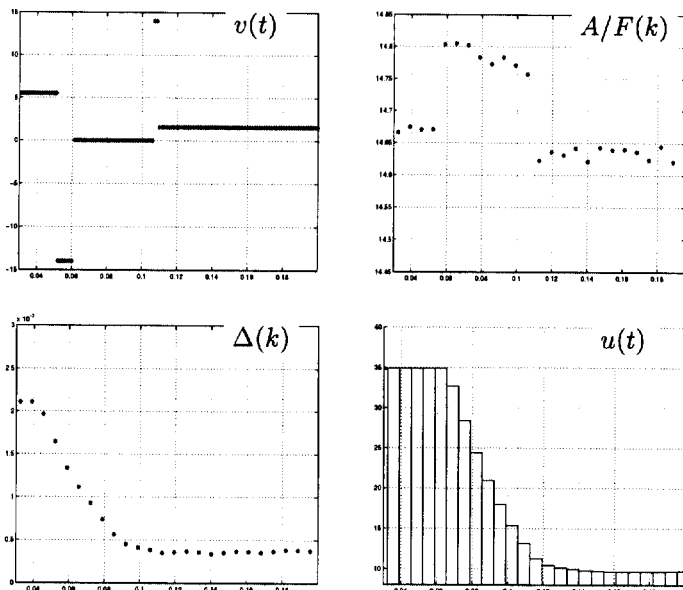
The DC motor voltage control  $v$  is obtained from (31). The proposed injection control (32) needs the estimate  $\tilde{p}(k)$  of the manifold pressure  $p(t_{k+2})$ . To simplify this computation in the engine control unit, control  $v$  is also applied on a discretized time axis  $t_h = hT$ , with  $T = 1.5$  ms. By this choice, samples  $\hat{\alpha}(h)$  and  $\hat{p}(h)$  can be obtained from the discretized version of (1–2), i.e.

$$[\hat{\alpha}(h+1), \hat{p}(h+1)]^T = A_d^{-1} [\hat{\alpha}(h), \hat{p}(h)]^T + b_d v(h) \quad (33)$$

(with sample period  $T$ ), with initial conditions  $(\hat{\alpha}(0), \hat{p}(0)) = (\alpha(t_k), p(t_k))$ , and feedback  $v$  given by (31). Manifold pressure estimate is set to

$$\tilde{p}(k) = (a_p \hat{p}(\bar{h}) + b_p \hat{\alpha}(\bar{h})) [(2\pi)/\omega(t_k) - \bar{h}T] \quad (34)$$

where  $\bar{h} = \arg \min_h [(2\pi)/\omega(t_k) - \bar{h}T]$ . Due to time discretization switching points cannot be precisely located in the state space on the switching lines  $\alpha = 0$  and  $\sigma(\alpha, p) = 0$ . A study based on modal analysis of dynamics (1–2) reveals that satisfaction of problem

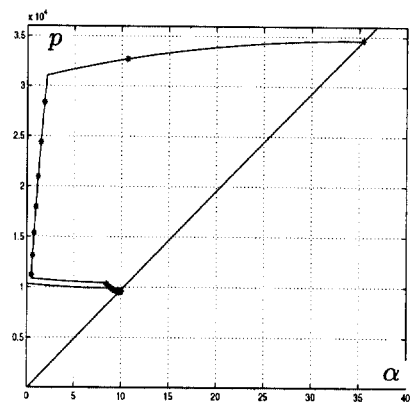


**Figure 2:** Simulation results: DC motor voltage  $v$  and injection times  $\Delta$  (on the left); air-fuel ratio  $A/F$  and generated torque  $u$  (on the right).

constraints and convergence to  $(\alpha^1, p^1)$  is guaranteed when switchings are always anticipated with respect to ideal switching point. It can be shown that

**Proposition 3** *Applying feedback (32) and (31), with estimates (33–34), to the engine hybrid model  $\mathcal{E}$ , the evolution of  $(\alpha, p)$  converges to the rectangular set  $R = [\alpha^1, \alpha^2] \times [p^1, p^2]$ , with  $[\alpha^2, p^2] = A_d^{-1}([\alpha^1, p^1]^T - b_d V)$ . Once in  $R$ , control  $v = -(a_\alpha/b_\alpha)\alpha^1$  produces asymptotic convergence to  $(\alpha^1, p^1)$ . The torque error  $|u - u^1|$ , when  $(\alpha, p) \in R$ , can be made arbitrarily small by a proper choice of the sampling time  $T$ .*

The proposed hybrid control law has been evaluated in a number of simulations. Figures 2 show the results of the proposed approach when the engine is requested to change the produced torque from the initial constant value  $u^0 = 35$  Nm to the final constant value  $u^1 = 10$  Nm, with initial crankshaft velocity 4000 rpm. The air-to-fuel ratio is requested to remain within a 1% band centered around the stoichiometric value. The control starts at  $t_0 = 0.45$  s from the steady state corresponding to  $u^0$ . The throttle valve is forced to close as fast as possible under the initial control  $v(t) = -V$  (see Figures 3), while, according to (32), fuel injection is controlled so that  $A/F$  is maintained on the maximum feasible value. When the valve is about to close, at time  $t = 0.064$  s, the control  $v(t)$  is set to 0. At time  $t = 0.107$  s the switching surface  $\Sigma$  is approached and the control  $v(t)$  is set to  $V$ . Then, starting from time  $t = 0.11$  s, the valve is controlled so that the air trapped in the cylinders is regulated in proportion to the evaporated fuel trapped in the cylinders. At time  $\bar{t} = 0.126$  s the requested torque  $u^1$  is generated. From



**Figure 3:** Switching surfaces for DC motor voltage  $v$  and a trajectory of the hybrid closed-loop system.

this time, the throttle valve position is constant and the fuel system is controlled so that the fuel puddle dynamics is compensated.

## 5 Conclusions

The problem of delivering as quickly as possible a requested torque produced by a spark ignition engine equipped with a multi-point port injection manifold and with drive-by-wire electronics to control the throttle valve position has been addressed. The optimization problem subject to the constraint that the air/fuel ( $A/F$ ) ratio stays as close as possible to stoichiometric ratio has been solved using hybrid system modeling and control approaches.

## References

- [1] C. F. AQUINO, *Transient A/F characteristics of the 5 liter central fuel injection engine*, Tech. Rep. No. 910494, SAE, 1991.
- [2] A. BALLUCHI, M. D. BENEDETTO, C. PINELLO, C. ROSSI, AND A. SANGIOVANNI-VINCENTELLI, *Hybrid control in automotive applications: the cut-off control*, *Automatica*, 35 (1999), pp. 519–535.
- [3] A. E. BRYSON AND Y.-C. HO, *Applied optimal control, optimization, estimation, and control*, John Wiley and Sons, Inc., New York, N.Y., 1975.
- [4] E. HENDRICKS AND T. VESTERHOLM, *The analysis of mean value SI engine models*, Tech. Rep. No. 920682, SAE, 1992.
- [5] E. HENDRICKS, T. VESTERHOLM, P. K Aidantzis, P. Rasmussen, AND M. Jensen, *Nonlinear transient fuel film compensation*, Tech. Rep. No. 930767, SAE, 1993.
- [6] P. MORALL, *Adaptive compensation of fuel dynamics in an SI engine using a switching EGO sensor*, in Proc. CDC95, New Orleans, LA, 1995, pp. 661–666.