
cratome
 and to the Omce of Management and Bucget. Paperwork Redscticn Profect 107040: ©es Washengen
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Command, Control and Ocean Surveillance Center (NCCOSC)
RDT\&E Division
San Diego, CA 92152-5001
2. SPCNSOANGMONTTOAING AGENCY NAME(S) AND ADORESS(ES)

RDT\&E Division
Block Programs
San Diego, CA 92152-5001
11. SUPPL EMENTARY NOTES
3. ABSTRACT (Maxdmum 200 words)

The use of parabolic equation (PE) methods has become very popular in recent years for modeling radar propagation effects in the lower atmosphere, especially for cases in which the vertical refractive index profile changes along the propagation path. The PE method normally used is the split-step method described by Tappert, which has been mplemented by Dockery, Craig and Levy, and others. An advantage of the PE method is its ability to compute propagation is that it requires extensive computation. Computational requirements increase with higher fiequencies, larger antenna beamwidths, and higher altitudes for which results are desired. For many practical combinations of these parameters, the use of PE models on personal computers is impractical without extra hardware such as transputers.

Published in IEE International Conference No. 365 "Radar 92," October 1992.

- UNCLASSIFIED

| 21a NaME OF RESPONSIBLE INDNIDUAL <br> H. V. Hitney |  <br> (619)553-1438 | aramenem Cinis 5:ts |
| :---: | :---: | :---: |

Berbert v. Hitney

Naval Comand, Control and ocean survaillance Centar, RDT\& Division, den

## LNTRODOCTION

The use of parabolic equation (PE) wethods has become very popular in recent years tor modeling radar propagation effects in the lower atmosphere, especially for cases in which the vertical refractive index profile changes along the propayation path. The PE method normally used is the split-step method described by tappert (1), which has been implemented by Dockery (2), Craig and Levy (3), and others. An advantage of the PE method is its ability to compute propagation effects within the horizon as well as beyond the horizon, thereby allowing computations to be made in all regions of practical interest to radar engineers or operators with just one model. However, a significant disadvantage of the split-step PE method is that it requires extensive computation. Computational requirements increase with higher frequencies, larger antenna beamwidths, and higher altitudes for which results are desired. For many practical combinations of these parameters, the use of PE models on personal computers is impractical without extra hardware such as transputers.

This paper presents a hybrid propagation model called the Radio Physical optics (RPO) model that uses a combination of ray optics (RO) and split-step PE methods to overcome the high computational burden of pure splitstep PE methods. RPO considers four regions shown in Figure 1. At ranges less than 2500 meters and for all elevation angles above 5 degrees, RPO uses a flat earth (FE) model that ignores refractive and earth-curvature effects. For the region beyond the FE region where the grazing angles of reflected rays from the transmitter are above a small limiting value, a full Ro model is used that accounts for the effects of refraction and earth curvature. The PE model is used for ranges beyond the Ro region, but only for altitudes below a maximum PE altitude determined by the maximum 1024-point fast-Fourier transform (FFT) allowed. For ranges beyond the RO region and heights above the PE region, an extended-optics ( $x a$ ) method is used that is initialized by the $p e$ model at the maximum PE altitude, and uses ray-optics methods to propagate the signal to higher altitudes. Continuity of the solutions across each region's boundaries is kept to icss than 0.1 dB by careful selection of the limiting Ro grazing angle and the maximum PE propagation angle.

Figure 2 shows an RPO example for 3300 MHz with a transmitter at 30.5 meters above sea level. The figure shows propagation loss in d8 on gray-shade scale versus range and height. Propagation loss is the ratio of transmitted to received power using the actuai transaitter antenna pattern but normalized to 0 dB antenna gain. The environment is a measured range-dependent case from point toma in San Diego, California on a path toward Guadalupe Isle. Mexico. The
refractivity profiles along this path indicated a trapping layer that inareased in height with incressing range, such that a surface-based duct extsted at Point Loma and a low-elevated duct existed at 250 km . As a reference point to figure i, the lowes let: corner of the xo region in this example is at a range of 95 km and a height of 1260 m .

## RAY ORTLCS KODRL

Refractive index protiles are specified in Rpo at one or more ranges, and are entered in terms of modified refractivity. M, defined is

$$
M *(n-1 \cdot 2 / a) \times 10^{\circ}
$$

(d)
where $n$ is the radio refractive index, $z$ is height, and a is the earth's radius. computations in the ro region do not considet range-dependent environmental effects, but rather depend only on the vertical mprofile at the transmitter.

The key to RPO's efficiency is keeping the angles considered in the pe model as small as possible by maximizing the ro region. A limiting grazing angle $\mathrm{C}_{0}$ for reflected rays is determined that defines the maximum range and altitude to which the ro method can be applied. Ho is first computed as

$$
\begin{equation*}
\psi=0.04443,1^{1 / 3} \tag{2}
\end{equation*}
$$

where to is in radians and $f$ is frequency in MHz. Fofrom equation ( 2 ) is 2.5 times the limit given by Reed and Russell (4). The factor 2.5 was chosen to ensure that error: in the RO solution would be less than 0.1 ? * from equation (2) is limited to values above 0.002 radians, and then doubled if a range-dependent environment has been specified. Finally, to is increased by an amount $\delta$ to account for ducting given by

$$
\begin{equation*}
\delta \psi=\sqrt{2 \times 10^{6}\left(\mathrm{M}_{0}-\mathrm{M}_{\infty}\right)} \tag{3}
\end{equation*}
$$

where $M_{0}$ is $M$ at the surface and $M_{0}$ is the minimum value of $M$ at all heights.

The Ro method consists of tracing a serzes of direct and reflected rays through selected control points, and then interpolating tho magnitudes of the direct and reflected rays and the phase angle between them at each desired RFO output point. The magnitude of each ray is computed from a spreading term relative to free-space spreading, and the phase angle is determined from the optical path length differences from the ground range for each ray. The direct and reflected ray through a given point are characterized by their elevation angles at the transmitter. and $a_{r}$, respectively. The raytrace method is summarized below for a single rtep.

The elevation angle at the end of the stop. a, is qiven by

$$
\begin{equation*}
a_{1}=\sqrt{a_{0}^{2}+2 \times 10^{6}\left(M_{1,1}-M_{1}\right)} \tag{4}
\end{equation*}
$$

where $a_{0}$ is the elevation angle at the beginning of the step, and $M_{1}$ and $M_{1-1}$ represent $M$ at the beginning and end of the step. The range increment $\Delta x$, spreading increment $\Delta S$, and optical path length difference Increment $\Delta D$ over the step are qiven as

$$
\Delta x=\left(a_{1}-\alpha_{p}\right) / g_{1}
$$

$$
\begin{equation*}
\Delta S=\left(\alpha / \alpha,-\alpha / \alpha_{0}\right) / g_{1} \tag{5}
\end{equation*}
$$

$\Delta 0=\left[\left(10{ }^{-} M_{1}-a_{0}^{2} / 2\right)\left(a_{1}-a_{0}\right] \cdot\left(a_{1}^{3}-a_{0}^{3}\right) / 3\right) / g_{1}$

$$
g_{1}=10^{\circ}\left(M_{i, 1}-M_{1}\right) /\left(z_{1.1}-z_{1}\right)
$$

where a is the elevation angle at the transmitter, and $z_{1}$ and $z_{1,1}$ are the heights at the beginning and end of the step. The total range, $x$, total spreading term, $S$, and total optical path length difference, $D$, are given by the sums of $\Delta x, \Delta S$, and $\Delta D$ over all steps along each ray. The propagation factors, $F_{d}$ and $F_{r}$, for the direct and reflected rays and the total phase angle between the rays in radians, $n$, are given by

$$
\begin{gather*}
F_{d}^{2}=f_{d}^{2}\left|\frac{x}{\left(\beta_{d} S_{d}\right)}\right| \\
F_{r}^{2}=f_{r}^{2} R^{2}\left|\frac{x}{\left(\beta_{r} S_{r}\right.}\right|  \tag{6}\\
n=\left(D_{r}-D_{d}\right) k+\phi
\end{gather*}
$$

where $f_{d}$ and $f_{r}$ are antenna pattern factors corresponding to the elevation angles at the transmitter, $\alpha_{d}$ and $\alpha_{r}$, respectively. $\beta_{d}$ and Brare elevation angles at the terminal point of each ray for the direct and reflected rays. $k$ is the wave number $2 \pi / \lambda$ where $\lambda$ is wavelength in meters. $F$ and $\phi$ are the magnitude and phase lag of the reflection coefficient, which are computed for horizontal, vertical, or circular polarization and include the effect of sea roughness based on wind speed in the same manner as described by Patterson et al. (5). The sum of the two ray components is given by

$$
\begin{equation*}
F^{2}=F_{d}^{2}+F_{r}^{2}+2 F_{d} F_{r} \cos n \tag{7}
\end{equation*}
$$

where $P$ is the propagation factor defined as the ratio of the field strength to the freespace field strength. Propagation loss $L$ in dB is computed as

$$
\begin{equation*}
L=20 \log t+20 \log x-10 \log F^{2}-27.56 \tag{8}
\end{equation*}
$$

where log is base 10 , f is frequency in maiz and $x$ is range in meters.

## FLAT EARTH YODEL

The flat earth model ignores all effects from refraction and earth curvature, and computes the direct and reflected ray interference pattern using straight line paths. Full account is given to the antenna pattern and sea-surface reflection coefficient, including rough surface effects. The direct path length, $r_{1}$, and the reflected path length, $r_{2}$, are given by

$$
\begin{align*}
& r_{1}=\sqrt{\left(z-z_{1}\right)^{2}+x^{2}} \\
& r_{2}=\sqrt{\left(z+z_{t}\right)^{2}+x^{2}} \tag{9}
\end{align*}
$$

 the transmitcer and 2 , 1 s the transtajetes height above the surface. The ufseirg argie t, and the direct ray elevation argle at ris


$$
\begin{align*}
& * \tan ((z+z,) ; x)  \tag{10}\\
& a_{0} \cdot \tan ^{\prime}\left(\left\{z+z_{1}\right) / x\right)
\end{align*}
$$

and the reflected ray eievation angle is given by $a$, - . The total phase lay. $n$, in radians is

$$
\begin{equation*}
\Omega *\left\langle\Gamma_{2} \cdot r_{1}\right\} \times \cdots \tag{11}
\end{equation*}
$$

The propagation factor, F, for the sohereft sum of the two ray components is computed by equation (7), where $F_{a}=f_{s}$ and $F$, $f$ f. Propagation loss is computed fróm equation (8), substituting $F$, for $x$.

## PARABOLIC EOQATION HQOEY

The split-step pe wodel tollows Dockery (2). where the complex field u(x,z\} is advanced io $u(x+\delta x, z) b y$
where $M$ is the moditled refractivity as defined in equation (1), The fourier transform $g$ of $u(x, z)$ is defined as

$$
u(x, p)=S[u(x, z)]=\int_{-}^{-} u(x, z) e^{\cdot \operatorname{pax}(23)}
$$

Where $p=k \sin \theta$, and $\theta$ is the angle trom the horizontal. In RPO, only real-valued sine FFTs are used, with which the real and imaginary parts of u are transtormed separately. A filter is applied to the upper $1 / 4$ of the field in both $z$-and p-spaces at each step to ensure that the field reduces to zero at the top of the transform. Transtorm size varies, but never exceeds 1024 points.

The starting field at $x=0$ is constructed in p-space based on imaqe theory and far-ifeld approximations. Thus

$$
\begin{equation*}
U(0, p)=G\left[t_{d} e^{i p x_{t}} \cdot f, R e^{((p,-\infty)}\right] \tag{14}
\end{equation*}
$$

Which is normalized by $G$ such that

$$
\begin{equation*}
F^{2}=x|u(x, 2)|^{2} \tag{15}
\end{equation*}
$$

Which is used in equation (8) to compute propagation loss in $d B$. The magnitude $R$ and phase lag of the reflection coefficient. include polarization and rough surface effects in the same manner as used in the $R O$ region. A Gaussian taper function, much stronger than the filter reforred to above, is applied to the upper $1 / 4$ of the starting solution. This flitstiun reduces the field up to 70 dB , in addition to any reduction by the antenna pattern.

The selection of the height increment $\delta z$ between adjacent values of $u$ and the range increment ix follows methods outlined by Tappert (1), and summarized as

$$
\begin{gather*}
\delta z=\lambda_{i} i_{i} \\
\theta_{\max }+\frac{1}{2 i_{i}}  \tag{16}\\
\delta x=2 k(\delta z)
\end{gather*}
$$

"where $N_{2}$ is an appropriate integer number, and $\theta_{\text {en }}$ is the maximum angle that can te accommodated in the PE solution. In Ryo, N is determined as the maximum integer such chat $3 / 4$ of $\theta$ just exceeds the maximum elevation angle of the RO 11 miting ray at any height below $1 / 4$ of the maximum PE neight

## EXTENDED OPTICS MODEL

The propagation factor $F$ is computed frow the PE solution at the top of the PE region and is used to initialize an Ro model in the xo region. This model traces rays, along which s from the $P E$ solution is held constant, taking account of the full range-dependent refractivity environment, and basea on initial angles $\beta$ at the PE/XO boundary. If a reflected ray from the transmitter exists at the pe/xo boundary, then $f$ is the local elevation angle of this ray at the $\mathrm{PE} / \mathrm{XO}$ boundary. The greatest range at which reflected rays exist along the $p E / X O$ boundary is called the optical limit. For ranges beyond the optical limit, $\beta$ is given by

$$
\begin{equation*}
\beta=\sqrt{\theta_{6}^{2}+2 \times 10^{4}\left(M-M_{t}\right)} \tag{17}
\end{equation*}
$$

where $\beta_{1}$ and $Y_{1}$ are $\beta$ and modified refractivity at the optical 1 imit, and $M$ is the modified refractivity at the desired range. Using linear interpolation techniques. $F$ is defined at all puints within the xo region, and propagation loss is calculated using equation ( 8 ).

## COMPARISON TO QTHER METHODS

Figure 3 compares RPO results to results from a waveguide prograr and a split-step PE model. The case presented is a homogeneous surface based duct defined by the modifled refractivity profile of Table 1 . The frequency is 3000 kHz , the antenna height is 30.5 meters, the polarization is horizontal. the antenna pattern is omni-directional, and a smooth sea surface is assumed. Figure 3 plots propagation factor in $d B$ versus altitude at a range of 185 km . Figure 3 a shows the RPO results for the ducting case by a solid curve, and the corresponding standard atmosphere case by a dot-dash curve. The dotted horizontal lines in Figure 3a indicate the boundaries of the PE, XO, and RO regions for the ducting case at the is5 km range. The effects of this strong duct in all three of the RPO regions is clear from a comparison of the ducting and standard curves. Figure 3b presents wayeguide results for this same case previously reported by Hitney et al. (6). The wavequide results are practicaliy identical to the RPO results. Figure 3 c presents results from a split-step PE model described by Barrios (7). These results are also virtually identical to the Rpo results.

The times required to compute the ducting cases of Figure 3 on a $25 \mathrm{MHz} \mathrm{IBM/PC-}$ compatible 486 computer were 69,381 , and 310 seconds for the RPO, waveguide, and PE models, respectively. In more stressful cases. RPO has proven to be 25 to 100 times faster than split-step PE models. For some practical combinations of higher heights and frequencies and using beawwidths above a few degrees. PE models are usually impractical due to large transform size requirements. RPC has no difficulty with these applications.

One limitation of the current implementrition of fyo is the use of a range-independent ko wodel in range-dependent environment:, which
in extreme cases can coust sumempintitap. along the RO/PE boundary. However, expesienc. has shown that for realistic envirotments. such as the case of ligute 2, the imitin; grazing angle is high enough to esture acceptable Ro results. A second lisatation concerns rough surface eftects in the ph model, which are includes in the siatting solution to account for higher amale ettect: but are fqnored thereafter. Compar:sons ot waveguide and roo results indscite this simplification can affect come hagn frequen, applications in ducting enufronmerts

## CONCLUEIONS

Based on comparisons of results, ko is as accurate as vaveguide or split-step pe models. RPO can compute results for rargedependent environments in the some way pe models can, but is applicable to wiser antenna patterns than typleal PE models. Min has proven to be auch faster than split-stef: PE models, yet it zequires far lesis memory than typical PE models. The accurate result wider applicability, faster computation times, and smaller memory requirements make Rpo an ldes model for use in all propagation assessment or engineering-aid programs. including those hosted on personal computers

## ACRHOMLEDGEMENTS

Special thanks are due to F.D. Tappert for providing a compact working PE wodel in August 1990, frow which RFO's PE model was derived. The development of RPO was sponsorm by the office of Naval Technology. Patent protection is pending on this work. This paper is in the public domain.

## REPRRENCER

1. Tappert, F.D., 1977, "The Parabolic Approximation Method", in "Wave Propagation and Underwater Acoustics" J.B. Keller and J.S. Papadakis. Eds. Now York, Springer-Verlag, 224-285.
2. Dockery, G.D.. 1988, LEFE Trans. Antcnns: Rropagat. 35 . 1464-1470.
3. Craig, K.H. and Levy, M.F., i9a7, "The Modelling of Transhorizon anomalous Propagation Conditions". IEE conf. Pub, 274 (ICAP '87), 347-35
4. Reed, H.R., and Rus? ell, C.M., 1966. "ultra High Frequer if Propagation," Boston Technical wishers. Inc.. Camistidge, MA, *s.
5. Patterson, W.1., Hattan, C.F., Hitmey, H.V., Paulus, R.A., Barrios, A.E., Lindem, G.E., and Anderson, K.D.. 1970 , "Engineer's Refractive Effects Prediction System (EREPS) Revision 2.0," Naval Ocean System: Center Tech. Doc. 1342, San die $O$, CA, Usa.
6. ittney, H.V., Richter, J. H., Pappert, R.A., Anderson, K.D., and mumgartner. G.8., 1985. Eroc. IEEE, 3], 255-283
7. Barrios, A.E., 1991, "Radio Wave Propagation in Horizontally Intomogeneous Environments by Using the Parabolic Equation Method," Naval Oeman Systems Center Tech. Rep. 1430, San Diego, ca, USA.

| Height | Reiractivity <br> M units |
| :---: | :---: |
| 0 | 367 |
| 267 | 401 |
| 165 | 354 |
| 3000 | 703 |

rable 1. Modilied refrdetivity versus helght profile for the ducting case of figure 3 .


Piquy 1. The four RPO regions.


Figure 3. Propagation factor versus height for a surface-based duct from: (a) RPO, (b) waveguide, and (c) PE models. Dot-dash curve is a $s t a n d a r d$ atmosphere reference from RPO.


