Malaysian Journal of Mathematical Sciences 7(2): 219-246 (2013)

MALAYSIAN JOURNAL OF MATHEMATICAL SCIENCES

Journal homepage: http://einspem.upm.edu.my/journal

Hybrid Synchronization of *n*-scroll Chaotic Chua Circuits using Adaptive Backstepping Control Design with Recursive Feedback

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ABSTRACT

In this paper, the hybrid synchronization is investigated for n-scroll chaotic Chua circuit (Wallace *et al.* (2001)) using adaptive backstepping control design based on recursive feedback control. Our theorems on hybrid synchronization for n-scroll chaotic Chua circuits are established using Lyapunov stability theory. The adaptive backstepping control links the choice of Lyapunov function with the design of a controller and guarantees global stability performance of strict-feedback chaotic systems. The adaptive backstepping control maintains the parameter vector at a predetermined desired value. The adaptive backstepping control method is effective and convenient to synchronize and estimate the parameters of the chaotic systems. Mainly, this technique gives the flexibility to construct a control law and estimate the parameter values. Numerical simulations are also given to illustrate and validate the synchronization results derived in this paper.

Keywords: Chaos, hybrid synchronization, adaptive backstepping control, *n*-scroll chaotic chua circuit.

1. INTRODUCTION

Dynamics systems described by nonlinear differential equations can be strongly sensitive to initial conditions. This phenomenon is known as deterministic chaos, which means that the mathematical description of the system is deterministic but behavior of the system is unpredictable. Chaos refers to one type of complex dynamical behaviors that possess extreme sensitivity to tiny variations of initial conditions, bounded trajectories in phase space and fractional topological dimensions. The fundamental characteristic of a chaotic system is its sensitivity to the initial state. That is to say, chaotic systems starting off from very similar initial states can develop into radically divergent ways. Such sensitive dependence is often referred to as the Butterfly effect. In general, synchronization research has been focused on two areas. The first one works with the state observers, where the main applications pertain to the synchronization of nonlinear oscillators. The second one is the use of control laws, which allows achieving the synchronization between nonlinear oscillators, with different structures and orders.

The synchronization of chaotic system was first researched by Yamada and Fujisaka (Fujisaka and Yamada (1983)) with subsequent work by Pecora and Carroll (Pecora and Carroll (1990), Pecora and Carroll, (1991)). The synchronization of chaos is one way of explaining sensitive dependence on initial conditions (Alligood *et al.* (1997), Edward (2002)). It has been established that the synchronization of two chaotic systems, that identify the tendency of two or more systems are coupled together to undergo closely related motions. The problem of chaos synchronization is to design a coupling between the two systems such that the chaotic time evaluation becomes ideal. The output of the response system asymptotically follows the output of the drive system i.e. the output of the drive system controls the response system.

The synchronization for chaotic systems has been widespread to the scope, such as generalized synchronization (Wang and Zhu (2006)), phase synchronization (Ge and Chen (2004)), lag synchronization, projective synchronization (Qiang (2007)), generalized projective synchronization (Jian-Ping and Chang-Pin (2006), Li *et al.* (2007), Sundarapandian and Sarasu (2012), Sarasu and Sundarapandian (2012)) and even anti-synchronization. The property of anti-synchronization establishes a predominating phenomenon in symmetrical oscillators, in which the state vectors have the same absolute values but opposite signs. When synchronization and anti synchronization coexist, simultaneously, in chaotic systems, the synchronization is called hybrid synchronization (Li (2008), Sundarapandian and Suresh (2012), Sundarapandian and Sivaperumal (2012)). A variety of schemes for ensuring the control and synchronization of such systems have been demonstrated based on their potential applications in various fields including chaos generator design, secure communication (Murali and Laksmanan (2003a), Yang and Chua (1999)), physical systems (Murali and Laksmanan (1996)), chemical reaction (Han *et al.* (1995)), ecological systems (Blasius *et al.* (1999)), information science (Kocarev and Parlitz (1995)), energy resource systems (Zuolei Wang (2010)), ghostburster neurons (Jiang Wang (2009)), bi-axial magnet models (Moukam Kakmeni *et al.* (2006)), neuronal models (Hindmarsh and Rose (1984) and Yan-Qiu Che *et al.* (2010)), IR epidemic models with impulsive vaccination (Guang Zhao Zeng *et al.* (2005)) and predicting the influence of solar wind to celestial bodies (Junxa Wang *et al.* (2006)), etc. So far a variety of impressive approaches have been proposed for the synchronization of the chaotic systems such as the OGY method (Ott (1990)), sampled feedback synchronization method (Murali and Laksmanan (2003b)), time delay feedback method (Park and Kwon (2003)), adaptive design method (Lu *et al.* (2004), Park *et al.* (2003), Park (2008)), sliding mode control method (Yau (2004), Sundarapandian (2011)), active control method (Sundarapandian and Suresh (2010a); Sundarapandian and Suresh (2010b)) and backstepping control design (Wu and Li (2003), Yu and Zhang (2006), Suresh and Sundarapandian (2012a, 2012b, 2013)) etc.

Adaptive control design is a direct aggregation of a control methodology with some form of a recursive system identification and the system identification could be aimed to determining the system to be controlled is linear or nonlinear systems. The system identification is only the parameters of a fixed type of model that need to be determined and limiting the parametric system identification and parametric adaptive control. Adaptive control design is studied and analyzed in theory of unknown but fixed parameter systems. In this paper, Adaptive control design with feedback input approach is proposed. This approach is a systematic design approach and guarantees global stability of the n-scroll Chua chaotic circuit (Wallace *et al.* (2001)). Based on the Lyapunov function, the adaptive update control is determined to tune the controller gain based on the precalculated feedback control inputs. We organize this paper as follows. In Section 2, we present the methodology of hybrid chaos synchronization by adaptive control method. In Section 3, we give a description of the chaotic systems discussed in this paper. In Section 4, we demonstrate the hybrid synchronization of identical n–scroll chaotic Chua circuits. In Section 5, we summarize the results obtained in this paper.

2. PROBLEM STATEMENT AND METHODOLOGY

In general, the two dynamic systems in synchronization are called the master and slave system respectively. A well designed controller will make the trajectory of the slave system track the trajectory of the master system, which are the two systems will be synchronous.

Consider the master system described by the dynamics

$$\begin{aligned} \dot{x}_1 &= F_1(x_1, x_2, \ldots x_n, \alpha_i) \\ \dot{x}_2 &= F_2(x_1, x_2, \ldots x_n, \alpha_i) \\ \dot{x}_3 &= F_3(x_1, x_2, \ldots x_n, \alpha_i) \\ &\vdots \\ \dot{x}_n &= F_n(x_1, x_2, \ldots x_n, \alpha_i) \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is a state vectors of the system and α_i are positive unknown parameters, $\hat{\alpha}_i$ are estimates of the parameters α_i.

Consider the slave system with the controller $u_i, i = 1, 2, 3 \ldots n$ described by the dynamics

$$\begin{aligned} \dot{y}_1 &= G_1(y_1, y_2, \ldots y_n, \alpha_i) + u_1 \\ \dot{y}_2 &= G_2(y_1, y_2, \ldots y_n, \alpha_i) + u_2 \\ \dot{y}_3 &= G_3(y_1, y_2, \ldots y_n, \alpha_i) + u_3 \\ \vdots & \qquad \vdots \qquad\qquad \vdots \\ \dot{y}_n &= G_n(y_1, y_2, \ldots y_n, \alpha_i) + u_n \end{aligned} \tag{2}$$

where $u_i, i = 1, 2, 3 \ldots n$ is the input to the system with parameter estimator $\hat{\alpha}_i$, i = 1, 2, 3...n , $y(t) \in R^n$ is state vectors of the system including the controller and identifier. F_i, G_i, i = 1, 2, 3...n are linear and nonlinear functions with inputs from system (2) and (1).

F_i Equals to G_i , then the systems states are identical chaotic hybrid synchronization otherwise the systems states are non identical chaotic hybrid synchronization. The chaotic systems (1) and (2) depend not only on state variables but also on time t and the parameters.

The hybrid synchronization error is defined as

$$e_i = \begin{cases} y_i - x_i & \text{if } i \text{ is odd} \\ y_i + x_i & \text{if } i \text{ is even} \end{cases} \tag{3}$$

Then the error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= G_1(y_1, y_2, \ldots y_n, \alpha_i) - F_1(x_1, x_2, \ldots x_n, \alpha_i) + u_1 \\ \dot{e}_2 &= G_2(y_1, y_2, \ldots y_n, \alpha_i) + F_2(x_1, x_2, \ldots x_n, \alpha_i) + u_2 \\ \dot{e}_3 &= G_3(y_1, y_2, \ldots y_n, \alpha_i) - F_3(x_1, x_2, \ldots x_n, \alpha_i) + u_3 \\ &\vdots \\ \dot{e}_n &= G_n(y_1, y_2, \ldots y_n, \alpha_i) - F_n(x_1, x_2, \ldots x_n, \alpha_i) + u_n \end{aligned} \tag{4}$$

where $u_i, i = 1, 2, 3 \ldots n$ are controllers to the system with parameter estimator $\hat{\alpha}_i$.

The parameter estimation error is defined as

$$e_{\alpha_i} = \alpha_i - \hat{\alpha}_i, \text{ i = 1, 2, 3...n} \tag{5}$$

The synchronization error systems control a controlled chaotic system with control input $u_i, i = 1, 2, 3 \ldots n$ with adaptive update law $\hat{\alpha}_i$ as a function of the parameter estimator error states e_{α_1}, e_{α_2}, $e_{\alpha_3} \ldots e_{\alpha_n}$. That means the systematic adaptive feedback so as to stabilize the error dynamics (4) converge to zero as time $t \to \infty$. This implies that the controller $u_i, i = 1, 2, 3 \ldots n$ and adaptive update law $\hat{\alpha}_i$ should be designed so that the two chaotic systems can be synchronized. In mathematically

$$\lim_{t \to \infty} \| e(t) \| = 0 \tag{6}$$

Adaptive backstepping control design is systematic and guarantees global stabilities performance of strict–feedback chaotic systems. By using the adaptive backstepping control design, the chaotic system is stabilized with respect to Lyapunov function. The Lyapunov stability approach consists in finding an update law. The Lyapunov stability function technique as our methodology, the controller design can be divided into two steps. The first one need the derivation of control Lyapunov function and the second step involves using an existing control Lyapunov function to be synchronizing the chaotic systems

We consider the stability of the system

$$\dot{e}_1 = G_1(y_1, y_2, \ldots y_n, \alpha_i) - F_1(x_1, x_2, \ldots x_n, \alpha_i) + u_1 \tag{7}$$

where $x(t) \in R^n$, $y(t) \in R^n$ are state variables and α_i, i = 1, 2, 3...n are positive unknown parameters, $\hat{\alpha}_i$, i = 1, 2, 3...n are estimates of the parameter α_i, i = 1, 2, 3...n. u_1 is control as long as this feedback stabilize the system (7) converges to zero as $t \to \infty$, where $e_2 = \alpha_1(e_1)$ is regarded as an virtual controller.

We consider the Lyapunov function defined by

$$V_1(e_1) = \mathrm{e}_1^{\mathrm{T}} P_1 e_1 + \sum_{i=1}^{i=k} \mathrm{e}_{\alpha_i}^{\mathrm{T}} Q_1 e_{\alpha_i} \tag{8}$$

where P_1 and Q_1 are positive matrices.

Let us define the parameter estimation error as

$$e_{\alpha_i} = \alpha_i - \hat{\alpha}_i, \text{ i} = 1, 2, 3\ldots\text{k} \tag{9}$$

Differentiating equation (8) along the trajectories (7) and using

$$\dot{e}_{\alpha_i} = -\hat{\alpha}_i, \text{ i} = 1, 2, 3\ldots\text{n}.$$

The derivative of $V_1(e_1)$ is

$$\dot{V}_1(e_1) = -\mathrm{e}_1^{\mathrm{T}} R_1 e_1 - \sum_{i=1}^{i=k} \mathrm{e}_{\alpha_i}^{\mathrm{T}} S_1 e_{\alpha_i}. \tag{10}$$

where R_1 and S_1 are positive definite matrices.

Then $\dot{V}_1$ is a negative definite function on R^n. Thus by Lyapunov stability theory (Che *et al.* (2010)) the error dynamics (7) is asymptotically stable. The virtual control is $e_2 = \alpha_1(e_1)$ and the state feedback input u_1 makes the system (7) asymptotically stable.

The function $\alpha_1(e_1)$ is estimative when e_2 considered as controller. The error between e_2 and $\alpha_1(e_1)$ is

$$w_2 = e_2 - \alpha_1(e_1) \tag{11}$$

Consider the (e_1, w_2) subsystem given by

$$\begin{aligned} \dot{e}_1 &= G_1(y_1, y_2, \ldots y_n, \alpha_i) - F_1(x_1, x_2, \ldots x_n, \alpha_i) + u_1 \\ \dot{w}_2 &= G_2(y_1, y_2, \ldots y_n, \alpha_i) - F_2(x_1, x_2, \ldots x_n, \alpha_i) - \dot{\alpha}_1(e_1) + u_2 \end{aligned} \tag{12}$$

Let e_3 be a virtual controller in system (12). Assume when

$$e_3 = \alpha_2(e_1, w_2)$$

the system (12) is made asymptotically stable.

Consider the Lyapunov function defined by

$$V_2(e_1, w_2) = V_1(e_1) + w_2^T P_2 w_2 + \sum_{i=k+1}^{i=m} \mathrm{e}_{\alpha_i}^T Q_2 e_{\alpha_i} \tag{13}$$

where P_2 and Q_2 are positive matrices.

Let us define the parameter estimation error as

$$e_{\alpha_i} = \alpha_i - \widehat{\alpha}_i, \ \mathrm{i = k+1, 2, 3...m} \tag{14}$$

Differentiating equation (13) along the trajectories (12) and using

$$\dot{e}_{\alpha_i} = -\widehat{\alpha}_i, \ \mathrm{i = k+1, 2, 3...m} \tag{15}$$

Suppose the derivative of $V_2(e_1, w_2)$ is

$$\dot{V}_2(e_1, w_2) = -\mathrm{e}_1^{\mathrm{T}} R_1 e_1 - w_2^{\mathrm{T}} R_2 w_2 - \sum_{i=1}^{i=k} \mathrm{e}_{\alpha_i}^{\mathrm{T}} S_1 e_{\alpha_i} - \sum_{i=k+1}^{i=m} \mathrm{e}_{\alpha_i}^{\mathrm{T}} S_2 e_{\alpha_i} \tag{16}$$

where R_1, R_2, S_1, S_2 are positive definite matrices.

Then $\dot{V}_2$ is a negative definite function on R^n.

Thus by Lyapunov stability theory (Hahn (1967)) the error dynamics (12) is globally asymptotically stable. For the n th state of the error dynamics, define the error variable w_n as

$$w_n = e_n - \alpha_{n-1}(e_1, w_2, w_3 \ldots w_{n-1}) \tag{17}$$

Consider the $(e_1, w_2, w_3 \ldots w_n)$ subsystem given by

$$\begin{aligned}
\dot{e}_1 &= G_1(y_1, y_2, \ldots y_n, \alpha_i) - F_1(x_1, x_2, \ldots x_n, \alpha_i) + u_1 \\
\dot{w}_2 &= G_2(y_1, y_2, \ldots y_n, \alpha_i) + F_2(x_1, x_2, \ldots x_n, \alpha_i) - \dot{\alpha}_1(e_1) + u_2 \\
\dot{w}_3 &= G_3(y_1, y_2, \ldots y_n, \alpha_i) - F_3(x_1, x_2, \ldots x_n, \alpha_i) - \dot{\alpha}_2(e_1, w_2) + u_3 \\
&\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
\dot{e}_n &= G_n(y_1, y_2, \ldots y_n, \alpha_i) - F_n(x_1, x_2, \ldots x_n, \alpha_i) - \dot{\alpha}_{n-1}(e_1, w_2, w_3 \ldots w_n) + u_n
\end{aligned} \tag{18}$$

Consider the Lyapunov function defined by

$$V_n(e_1, w_2, w_3 \ldots w_n, e_{\alpha_i}) = V_{n-1}(e_1, w_2, w_3 \ldots w_n) + w_{\mathrm{n}}^{\mathrm{T}} P_n w_n + \sum_{i=m+1}^{i=n} \mathrm{e}_{\alpha_i}^{\mathrm{T}} Q_n e_{\alpha_i} \tag{19}$$

where P_n and Q_n are positive matrices.

Let us define the parameter estimation error as

$$e_{\alpha_i} = \alpha_i - \hat{\alpha}_i, \ \mathrm{i = m+1, 2, 3 \ldots n} \tag{20}$$

Differentiating equation (19) along the trajectories (18) and using

$$\dot{e}_{\alpha_i} = -\widehat{\alpha}_i,\ \mathrm{i = m+1, 2, 3...n} \tag{21}$$

Suppose the derivative of $V_2(e_1, w_2)$ is

$$\begin{aligned} \dot{V}_n(e_1, w_2, w_3 ... w_n) = & -\mathrm{e}_1^{\mathrm{T}} R_1 e_1 - w_2^{\mathrm{T}} R_2 w_2 - w_3^{\mathrm{T}} R_3 w_3 - ... - w_{\mathrm{n}}^{\mathrm{T}} R_n w_n \\ & - \sum_{i=1}^{i=k} \mathrm{e}_{\alpha_i}^{\mathrm{T}} S_1 e_{\alpha_i} - \sum_{i=k+1}^{i=m} \mathrm{e}_{\alpha_i}^{\mathrm{T}} S_2 e_{\alpha_i} - ... - \sum_{i=q+1}^{i=n} \mathrm{e}_{\alpha_i}^{\mathrm{T}} S_n e_{\alpha_i} \end{aligned} \tag{22}$$

where $R_1, R_2, R_3 ... R_n, S_1, S_2, S_3 ... S_n$ are positive definite matrices.

Then $\dot{V}_n$ is a negative definite function on R^n.

Thus by Lyapunov stability theory (Hahn (1967)) the error dynamics (14) is asymptotically stable. The virtual control is

$$e_n = \alpha_{n-1}(e_1, w_2, w_3 ... w_{n-1})$$

and the state feedback input u_n make the system (18) globally asymptotically stable.

Thus by Lyapunov stability theory (Hahn (1967)), the error dynamics (4) is globally asymptotically stable for all initial condition $e(0) \in R^n$. Hence, the states of master and slave systems are globally and asymptotically hybrid synchronized and the adaptive control law is given by

$$\widehat{\alpha}_i = G(e) + k_i e_{\alpha_i} \tag{23}$$

where k_i is positive constant, $e_i = \begin{cases} y_i - x_i & \text{if } i \text{ is odd} \\ y_i + x_i & \text{if } i \text{ is even} \end{cases}$ is the error vector, and $G : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous vector function with the error as its arguments.

Theorem 1. The chaotic system (1) and (2) are globally exponentially hybrid synchronized with adaptive backstepping control with recursive feedback inputs $u_1 = u_2 = u_3....u_n = \mu(x, y, e_i, w_i)$. The adaptive control law is updated by

$\hat{\alpha}_i = G(e_i) + k_i e_{\alpha_i}$, where $e_i = \begin{cases} y_i - x_i \text{ if } i \text{ is odd} \\ y_i + x_i \text{ if } i \text{ is even} \end{cases}$ is an error and

$\mu = R^n \to R^n$, $G : \mathbb{R}^n \to \mathbb{R}^n$ are continuous vector function with x, y and e as its arguments.

3. SYSTEM DESCRIPTION

Recently, theoretical design and hardware implementation of different kinds of chaotic oscillators have attracted increasing attention, aiming real world applications of many chaos based technologies and information systems. In current research interest is creating various complex multi scroll chaotic attractors by using simplified and generic electrical circuit. Here which we are interested is the n– scroll Chua circuit which is an improved model of chaotic system introduced by Wallace *et al.* (2001). In fact, it is now obvious that can be derived from simplified and generic electrical circuit.

a) The n-Scroll Chua system

Chua's system is utilized for the investigation. The dynamical equation of *n*–scroll Chua system with sine function (Wallace *et al.* (2001)) is given by

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - f(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \end{aligned} \tag{24}$$

where $f(x_1)$ is given by

$$f(x_1) = \begin{cases} \frac{b}{2a}(x_1 - 2ac) & if \quad x_1 \geq 2ac \\ -b\sin(\frac{\pi x_1}{2a} + d) & if \quad -2ac \leq x_1 \leq 2ac \\ \frac{b}{2a}(x_1 + 2ac) & if \quad x_1 \leq 2ac \end{cases} \tag{25}$$

where a, b, c, and d are positive real constants.

The piecewise linear function is only nonlinearity in the system. A sine function is couched to obtain the nonlinearity needed for generating chaos in Chua system. For the chaotic case, the parameter values are taken in equations as

$$\alpha = 10.814,\ \beta = 14.0,\ a=1.3,\ b=0.11 \text{ and } d=0$$

Furthermore, if we choose $c = 1, 2, 3$ and 5, then we obtain 2-scroll, 3-scroll, 4-scroll and 6-scroll attractors respectively, as depicted in Figure 1(a)–(d). A maximum of six scrolls can be observed.

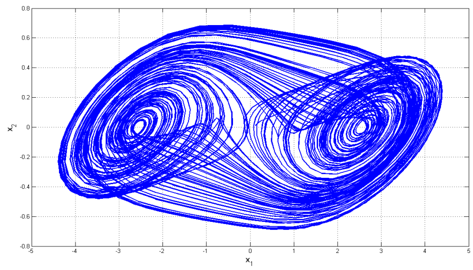

Figure 1(a): 2- Scroll chaotic attractor

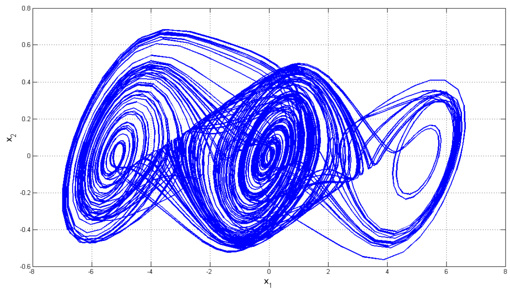

Figure 1(b): 3- Scroll chaotic attractor

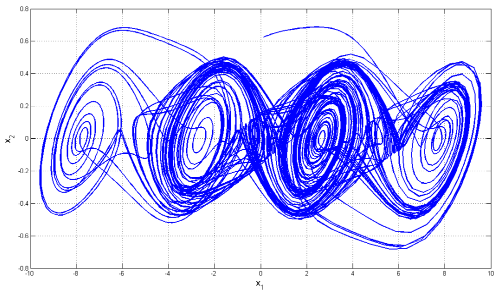

Figure 1(c): 4- Scroll chaotic attractor

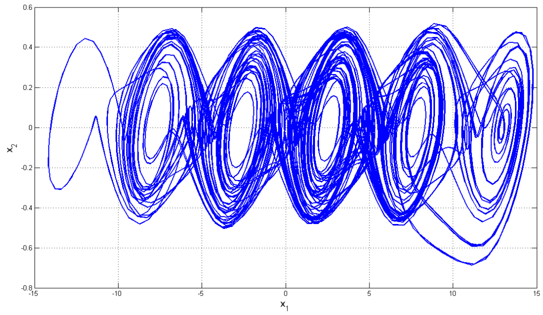

Figure 1(d): 6- Scroll chaotic attractor

4. HYBRID SYNCHRONIZATION OF IDENTICAL *N*-SCROLL CHUA SYSTEMS USING ADAPTIVE BACKSTEPPING CONTROL DESIGN BASED ON RECURSIVE FEEDBACK CONTROL

In this section we apply the adaptive backstepping method is applied for the hybrid synchronization of two identical *n*-scroll chaotic Chua circuits (Wallace *et al.* (2001)) when the parameter values are unknown. Thus, the master system is described by the *n*-scroll chaotic Chua circuit dynamics

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - f(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \end{aligned} \tag{26}$$

where $f(x_1)$ is given by

$$f(x_1) = \begin{cases} \dfrac{b}{2a}(x_1 - 2ac) & if \quad x_1 \geq 2ac \\ -b\sin(\dfrac{\pi x_1}{2a} + d) & if \quad -2ac \leq x_1 \leq 2ac \\ \dfrac{b}{2a}(x_1 + 2ac) & if \quad x_1 \leq 2ac \end{cases} \tag{27}$$

x_1, x_2, x_3 are state variables and α, β, a, b, c are positive unknown parameters, $\hat{\alpha}, \hat{\beta}, \hat{a}, \hat{b}, \hat{c}$ are estimates of the parameters α, β, a, b, c.

The slave system is also described by the n- scroll chaotic Chua circuit dynamics

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - f(y_1)) + u_1 \\ \dot{y}_2 &= y_1 - y_2 + y_3 + u_2 \\ \dot{y}_3 &= -\beta y_2 + u_3 \end{aligned} \tag{28}$$

where $f(y_1)$ is given by

$$f(y_1) = \begin{cases} \dfrac{b}{2a}(y_1 - 2ac) & if \quad y_1 \geq 2ac \\ -b\sin(\dfrac{\pi y_1}{2a} + d) & if \quad -2ac \leq y_1 \leq 2ac \\ \dfrac{b}{2a}(y_1 + 2ac) & if \quad y_1 \leq 2ac \end{cases} \tag{29}$$

y_1, y_2, y_3 are state variables and u_1, u_2, u_3 are the backstepping controller to be designed.

The synchronization error is defined by

$$e_1 = y_1 - x_1; e_2 = y_2 + x_2; e_3 = y_3 - x_3 \tag{30}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= \alpha y_2 - \alpha x_2 - \alpha[f(y_1) - f(x_1)] + u_1 \\ \dot{e}_2 &= y_1 + x_1 - e_2 + y_3 + x_3 + u_2 \\ \dot{e}_3 &= -\beta y_2 + \beta x_2 + u_3 \end{aligned} \tag{31}$$

a) When $[f(y_1) - f(x_1)] \geq 2ac$ and $[f(y_1) - f(x_1)] \leq -2ac$

The objective is to find the control law and adaptive update law, so the that the system (21) is asymptotically stabilized at the origin and estimates the unknown parameters α, β, a, b, c .We introduce the backstepping procedure to design the controller u_1, u_2, u_3 , where u_1, u_2, u_3 are recursive control feedback, as long as these recursive feedback stabilize system (21) converge to zero as the time $t \to \infty$.

First we consider the stability of the system

$$\dot{e}_3 = -\beta y_2 + \beta x_2 + u_3 \tag{32}$$

where e_2 is regarded as virtual controller.

Consider the Lyapunov function defined by

$$V_1(e_1, e_\beta) = \frac{1}{2}e_1^2 + \frac{1}{2}e_\beta^2 \tag{33}$$

Let us define the parameter estimation error as

$$e_\beta = \beta - \hat{\beta} \tag{34}$$

Differentiating equation (33) along the trajectories (32) and using

$$\dot{e}_\beta = -\dot{\hat{\beta}}$$

We find $\dot{V}_1(e_3, e_\beta)$ is as following

$$\dot{V}_1 = e_3(-\beta y_2 + \beta x_2 + u_3) + e_\beta(-\dot{\hat{\beta}}) \tag{35}$$

Assume the controller $e_2 = \alpha_1(e_3)$. If

$$\alpha_1(e_3) = -k_1 e_3 \tag{36}$$

and

$$u_3 = -2\beta x_2 + 2\hat{\beta} e_2. \tag{37}$$

In equation (35), the parameters are updated by the update law

$$\dot{\hat{\beta}} = -2e_2 \mathrm{e}_3 + \mathrm{k}_2 \mathrm{e}_\beta \tag{38}$$

substituting equations (36), (37) and (38) into equation (35), then we have

$$\dot{\mathrm{V}}_1 = -k_1 \beta e_3^2 - k_2 e_\beta^2. \tag{39}$$

Which is a negative definite function on R^3 since $k_1,\ k_2 > 0$.

The recursive feedback u_3 and the virtual control is $e_2 = \alpha_1(e_3)$ makes the system (32) globally asymptotically stable. Function $\alpha_1(e_3)$ is an estimative function when e_2 is considered as a controller.

The error between e_2 and $\alpha_1(e_3)$ is

$$w_2 = e_2 - \alpha_1(e_3) \tag{40}$$

Consider (e_3, w_2) subsystem given by

$$\begin{aligned} \dot{e}_3 &= \beta w_2 - \beta k_1 e_3 - 2e_\beta e_2 \\ \dot{w}_2 &= e_1 + [k_1(\beta - 2e_\beta) - 1]e_2 + 2x_1 + y_3 + x_3 + u_2 \end{aligned} \tag{41}$$

Let e_1 as a virtual controller in system (41).

Assume that when $e_1 = \alpha_2(e_3, w_2)$, the system (41) is made globally asymptotically stable.

Let us define the Lyapunov function as

$$V_2(e_3, w_2) = V_1(e_3) + \frac{1}{2}w_2^2 \tag{42}$$

The derivative of $V_2(e_3, w_2)$ is

$$\begin{aligned} \dot{V}_2 = &-k_1\beta e_3^2 - k_2 e_\beta^2 + w_2(e_1 + [k_1(\beta - 2e_\beta) - 1]e_2 \\ &+ 2x_1 + y_3 + x_3 + u_2) \end{aligned} \tag{43}$$

If we choose

$$\alpha_2(e_3, w_2) = -\beta e_3 \tag{44}$$

and

$$u_2 = -[k_1(\beta - 2e_\beta) - 1]e_2 - 2x_1 - y_3 - x_3 - k_3 w_2 \tag{45}$$

Then it follows

$$\dot{V}_2 = -k_1\beta e_3^2 - k_2 e_\beta^2 - k_3 w_2^2, \tag{46}$$

which is a negative definite function on R^3 since $k_1,\ k_2,\ k_3 > 0$.

Thus $\dot{V}_2$ is negative definite function and hence the system (41) is globally asymptotically stable.

Function $\alpha_2(e_3, w_2)$ is an estimative function when e_1 is considered as a controller. The error between e_1 and $\alpha_2(e_3, w_2)$ is

$$w_3 = e_1 - \alpha_2(e_3, w_2) \tag{47}$$

Considering (e_3, w_2, w_3) subsystem given by

$$\begin{aligned} \dot{e}_3 &= \beta w_2 - \beta k_1 e_3 - 2e_\beta e_2 \\ \dot{w}_2 &= w_3 - \beta e_3 - k_3 w_2 \\ \dot{w}_3 &= \alpha y_2 - \alpha x_2 - \frac{\alpha b \pi}{2a} e_1 + \beta(\beta w_2 - \beta k_1 e_3 - 2e_2 e_\beta) + u_1 \end{aligned} \tag{48}$$

Consider the Lyapunov function defined by

$$\begin{aligned} V_3(e_3, w_2, w_3, e_\beta) = V_2(e_3, w_2) &+ \frac{1}{2} w_3^2 \\ &+ \frac{1}{2} e_\alpha^2 + \frac{1}{2} e_a^2 + \frac{1}{2} e_b^2 + \frac{1}{2} e_c^2 \end{aligned} \tag{49}$$

Let us define the parameter estimation error as

$$e_\alpha = \alpha - \hat{\alpha}; \ \ e_a = a - \hat{a}; \ \ e_b = b - \hat{b}\ ; \ e_c = c - \hat{c}$$

Differentiating equation (49) along the trajectories (48) and using (50)

$$\dot{e}_\alpha = -\dot{\hat{\alpha}}\ ; \ \ \dot{e}_a = -\dot{\hat{a}}; \ \ \dot{e}_b = -\dot{\hat{b}}\ ; \ \dot{e}_c = -\dot{\hat{c}}. \tag{50}$$

We find $\dot{V}_3(e_3, w_2, w_3, e_\alpha, e_a, e_b, e_c)$ is as following

$$\begin{aligned} \dot{V}_3(e_3, w_2, w_3, e_\beta) = &-\beta k_1 e_3^2 - k_2 e_\beta^2 - k_3 w_2^2 + w_3[w_2 + \alpha y_2 - \alpha x_2 \\ &+ \frac{\pi \alpha b}{2a} e_1 + \beta(\beta w_2 - \beta k_1 e_3 - 2e_2 e_\beta) + u_1] \\ &+ e_\alpha(-\dot{\hat{\alpha}}) + e_a(-\dot{\hat{a}}) + e_b(-\dot{\hat{b}}) + e_c(-\dot{\hat{c}}) \end{aligned} \tag{51}$$

We choose

$$\begin{aligned} u_1 &= -w_2 + 2\alpha x_2 - \hat{\alpha} e_2 + \frac{\alpha \hat{b} \pi}{2a} e_1 - \beta(\beta w_2 - \beta k_1 e_3 - 2e_2 e_\beta) \\ &+ e_a + e_b + e_c - k_4 w_3 \end{aligned} \quad (52)$$

In equation (51), the parameter updated by the update law

$$\dot{\hat{\alpha}} = \mathrm{w}_3 \mathrm{e}_2 + \mathrm{k}_5 \mathrm{e}_\alpha; \quad \dot{\hat{a}} = \mathrm{w}_3 + \mathrm{k}_6 \mathrm{e}_a; \quad \dot{\hat{b}} = \frac{\alpha \pi}{2a} \mathrm{e}_1 \mathrm{w}_3 + \mathrm{k}_7 \mathrm{e}_b; \quad \dot{\hat{c}} = \mathrm{w}_3 + \mathrm{k}_8 \mathrm{e}_c \quad (53)$$

Substituting equation (52) and (53) into equation (51), then we have

$$\begin{aligned} \dot{V}_3(e_1, w_2, w_3, e_\beta) = &- k_1 e_3^2 - k_2 e_\beta^2 - k_3 w_2^2 - k_4 w_3^2 - k_5 e_\alpha^2 - k_6 e_a^2 \\ &- k_7 e_b^2 - k_8 e_c^2 \end{aligned} \quad (54)$$

Which is a negative definite function on R^3 since $k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8 > 0$. and hence the system (48) is globally asymptotically stable.

Thus by a Lyapunov stability theory (Hahn (1967)), the error dynamics (31) is globally asymptotically hybrid synchronized.

Theorem 2. The *n*-scroll chaotic Chua circuit (26) and (28) are globally asymptotically hybrid synchronized for any initial conditions with the recursive controller u_1, u_2, u_3 defined by

$$\begin{aligned} u_1 &= -w_2 + 2\alpha x_2 - \hat{\alpha} e_2 + \frac{\alpha \hat{b} \pi}{2a} e_1 - \beta(\beta w_2 - \beta k_1 e_3 - 2e_2 e_\beta) \\ &\quad + e_a + e_b + e_c - k_4 w_3 \\ u_2 &= -[k_1(\beta - 2e_\beta) - 1] e_2 - 2x_1 - y_3 - x_3 - k_3 w_2 \\ u_3 &= -2\beta x_2 + 2\hat{\beta} e_2 \end{aligned}$$

and the parameter updated by the update law

$$\dot{\hat{\alpha}} = \mathrm{w}_3 \mathrm{e}_2 + \mathrm{k}_5 \mathrm{e}_\alpha; \quad \dot{\hat{\beta}} = -2e_2 \mathrm{e}_3 + \mathrm{k}_2 \mathrm{e}_\beta$$

$$\dot{\hat{a}} = \mathrm{w}_3 + \mathrm{k}_6 \mathrm{e}_a; \quad \dot{\hat{b}} = \frac{\alpha \pi}{2a} \mathrm{e}_1 \mathrm{w}_3 + \mathrm{k}_7 \mathrm{e}_b; \quad \dot{\hat{c}} = \mathrm{w}_3 + \mathrm{k}_8 \mathrm{e}_c$$

b) When $-2ac \le [f(y_1) - f(x_1)] \le 2ac$

The objective is to find the control law and adaptive update law, so the that the system (31) is asymptotically stabilized at the origin and estimates the unknown parameters $\alpha, \beta, a, b, c.$ We introduce the back stepping procedure to design the controller $u_1, u_2, u_3,$ where u_1, u_2, u_3 are recursive control feedback, as long as these recursive feedback stabilize system (31) converge to zero as the time $t \to \infty$.

First we consider the stability of the system

$$\dot{e}_3 = -\beta y_2 + \beta x_2 + u_3 \tag{55}$$

where e_2 is regarded as virtual controller.

Consider the Lyapunov function defined by

$$V_1(e_1, e_\beta) = \frac{1}{2}e_1^2 + \frac{1}{2}e_\beta^2 \tag{56}$$

Let us define the parameter estimation error as

$$e_\beta = \beta - \hat{\beta} \tag{57}$$

Differentiating equation (56) along the trajectories (55) and using (58)

$$\dot{e}_\beta = -\dot{\hat{\beta}} \tag{58}$$

The derivative of $\dot{V}_1(e_3, e_\beta)$ is

$$\dot{V}_1 = e_3(-\beta y_2 + \beta x_2 + u_3) + e_\beta(-\dot{\hat{\beta}}) \tag{59}$$

Assume the controller $e_2 = \alpha_1(e_3)$.

If we choose

$$\alpha_1(e_3) = -k_1 e_3 \tag{60}$$

and

$$u_3 = -2\beta x_2 + 2\hat{\beta} e_2 \tag{61}$$

In equation (59), the parameters are updated by the update law

$$\dot{\hat{\beta}} = -2e_2 e_3 + k_2 e_\beta \tag{62}$$

Substituting equation (60), (61) and (62) into equation (59), then we have

$$\dot{V}_1 = -k_1 \beta e_3^2 - k_2 e_\beta^2 \tag{63}$$

Which is a negative definite function, since $k_1,\ k_2,\ k_3 > 0$.

Hence the system (55) is globally asymptotically stable.

Function $\alpha_1(e_3)$ is an estimative function when e_2 is considered as a controller.

The error between e_2 and $\alpha_1(e_3)$ is

$$w_2 = e_2 - \alpha_1(e_3) \tag{64}$$

Consider (e_1, w_2) subsystem given by

$$\begin{aligned} \dot{e}_3 &= \beta w_2 - \beta k_1 e_3 - 2e_\beta e_2 \\ \dot{w}_2 &= e_1 + [k_1(\beta - 2e_\beta) - 1]e_2 + 2x_1 + y_3 + x_3 + u_2 \end{aligned} \tag{65}$$

Let e_1 be a virtual controller in system (65).
Assume that when $e_1 = \alpha_2(e_3, w_2)$, the system (65) is made globally asymptotically stable.

Let us define the Lyapunov function as

$$V_2(e_3, w_2) = V_1(e_3) + \frac{1}{2} w_2^2 \tag{66}$$

The derivative of $V_2(e_3, w_2)$ is

$$\begin{aligned} \dot{V}_2 = &-k_1 \beta e_3^2 - k_2 e_\beta^2 + w_2(e_1 + [k_1(\beta - 2e_\beta) - 1]e_2 \\ &+ 2x_1 + y_3 + x_3 + u_2) \end{aligned} \tag{67}$$

We choose

$$\alpha_2(e_3, w_2) = -\beta e_3 \tag{68}$$

and

$$u_2 = -[k_1(\beta - 2e_\beta) - 1]e_2 - 2x_1 - y_3 - x_3 - k_3 w_2 \tag{69}$$

Then it follows that

$$\dot{V}_2 = -k_1 \beta e_3^2 - k_2 e_\beta^2 - k_3 w_2^2. \tag{70}$$

Thus $\dot{V}_2$ is a negative definite function, since $k_1,\ k_2,\ k_3 > 0,$ and hence (65) is globally asymptotically stable.

Function $\alpha_2(e_3, w_2)$ is an estimative function when e_1 is considered as a controller.

The error between e_1 and $\alpha_2(e_3, w_2)$ is

$$w_3 = e_1 - \alpha_2(e_3, w_2) \tag{71}$$

Consider (e_3, w_2, w_3) subsystem given by

$$\begin{aligned} \dot{e}_3 &= \beta w_2 - \beta k_1 e_3 - 2e_\beta e_2 \\ \dot{w}_2 &= w_3 - \beta e_3 - k_3 w_2 \\ \dot{w}_3 &= \alpha y_2 - \alpha x_2 + \alpha b \sin(\frac{\pi y_1}{2a} + d) \\ &\quad - \alpha b \sin(\frac{\pi x_1}{2a} + d) + \beta(\beta w_2 - \beta k_1 e_3 - 2e_2 e_\beta) + u_1 \end{aligned} \tag{72}$$

Consider the Lyapunov function defined by

$$\begin{aligned} V_3(e_3, w_2, w_3, e_\beta) &= V_2(e_3, w_2) \\ &\quad + \frac{1}{2}w_3^2 + \frac{1}{2}e_\alpha^2 + \frac{1}{2}e_a^2 + \frac{1}{2}e_b^2 + \frac{1}{2}e_c^2 \end{aligned} \tag{73}$$

Let us define the parameter estimation error as

$$e_\alpha = \alpha - \hat{\alpha};\ \ e_a = a - \hat{a};\ \ e_b = b - \hat{b}\ ;\ e_c = c - \hat{c} \tag{74}$$

Differentiating equation (73) along the trajectories (72) and using (75),

$$\dot{e}_\alpha = -\dot{\hat{\alpha}} \; ; \; \dot{e}_a = -\dot{\hat{a}}; \; \dot{e}_b = -\dot{\hat{b}} \; ; \dot{e}_c = -\dot{\hat{c}} \tag{75}$$

The derivative of $\dot{V}_3(e_3, w_2, w_3, e_\alpha, e_a, e_b, e_c)$ is

$$\begin{aligned}\dot{V}_3(e_3, w_2, w_3, e_\beta) = & -\beta k_1 e_3^2 - k_2 e_\beta^2 - k_3 w_2^2 + w_3[w_2 + \alpha y_2 - \alpha x_2 \\ & +\alpha b \sin(\frac{\pi y_1}{2a} + d) - \alpha b \sin(\frac{\pi x_1}{2a} + d) \\ & +\beta(\beta w_2 - \beta k_1 e_3 - 2e_2 e_\beta) + u_1] \\ & + e_\alpha(-\dot{\hat{\alpha}}) + e_a(-\dot{\hat{a}}) + e_b(-\dot{\hat{b}}) + e_c(-\dot{\hat{c}})\end{aligned} \tag{76}$$

We choose

$$\begin{aligned}u_1 = & -w_2 + 2\alpha x_2 - \hat{\alpha} e_2 + \frac{\alpha \hat{b} \pi}{2a} e_1 \\ & -\alpha b \sin(\frac{\pi y_1}{2a} + d) + \alpha b \sin(\frac{\pi x_1}{2a} + d) \\ & -\beta(\beta w_2 - \beta k_1 e_3 - 2e_2 e_\beta) + e_a + e_b + e_c - k_4 w_3\end{aligned} \tag{77}$$

In equation (49), the parameter updated by the update law

$$\begin{aligned}&\dot{\hat{\alpha}} = w_3 e_2 + k_5 e_\alpha; \quad \dot{\hat{a}} = w_3 + k_6 e_a, \\ &\dot{\hat{b}} = w_3 + k_7 e_b; \quad \dot{\hat{c}} = w_3 + k_8 e_c.\end{aligned} \tag{78}$$

Substituting equation (77) and (78) into equation (76), then we have

$$\begin{aligned}\dot{V}_3(e_1, w_2, w_3, e_\beta) = & -k_1 e_1^2 - k_2 e_\alpha^2 - k_3 e_a^2 - k_4 e_b^2 - k_5 e_c^2 \\ & -k_6 w_2^2 - k_7 w_3^2 - k_8 e_\beta^2.\end{aligned} \tag{79}$$

Thus $\dot{V}_3$ is a negative definite function, since $k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8 > 0$, and hence the system (72) is globally asymptotically stable.

Thus by a Lyapunov stability theory (Hahn (1967)), the error dynamics (31) is globally asymptotically hybrid synchronized.

Theorem 3. The *n*-scroll chaotic Chua circuit (26) and (28) are globally asymptotically hybrid synchronized for any initial conditions with the recursive controller u_1, u_2, u_3 defined by

$$\begin{aligned} u_1 &= -w_2 + 2\alpha x_2 - \hat{\alpha} e_2 + \frac{\alpha \hat{b} \pi}{2a} e_1 \\ &\quad - \alpha b \sin(\frac{\pi y_1}{2a} + d) + \alpha b \sin(\frac{\pi x_1}{2a} + d) \\ &\quad - \beta(\beta w_2 - \beta k_1 e_3 - 2e_2 e_\beta) + e_a + e_b + e_c - k_4 w_3 \\ u_2 &= -[k_1(\beta - 2e_\beta) - 1]e_2 - 2x_1 - y_3 - x_3 - k_3 w_2 \\ u_3 &= -2\beta x_2 + 2\hat{\beta} e_2 \end{aligned}$$

and the parameter updated by the update law

$$\dot{\hat{\alpha}} = w_3 e_2 + k_5 e_\alpha; \quad \dot{\hat{\beta}} = -2e_2 e_3 + k_2 e_\beta$$

$$\dot{\hat{a}} = w_3 + k_6 e_a; \quad \dot{\hat{b}} = w_3 + k_7 e_b; \quad \dot{\hat{c}} = w_3 + k_8 e_c.$$

5. NUMERICAL SIMULATION

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the system of differential equations (26) and (28) with the feedback controls u_1, u_2, u_3.

The parameters (Wallace *et al* (2001), Suyken *et al* (1997)) of the systems (26) and (28) are taken in the case of chaotic case as

$$\alpha = 10.814,\ \beta = 14.0,\ a = 1.3,\ b = 0.11,\ c = 3,\ d = 0$$

The initial values of the master system (26) are chosen as

$$x_1(0) = 0.125, x_2(0) = 0.625, x_3(0) = 0.941$$

The initial values of the slave system (28) are chosen as

$$y_1(0) = 0.321\ y_2(0) = 0.487, y_3(0) = 0.965$$

The initial values of the estimated parameters are

$$\hat{\alpha}(0) = 2 \; \hat{\beta}(0) = 0.3, \hat{a}(0) = 6, \hat{b}(0) = 8, \hat{c} = 10$$

We take the parameters $k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = k_7 = k_8 = 2$.

Figure 2 (a), (b) and (c) depict the hybrid synchronization of identical *n*-scroll Chua's circuit (26) and (28).

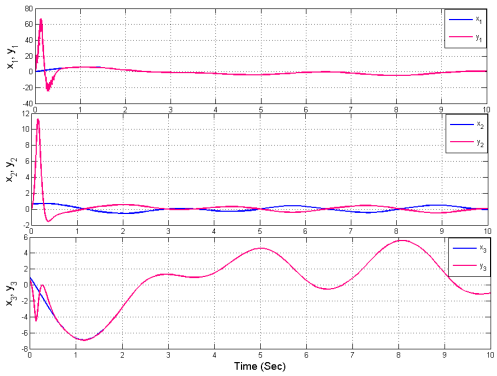

Figure 2(a): Hybrid Synchronization of *n*-scroll chaotic attractor

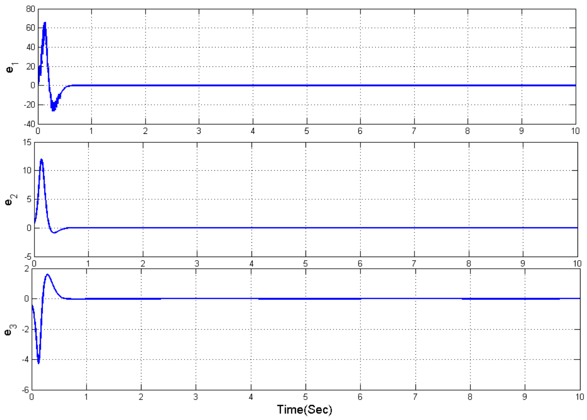

Figure 2(b): Error plot for *n*-scroll chaotic attractor

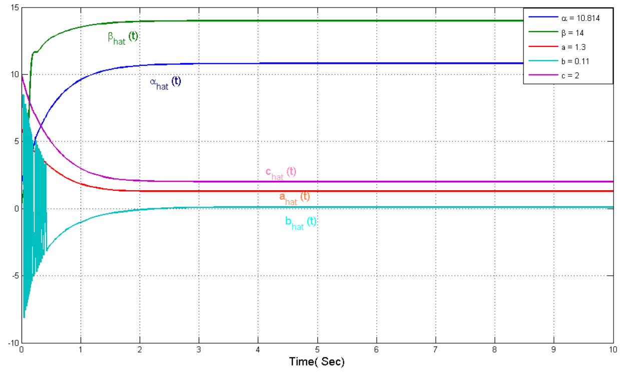

Figure 2(c): Parameter Estimation of *n*- scroll chaotic attractor

6. CONCLUSION

In this paper, adaptive backstepping control method has been applied to estimate the fixed but unknown parameter and achieve hybrid synchronization for a family of *n*-scroll chaotic Chua circuit. The advantage of this method is a recursive procedure for synchronizing chaotic system and there is no derivative in controller. The adaptive backstepping control design has been demonstrated to family of n-scroll chaotic Chua circuit. Numerical simulations have been given to illustrate and validate the effectiveness of the proposed synchronization schemes of the chaotic circuit. The adaptive backstepping control design is very effective and convenient to achieve global chaos hybrid synchronization.

REFERENCES

Alligood, K. T., Sauer, T. and Yorke, J. A. 1997. *Chaos: an Introduction to Dynamical Systems*. Berlin, Germany: Springer–Verlag.

Blasius, B., Huppert, A. and Stone, L. 1999. Complex dynamics and phase synchronization in spatially extended ecological system. *Nature*. **399**: 354–359.

Ott, E. 2002. *Chaos in Dynamical Systems*. United Kingdom: Cambridge University Press.

Fujisaka, H and Yamada, T. 1983. Stability theory of synchronized motion in coupled-oscillator systems. *Progress of Theoretical Physics*. **63**: 32–47.

Ge, Z. M. and Chen, C. C. 2004. Phase synchronization of coupled chaotic multiple time scales systems. *Chaos, Solitons and Fractals*. **20**: 639–647.

Guang, Z. Z, Lan, S. C and Li, H. S. 2005. Complexity of an SIR epidemic dynamics model with impulsive vaccination control. *Chaos, Solitons and Fractals*. **26**: 495–505.

Hahn, W. 1967. *The Stability of Motion*. Berlin, Germany: Springer-Verlag.

Han, S. K., Kerrer, C. and Kuramoto, Y. 1995. D-phasing and bursting in coupled neural oscillators. *Phys. Rev. Lett.* **75**: 3190–3193.

Hindmarsh, J. L., Rose, R. M. 1984. A model of neuronal bursting using 3-coupled 1st order differential equations. *Proc. Roy. Soc. Lond. B. Biol.* **221**: 81–102.

Jiang Wang, Lisong Chen, Bin Deng. 2009. Synchronization of ghost burster neurons in external electrical stimulation via H1 variable universe fuzzy adaptive control. *Chaos, Solitons and Fractals*. **39**: 2076–2085.

Jian-Ping, Y. and Chang-Pin, L. 2006. Generalized projective synchronization for the chaotic Lorenz systems and the chaotic Chen system. *Journal of Shanghai University*. **10**: 299–304.

Junxa. W., Dianchen, L. and Lixin, T. 2006. Global synchronization for time delay WINDMI system. *Chaos, Solitons and Fractals*. **30**: 629–635.

Kocarev, L. and Parlitz, U. 1995. General approach for chaotic synchronization with applications to communications. *Phys. Rev. Lett.* **74**: 5028–5030.

Li, R. H. 2008. A special full-state hybrid projective synchronization in symmetical chaotic systems. *Applied Math. Comput.* **200**: 321–329.

Li, R. H., Xu, W. and Li, S. 2007. Adaptive generalized projective synchronization in different chaotic systems based on parameter identification. *Phys. Lett. A*. **367**: 199–206.

Lu, J., Wu, X., Han, X. and Lu, J. 2004. Adaptive feedback synchronization of a unified chaotic system. *Phys. Lett. A*. **329**: 327–333.

Moukam, K. F. M., Nguenang, J. P. and Kofane, T. C. 2006. Chaos synchronization in bi-axial magnets modeled by bloch equation. *Chaos, Solitons and Fractals*. **30**: 690–699.

Murali, K and Lakshmanan, M. 1996. *Chaos in Nonlinear Oscillators: Controlling and Synchronization*. Singapore: World Scientific.

Murali, K. and Lakshmanan, M. 2003. Secure communication using a compound signal using sampled-data feedback. Applied Mathematics and Mechanics. **11**: 1309–1315.

Ott, E., Grebogi, C. and Yorke, J. A. 1990. Controlling chaos. *Phys. Rev. Lett.* **64**: 1196–1199.

Park, J. H. 2008. Adaptive control for modified projective synchronization of a four-dimensional chaotic system with uncertain parameters. *J. Computational and Applied Math*. **213**: 288–293.

Park, J. H. and Kwon, O. M. 2003. A novel criterion for delayed feedback control of time-delay chaotic systems. *Chaos, Solitons and Fractals*. **17**: 709–716.

Park, J. H., Lee, S. M. and Kwon, O. M. 2007. Adaptive synchronization of Genesio-Tesi chaotic system via a novel feedback control. *Physics Letters A*. **371**: 263-270.

Pecora. L. M. and Carroll. T. L. 1991. Synchronizing chaotic circuits. *IEEE Trans. Circ. Sys.* **38**: 453–456.

Pecora. L. M. and Carroll, T. L. 1990. Synchronization in chaotic systems. *Phys. Rev. Lett.* **64**. 821-824.

Qiang, J. 2007. Projective synchronization of a new hyper chaotic Lorenz systems. *Phys. Lett. A*. **370**: 40–45.

Sarasu, P. and Sundarapandian, V. 2012. Generalized projective synchronization of three-scroll chaotic systems via active control. *CCSIT 2012, Part-I, LNICST*. 84: 124–133.

Sundarapandian, V. and Sarasu, P. 2012. Generalized projective synchronization of double-scroll chaotic systems using active feedback control. *CCSIT 2012, Part-I, LNICST*. **84**: 111–118.

Sundarapandian, V. and Suresh, R. 2010. Global chaos synchronization for Rossler and Arneodo chaotic systems by nonlinear control. *Far East Journal of Applied Mathematics*. **44**: 137-148.

Sundarapandian, V. 2011. Global chaos synchronization of the Pehlivan systems by sliding mode control. *International J. Computer Science and Engineering*. **03**: 2163-2169.

Sundarapandian, V. and Sivaperumal, S. 2012. Hybrid synchronization of hyper chaotic Chen system via sliding mode control. *CCSIT 2012, Part-I, LNICST*. **84**: 73–82.

Sundarapandian, V. and Suresh, R. 2010. New results on the global chaos synchronization for Liu-Chen-Liu and Lu chaotic systems. *PEIE 2010, CCIS*. 102: 20–27.

Sundarapandian, V. and Suresh, R. 2012. Hybrid synchronization of Arneodo and Rossler chaotic systems by active nonlinear control. *CCSIT 2012, Part-I, LNICST*. **84**: 257–266.

Suresh, R. and Sundarapandian, V. 2012. Global Chaos synchronization of hyperchaotic Chen-Lee system using backstepping contol via recursive feedback. *Lecture Notes in Computer Science*. **305**: 212–221.

Suresh, R. and Sundarapandian, V. 2012b. Hybrid synchronization of *n*-scroll Chua and Lure chaotic system using backstepping control via novel feedback. *Archives of Control Science*. **22(LVIII)**: 255-278.

Suresh, R. and Sundarapandian, V. 2013. Global chaos synchronization of a family of *n*-scroll chua circuits using backstepping control with recursive feedback. *Far East Journal of Mathematical Sciences*. **73**: 73-95.

Suykens, J., Vandewalle, J and Chua, L. 1997. Nonlinear H_∞ synchronization of chaotic Lure system. *Int. J. Bifurcation Chaos.* **7**: 1323-1325.

Wallace, K. S., Tang, G., Zhong, Q., Chen, G. and Man, K. F. 2001. Generation of *n*–scroll Attractors via sine Function. *IEEE Trans. Circ. Sys-I Fundamental Theory and Applications*. **48**: 1369–1372.

Wang, Y. M. and Zhu, H. 2006. Generalized synchronization of continuous chaotic systems. *Chaos, Solitons and Fractals*. **27**: 97–101.

Wu, X. and Lu Jun-An 2003. Parameter identification and backstepping control of uncertain Lu system. *Chaos, Solitons and Fractals*. **18**: 721–729.

Yang, T. and Chua, L. O. 1999. Generalized synchronization of chaos via linear transformations. *Internat. J. Bifur. Chaos*. **9**: 215–219.

Che, Y. Q., Wang, J., Tsang, K. M., and Chen, W. L. 2010. Unidirectional synchronization for Hindmarsh-Rose neurons via robust adaptive sliding mode control. *Nonlinear Analysis: Real world Application.* **11**: 1096–1104.

Yau, H. T. 2004. Design of adaptive sliding mode controller for chaos synchronization with uncertainties. *Chaos, Solitons and Fractals*. **22**: 341–347.

Yu, Y. G. and Zhang, S. C. 2006. Adaptive backstepping synchronization of uncertain chaotic systems. *Chaos, Solitons and Fractals*. **27**: 1369–1375.

Wang, Z. 2010. Chaos synchronization of an energy resource system based on linear control. *Nonlinear Analysis: Real world Application.* **11**: 3336-3343.