Hybridization of acoustic waves in piezoelectric plates

Andrei Teplykh, Boris Zaitsev, Iren Kuznetsova, Irina Borodina

Institute of Radio Engineering and Electronics, Saratov Department,

Russian Academy of Science, 410019 Saratov, Russia, **teplykhaa@mail.ru, zaitsev@ire.san.ru**

The investigation of hybridization effect was carried out for different types of plate acoustic waves. We analyzed Lamb and shear-horizontal waves of zero and higher orders for number of crystallographic orientations of plate and different electrical boundary conditions. The purpose of paper is the clarification of mechanism of hybridization and definition of numerical criteria for estimation of hybrid waves coupling. For the search of parameter ranges, where hybridization is possible, analysis of dependences of phase velocity on parameter hf was performed (h = plate thickness, f = wave frequency). For determination of the touch type ("contiguity" or "intersection") behavior of "contact points" of dispersion curves by infinitesimal change of plate orientation was also analyzed. The analysis showed, that for electrically open surfaces of plate curves can be contiguous or repulse and the point of contact can exist only for certain values of hf, crystallographic orientations and wave types. However, the infinitesimal change of propagation direction leads to the finite difference between phase velocities and one can see smooth "interchange" of polarization directions, i.e. waves changes their types. As for plates with electrically shorted surface these curves always repulse. The analysis showed that there exist two mechanisms of hybridization: electrical or mechanical coupling. It means that one wave can generate another wave if their phase velocities are close each other. In this case there exists the energy exchange between these waves. We propose to use the mutual electrical and mutual mechanical energies of coupling waves as numerical criteria of wave hybridization.

1 Introduction

The dispersion dependencies (the frequency versus wave number, phase velocity versus frequency, etc) are very important characteristics of waves of various natures: electromagnetic, acoustic, magnetostatic, plasmic, etc [1]. At that waves of different types (or different modes) having the small amplitude and propagating in linear wave-guides are linearly independent, i.e. they propagate without interaction each other and without exchange by energy [2]. Aside from waves of different types are mutually orthogonal and the integral of the mutual energy of any pair of waves over the cross-section of wave-guide is equal always zero [3]. But there exist the situations corresponding to spatially - time synchronism when the separate own modes of wave-guide cease to de independent and become as coupling waves. Such coupling waves are called as hybrid. They are well-known in dielectric electromagnetic wave-guides [3], in magnetic materials [4,5], optical fibers [3], etc. The existence of hybrid Lamb waves and shear horizontal (SH) waves of zero order in piezoelectric plates also was predicted [6]. It is known that in such cases the appropriate dispersion dependencies of coupling waves are contiguous or are close each other. At that the points of contact or rapprochement of dispersion dependencies are very interesting not only from fundamental point of view but for various practical applications. It is connected with the possibility of effective excitation of non-piezoactive acoustic waves. This paper is devoted to further study of hybridization effect of acoustic waves of various types of zero and higher orders (Lamb and SH waves) in piezoelectric plates. We have studied the various crystallographic orientations of plates of lithium niobate and lithium tantalate with different electrical boundary conditions. In the course of investigation we have revealed two mechanisms of existence of hybrid waves in piezoelectric plates and proposed to use the mutual electrical and mechanical energies of these waves as numerical criteria of their hybridization.

The investigation was carried out by two steps. On the first step we calculated the dependencies of phase velocities of plate waves on parameter hf. Here h is the plate thickness, f is the wave frequency. These dependencies were calculated for different plate orientations, propagation directions and plate thickness. The second step was devoted to search and detailed analysis of points of contact or rapprochement of dispersion dependencies. Below we consider shortly the results of these steps.

2 The calculation of dependencies of phase velocities on parameter hf

Consider the propagation of acoustic wave along the axis x_1 in piezoelectric plate occupying the region $0 < x_3 < h$. Let vacuum occupies the residual space. We used the equation of motion of elastic medium, Laplace's equation and constitutive equations for elastic medium and vacuum. These equations together with electrical and mechanical boundary conditions were solved by standard numerical method, which is described in detail in [7]. As a result of solution of this problem the dependencies of phase velocity of Lamb and SH waves on parameter hf and wave structures (amplitudes of all electrical and mechanical variables in cross-section of plate) have been found. As materials under study we used lithium niobate and lithium tantalate. Their material constants were taken from [8]. We analyzed all propagation directions in the plates of the main crystallographic cuts X, Y, and Z, the number of order was changed from 0 to 7. Figure 1 shows the typical dispersion dependencies for plate of Y-X lithium niobate.

3 The analysis of the hybridization effect

On this step we analyzed in detail the behaviors of dispersion dependencies near the points of their contact or rapprochement. The analysis of hybridization effect was carried out by the next way. For given crystallographic orientation of plate determined by three Euler's angles (φ, θ, ψ) we found the points of contacts of dispersion dependencies. In other words we defined those values of parameter hf, for which the waves 1 and 2 had the same phase velocities $V_1 = V_2$. From Figure 1 one can see that there exist the points of contact for the following pairs of waves S_0 - SH_0, A_1 - S_1, etc.

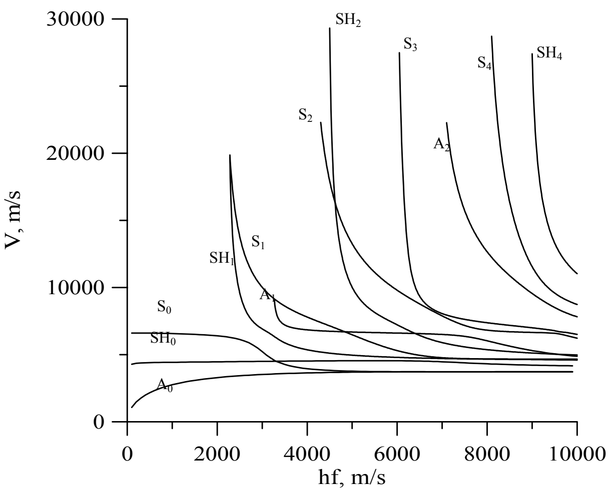

Figure 1: $LiNbO_3$ (Euler's angles 0,90,180), electrically free plate, typical V(hf) dependencies

The appropriate values of parameter hf, which correspond to these points were found. Then we analyzed the same dependencies at small change of propagation direction of wave (Euler's angles φ+δ,θ,ψ). At that the velocities of analyzed wave pairs began to differ on finite value of ΔV. The increase of the angle change δ leaded to increasing the degree of repulsion of dispersion dependencies ΔV. At that with increasing the parameter hf the interacted waves smoothly exchanged by their polarizations and changed the type. Figures 2 - 4 and 5 - 7 show the dependencies of phase velocity on parameter hf for electrically free plate for the aforementioned wave pairs S_0 - SH_0 and A_1 - S_1, respectively. The numbers in braces designate the normalized components of mechanical displacement.

These figures confirm that with increasing the angle change δ the value of repulsion ΔV increases. Such behavior of dispersion dependencies acts as qualitative criterion of wave hybridization. For example the Table 1 demonstrates the characteristics of all hybrid wave pairs existing in range hf = 3000-13000 m/s for Y-X+0° $LiNbO_3$.

Table 1. Examined V(hf) curves contact points for Y-X+0° $LiNbO_3$ (Euler's angles 0, 90, 180)

hf, m/s	velocity, m/s,	Wave 1, type,K^2 %	wave 2, type,K^2 %
3316.8382	4503.4128	SH_0, 16.2	S_0, 0.0005
4836.4450	6696.6392	A_1, 0.01	S_1, 1.83
6399.5497	6571.0770	SH_2, 0.02	S_1, 0.72
9493.1836	6788.8033	S_3, 1.97	SH_4, 0.01
9744.1505	6658.1970	S_3, 0	S_2, 0.016
9559.5160	5026.2621	A_3, 0.0005	SH_2, 3.65
6296.9217	3717.6441	A_0, 0.004	S_0, 1.46
11688.7379	5445.2212	A_5,1.7×10^{-5}	S_4, 4.71

In a whole the analysis of the big amount of hybrid waves showed that for piezoelectric plates there are at least two mechanisms of wave hybridization, namely due to mechanical or/and electrical coupling. It means that at the certain value of parameter hf when waves have the close values of phase velocity it is possible the exciting of another wave due to mechanical or/and mechanical oscillations of initial wave. Apparently that in this case the coupled waves should have the common components of mechanical displacement or/and strength of electrical field. At that there exist the exchange by energies of these interacted waves and they cease to be as independent.

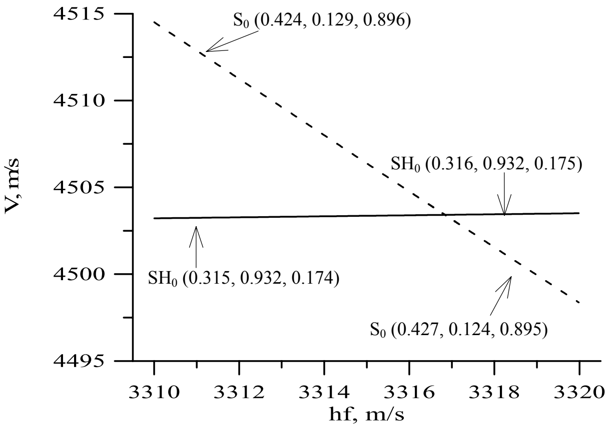

Figure 2: Y-X+0° $LiNbO_3$ (Euler's angles 0, 90, 180), S_0-SH_0 wave pair. Curves contact at hf=3316.8382 m/s, V=4503.4128 m/s, ΔV=0 m/s

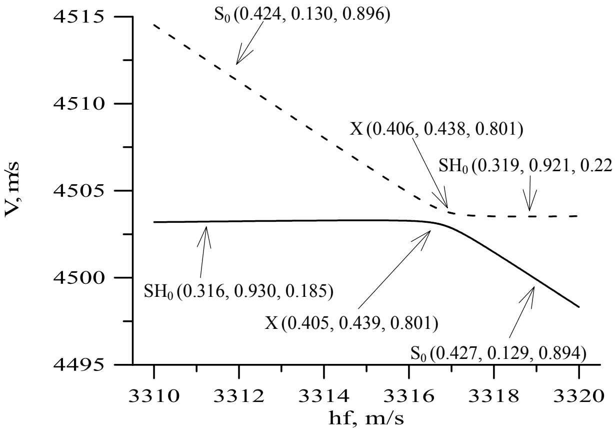

Figure 3: Y-X+0.1° $LiNbO_3$ (Euler's angles 0.1, 90, 180), S_0-SH_0 wave pair. Curves repulse and change type at hf=3316.8417 m/s, Vsh_0=4502.9950 m/s, Vs_0=4503.8184 m/s, ΔV=0.82342 m/s

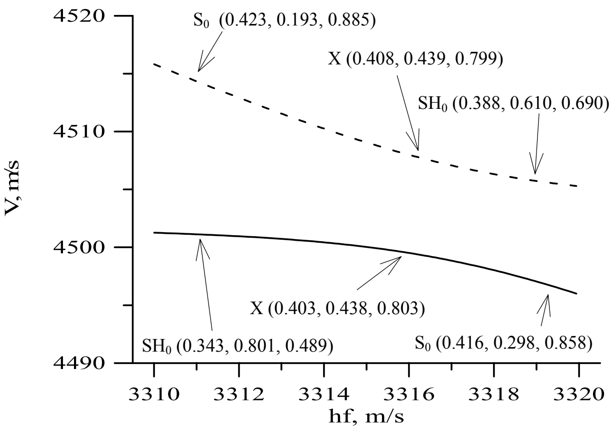

Figure 4: Y-X+1° $LiNbO_3$ (Euler's angles 1, 90, 180), S_0-SH_0 wave pair. Curves repulse and change type at hf=3317.2879 m/s, Vsh_0=4498.6223 m/s, Vs_0=4506.8351 m/s, ΔV=8.2127 m/s

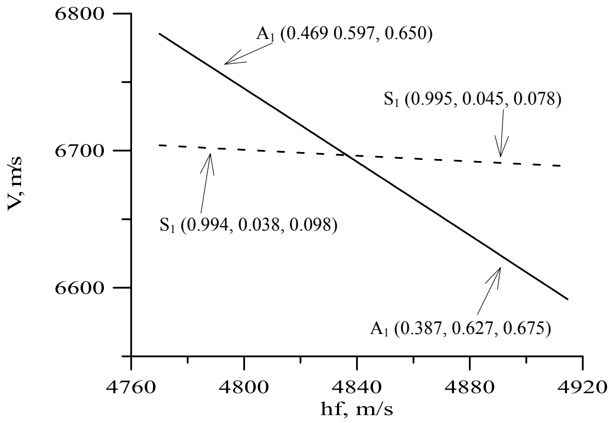

Figure 5: Y-X+0° $LiNbO_3$ (Euler's angles 0, 90, 180), A_1-S_1 wave pair. Curves contact at hf=4836.4450 m/s, V=6696.6391 m/s, ΔV=0 m/s

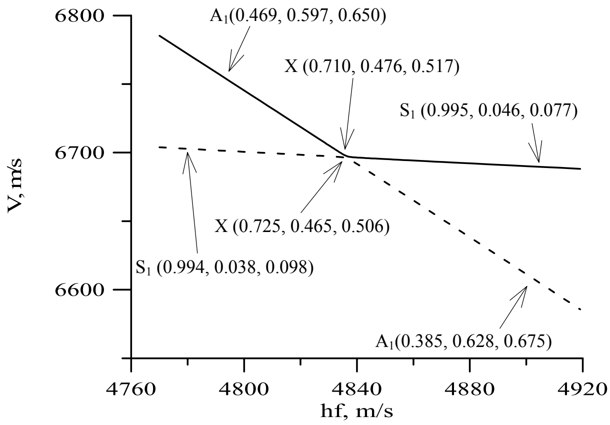

Figure 6: Y-X+0.1° $LiNbO_3$ (Euler's angles 0.1, 90, 180), A_1-S_1 wave pair. Curves repulse and change type at hf=4836.4144 m/s, Va_1=6697.3809 m/s, Vs_1=6695.9545 m/s, ΔV=1.42644 m/s

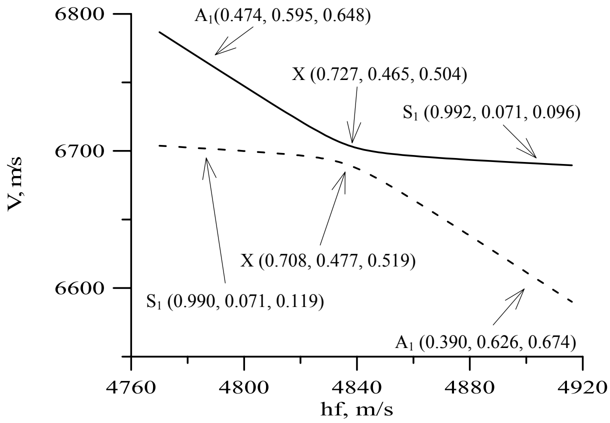

Figure 7: Y-X+1° $LiNbO_3$ (Euler's angles 1, 90, 180), A_1-S_1 wave pair. Curves repulse and change type at hf=4837.0428 m/s, Va_1=6703.9741 m/s, Vs_1=6689.7320 m/s, ΔV=14.242 m/s

Analysis showed that by analogy with magnetoelastic waves [5] the coefficient M determined as

$$M = \frac{W^{12} + W^{21}}{W^1 + W^2} \quad (1)$$

may act as numerical criterion of wave hybridization

Here $W^{12} = W_M^{12} + W_E^{12}$ is the averaged in time total mutual energy of two coupled waves 1 and 2. For one's turn

$$W_M^{12} = \frac{1}{2T}\int_{-h}^{0}\int_{0}^{T} T_{ij}^1 S_{ij}^2 dt dx_3 ,$$

and

$$W_E^{12} = \frac{1}{2T}\int_{-h}^{0}\int_{0}^{T} D_j^1 E_j^2 dt dx_3$$

are the mechanical and electrical mutual energies of waves 1 and 2 for unit aperture, respectively. As for total energy of wave 1 on unit aperture $W^1 = W_S^1 + W_V^1$ it consists of mechanical and electrical contributions in plate

$$W_S^1 = \frac{1}{2T}\int_{-h}^{0}\int_{0}^{T} (T_{ij}^1 S_{ij}^1 + D_j^1 E_j^1) dt dx_3 ,$$

and electrical contribution in vacuum

$$W_V^1 = \frac{1}{2T}\left(\int_{0}^{\infty}\int_{0}^{T} D_j^1 E_j^1 dt dx_3 + \int_{-\infty}^{-h}\int_{0}^{T} D_j^1 E_j^1 dt dx_3\right)$$

The total energy of wave 2 is defined similarly. Here T is the wave period, T_{ij} is the stress tensor, S_{ij} is the strain tensor, D_j is the electrical displacement, E_j is the strength of electrical field. The magnitudes refer to waves 1 and 2 are designated by appropriate indices. It is easy to prove that in general case always $W^{12} = W^{21}$. Thus the coefficient M is the dimensionless magnitude, which is maximum at equal values of energies of coupled waves, does not exceed in absolute value 1 ($|M| \le 1$), and is equal zero for any two waves excluding regions of hybridization (the condition of modes orthogonality). The dependencies of coefficient M on parameter hf at different values of angle change δ and repulsion ΔV are presented in Figures 8 - 10 and 11 - 13 for electrically free plate S_0 - SH_0 and A_1 - S_1 wave pairs, respectively. These dependencies have the form of resonant curve and with increasing the value of repulsion ΔV coefficient M practically does not change, but the width of this curve increases.

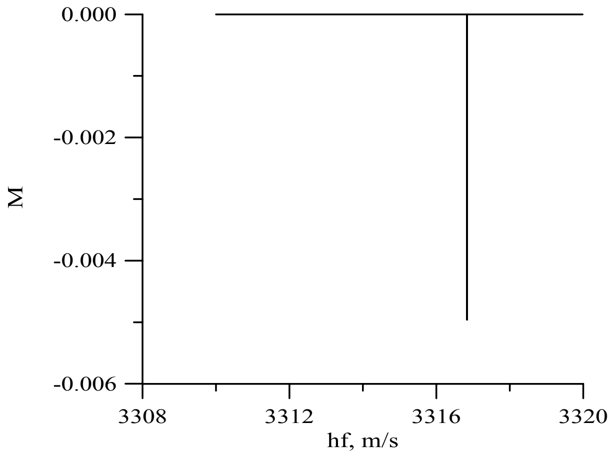

Figure 8: Y-X+0° $LiNbO_3$ (Euler's angles 0, 90, 180), S_0-SH_0 wave pair. Maximum at hf=3316.8382 m/s, M=-0.0049.

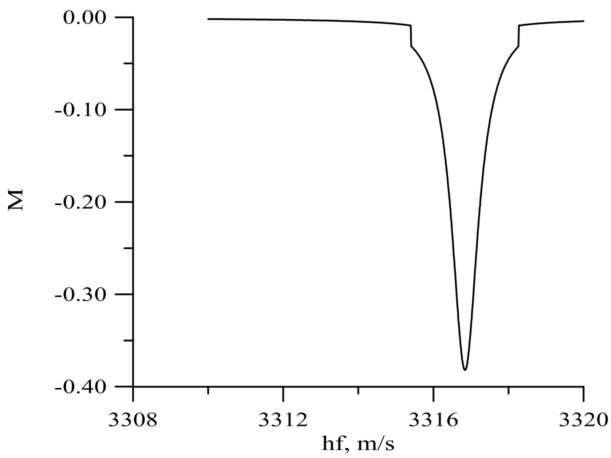

Figure 9: Y-X+0.1° $LiNbO_3$ (Euler's angles 0.1, 90, 180), S_0-SH_0 wave pair. Maximum at hf=3316.8417 m/s, M=-0.382.

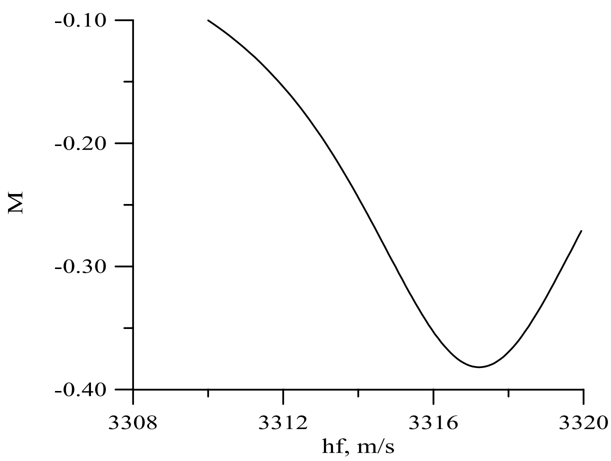

Figure 10: Y-X+1° $LiNbO_3$ (Euler's angles 1, 90, 180), S_0-SH_0 wave pair. Maximum at hf=3317.2879 m/s, M=-0.381

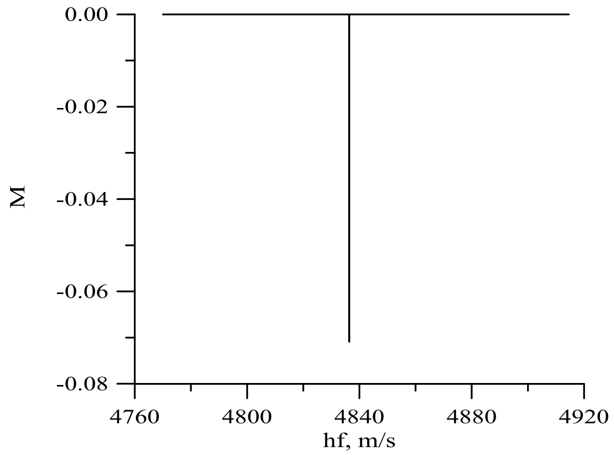

Figure 11: Y-X+0° $LiNbO_3$ (Euler's angles 0,90,180), A_1-S_1 wave pair. Maximum at hf=4836.4450 m/s, M=-0.070

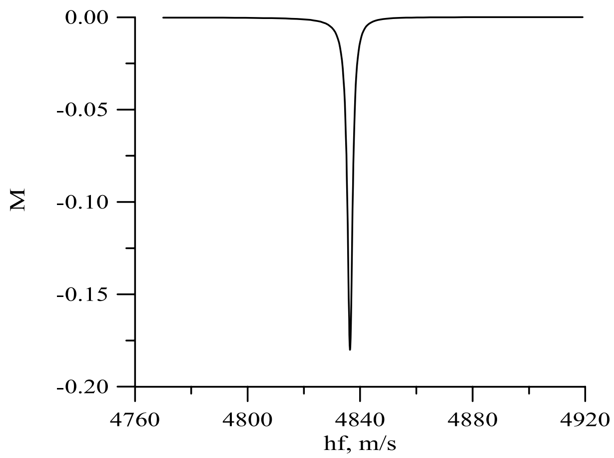

Figure 12: Y-X+0.1° $LiNbO_3$ (Euler's angles 0.1, 90, 180), A_1-S_1 wave pair. Maximum at hf=4836.4144 m/s, M=-0.180

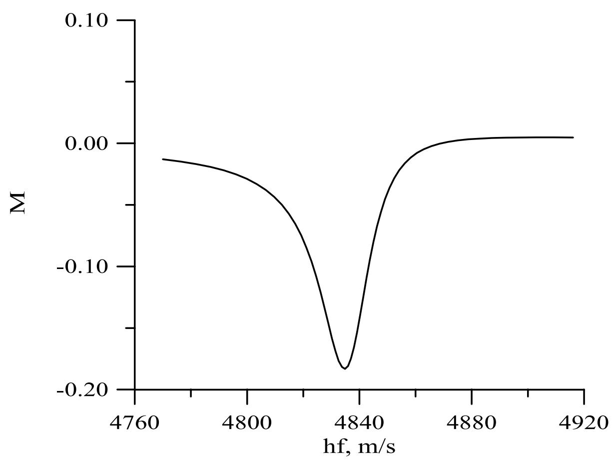

Figure 13: Y-X+1° $LiNbO_3$ (Euler's angles 1, 90, 180), A_1-S_1 wave pair. Maximum at hf=4834.8916 m/s, M=-0.183

We have analyzed a big amount of hybrid pairs in piezoelectric plates and have not found the situations when there exists only pure mechanical or pure electrical coupling. It has been established that mutual electrical and mechanical energies may have the same or opposite signs. It means that for phasing-in the mechanical variables of coupled waves the electrical variables may be in phase as well as in antiphase. The existence of pure mechanical coupling was confirmed on example of hybrid waves in plates of such non-piezoelectric material as silicon. In order to weaken the influence of electrical coupling and to analyze the influence of pure mechanical coupling in piezoelectric material we considered one more case when both surfaces of the plate were electrically shorted. The electrical shorting of surface leaded as a rule to increase of coefficient *M*.

4 Conclusions

The carried out investigation showed, that in piezoelectric plates at certain crystallographic orientations and values of parameter hf, there exist the hybridization of plate acoustic waves. It has been found, that the mechanisms of this effect are the electrical and mechanical coupling of waves. The obtained results besides the fundamental significance may be useful for practical applications, for example for effective excitation of non-piezoactive acoustic waves.

Acknowledgments

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