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Hybridization of Grey Wolf Optimizer and Crow Search Algorithm Based on Dynamic Fuzzy Learning Strategy for Large-Scale Optimization

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ABSTRACT A novel optimization algorithm named hybrid grey wolf optimizer with crow search algorithm (GWO-CSA) is developed in this paper for handling large-scale numerical optimization problems. The proposed GWO-CSA algorithm combines the strong points of both grey wolf optimizer (GWO) and crow search algorithm (CSA) with the aim to escape from local optima with faster convergence than the standard GWO and CSA. In this algorithm, GWO operates in enhancing the exploration ability while CSA works as a local searching scheme to emphasize the exploitation capability to achieve global optimal solutions. In this sense, the movement direction and speed of leader grey wolf (alpha) is improved by incorporating the CSA phase. Also, a dynamic fuzzy learning strategy (DFLS) is introduced to enable the occurring of tiny changes in the neighborhood of the best solution to avoid the caught in the local optima and refine the quality of the obtained solution. The robustness and efficiency of the proposed GWO-CSA algorithm are investigated on fifteen CEC 2015 benchmark problems in addition to four large-scale problems and four real applications related to engineering design optimization taken from the literature. The comprehensive comparisons with other algorithms have demonstrated the effectiveness of GWO-CSA to address optimization tasks.

INDEX TERMS Grey wolf optimizer, crow search algorithm, numerical optimization, hybridization.

I. INTRODUCTION

Nowadays optimality concepts have appeared frequently in several real-world applications such as engineering designs [1], [2], statistical physics [3], economics [4], chemistry [5], power system [6] and information theory [7]. In this regard, optimization methodologies have a significant role in searching for the optimal solution among all the reasonable solutions that minimize or maximize the output of a given system [8]. However, obtaining optima in numerous complex optimization fields require notable evaluations and computations. Because of the limitations such as time-consuming, the dependency of the initial point, higher dimensionality, and non-convexity and non-differentiability of the cost function, the solely relying on traditional optimization algorithms (TOAs) is unreliable.

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To overcome the lacks of TOAs and meet the ever increasing of optimal industrialization, meta-heuristic optimization algorithms (MOAs) [9]–[15] have flourished and attracted the attention of many researchers and scientists during the past two decades. In this context, researchers have proposed a sequence of intelligent methods inspired by certain rules. Particle swarm optimization (PSO) [16], sine cosine algorithm (SCA) [17], [18], moth-flame optimization algorithm (MFO) [19], ant colony system (ACS) [20], artificial bee colony (ABC) [21], firefly algorithm [22], [23], and gravitational search algorithm (GSA) [24]. These optimization algorithms have been investigated by several researchers to deal with optimization tasks at various fields such as design optimization [25], resource allocation [26], economic dispatch [27], and multi-objective optimization [28].

Grey wolf optimizer (GWO) is one of the recent MOAs, which is developed by Mirjalili *et al.* [29]. The main inspiration is introduced based on the strategy of hunting and the hierarchy of leadership of the grey wolves in nature. Due to its simple structure and easiness of implementation, it has been successfully employed to deal with a wide area of optimization problems including feature subset selection [30], economic dispatch problems [31], optimal power flow problem [32] and flow shop scheduling problem [33]. However, as a new intelligent technique, GWO acquires some disadvantages. The first one is its guidance towards the three wolves at each iteration hampers the search diversity and leads to a local optimum. The second one is that no mechanism is employed to enhance the best position of the alpha grey wolf during each generation which may yield a poor quality of the final solution.

Apart from the previously introduced GWO algorithms, many attentions have been developed in the literature to realize and achieve the optimal solutions for numerous contemporary tasks. However, several experiments with high dimensional, complex, and multimodal optimization problems have confirmed that GWO acquires a mediocre convergence trends and still easily be stuck at local optima. Consequently, many researchers have attempted to improve/modify the performance of GWO in recent year. In [34], Heidari *et al.* developed a novel modified GWO by integrating the Levy flight pattern, named LGWO, to solve unconstrained optimization tasks. In [35], Long *et al.* proposed a random opposition-based learning GWO (ROGWO) for solving benchmark problems as well as optimization of engineering designs. In [36], Gupta and Deep introduced a memory-based GWO (mGWO) to deal with global optimization tasks. In [37], Long *et al.* suggested a novel GWO based on refraction learning (RLGWO) to enhance the original mode of GWO for solving benchmark test functions, while Long *et al.* [38] proposed an improved GWO (IGWO) by introducing a nonlinear adjustment strategy for controlling the exploration and exploitation searches, and also an updating strategy for position is presented. In [39], Long *et al.* suggested a novel exploration-enhanced GWO (EEGWO), which employs the nonlinear strategy of control parameter and modified formulation for position-updating strategy to improve the exploration ability as well as balance the convergence among the convergence speed and precision of solution. In [40], Long *et al.* developed an efficient and robust GWO (ERGWO) with an enhanced framework to balance the exploration and exploitation engines while dealing with numerical optimization problems. In [41], Gupta and Deep proposed an enhanced leadership of GWO with Levy-flight, named GLFGWO, with the aim to accelerate the search process and improve the convergence trends while dealing with unconstrained and constrained benchmark optimization problems. In [42], Yan *et al.* developed a novel weighted distance-based GWO (GWOWD) for improving the capability of the algorithm as well as escaping from the local optima when tackling with the benchmark optimization suits and engineering designs.

Although, the above improvements have been tried to enhance the solution accuracy and performance of the GWO, but still some difficult cases such as for more complex multimodal tasks the algorithm can suffer from the stagnation at local optima (LO) and thus the obtained solutions cannot be accepted on the global scale [43]. Another reason for improving the performance of the GWO can be answered through the fact recognized by the ''No Free Lunch'' theorem [44], that states that there is no unique optimization method can claim the best performance for all optimization natures. Hence, this theorem logically opens the room of research to propose new algorithms or improve the searching mechanism of the existing ones. Thus by motivating these facts, the present work proposes a hybrid sequential variant based on GWO and crow search algorithm (CSA) aiming to exhibit more robust performance and greater flexibility against the difficult and complicated optimization problems. To the best of our information, this proposed hybrid variant is proposed for the first time.

Crow search algorithm (CSA) is a new intelligent meta-heuristics method that is developed by Askarzadeh [45]. It imitates the social, intelligent behavior of the crows during the storing and restoring processes of the excess food. CSA has a simple structure, and it is applied for dealing with optimization problems such as the economic load dispatch problem [46], magnetic resonance brain images [47] and engineering optimization [48]. However, CSA does not have the specific domain knowledge to each problem and may face the dilemma of trapping in a local optimum. To address the above issues, GWO is hybridized with CSA in a novel strategy with the aim to refine the diversity of solutions and evade the falling in the local optimum.

In this work, a newly developed hybrid meta-heuristic algorithm named hybrid grey wolf optimizer with crow search algorithm (GWO-CSA) is implemented to solve different natures of benchmark problems and real-world applications. GWO-CSA combines the desirable properties of both GWO and CSA to mitigate their weaknesses. In GWO-CSA, CSA is embedded to improve the movement of grey wolves of the GWO. Also, the serialized scheme among the GWO and CSA can enhance the diversity of the solution efficiently. A novel dynamic fuzzy learning strategy (DFLS) is introduced to preserve the quality of the best solution for each iteration. To investigate and validate the efficacy of the proposed GWO-CSA, it is benchmarked on different optimization tasks and compared with other well-established techniques. Simulation results exhibit a superior performance of the proposed GWO-CSA regarding quality and reliability. Therefore, GWO-CSA can be an efficient alternative to deal with complex optimization tasks.

The main contributions regarding this work are outlined as follows:

(1) GWO-CSA algorithm is introduced to solve different optimization tasks. In GWO-CSA, the CSA is embedded into GWO to exhibit two features, namely, to improve the movement of the leader wolf in its hierarchical structure (i.e., alpha grey wolf) and exchange the information that enhances the diversity of solutions.

(2) Dynamic fuzzy learning strategy (DFLS) is designed and implemented to enhance the quality of the best so far solution and improve the convergence performance.

(3) A modified updating strategy based on elite-opposition is introduced to balance the search among the diversification and intensification capabilities.

(4) The effectiveness of GWO-CSA is proved through different natures of benchmark problems as well as the comprehensive comparisons with other algorithms from the literature.

The remainder of the paper is organized using some sections. Section II presents the overview of the grey wolf optimizer (GWO) and crow search algorithm (CSA), Section III develops the motivation behind the hybridization, Section IV introduces in detail the proposed hybrid GWO-CSA. In Section V, the simulation results and comparisons are demonstrated. Finally, Section VI provides the conclusions and future research.

II. OVERVIEW OF GWO AND CSA

This section is devoted to overview the basics of GWO and CSA, respectively.

A. BASIS OF GWO

Grey wolf optimizer (GWO) [29] is developed as a cooperative algorithm based on the hunting behavior of grey wolves and the social leadership among them in nature. The hierarchical leadership is simulated by employing four grey wolves such as alpha, beta, delta, and omega. The first three best wolves positions are denoted as α, β, and δ while the rest of all wolves are supposed to be omega (ω) and ω wolves are guided by these three best wolves. The updating position of each wolf is executed employing some mathematical equations [29].

During the hunting process, grey wolves attempt to encircle the prey that is modeled mathematically as follows:

$$\Delta\,(Iter+1)=\Delta_p\,(Iter)-A\circ|C\circ\Delta\,(Iter)-\Delta\,(Iter)\,| \quad (1)$$

where $Iter$ denotes the current iteration, $\circ$ presents the Hadamard product operation, whereas Δ_p and Δ represent respectively, the position of prey and the position of the grey wolf. The vectors A and C are determined as follows:

$$A = 2\cdot a\circ r_1 - a \quad (2)$$

$$C = 2\cdot r_2 \quad (3)$$

$$a\,(Iter) = 2-2\cdot\frac{Iter}{T} \quad (4)$$

where a is a linearly decreasing parameter from 2 to 0, and it aims to preserve the exploration and exploitation capabilities, r_1 and r_2 are random vectors from the interval [0, 1]. Here, T is a maximal number of iterations.

The hunting process is often managed by the alpha grey wolf, and also beta and delta grey wolves might join in this process. However, the prey location (optimum) is unknown over the search area; it is supposed that the wolves, alpha, beta, and delta, exhibit better perception regarding the probable location of the prey. Thus, these three wolves (fittest) are maintained to guide the other wolves towards the probable location of the prey. Thus scenario of hunting is modeled as follows.

$$\begin{aligned}\Delta_1 &= \Delta_\alpha - A_1\circ|C_1\circ\Delta_\alpha-\Delta|\\ \Delta_2 &= \Delta_\beta - A_2\circ|C_2\circ\Delta_\beta-\Delta|\\ \Delta_3 &= \Delta_\delta - A_3\circ|C_3\circ\Delta_\delta-\Delta|\end{aligned} \quad (5)$$

The updating process of a candidate's position through using the alpha, beta, and delta wolves is as follows.

$$\Delta\,(Iter+1)=\frac{\Delta_1\,(Iter)+\Delta_2\,(Iter)+\Delta_3\,(Iter)}{3} \quad (6)$$

where A_1, A_2, and A_3 are similar to A, and C_1, C_2, and C_3 are similar to C. The practical steps of the GWO are provided in Algorithm 1. The updating process of a candidate's position through using the alpha, beta, and delta wolves in 2-dimension is provided in Figure 1. Figure 1 shows that the three best wolves (α, β, and δ) can obtain the location of the prey as well as the rest wolves update their location in the vicinity of the prey, randomly.

Algorithm 1 Pseudo-Code of the GWO

Input : T - number of iterations; N - population size.
Output: Δ_α - the best wolf (solution)

```
Initialize the location of each wolf randomly to
  constitute the population
Evaluate each wolf and obtain the Δα, Δβ, and Δδ (first
  three best wolves positions) using the objective function
while Iter≤T do
    for i=1 to N do
        Update the position of each wolf as
        Δ (Iter + 1) = (Δ1(Iter)+Δ2(Iter)+Δ3(Iter))/3
    end
    Update a : a (Iter) = 2 − 2 · Iter/T
    Update A : A = 2 · a ∘ r1 − a, and C : C = 2 · r2
    Compute the fitness of each grey wolf
    Update the Δα, Δβ, and Δδ using the objective
      function
end
Output: obtain the best individual Δα
```

B. BASICS OF CSA

Crow search algorithm (CSA) is developed by Alireza Askarzadeh [45] based on the nature intelligent of crows. Crows are intelligent birds as their behaviors exhibit a high level of cleverness, such as self-awareness in mirror test and tool making ability. One of their unusual behaviors is that they follow the other birds to observe the food hiding places and steal their food. Each crow acquires a hiding place to store its surplus food, and it considers awareness to safeguard it from probable followers. Also, the crow can make fool by going to other location if another crow follows it. CSA' behavior is formulated through the following assumptions [45]:

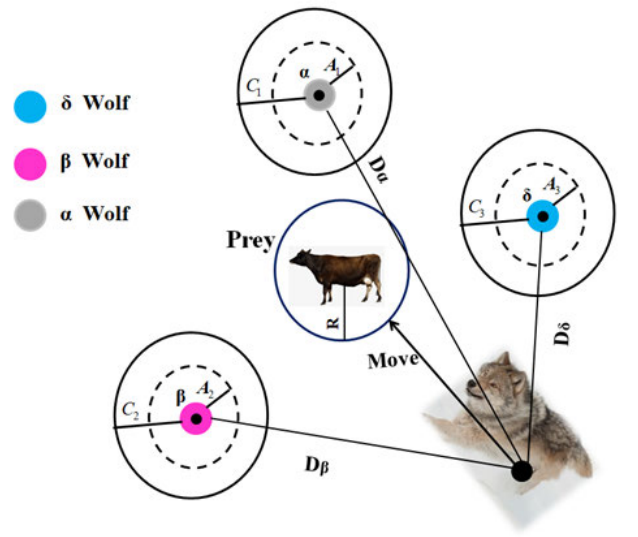

FIGURE 1. Updating strategy in GWO.

1. Crows are found together as a flock.
2. Crows can memorize the locations of their hiding places.
3. Crows recognize victim's hiding place by following each other.
4. Each crow protects food stores by a probability.

It is assumed that crows store their food in an n-dimensional search environment and N is the number of crows. The current location of the crow j at $Iter$-th iteration is defined as a vector:

$$x_{j,Iter} = [x^1_{j,Iter}, x^2_{j,Iter}, \ldots, x^n_{j,Iter}] \quad (7)$$

where $k = 1, 2, \ldots, N$, and $Iter = 1, 2, \ldots, T$. Each crow in the flock has its memory where its food hiding place is saved. The food hiding location of crow j at $Iter$-th iteration is defined by $m_{j,Iter}$ which is the best position obtained by crow j till now.

Suppose that at $Iter$-th iteration, crow j needs to go to its food hiding position $m_{j,Iter}$. At the same time (iteration) crow i attempts to follow crow j in order to its the food hiding position. In this situation, two cases may have occurred:

Case 1: Crow j does not become aware that another crow i is tracking it. In this situation, the crow i can reach the food hiding location of crow j and the crow i will update its position as follows.

$$x_{i,Iter+1} = x_{i,Iter} + r_i \cdot fl_{i,Iter} \cdot \left(m_{j,Iter} - x_{i,Iter}\right) \quad (8)$$

where r_i is a random number that distributed uniformly in the interval [0, 1] and $fl_{i,Iter}$ is the flight length of crow i at $Iter$-th iteration that has a significant effect on the searching capability of algorithm, where lower values of fl enhances the local search (closer to $x_{i,Iter}$), while higher values of fl promotes the exploration that is denoted as global search (far away from $x_{i,Iter}$) (i.e. see Figure 2).

Case 2: Crow j finds out that the crow i is tracking it. Therefore, the crow j will fool crow i by going to another

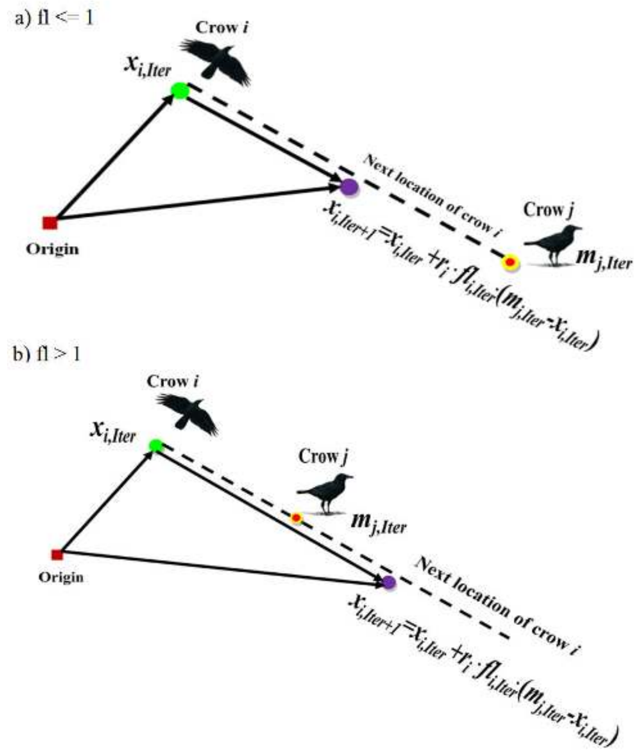

FIGURE 2. The strategy of searching by the crow regarding the two cases: $fl \leq 1$ (a), and $fl > 1$ (b).

location in the search region. In general, the two cases can be considered as follows:

$$x_{i,Iter+1} = \begin{cases} x_{i,Iter} + r_i \cdot fl_{i,Iter} \cdot \\ \cdot \left(m_{j,Iter} - x_{i,Iter}\right) & \text{if } r_j \geq AP_{j,Iter} \\ \text{a random position,} & \text{if otherwise} \end{cases} \quad (9)$$

$$m_{i,Iter+1} = \begin{cases} x_{i,Iter+1}, & \text{if } f\left(x_{i,Iter+1}\right) > f\left(m_{i,Iter}\right) \\ m_{i,Iter}, & \text{if otherwise} \end{cases} \quad (10)$$

where r_j is a random number that uniformly distributed in the range [0, 1] and $AP_{j,Iter}$ represents the awareness probability of crow j at $Iter$-th iteration. The main steps of the CSA are introduced in Algorithm 2.

III. THE MOTIVATION FOR THIS WORK

The standards of the grey wolf optimizer (GWO) and crow search algorithm (CSA) exhibit good performances on some unimodal benchmark function problems. However, they deal with complex multimodal functions, the trapping in local optima, as well as the premature convergence, may be occurred. Furthermore, dealing with large-scale dimensions may deteriorate the performances of simple algorithms. To overcome these shortages and improve the searching capability, a new hybrid algorithm based on GWO and CSA is introduced to solve complex life problems as well as large-scale dimensions. The proposed algorithm is called GWO-CSA. In the GWO-CSA, the movements of all grey

Algorithm 2 Pseudo-Code of the CSA

Input : T - number of iterations; N - number of crows, fl - flight lenght, AP - awareness probability
Output: Best crow location

Initialize randomly the location of a flock of N crows in the search region
Evaluate the crows' locations
Fill the own memory of each crow with its initial location
while $Iter \leq T$ **do**
  for $i=1$ *to* N **do**
    Elicit one of the crows at random to track it (for example j) Generate $r \in [0, 1]$
    if $r < AP$ **then**
      $x_{i,Iter+1} = x_{i,Iter} + r_i \cdot fl_i \cdot \left(m_{j,Iter} - x_{i,Iter}\right)$
    else
      $x_{i,Iter+1}$=random location
    end
  end
  end
  Check the feasibility of new locations
  Evaluate the new locations of the crows
  Perform the updating of the memory:

$$m_{i,Iter+1} = \begin{cases} x_{i,Iter} & \text{if } f\left(x_{i,Iter}\right) > f\left(m_{i,Iter}\right) \\ m_{i,Iter}, & \text{if otherwise} \end{cases}$$

end
Output: best crow location

wolves, as well as an alpha grey wolf, are improved based on CSA to enhance the diversity of solutions, efficiently.

Further, a dynamic fuzzy learning strategy (DFLS) based on the information of the best solution is introduced to enable the tiny perturbation in the neighborhood of the best so far outcome and then refine the quality of the solution. By this methodology, the balance among exploration and exploitation can be enhanced and the sucking in local optima can be avoided. The hybrid variant has been tested on numerous benchmark problems with different dimensions and some of engineering design applications. Simulation results affirm its robustness of searching when dealing with numerous problems.

IV. THE PROPOSED HYBRID ALGORITHM

A hybrid grey wolf optimizer with crow search algorithm (GWO-CSA) is presented with the aim to integrate the searching merits of both algorithms. In this sense, GWO aims to enhance the exploration search in the first stage of the searching scheme, while CSA aims to preserve the exploitation capability in the final stage of this scheme. Further, a dynamic fuzzy learning strategy (DFLS) is presented to enable the occurring of tiny changes in the neighborhood of the best solution to mitigate the trapping in the local solutions and refine the quality of solutions. Therefore, the proposed GWO-CSA involves three main improvements. Firstly, a learning strategy based on opposition searching is introduced to preserve the diversity of crows. Secondly, an iterative level hybridization with CSA is presented to accelerate the approaching of the best solution. Thirdly, a dynamic fuzzy learning strategy (DFLS) is developed as a neighborhood searching strategy for achieving top-quality of solutions in each generation. The kernel idea behind GWO-CSA is demonstrated as follows.

A. UPDATING OF CSA-BASED OPPOSITION LEARNING

In CSA, the crow is updated by considering the awareness probability, when crow j does aware that another crow i is following it, then crow i will update its position randomly. This may lack the diversity of solution and may be deteriorated with the immediate convergence rate. Thus, instead of updating randomly, a strategy based on the opposition learning is developed to preserve the crow's diversity and increase the exploration capability. The updating strategy is as follows.

$$x_{i,Iter+1} = \begin{cases} x_{i,Iter} + r_i \cdot fl_{i,Iter} \cdot \\ \cdot \left(m_{j,Iter} - x_{i,Iter}\right) & \text{if } r_j \geq AP_{j,Iter} \\ q \cdot (ub + lb) - x_{i,Iter}, & \text{if otherwise} \end{cases} \tag{11}$$

where ub and lb illustrate the limits of the search space, and q denotes a random number in [0, 1].

B. ITERATIVE HYBRIDIZATION-BASED GWO WITH CSA

This stage aims to execute both algorithms in sequence iteratively to enhance the optimization performance. Here GWO is used as explore tool to attain the promising areas and CSA is then allowed to exploit these areas to find better solutions. In this sense, GWO starts the search procedures using its mechanism, and then CSA is initialized with the alpha grey wolf and the other wolves to improve the location of an alpha grey wolf.

The updating process of the candidate's position through using the alpha, beta, and delta wolves is as follows. The three best crows denoted by Δ_{Crow1}, Δ_{Crow2} and Δ_{Crow3} are obtained using the fitness function then they are compared with those produced by GWO (Δ_α, Δ_β, and Δ_δ) to attain the survival ones as follows.

$$\begin{aligned} \Delta_\alpha &= \arg\min\{f\left(\Delta_\alpha\right), f\left(\Delta_{Crow1}\right)\} \\ \Delta_\beta &= \arg\min\{f\left(\Delta_\beta\right), f\left(\Delta_{Crow2}\right)\} \\ \Delta_\delta &= \arg\min\{f\left(\Delta_\delta\right), f\left(\Delta_{Crow3}\right)\} \end{aligned} \tag{12}$$

C. DYNAMIC FUZZY LEARNING STRATEGY (DFLS)

Zadeh developed the main concept of a fuzzy set (FS) in 1965 [49]. The FS is different from the ordinary set in which the element or the object is characterized by two values (i.e., 0 or 1), where 1 and 0 indicate the element which belongs and does not belongs to S, respectively, where S is the FS in U (i.e., the universe of discourse) is recognized by a membership (characteristic) $\mu_s(x)$ that specifies a real

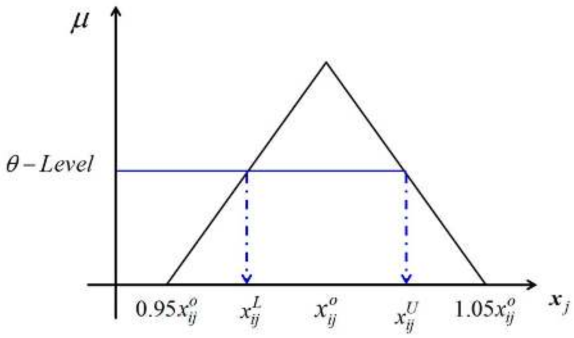

FIGURE 3. Fuzzy numbers representation.

number from the interval [0, 1] for each class (point) x in U. Also the value of $\mu_A(x)$ elucidates the degree of membership of x in S, where the nearer value of $\mu_S(x)$ to unity the higher grade of membership of x in S.

Definition 1: Let U represents a group (collection) of objects or elements defined generically by x, then the set $S^{\%}$ of ordered pairs represents the fuzzy set:

$$S^{\%} = \left\{\left(x, \mu_{S^{\%}}(x)\right) | x \in U\right\} \tag{13}$$

where $\mu_{S^{\%}}(x)$ indicates the membership function (generalized characteristic function).

To implement the DFLS, the approximated optimal solution $x^{\circ} = \left(x_1^{\circ}, x_2^{\circ}, \ldots, x_n^{\circ}\right) = \Delta_{\alpha}$ is obtained through the scenarios of GWO and CSA. In this sense, DFLS aims to make a tiny perturb around the approximated optimal solution by constructing the membership function, as in Equation 16, which assigns different grads for the local region of optimal solution that can reside. The bounds of the local region are determined based on θ-the cut level that aims to siege the optimal solution, where the bounds (i.e., upper and lower bounds) of the local region can be depicted in Figure 3.

$$\mu\left(x_{ij}^{\circ}\right) = \begin{cases} 1 & x = x_{ij}^{\circ} \\ \dfrac{20x}{x_{ij}^{\circ}} - 19 & 0.95x_{ij}^{\circ} \le x \le x_{ij}^{\circ} \\ 21 - \dfrac{20x}{x_{ij}^{\circ}} & x_{ij}^{\circ} \le x \le 1.05x_{ij}^{\circ} \\ 0 & x < 0.95x_{ij}^{\circ} \text{ or } x > 1.05x_{ij}^{\circ} \end{cases} \tag{14}$$

Consider the optimal solution x_j° in the j-th dimension equals 1. In this sense, when $\theta = 1$, the value of $x = x_j^{\circ}$ remains as it is (see Figure 4a), while for $\theta = 0$, the value of x_j° having the ends, $x \in [0.95, 1.05]$ (i.e., $x_j^l = 0.95$ and $x_j^u = 1.05$ as in Figure 4b). Further for any θ such that $\theta = 0.6$, the value of x_j^o gets the bounds 0.98 and 1.02, $x_j^l = 0.98$ and $x_j^u = 1.02$ as in Figure 4c. The main procedures of DFLS can be stated as follows.

Step 1. Formulate the membership function and its width for each dimension as in Figure 3 and Equation 16.

Step 2. Generate the value of θ-cut level randomly to obtain dynamic bounds for the searching process.

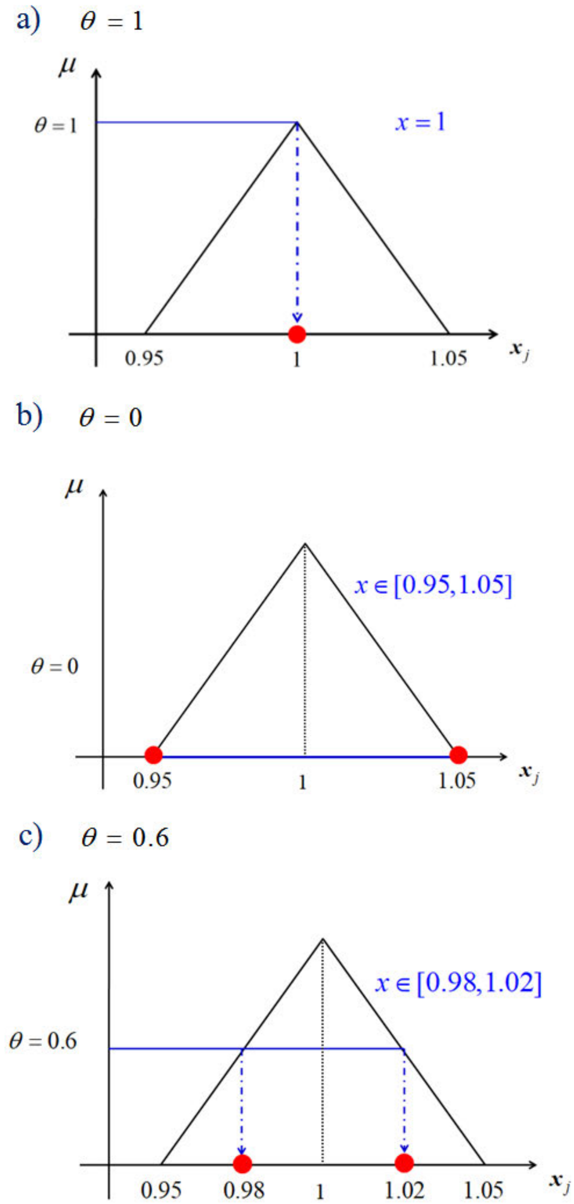

FIGURE 4. θ – Levels schemes.

Step 3. After applying the θ-cut level, the crisp bounds for the j-th dimension is determined as follows.

$$x_j^{LF} = \frac{\theta x_j^{\circ}}{20} + 0.95x_j^{\circ}, x_j^{UF} = 1.05x_j^{\circ} - \frac{\theta x_j^{\circ}}{20} \tag{15}$$

Step 4. Map the crisp bounds-based fuzzy technique into optimization search as follows.

$$\overline{x_j} = \begin{cases} x_j^{LF} + r_{f1} \cdot \left(x_j^{UF} - x_j^{LF}\right) & rand < 0.5 \\ x_j^{UF} + r_{f2} \cdot \left(x_j^{UF} - x_j^{LF}\right) & otherwise \end{cases} \tag{16}$$

where r_{f1}, r_{f2} are random numbers in [0, 1].

Step 5. If $f(\overline{x}) < f(x^{\circ})$ then put $x^{\circ} = \overline{x}$. The working code of the introduced DFLS is shown in Algorithm 3.

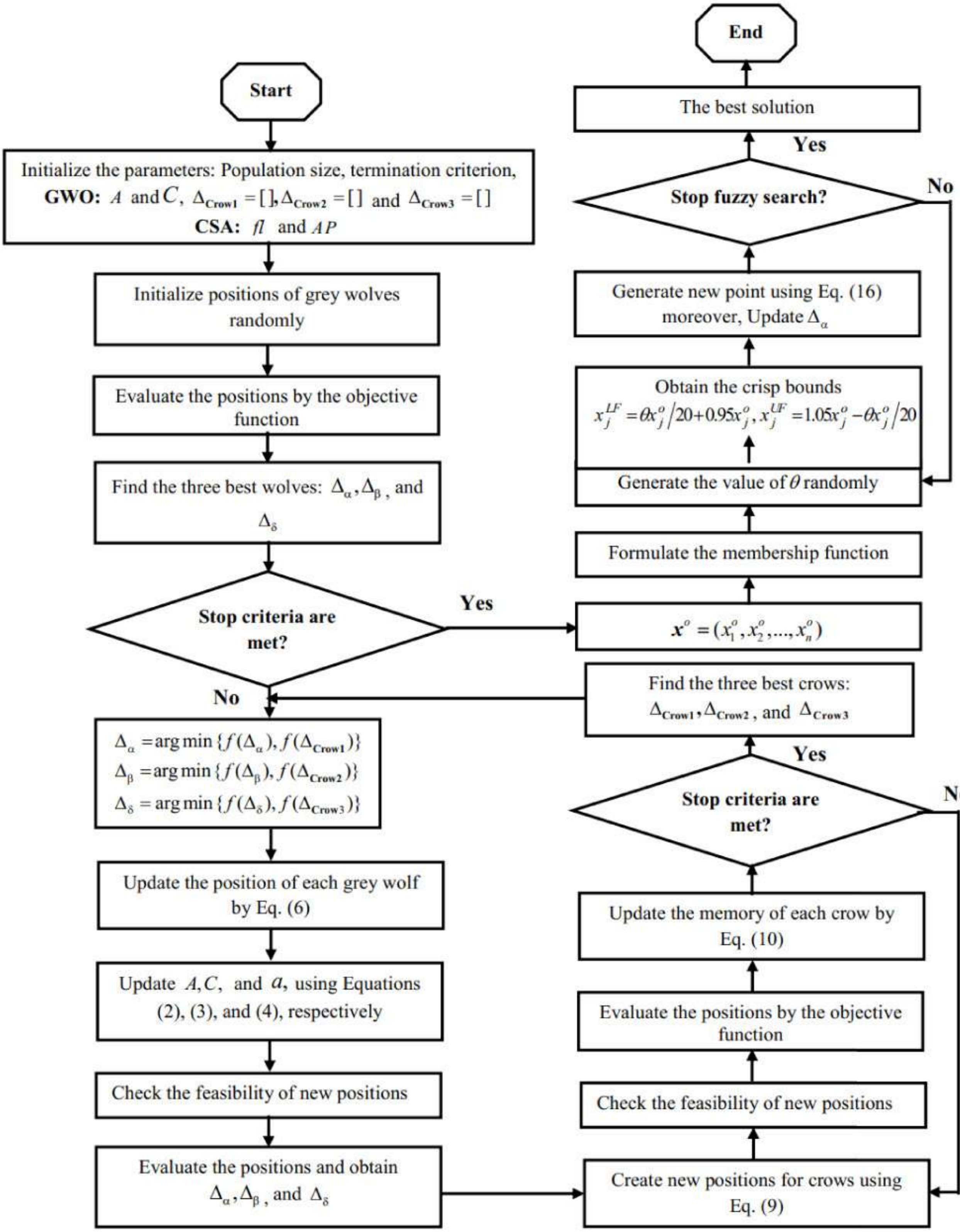

FIGURE 5. The flowchart of the proposed GWO-CSA.

Algorithm 3 Pseudo-Code of the DFLS

Input : $x^\circ = (x_1^\circ, x_2^\circ, \ldots, x_n^\circ)$
Output: x°

1 Formulate the membership function
2 $k \leftarrow 1, j = 1 : n$
3 $\theta \leftarrow rand$
4 Fuzzy bounds: $x_j^{LF} = \frac{\theta x_j^\circ}{20} + 0.95 x_j^\circ, x_j^{UF} = 1.05 x_j^\circ - \frac{\theta x_j^\circ}{20}$
5 $\overline{x_j} = \begin{cases} x_j^{LF} + rand \cdot \left(x_j^{UF} - x_j^{LF}\right) & \text{if rand<0.5} \\ x_j^{UF} - rand \cdot \left(x_j^{UF} - x_j^{LF}\right) & \text{if otherwise} \end{cases}$
6 $f(\overline{x}) < f(x^\circ) \Rightarrow x^\circ = \overline{x}$; then set $\Delta_\alpha = x^\circ$
7 $k \leftarrow k + 1$
8 Output: $x_\circ$

TABLE 1. Test functions.

Function formula	Range
$F_1 = \sum_{i=1}^{n} i \cdot x_i^4 + random[0, 1)$	[-1.28, 1.28]
$F_2 = \sum_{i=1}^{n} [x_i^2 - 10 \cdot cos(2 \cdot \pi \cdot x_i) + 10]$	[-100, 100]
$F_3 = 1 - 10 \cdot cos\left(2 \cdot \pi \cdot \sqrt{\sum_{i=1}^{n} x_i^2}\right) + 0.1 \cdot \sqrt{\sum_{i=1}^{n} x_i^2}$	[-100, 100]
$F_4 = 0.5 + \frac{sin^2\left(\sqrt{\sum_{i=1}^{n} x_i^2}\right) - 0.5}{1 + 0.001 \cdot \left(\sum_{i=1}^{n} x_i^2\right)^2}$	[-100, 100]

Thus the GWO-CSA improves the exploration searches by GWO in the initial stage and enhances the exploitation capabilities of CSA in the final stage to achieve global optimal solutions. Further, the DFLS is introduced to achieve a high quality of the final solution. The flowchart of the GWO-CSA is showed in Figure 5.

V. EXPERIMENTS AND RESULTS

A. BENCHMARK PROBLEMS

In this section, four test functions (F_1: Quartic, F_2: Rastrigin, F_3: Salmon, F_4: Schaffer) are selected from [19] and listed in Table 1. These problems are typical high-complicated test problems, where they involve different natures, such as unimodal, multi-modal, separable, non-separable, regular, irregular, and multi-dimensional problems. The characteristics of these problems such as formulas, and ranges are recorded in Table 1. The optimal value for each problem is equal to 0. For all test instances, we attempt to investigate the performance of the proposed method as well as the comparative algorithms with three experiments meanwhile three

TABLE 2. The parameter settings for all algorithms.

Algorithm	Parameters
GWO [29]	Rand parameters: $r_1 = rand$, $r_2 = rand$; $a \in [2, 0]$ (linear decreasing parameter)
CSA [45]	Awareness probability: $AP = 0.1$; Flight length: $fl = 2$
SCA [17]	$r_1 \in [2, 0]$ (linear decreasing parameter); $r_2 = 2 \cdot \pi \cdot rand$, $r_3 = 2 \cdot rand$, $r_4 = rand$, $a = 2$
GWO-SCA [50]	Used the same settings of both GWO and SCA
MHDA [51]	Inertia weight: $w_{min} = 0.2$, $w_{max} = 0.9$; Cognitive and social parameters: $c_1 = 1$, $c_2 = 1$; Levy flight constant: 1.5
GWO-CSA	Used the same settings of both GWO and CSA

TABLE 3. The PC configuration.

Name	Detailed settings
Hardware	CPU: Intel Core i5 3337U 1.8 GHz; RAM: 4 GB; Hard drive: 500 GB
Software	Operating system: Windows 7 (64 bit); Programming environment: Matlab R2014a

dimensions, i.e., $D = 100$, 500 and 1000, are investigated. It is noted that the difficulty of searching process grows exponentially with the dimension.

B. PARAMETER SETTINGS

In all experiments, the parameters of the proposed GWO-CSA and the comparative algorithms are adjusted after running a few trials as follows. The population size (PS) is set to 30 while the maximum number of iterations (T) is set to 300, (i.e., the maximum number of function evaluation is set to 9000) for all test problems with the employed dimensions. To obtain a fair comparison, each algorithm is executed 20 independent runs for each test problem, with the same set of random seeds. The other control parameters configurations of all comparative algorithms are presented using the suggestions in their corresponding literature and they reported in Table 2. To get unbiased comparisons of CPU times, all the experiments are carried out utilizing the same PC, where its configuration is provided in Table 3.

C. EXPERIMENTAL RESULTS

To validate the proposed GWO-CSA for large-scale global optimization problems, it is tested on some benchmark problems that have different natures, which are listed in Table 1. The proposed GWO-CSA is compared with classical algorithms and hybrid ones such as GWO [1], CSA [45], SCA [17], GWO-SCA [50], and MHDA [51]. Three experiments are conducted with three dimensions, respectively, $D = 100$, 500 and 1000, where in each one, the results such as the best value, average (mean), worst, and standard deviation (st. dev.) are reported. In addition, the convergence curves that describe the convergence rate of all algorithms on all test functions are provided for $D = 1000$ only due to the space limitation. Based on the depicted convergences, GWO-CSA

TABLE 4. Results of GWO, CSA, SCA, GWO-SCA, MHDA, and GWO-CSA with $D = 100$.

Fun.	Measures	GWO	CSA	SCA	GWO-SCA	MHDA	GWO-CSA
F_1	Best	8.26E-03	1.72E-04	2.85E+01	1.18E-02	12.75766	1.17E-06
	Mean	1.75E-02	1.54E-03	2.02E+02	2.16E-02	4.28E+01	1.31E-04
	Worst	3.14E-02	2.86E-03	3.95E+02	3.32E-02	9.86E+01	3.26E-04
	Std. dev.	7.15E-03	8.65E-04	1.26E+02	6.41E-03	2.67E+01	1.12E-04
F_2	Best	2.08E+01	0	1.41E+02	4.01E+01	1.89E+02	0
	Mean	3.33E+01	0	2.77E+02	6.81E+01	6.83E+02	0
	Worst	4.71E+01	0	4.22E+02	1.10E+02	9.86E+02	0
	Std. dev.	1.01E+01	0	9.28E+01	1.91E+01	3.03E+02	0
F_3	Best	5.00E-01	1.57E-06	6.36E+00	7.00E-01	5.87E+00	0
	Mean	5.60E-01	5.54E-02	1.21E+01	7.89E-01	1.47E+01	0
	Worst	6.00E-01	9.99E-02	1.89E+01	8.99E-01	2.59E+01	0
	Std. dev.	5.16E-02	4.56E-02	4.42E+00	7.38E-02	7.04E+00	0
F_4	Best	4.43E-01	9.40E-11	5.00E-01	4.92E-01	5.00E-01	0
	Mean	4.62E-01	1.01E-02	5.00E-01	4.95E-01	5.00E-01	0
	Worst	4.80E-01	4.37E-02	5.00E-01	4.97E-01	5.00E-01	0
	Std. dev.	1.99E-02	1.81E-02	9.07E-08	2.22E-03	4.54E-07	0

TABLE 5. Results of GWO, CSA, SCA, GWO-SCA, MHDA, and GWO-CSA with $D = 500$.

Fun.	Measures	GWO	CSA	SCA	GWO-SCA	MHDA	GWO-CSA
F_1	Best	1.74E-01	2.17E-04	1.38E+04	2.69E-01	1.01E+03	3.00E-06
	Mean	2.24E-01	2.41E-03	2.03E+04	3.75E-01	2.62E+03	1.13E-04
	Worst	3.01E-01	9.25E-03	2.73E+04	4.88E-01	4.52E+03	2.68E-04
	Std. dev.	5.19E-02	3.21E-03	4.70E+03	8.31E-02	1.44E+03	9.23E-05
F_2	Best	3.08E+02	0	7.96E+02	5.13E+02	1.63E+03	0
	Mean	3.96E+02	0	1.59E+03	6.98E+02	3.69E+03	0
	Worst	4.94E+02	0	2.58E+03	8.14E+02	5.72E+03	0
	Std. dev.	6.20E+01	0	7.08E+02	1.16E+02	1.68E+03	0
F_3	Best	2.39E+00	8.65E-07	4.19E+01	3.29E+00	3.03E+01	0
	Mean	2.83E+00	1.75E-02	5.46E+01	3.76E+00	3.40E+01	0
	Worst	3.29E+00	9.98E-02	6.31E+01	4.09E+00	3.99E+01	0
	Std. dev.	2.94E-01	4.03E-02	7.41E+01	2.80E-01	3.43E+00	0
F_4	Best	4.99E-01	6.70E-07	5.00E-01	4.99E-01	5.00E-01	0
	Mean	4.99E-01	2.91E-02	5.00E-01	4.99E-01	5.00E-01	0
	Worst	4.99E-01	4.36E-02	5.00E-01	4.99E-01	5.00E-01	0
	Std. dev.	3.50E-06	2.25E-02	6.09E-10	9.19E-07	4.48E-09	0

TABLE 6. Results of GWO, CSA, SCA, GWO-SCA, MHDA, and GWO-CSA with $D = 1000$.

Fun.	Measures	GWO	CSA	SCA	GWO-SCA	MHDA	GWO-CSA
F_1	Best	9.19E-01	3.12E-4	7.20E+043	2.20E+00	6.08E+03	4.34E-06
	Mean	1.33E+00	1.95E-3	8.82E+04	2.94E+00	1.26E+04	1.25E-04
	Worst	1.91E+00	6.43E-3	1.16E+05	3.67E+00	1.94E+04	3.55E-04
	Std. dev.	3.27E-01	2.02E-3	1.47E+04	4.48E-01	4.22E+03	1.07E-04
F_2	Best	7.26E+02	0	1.32E+03	1.57E+03	6.92E+03	0
	Mean	1.15E+03	0	2.74E+03	1.89E+03	9.94E+03	0
	Worst	1.42E+03	0	4.40E+03	2.24E+03	1.18E+04	0
	Std. dev.	2.35E+02	0	1.24E+03	2.51E+02	1.71E+03	0
F_3	Best	5.49E+00	1.07E-05	5.05E+01	8.39E+00	4.18E+01	0
	Mean	6.25E+00	3.33E-2	7.57E+01	9.31E+00	5.37E+01	0
	Worst	7.04E+00	9.98E-2	8.94E+01	10.09E+0	6.69E+01	0
	Std. dev.	5.97E-01	5.15E-2	1.40E+01	7.11E-01	9.11E+00	0
F_4	Best	4.99E-01	4.08E-02	5.00E-01	5.00E-01	5.00E-01	0
	Mean	5.00E-01	4.30E-2	5.00E-01	5.00E-01	5.00E-01	0
	Worst	5.00E-01	4.36E-2	5.00E-01	5.00E-01	5.00E-01	0
	Std. dev.	1.53E-07	1.29E-3	1.10E-10	2.81E-08	1.19E-09	0

can provide faster convergence rate and has higher precision than other algorithms.

In Experiment 1, the proposed algorithm and five comparative algorithms are conducted with dimension $D = 100$, where the statistical measures are presented in Table 4. Based on the reported results of Table 4, it is evident that, overall, GWO-CSA gives the best result among all compared algorithms from the statistical view. Compared to the other algorithms, GWO, CSA, SCA, GWO-SCA, and MHDA, GWO-CSA finds dominant results for all test functions.

TABLE 7. Results of various GWO variants on the studied benchmark function with $D = 1000$.

Fun.	Measures	LGWO	ROGWO	mGWO	RLGWO	IGWO
F_1	Best	9.02E+02	1.54E-05	5.31E+01	4.95E-04	4.99E-04
	Mean	2.24E+03	2.72E-04	1.34E+02	1.36E-02	2.76E-03
	Worst	6.13E+03	8.37E-04	2.49E+02	4.85E-02	8.00E-03
	Std. dev.	1.47E+03	2.99E-04	6.30E+01	1.60E-02	2.39E-03
F_2	Best	3.39E+03	**0.00E+00**	1.99E+03	3.30E+00	**0.00E+00**
	Mean	3.67E+03	0.00E+00	2.15E+03	2.43E+01	0.00E+00
	Worst	4.38E+03	0.00E+00	2.36E+03	6.33E+01	0.00E+00
	Std. dev.	4.12E+02	0.00E+00	1.64E+02	2.61E+01	0.00E+00
F_3	Best	2.67E+01	9.99E-02	1.67E+01	9.99E-02	1.00E-01
	Mean	3.09E+01	9.99E-02	1.90E+01	1.41E-01	1.11E-01
	Worst	3.64E+01	9.99E-02	2.33E+01	3.00E-01	1.50E-01
	Std. dev.	3.57E+00	1.54E-06	2.59E+00	8.89E-02	2.18E-02
F_4	Best	5.00E-01	**0.00E+00**	5.00E-01	5.00E-01	4.37E-02
	Mean	5.00E-01	1.09E-02	5.00E-01	5.00E-01	4.37E-02
	Worst	5.00E-01	4.37E-02	5.00E-01	5.00E-01	4.38E-02
	Std. dev.	8.94E-11	2.18E-02	2.40E-09	1.67E-09	4.48E-05

Fun.	Measures	EEGWO	ERGWO	GLFGWO	GWOWD	
F_1	Best	**3.94E-05**	2.44E+05	2.81E+01	3.23E-04	
	Mean	2.74E-04	2.51E+05	5.40E+01	1.10E-03	
	Worst	7.81E-04	2.58E+05	9.57E+01	3.00E-03	
	Std. dev.	2.19E-04	4.41E+03	2.31E+01	8.02E-04	
F_2	Best	**0.00E+00**	1.76E+04	1.94E+03	**0.00E+00**	
	Mean	0.00E+00	1.79E+04	2.15E+03	0.00E+00	
	Worst	0.00E+00	1.81E+04	2.34E+03	0.00E+00	
	Std. dev.	0.00E+00	1.77E+02	1.57E+02	0.00E+00	
F_3	Best	**1.06E-90**	1.75E+02	1.19E+01	1.00E-01	
	Mean	1.81E-88	1.79E+02	1.31E+01	1.34E-01	
	Worst	8.62E-88	1.82E+02	1.40E+01	2.07E-01	
	Std. dev.	3.81E-88	2.57E+00	7.83E-01	4.51E-02	
F_4	Best	**0.00E+00**	5.00E-01	5.00E-01	4.59E-02	
	Mean	0.00E+00	5.00E-01	5.00E-01	8.48E-02	
	Worst	0.00E+00	5.00E-01	5.00E-01	1.63E-01	
	Std. dev.	0.00E+00	7.90E-12	5.23E-09	5.39E-02	

TABLE 8. Characteristic of CEC 2015 benchmark problems.

Function no.	Description	Global optimal (F^*)	Function nature
F_{C1}	Rotated Bent Cigar Function	100	Unimodal functions
F_{C2}	Rotated Discus Function	200	
F_{C3}	Shifted and Rotated Weierstrass Function	300	Simple multimodal functions
F_{C4}	Shifted and Rotated Schwefel's Function	400	
F_{C5}	Shifted and Rotated Katsuura Function	500	
F_{C6}	Shifted and Rotated HappyCat Function	600	
F_{C7}	Shifted and Rotated HGBat Function	700	
F_{C8}	Shifted and Rotated Expanded Griewank's plus Rosenbrock's Function	800	
F_{C9}	Shifted and Rotated Expanded Scaffer's F6 Function	900	
F_{C10}	Hybrid Function 1 (N=3)	1000	Hybrid functions
F_{C11}	Hybrid Function 2 (N=4)	1100	
F_{C12}	Hybrid Function 3 (N=5)	1200	
F_{C13}	Composition Function 1 (N=5)	1300	Composition functions
F_{C14}	Composition Function 2 (N=3)	1400	
F_{C15}	Composition Function 3 (N=5)	1500	

In experiment 2, the performance of the GWO-CSA is investigated and compared with that of the GWO, CSA, SCA, GWO-SCA, and MHDA on all test benchmark functions

TABLE 9. Comparative results among the proposed method versus different DE variants for CEC 2015 problems.

Fun.	Measures	DE	SHADE	LSHADE	GWO	EEGWO	GWO-CSA	DE1 [53]	DE2 [54]
F_{C1}	Best	1.0018E+02	1.0000E+02	1.0000E+02	1.0996E+06	7.5972E+09	1.0000E+02		
	Mean	6.2503E+02	1.0086E+02	1.0098E+02	1.2961E+07	9.4030E+09	**1.0000E+02**	3.4143E+09	6.09E+06
	Worst	1.5974E+03	1.0258E+02	1.0291E+02	2.8906E+07	1.2103E+10	1.0000E+02		
	Std. dev.	8.4298E+02	1.4887E+00	1.6773E+00	1.4346E+07	2.3821E+09	2.2366E-06		5.11E+06
F_{C2}	Best	5.1940E+02	2.7575E+02	2.1543E+02	8.5423E+03	5.4095E+04	2.0000E+02		
	Mean	1.5257E+03	5.7259E+03	2.7467E+02	9.2440E+03	2.4910E+06	**2.0000E+02**	7.4931E+04	4.40E+04
	Worst	3.5056E+03	1.6450E+04	3.7775E+02	1.0419E+04	5.0374E+06	2.0001E+02		
	Std. dev.	1.7147E+03	9.2876E+03	8.9605E+01	1.0239E+03	2.4935E+06	2.6285E-03		2.75E+04
F_{C3}	Best	3.0000E+02	3.0094E+02	3.0034E+02	3.0257E+02	3.1381E+02	3.0000E+02		
	Mean	**3.0000E+02**	3.0190E+02	3.0195E+02	3.0456E+02	3.1433E+02	3.0015E+02	3.1093E+02	3.20E+02
	Worst	3.0000E+02	3.0272E+02	3.0499E+02	3.0636E+02	3.1478E+02	3.0046E+02		
	Std. dev.	7.3969E-08	8.9441E-01	2.6327E+00	1.8972E+00	4.8862E-01	2.6517E-01		1.15E-03
F_{C4}	Best	4.1393E+02	4.0012E+02	4.0006E+02	5.4040E+02	2.6370E+03	4.0000E+02		
	Mean	4.6629E+02	4.6314E+02	4.2989E+02	8.9192E+02	2.8581E+03	**4.0003E+02**	2.2974E+03	4.91E+06
	Worst	5.6317E+02	6.3700E+02	5.1908E+02	1.1515E+03	3.1184E+03	4.0006E+02		
	Std. dev.	6.8909E+01	1.1613E+02	5.9456E+01	2.6347E+02	2.0067E+02	3.6058E-02		6.03E+00
F_{C5}	Best	5.0095E+02	5.0034E+02	5.0024E+02	5.0105E+02	5.0348E+02	5.0000E+02		
	Mean	5.0136E+02	5.0056E+02	5.0033E+02	5.0131E+02	5.0375E+02	**5.0000E+02**	5.0286E+02	3.34E+03
	Worst	5.0183E+02	5.0076E+02	5.0040E+02	5.0161E+02	5.0411E+02	5.0000E+02		
	Std. dev.	4.4271E-01	2.0926E-01	7.8866E-02	2.8385E-01	3.2390E-01	7.2694E-12		3.01E+02
F_{C6}	Best	6.0009E+02	6.0012E+02	6.0009E+02	6.0013E+02	6.0339E+02	6.0000E+02		
	Mean	6.0025E+02	6.0020E+02	6.0014E+02	6.0018E+02	6.0416E+02	**6.0001E+02**	6.0286E+02	5.39E+04
	Worst	6.0039E+02	6.0030E+02	6.0019E+02	6.0020E+02	6.0530E+02	6.0001E+02		
	Std. dev.	1.5102E-01	8.9762E-02	5.1317E-02	3.9077E-02	1.0095E+00	2.3603E-03		2.40E+04
F_{C7}	Best	7.0019E+02	7.0014E+02	7.0013E+02	7.0007E+02	7.5089E+02	7.0000E+02		
	Mean	7.0025E+02	7.0019E+02	7.0023E+02	7.0013E+02	7.6435E+02	**7.0001E+02**	7.2588E+02	7.06E+02
	Worst	7.0036E+02	7.0027E+02	7.0029E+02	7.0025E+02	7.7517E+02	7.0001E+02		
	Std. dev.	8.9741E-02	7.2801E-02	8.5234E-02	9.8834E-02	1.2355E+01	5.2237E-03		3.87E+00
F_{C8}	Best	8.0176E+02	8.0070E+02	8.0029E+02	8.0142E+02	5.9562E+04	8.0019E+02		
	Mean	8.0278E+02	8.0087E+02	8.0090E+02	8.0200E+02	1.1526E+05	**8.0025E+02**	4.1637E+03	5.60E+03
	Worst	8.0368E+02	8.0105E+02	8.0173E+02	8.0330E+02	2.6820E+05	8.0033E+02		
	Std. dev.	7.9259E-01	1.8162E-01	6.0580E-01	8.8828E-01	1.0207E+05	5.5944E-02		5.15E+04
F_{C9}	Best	9.0317E+02	9.0270E+02	9.0279E+02	9.0211E+02	9.0412E+02	9.0008E+02		
	Mean	9.0338E+02	9.0302E+02	9.0289E+02	9.0267E+02	9.0429E+02	**9.0010E+02**	9.0415E+02	1.00E+03
	Worst	9.0356E+02	9.0343E+02	9.0308E+02	9.0306E+02	9.0439E+02	9.0013E+02		
	Std. dev.	1.9781E-01	3.7239E-01	1.6332E-01	4.9583E-01	1.5274E-01	3.1763E-02		4.05E+00
F_{C10}	Best	1.0328E+03	1.2290E+03	1.2850E+03	2.2662E+03	1.2302E+06	1.0014E+03		
	Mean	1.2263E+03	1.4830E+03	1.4261E+03	4.8294E+03	1.6458E+07	**1.0024E+03**	1.3622E+06	3.10E+04
	Worst	1.3550E+03	1.7584E+03	1.5821E+03	9.1112E+03	3.5693E+07	1.0031E+03		
	Std. dev.	1.5756E+02	2.1811E+02	1.2260E+02	3.0186E+03	1.5143E+07	7.0058E-01		2.76E+04
F_{C11}	Best	1.1007E+03	1.1030E+03	1.1016E+03	1.1016E+03	1.1876E+03	1.1000E+03		
	Mean	1.1014E+03	1.1040E+03	1.1029E+03	1.1021E+03	1.2341E+03	**1.1000E+03**	1.1229E+03	1.65E+03
	Worst	1.1017E+03	1.1053E+03	1.1039E+03	1.1034E+03	1.3159E+03	1.1000E+03		
	Std. dev.	4.6240E-01	9.6664E-01	9.8196E-01	8.7912E-01	6.0401E+01	3.9372E-04		3.00E+01
F_{C12}	Best	1.2215E+03	1.2267E+03	1.2229E+03	1.3527E+03	1.6073E+03	1.2000E+03		
	Mean	1.2536E+03	1.3248E+03	1.2861E+03	1.3659E+03	1.7537E+03	**1.2000E+03**	1.5980E+03	1.30E+03
	Worst	1.3425E+03	1.3646E+03	1.3495E+03	1.3888E+03	1.9132E+03	1.2000E+03		
	Std. dev.	5.9317E+01	6.5728E+01	7.1353E+01	1.6180E+01	1.5974E+02	1.2359E-02		9.36E-01
F_{C13}	Best	1.6157E+03	1.6157E+03	1.6149E+03	1.6160E+03	1.9684E+03	1.6149E+03		
	Mean	1.6157E+03	1.6157E+03	1.6154E+03	1.6193E+03	2.5674E+03	1.6149E+03	1.7969E+03	**1.30E+03**
	Worst	1.6157E+03	1.6157E+03	1.6157E+03	1.6216E+03	3.1692E+03	1.6149E+03		
	Std. dev.	2.7847E-13	1.3160E-12	4.1134E-01	2.9451E+00	6.0040E+02	2.7847E-13		4.96E-02
F_{C14}	Best	1.5825E+03	1.5874E+03	1.5885E+03	1.5926E+03	1.6312E+03	1.5782E+03		
	Mean	1.5885E+03	1.5934E+03	1.5918E+03	1.5959E+03	1.6455E+03	**1.5783E+03**	1.6135E+ 03	6.08E+03
	Worst	1.5963E+03	1.6013E+03	1.5962E+03	1.5988E+03	1.6657E+03	1.5783E+03		
	Std. dev.	5.8953E+00	5.8470E+00	3.7203E+00	3.1166E+00	1.4515E+01	4.6161E-02		5.83E+03
F_{C15}	Best	1.5022E+03	1.5101E+03	1.5065E+03	1.5060E+03	2.1053E+03	1.5000E+03		
	Mean	1.6360E+03	1.7529E+03	1.6388E+03	1.7736E+03	2.1401E+03	**1.5000E+03**	1.9452E+03	1.61E+03
	Worst	1.9003E+03	1.8958E+03	1.9003E+03	1.9126E+03	2.1586E+03	1.5000E+03		
	Std. dev.	2.2885E+02	2.1142E+02	2.2654E+02	2.3179E+02	3.0129E+01	1.0969E-02		1.08E+00

with $D = 500$ to show their scalability. The obtained results as in Table 5 affirm that with increasing the dimensionality, GWO-CSA continues to give the best result, which means that the GWO-CSA is still insensitive to increasing the dimension. Also, the GWO-CSA provides superior performance compared to the GWO, CSA, SCA, GWO-SCA, and MHDA on all test benchmark functions.

In Experiment 3, the scalability of the GWO-CSA algorithm and the other comparative algorithms is further verified with $D = 1000$ for all test instances. Also, statistical

TABLE 10. The information on the engineering designs.

Prob.	Description	Mathematical model
CB	CB is a well-known design application that has five hollow parts with a cross-section as a square shaped. Thus it involves five structural parameters in total. Also, there is a vertical load fixed at node 6 and node 1 comprises a rigid support. The main target of this application is to minimize the weight of the beam under the vertical displacement constraint to achieve an optimal design.	Min $f(x) = 0.06224(x_1 + x_2 + x_3 + x_4 + x_5)$, Subject to: $g(x) = \frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} \le 1, 0.01 \le x_i \le 100, i = 1, 2, ..., 5.$
TBT	TBT aims to minimize the volume of a statistically loaded three-bar truss as the main goal and subject to stress (σ) constraints on every of the truss members through adjusting cross-sectional areas (x_1 and x_2).	Min $f(x) = (2\sqrt{2}x_1 + x_2) \cdot l$, Subject to: $g_1(x) = \frac{\sqrt{2}x_1+x x_2}{\sqrt{2x_1^2+2x_1x_2}}P - \sigma \le 0$, $g_2(x) = \frac{x_2}{\sqrt{2x_1^2+2x_1x_2}}P - \sigma \le 0$, $g_3 = \frac{1}{x_1+\sqrt{2}x_2}P - \sigma \le 0$, $0 \le x_{1,x_2 \le 1}$, $l = 100cm$, $P = 2kN/cm^2$, $\sigma = 2kN/cm^2$
PV	The goal of this design is to minimize the total cost (i.e., the cost of material, forming and welding of a cylindrical vessel that is capped at both terminals by hemispherical heads. It involves four design parameters conducted with it, namely the thickness of the pressure vessel ($T_s = x_1$), the thickness of the head ($T_h = x_2$), the inner radius of the vessel ($R = x_3$), and length of the vessel without heads ($L = x_4$) i.e., the variable vectors are given (in inches) by $x = (T_s, T_h, R, L_s) = (x_1, x_2, x_3, x_4)$.	Min $f(x) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$, Subject to: $g_1(x) = -x_1 + 0.0193x_3 \le 0$, $g_2(x) = -x_3 + 0.00954x_3 \le 0$, $g_3(x) = -\pi x_3^2x_4 - 1.25\pi x_3^3 + 1296000 \le 0$, $g_4(x) = x_4 - 240 \le 0$, $1 \cdot 0.0625 \le x_1, x_2 \le 99 \cdot 0.0625$, $10 \le x_3, x_4 \le 200$
CSI	CSI design is a practical engineering application which aims to obtain the minimum value of the total weight under the presence of ten constraints and eleven design parameters.	Min $f(x) = 1.98 + 4.90x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 2.73x_7$, Subject to: $g_1(x) = 1.16 - 0.3717x_2x_4 - 0.00931x_2x_{10} - 0.484x_3x_9 + 0.01343x_6x_{10} \le 1$, $g_2(x) = 0.261 - 0.0159x_1x_2 - 0.188x_1x_8 - 0.019x_2x_7 + 0.0144x_3x_5 + 0.0008757x_5x_{10} + 0.080405x_6x_9 + 0.00139x_8x_{11} + 0.00001575x_{10}x_{11} \le 0.32$, $g_3(x) = 0.214 + 0.00817x_5 - 0.131x_1x_8 - 0.0704x_1x_9 + 0.03099x_2x_6 - 0.018x_2x_7 + 0.0208x_3x_8 + 0.121x_3x_9 - 0.00364x_5x_6 + 0.0007715x_5x_{10} - 0.0005354x_6x_{10} + 0.00121x_8x_{11} \le 0.32$, $g_4(x) = 0.074 - 0.061x_2 - 0.163x_3x_8 + 0.001232x_3x_{10} - 0.166x_7x_9 + 0.227x_2^2 \le 0.32$, $g_5(x) = 28.98 + 3.818x_3 - 4.2x_1x_2 + 0.0207x_5x_{10} + 6.63x_6x_9 - 7.7x_7x_8 + 0.32x_9x_{10} \le 32$, $g_6(x) = 33.86 + 2.95x_3 + 1.1792x_{10} - 5.057x_1x_2 - 11.0x_2x_8 - 0.0215x_5x_{10} - 9.98x_7x_8 + 22.0x_8x_9 \le 32$, $g_7(x) = 46.36 - 9.9x_2 - 12.9x_1x_8 + 0.1107x_3x_{10} \le 32$, $g_8(x) = 4.72 - 0.5x_4 - 0.19x_2x_3 - 0.0122x_4x_{10} + 0.009325x_6x_{10} + 0.000191x_{11}^2 \le 4$, $g_9(x) = 10.58 - 0.674x_1x_2 - 1.95x_2x_8 + 0.02054x_3x_{10} - 0.0198x_4x_{10} + 0.028x_6x_{10} \le 9.9$, $g_{10}(x) = 16.45 - 0.489x_3x_7 - 0.843x_5x_6 + 0.0432x_9x_{10} - 0.0556x_9x_{11} + 0.000786x_{11}^2 \le 15.7$, $0.5 \le x_1 : x_7 \le 1.5$, $0.192 \le x_8, x_9 \le 0.345$, $-30 \le x_{10}, x_{11} \le 30$

TABLE 11. Statistical measures, design parameters, constraints, and objective function value for CB design.

	GWO	CSA	SCA	GWO-SCA	MHDA	GWO-CSA
Statistical measures						
Best	1.33660318	1.38642144	1.38929628	1.33661177	1.3365219	1.33652063
Mean	1.33679208	2.40365237	1.43076126	1.33673621	1.33655092	1.33652091
Worst	1.33708465	5.68640568	1.50654786	1.33716046	1.33669895	1.33652129
Std	1.26E-4	1.29E+0	4.49E-2	1.70E-4	5.34E-05	2.02E-07
Design variables						
x_1	5.96944602	5.52241796	5.90758647	5.98834502	6.01722636	6.01657985
x_2	5.32566082	6.02522636	5.59149900	5.30379966	5.30771483	5.30842878
x_3	4.52043753	4.33565400	5.69813321	4.48439998	4.49946184	4.49515624
x_4	3.51539429	3.30048029	3.08625420	3.52757744	3.49777773	3.50137666
x_5	2.14404825	3.09163019	2.03812558	2.17100285	2.15150091	2.15211898
Constraints						
g(x)	-2.128e-06	-6.98E-03	-33.5E-03	-64.4E-06	00.00	-1.15E-09
Objective function						
f(x)	1.33660318	1.38642144	1.38929628	1.33661177	1.3365219	1.33652063

TABLE 12. Statistical measures, design parameters, constraints, and objective function value for TBT design.

	GWO	CSA	SCA	GWO-SCA	MHDA	GWO-CSA
Statistical measures						
Best	263.89904	263.89583	264.01202	263.89623	263.89583	263.89583
Mean	263.90642	263.89583	267.96019	263.90706	263.89595	263.89583
Worst	263.93034	263.89583	282.84271	263.92688	263.89641	263.89583
Std	9.1963E-3	5.679E-14	7.8451471	1.1774E-2	1.7584E-4	2.359E-14
Design variables						
x_1	0.7879283	0.78867508	0.78859646	0.78912722	0.78868853	0.78867508
x_2	0.4103925	0.40824826	0.40963261	0.40697354	0.40821022	0.40824825
Constraints						
$g_1(x)$	-2.11E-05	1.32E-07	-0.00088	-1.87E-06	1.32E-07	1.32E-07
$g_2(x)$	-1.46168	-1.4641	-1.46297	-1.46555	-1.46414	-1.4641
$g_3(x)$	-0.53834	-0.5359	-0.53791	-0.53445	-0.53586	-0.5359
Objective function						
f(x)	263.899044	263.895834	264.012024	263.896239	263.895834	263.895834

measures, best value of the candidate problem, mean (average), worst and standard deviation, are recorded in Table 6. It is noted GWO-CSA still continues to provide the superior results over GWO, CSA, SCA, GWO-SCA, and MHDA algorithms on all test functions. Also the convergence curves for the six algorithms on the test functions are depicted in Figure 6, where GWO-CSA has faster convergence speed and higher precision than the others.

D. COMPARISON WITH SOME GWO VARIANTS

In order to investigate the performance of the proposed GWO-CSA, nine variants of GWO are benchmarked on the studied benchmark problems. These variants include LGWO [34], ROGWO [35], mGWO [36], RLGWO [37], IGWO [38], EEGWO [39], ERGWO [40], GLFGWO [41], and GWOWD [42], where the values of parameters for these nine variants of GWO used for comparison are suggested as

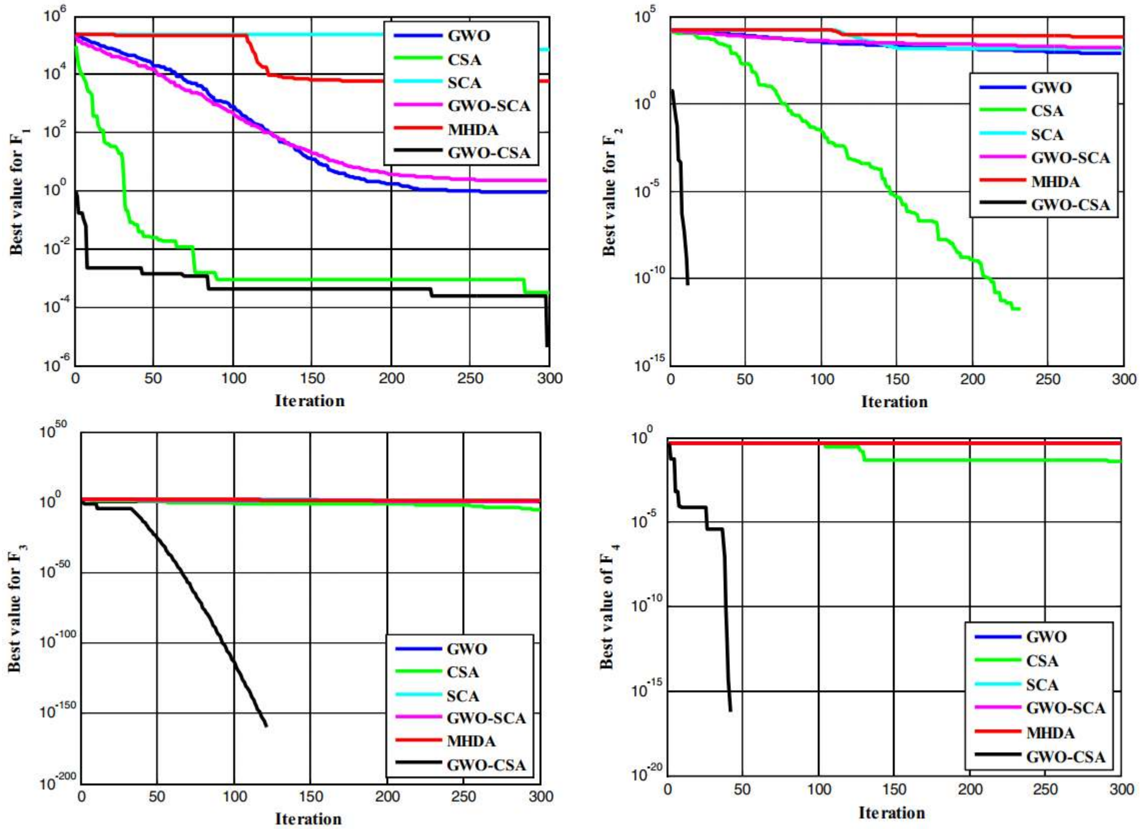

FIGURE 6. Convergence behaviors of the proposed GWO-SCA and the other algorithms for all problems with $D = 1000$.

TABLE 13. Statistical measures, design parameters, constraints, and objective function value for PV design.

	GWO	CSA	SCA	GWO-SCA	MHDA	GWO-CSA
Statistical measures						
Best	5908.5345	5880.8633	7031.1929	5900.6678	5881.2911	5880.6712
Mean	6059.3981	5966.1935	7798.9600	6103.7200	6295.6556	5880.6798
Worst	7151.5874	6568.2588	9216.6430	7215.9860	7299.4187	5880.6965
Std	384.5045	212.7941	651.6735	427.5887	550.4844	9.3103E-3
Design variables						
x_1	0.78225479	0.77822548	0.81298553	0.78037924	0.77823143	0.77816868
x_2	0.39016904	0.38306955	0.66236536	0.38772517	0.38307123	0.38303638
x_3	40.5055498	40.3221360	40.7679865	40.4232583	40.3200113	40.3196191
x_4	197.544224	199.966066	199.774505	198.762719	200	200
Constraints						
$g_1(x)$	-4.976E-4	-8.26E-06	-2.616E-2	-2.103E-4	-5.52E-05	-3.44E-08
$g_2(x)$	-5.366E-3	-9.26E-06	-0.2750	-3.704E-3	-3.11E-05	-1.68E-09
$g_3(x)$	-600.1346	-5.6511	-30928.26	-1028.988	-27.9130	-2.922E-2
$g_4(x)$	-42.4557	-40.0339	-40.2254	-41.2372	-40	-40
Objective function						
f(x)	5908.5345	5880.8633	7031.1929	5900.6678	5881.2911	5880.6712

TABLE 14. Statistical measures, design parameters, constraints, and objective function value for CSI design.

	GWO	CSA	SCA	GWO-SCA	MHDA	GWO-CSA
Statistical measures						
Best	22.857294	25.507469	23.939385	22.874437	22.846095	22.842988
Mean	22.968129	26.831227	24.809887	23.006052	23.089749	22.844126
Worst	23.156783	28.203675	25.456340	23.319260	23.466054	22.847309
Std	8.2185E-2	9.0509E-1	5.1983E-1	1.7481E-1	2.1511E-1	1.4135E-3
Design variables						
x_1	0.5010816	0.9929953	0.50000	0.5008358	0.50000	0.50000
x_2	1.1128018	1.0602686	1.14330361	1.11841009	1.1235094	1.11640162
x_3	0.50000	0.5001557	0.50000	0.50336968	0.5	0.5
x_4	1.3095957	1.2851489	1.50000	1.29848048	1.29109421	1.30214195
x_5	0.5010059	0.8730064	0.50000	0.50056488	0.50000	0.50000
x_6	1.50000	1.3577983	1.4999608	1.49981998	1.5	1.5
x_7	0.5004896	0.5096389	0.5452565	0.50150688	0.50000	0.50000
x_8	0.3449311	0.2750679	0.3053979	0.34108947	0.34498928	0.345
x_9	0.2134644	0.2732400	0.345000	0.24523493	0.345	0.192
x_{10}	-20.26586	-6.232694	-24.13208	-19.46179	-18.28564	-19.55508
x_{11}	-0.269344	-4.867085	4.37837216	-0.186058	0.3786174	0.0799473
Constraints						
$g_1(x)$	-0.631642	-0.464755	-0.790201	-0.628908	-0.639760	-0.617496
$g_2(x)$	-0.090523	-0.107387	-0.073804	-0.086039	-0.073758	-0.092703
$g_3(x)$	-31.74827	-31.78506	-31.73913	-31.74723	-31.74502	-31.74875
$g_4(x)$	-0.091111	-0.104871	-0.090002	-0.090753	-0.096013	-0.087277
$g_5(x)$	-4.253827	-4.809604	-4.277157	-4.058230	-3.575545	-4.278113
$g_6(x)$	-7.223413	-5.942318	-6.805583	-6.828977	-5.952624	-7.282889
$g_7(x)$	-0.008072	-0.005267	-0.264233	-0.000434	-3.49E-05	-6.29E-08
$g_8(x)$	-0.000179	-6.79E-07	-0.030873	-8.45E-05	-8.06E-12	-2.81E-09
$g_9(x)$	-0.978116	-0.740714	-0.930795	-0.959572	-0.942782	-0.965253
$g_{10}(x)$	-0.189637	-0.392157	-0.474267	-0.210004	-0.284404	-0.167556
Objective function						
f(x)	22.857294	25.507469	23.93938	22.874437	22.846095	22.842988

recommended in their corresponding literature. The obtained results of these variants are presented in Table 7. Based on the achieved results, it can be observed that the EEGWO provides the superior results among these variants but the proposed GWO-CSA still outperforms all the variants as it provides better results over the EEGWO for $F_1(x)$ and $F_3(x)$ and faster than it for $F_2(x)$ and $F_4(x)$. The best results among the presented variants are exhibited in boldface. On the other hand, the convergence graphs for all problems are provided in Figure 7 to exhibit the convergence rate towards the best solution during the searching process. Based on the Figure 7, the GWO-CSA still provides the faster rate than the other peers. Also the proposed GWO-CSA can achieve a stable

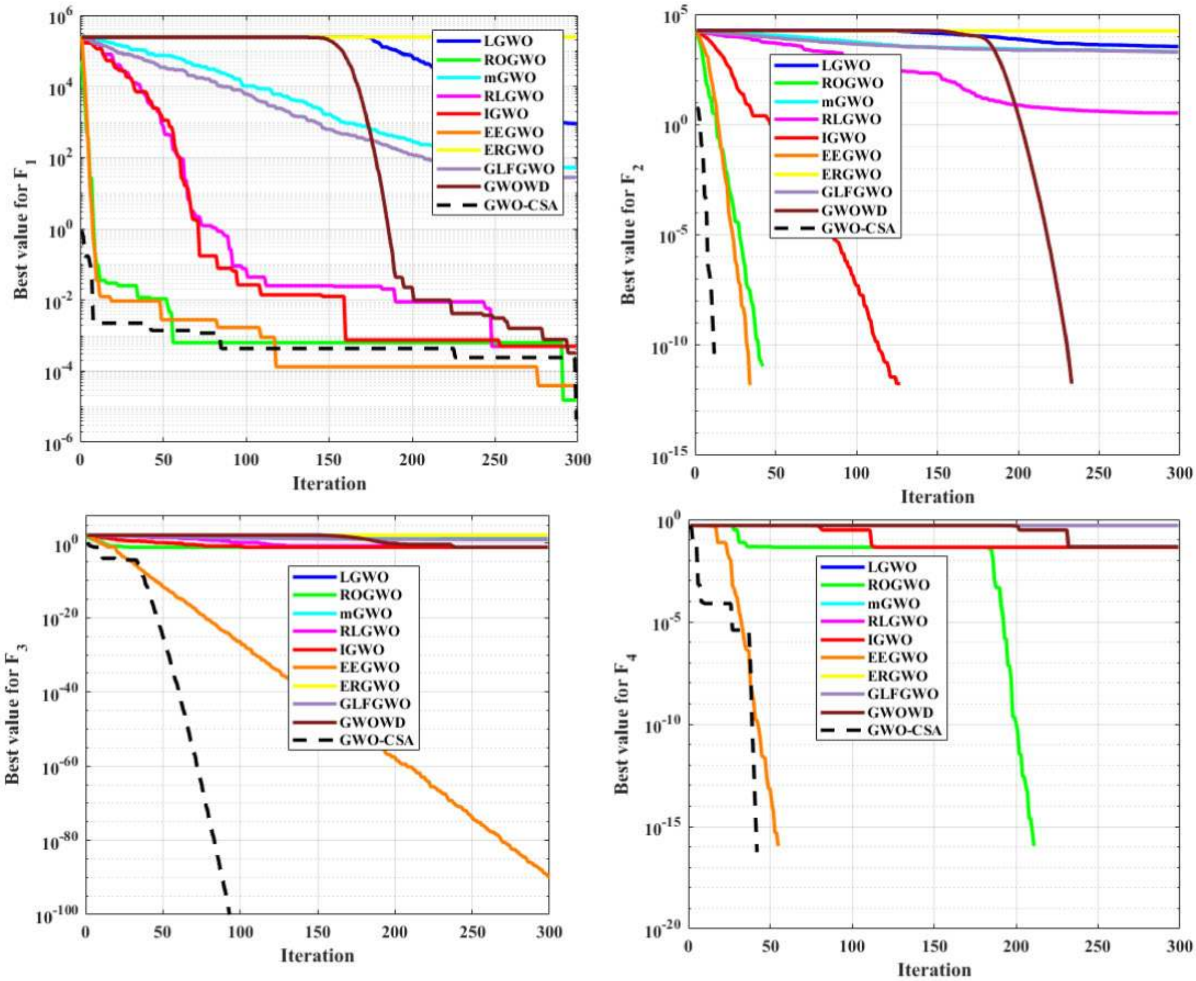

FIGURE 7. Convergence curves for various GWO variants and the proposed GWO-CSA with $D = 1000$.

performance than the other variants, where the statistical measures of the best, mean, worst, and standard deviation are seem to be coincident. Accordingly, it is evident that the proposed approach is more fruitful than other existing variants of GWO and thus the proposed algorithm can be considered a strongly suitable methodology for optimization sights.

E. INVESTIGATION ON CEC 2015 EXPENSIVE OPTIMIZATION PROBLEMS

For further validation regarding the performance of the proposed GWO-CSA, it is benchmarked on CEC 2015 benchmark problems which are more competitive suits and require robust optimizers to achieve a suitable accuracy of the obtained solutions with fast rates in limited allocated budgets. The CEC 2015 test suits represent the collection of 15 challenging expensive problems that involve highly complex composite and hybrid natures [52]. The natures of these problems involve the unimodal, multimodal, hybrid, and composition scenarios and they are listed in Table 8, where the global optimum value (F^*) is provided for each problem and also the range space for the variable bounds $\in [-100, 100]$. In this regard, the results of proposed GWO-CSA are compared with the traditional GWO, the most competitive variant of the GWO (i.e., EEGWO), DE and high performance variants of DE, including SHADE, and LSHADE. On the other the results of two other variants of DE, i.e. DE1 and DE2, are taken from [53] and [54], respectively. The results in terms of the statistical metrics are reported in Table 9 for 10 dimensional (10D) CEC 2015 problems. According to the achieved results, the proposed GWO-CSA can provide the better mean values and outperforms the other methods in most CEC 2015 problems. From Table 9, it can be observed that the results achieved by the proposed method are better in 13 cases, where the best results are highlighted with the bold values. Therefore, it can also conclude that the proposed methodology is better or competitive while the comparison with other methods. On the other hand, the convergence graphs for CEC 2015 problems are provided in Figure 8 to visualize the rate of convergence towards the better optima point during the searching process. Based on the depicted curves in Figure 8, mostly, GWO-CSA converges

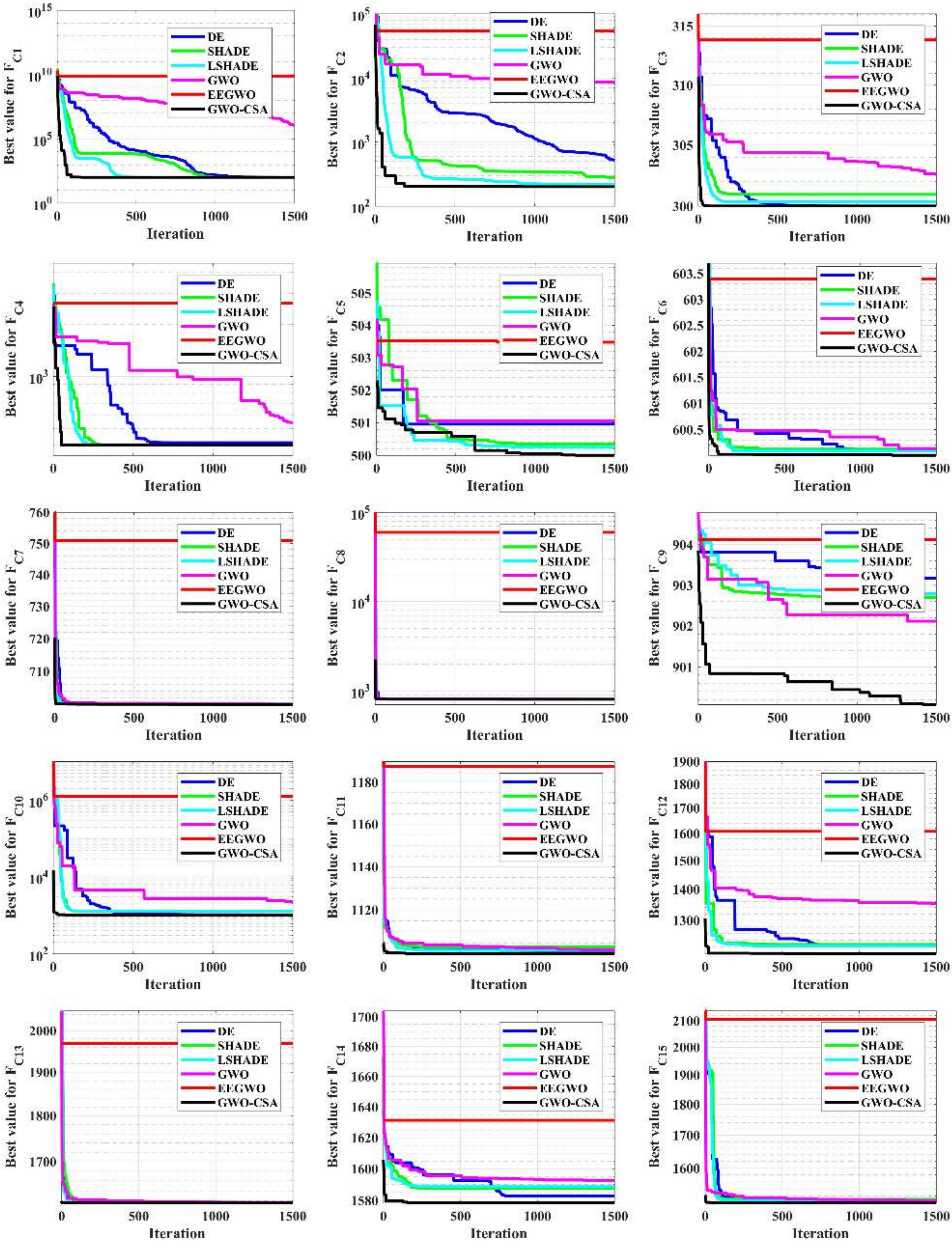

FIGURE 8. Convergence curves for the proposed method GWO-CSA versus different DE and GWO variants on CEC 2015 problems.

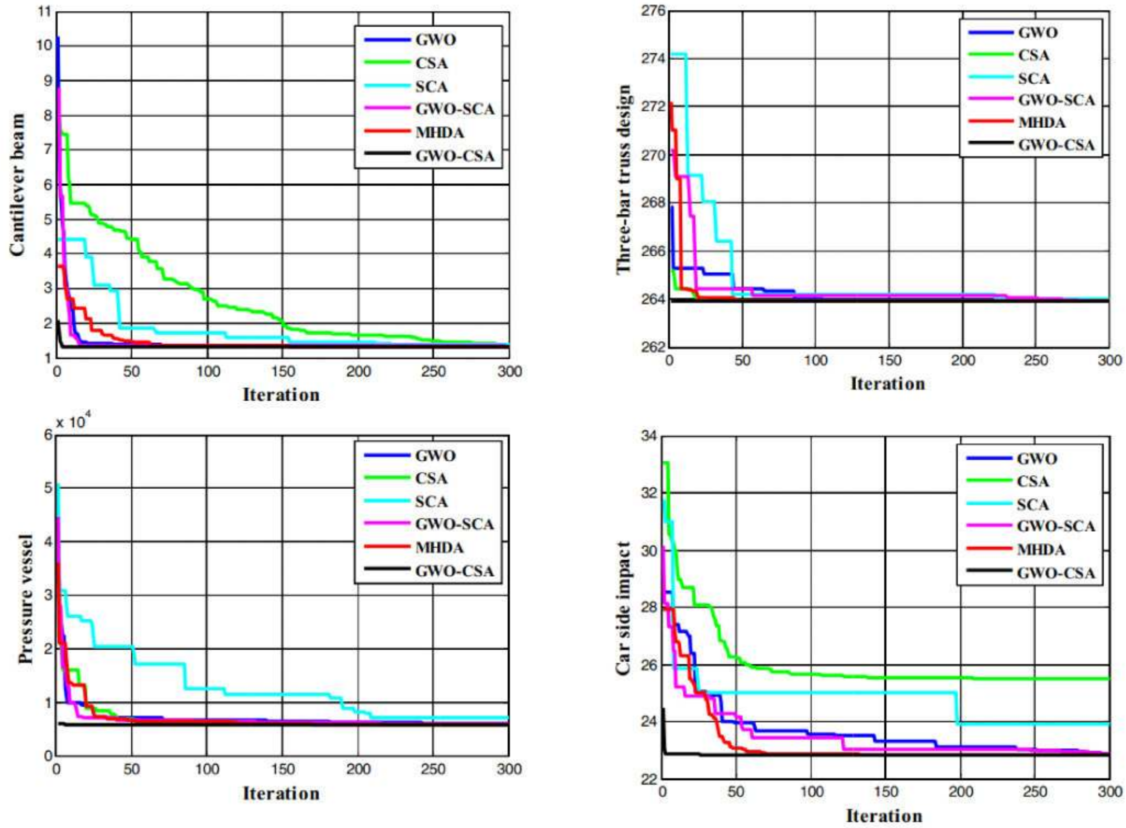

FIGURE 9. Convergence curves of the proposed GWO-CSA against the compared algorithms for the design applications.

with a faster rate towards the better optima point than other methods.

F. PRACTICAL APPLICATIONS IN ENGINEERING DESIGN PROBLEMS

In this subsection, further validation of the proposed GWO-CSA algorithm is conducted on some practical applications of engineering design problems. Four well-known practical applications, which are cantilever beam (CB), three-bar truss (TBT), pressure vessel (PV) and car side impact (CSI) [18], [48]. These design problems are widely employed in the literature to validate the efficiency of meta-heuristic algorithms. The details of these design problems as well as their mathematical models are presented in Table 10. On the other hand, the structure of each design problem is appended in Appendix.

The complexity of these engineering design optimization problems is contained behind the very tiny feasible region of the entire search space that is caused by a set of inequality and equality constraints. However, solving such problems is more challenging task not only due to the high nonlinearity of these problems, but also due to the complex search space shapes enclosed by various constraints. Additionally, in most practical tasks the optimal solution is found on the boundary between the feasible and infeasible regions. Therefore, developing a robust optimization algorithm to locate good feasible solution with acceptable accuracy is crucially important for engineering design fields. In this regard, the proposed GWO-CSA and other competitive algorithms are conducted to deal with some of engineering designs including CB, TBT, PV, and CSI.

Tables 11, 12, 13, 14 present the statistical results reported by GWO-CSA with the other compared algorithms for reported CB, TBT, PV, and CSI design problems, respectively. Also, the values of design parameters for all design problems associated with their constraints are reported as a counterparts as in Tables 11, 12, 13, 14. Based on the obtained results, we can conclude that the GWO-CSA gives superior results for designs over the other compared algorithms.

For the cantilever beam (CB) design problem, because of the best result, the proposed GWO-CSA achieves a better result than the other comparative algorithms where the overall results of the proposed GWO-CSA and the other algorithms are reported in Table 11. Also, the convergence curves for the proposed GWO-CSA and the comparative ones are displayed in Figure 9. Furthermore, the box plot diagram is presented in Figure 10 for all algorithms to exhibit the stability of the algorithms through the different runs.

For the three-bar truss (TBT) design application, the reported information in Table 12 provides that the result

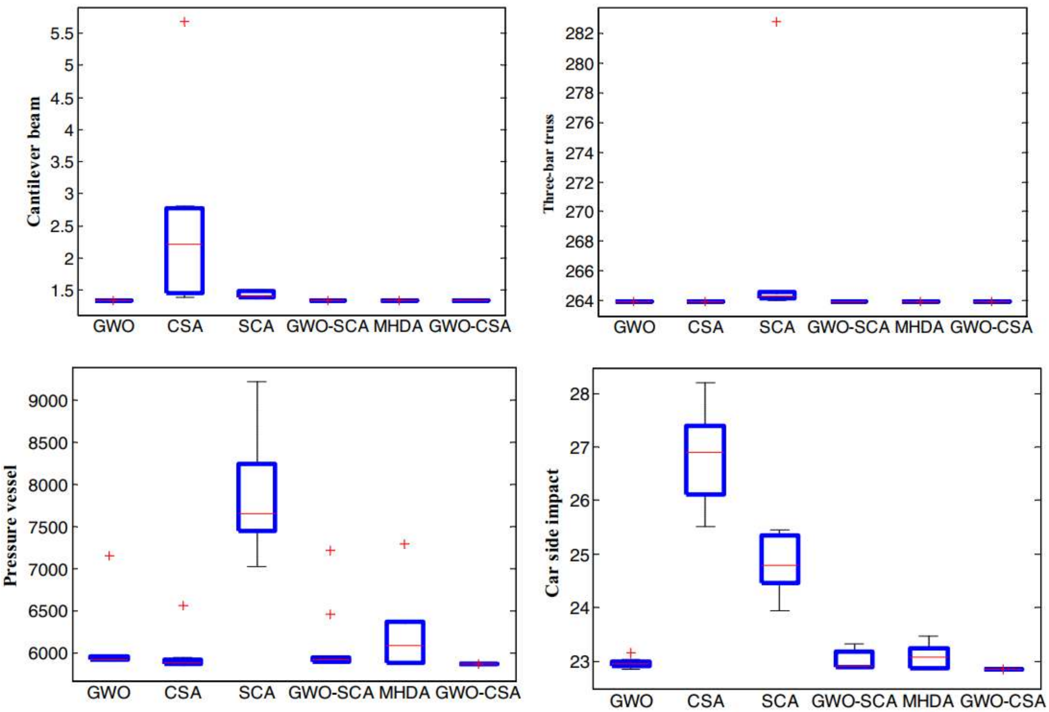

FIGURE 10. Box plot diagrams of the proposed GWO-CSA and different algorithms for the design applications.

obtained by GWO-CSA be similar to CSA regarding the best value and the mean result. Also, GWO-CSA gives faster convergence than the other comparative algorithms. In this sense, the convergence curves for the proposed GWO-CSA and the comparative ones are depicted in Figure 9 and the box plot diagram is showed in Figure 10 for all algorithms to exhibit the stability of the algorithms through the different runs.

For the pressure vessel (PV) design problem, Table 13 exhibits the results provided by GWO-CSA and the other comparative algorithms. Given mean value, the proposed GWO-CSA finds the better one over the other algorithms. Also, GWO-CSA gives faster convergence than the other algorithms, where convergence curves are portrayed in Figure 9 and the box plot diagram is presented in Figure 10 for all algorithms to exhibit the stability of the algorithms through the different runs.

For the car side impact (CSI) design application, the obtained results of the GWO-CSA and the other comparative ones are recorded in Table 14. Based on these results, the obtained one by GWO-CSA presents the superior result over the other comparative algorithms regarding statistical values. Also, GWO-CSA still affirms its robustness through achieving the faster rate of convergence performance over the other algorithms, where convergence curves are showed in Figure 9 and the box plot diagram is presented in Figure 10 for all algorithms to exhibit the stability of the algorithms through the different runs.

VI. CONCLUSION

This paper proposes a novel hybrid algorithm called GWO-CSA based on combining the features of both grey wolf optimizer (GWO) and crow search algorithm (CSA) to obtain balanced tradeoff among the exploration and exploitation capabilities. GWO-CSA works in sequence stages, where GWO operates in exploring the promising areas in the search region while CSA aims to exploit these areas with the aim to refine the positions of the grey wolves.

Further, a dynamic fuzzy learning strategy (DFLS) is developed to improve the quality of solution based on the alpha cut that sieges the promising solutions. Four benchmark test functions are conducted for large-scale dimensions, and also four engineering designed problems are investigated. Based on the reported results, it can conclude that the GWO-CSA has a superior performance that is caused by the integrating methodology of GWO, SCA, and DFLS. Simulations affirmed that the GWO-CSA could achieve very competitive outcomes compared to other comparative algorithms such as GWO, CSA, SCA, GWO-SCA, and MHDA. Finally, the GWO-CSA is an efficient methodology that can achieve the global optimum for most test instances and engineering applications.

However, even the proposed GWO-CSA approach has fulfilled competitive and progressive results while the comparisons with other methods in this work, the GWO-CSA may still have improved rooms to be competitive enough with more effective technologies. First, a novel parameter

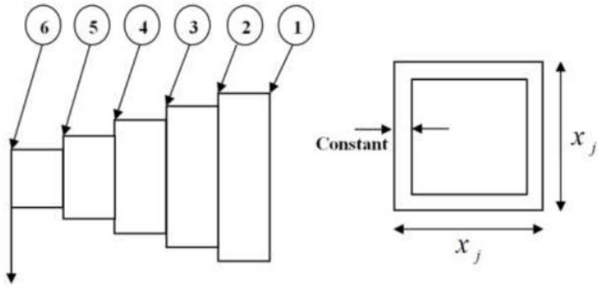

FIGURE 11. Cantilever beam structure (CB problem).

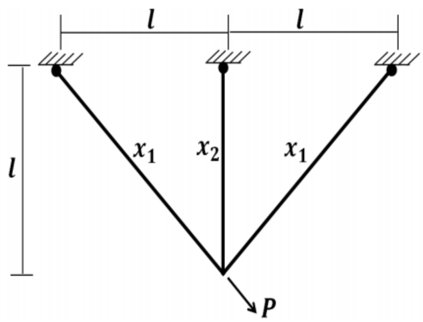

FIGURE 12. Three-bar truss structure (TBT problem).

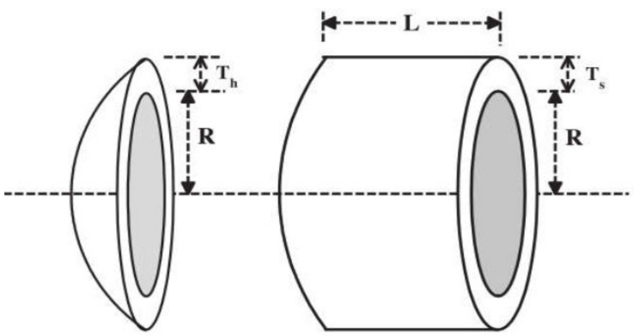

FIGURE 13. Pressure vessel structure (PV problem).

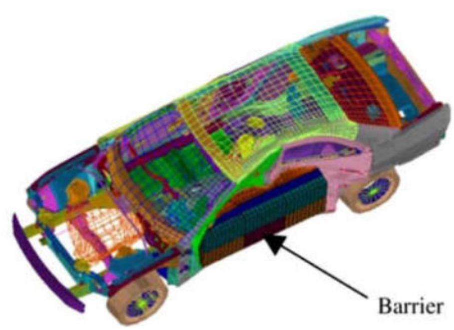

FIGURE 14. Model of car side impact (CSI problem).

adaptation scheme can be further explored rather than employing the parameters of initial works for GWO and CSA algorithms. Secondly, the effectiveness of the GWO-CSA still deserves further investigation on more harder realistic problems such as IEEE CEC 2017 test cases.

In future work, we intended to validate and analyze the GWO-CSA algorithm for solving many objectives optimization, combinatorial optimization and developing a binary version of the GWO-CSA.

APPENDIX

The structure of each design problem is presented as follows: cantilever beam structure (CB problem) – Figure 11, three-bar truss structure (TBT problem) – Figure 12, pressure vessel structure (PV problem) – Figure 13, model of car side impact (CSI problem) – Figure 14.

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