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RESEARCH ARTICLE

Hybridizing the Electromagnetism-like algorithm with Descent Search for Solving Engineering Design Problems (SI_CMMSE_08)

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In this paper, we present a new stochastic hybrid technique for constrained global optimization. It is a combination of the electromagnetism-like (EM) mechanism with a random local search, which is a derivative-free procedure with high ability of producing a descent direction. Since the original EM algorithm is specifically designed for solving bound constrained problems, the approach herein adopted for handling the inequality constraints of the problem relies on selective conditions that impose a sufficient reduction either in the constraints violation or in the objective function value, when comparing two points at a time. The hybrid EM method is tested on a set of benchmark engineering design problems and the numerical results demonstrate the effectiveness of the proposed approach. A comparison with results from other stochastic methods is also included.

Keywords: Hybrid method, electromagnetism-like mechanism, descent search, sufficient reduction, engineering design problems

AMS Subject Classification: 90C15; 90C56; 90C30

1. Introduction

The problem that is addressed in the paper considers finding a global solution of a nonlinear optimization problem in the following form:

$$\begin{array}{ll} \min\limits_{x\in\mathbb{R}^n} & f(x) \\ \text{subject to} & g(x) \le 0,\ x \in \Omega, \end{array} \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}^p$ are nonlinear continuous functions and $\Omega = \{x \in \mathbb{R}^n : l \le x \le u\}$. We do not assume that the objective function f is convex. This class of global optimization problems is very important and frequently encountered in engineering applications. Some algorithms for solving this type of problem require substantial gradient information and aim to improve the solution in a neighborhood of a given initial approximation. When the problem has more than one local solution, the convergence to the global solution may depend on the provided initial approximation. Stochastic-type methods with incorporated heuristics have been proposed to solve constrained global optimization problems

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with success. This paper presents a new stochastic hybrid technique for solving problems like (1).

A well-known approach for solving constrained optimization problems is based on penalty functions. The penalty techniques transform the constrained problem into an unconstrained problem by penalizing f when constraints are violated and then minimizing the penalty function using methods for unconstrained problems [4, 7, 9, 21, 27]. The main difficulty here is the updating of a positive penalty parameter. Large values give feasible solutions that have low accuracy, while small values generate infeasible with good accuracy solutions. An alternative to penalty functions in gradient-based or derivative-free methods for constrained optimization is the filter method. See, for example [1, 3, 11, 31]. Another approach for handling the constraints $g(x) \leq 0$ of problem (1) relies on a simple heuristic consisting on three selective rules, denoted by feasibility and dominance (FAD) rules [10, 17, 33]. We remark that these rules can be used for pairwise comparison with methods that have a population of points in every iteration. Other techniques that aim to preserve and force feasibility can be found in [15, 28–30].

In this paper, we are interested in the electromagnetism-like (EM) algorithm proposed in [5]. This is a population-based algorithm that simulates the electromagnetism theory of physics by considering each point in the population as an electrical charge. The method uses an attraction-repulsion mechanism to move a population of points towards optimality. The EM algorithm is specifically designed for solving optimization problems with bound constraints [4–6]. A natural extension to inequality and equality constrained optimization problems based on penalty and barrier functions is proposed in [4]. The integration of the FAD rules into the original EM algorithm is a simple task [26]. During a pairwise point comparison, these rules are used to select the best point as follows: (i) among two feasible points, the one that has better objective function value is preferred; (ii) any feasible point is preferred to any infeasible solution; and (iii) among two infeasible points, the one that has smaller constraints violation is preferred. However, convergence to optimality may not be guaranteed with these simple reduction conditions. Strong conditions may have to be imposed in order to consider a point preferred to any other point in the population. Imposing a sufficient reduction either in the constraints violation or in the objective function value is a challenge for constraint-handling techniques in stochastic type methods.

In this paper, we describe new selective conditions that aim to detect the best point of the population, to attract points to promising regions and to guarantee progress around the best point, imposing a sufficient reduction either in the constraints violation or in the objective function value. To improve accuracy of the solutions, we propose to use a derivative-free heuristic method to produce an approximate descent search direction to move the best point of the population, followed by a classical backtracking line search. We then test the new hybrid EM algorithm with a benchmark set of engineering design problems gathered from the literature. A comparison with the numerical results obtained by other stochastic methods from the literature is also included.

The remaining part of this paper is organized as follows. In Section 2 we briefly list the modifications that are introduced in the original EM algorithm to incorporate the FAD rules for constraint-handling. Section 3 presents the new hybrid EM algorithm that imposes sufficient reduction either in the constraints violation or in the objective function value, and uses a random descent search direction to improve accuracy around the best point of the population. Section 4 contains the results of the numerical experiments on a set of twelve benchmark engineering optimization problems. We conclude the paper in Section 5.

2. FAD rules for constraint-handling in the EM algorithm

In this section, we briefly present a simple extension of the EM algorithm proposed in [5, 6], to solve problem (1), that incorporates the FAD rules for constraint-handling. These rules are easily incorporated into the original algorithm, by modifying some of the main procedures of the algorithm. We use the following notation: x^i is the ith point of the population, x^i_k is the kth ($k = 1, \ldots, n$) coordinate of the point x^i, and p_{size} is the number of points in the population.

The original EM algorithm starts with a population of randomly generated points from the feasible set Ω. The FAD rules are then implemented to select the best point, x^{best}, of the population. Analogous to electromagnetism, each point is a charged particle that is released to the space. The charge of each point determines the magnitude of attraction of the point over the others in the population. Thus, the charge q^i considers now a measure of optimality and feasibility of the point x^i given by a fitness function,

$$q^i = \exp\left(\frac{-n(\text{fitness}(x^i) - \text{fitness}(x^{\text{best}}))}{\sum_{j=1}^{p_{\text{size}}}(\text{fitness}(x^j) - \text{fitness}(x^{\text{best}}))}\right), \tag{2}$$

with

$$\text{fitness}(x) = \begin{cases} f(x), & \text{if } x \text{ is feasible} \\ f_{\max} + \text{CV}(x), & \text{otherwise} \end{cases}, \tag{3}$$

being $f_{\max}$ the maximum function value of the feasible points of the population, and $\text{CV}(x) = \| \max\{0, g(x)\} \|_2$ is used to measure constraints violation. This way, a point x with $\text{CV}(x) = 0$ is feasible, whereas the point is infeasible if $\text{CV}(x) > 0$. If the current population has no feasible points, we set $f_{\max} = 0$.

The total force vector F^i exerted on each point x^i by the other $p_{\text{size}} - 1$ points is the sum of individual component forces, F^i_j, for $j = 1, \ldots, p_{\text{size}}$ and $j \neq i$, each depending on the charges q^i and q^j [5]. Then, the normalized total force vector exerted on the point x^i is used to move the point in the direction of the force by a random step size. Here, the best point, x^{best}, is not moved and is carried out to the subsequent iteration. The FAD rules are then used to select the best point of the new population. Finally, a local procedure performs a local refinement only to the best point in the population, see [5]. The reader is referred to [26] for the details concerning the implementation of the FAD rules in the EM algorithm, and the discussion of some experimental results with a set of academic problems [33].

3. Sufficient reduction conditions in a hybrid EM algorithm

We now present the new EM algorithm that imposes a sufficient reduction either in the constraints violation or in the objective function value, in order to detect the best point of the population, to attract points to promising regions and to guarantee progress around the best point. The hybridization relies on a local search that generates an approximate descent direction of a fitness function at the best point of the population.

First, we describe the sufficient reduction conditions to handle inequality constraints in the EM algorithm. Deterministic methods that use a penalty function ensure sufficient progress towards the solution by enforcing a sufficient reduction in the penalty function. To avoid the use of a merit function and the updating of the penalty parameter, while promoting global convergence from arbitrary initial approximations, Fletcher and Leyffer [11] proposed a filter technique in a deterministic optimization algorithm context. In a filter framework, for example in [3] and [31], a new point y might be considered to be acceptable, when compared with the current point x, if it leads to sufficient progress in one of the two measures (feasibility or optimality):

$$\mathrm{CV}(y) \leq (1-\gamma)\,\mathrm{CV}(x) \ \text{ or } \ f(y) \leq f(x) - \gamma\,\mathrm{CV}(x), \ \text{ for } \gamma \in (0,1).$$

To adapt the methodology of a filter as outline in [11] to this population-based stochastic framework, a point is preferred to any other point in the population if selective sufficient reduction conditions hold.

3.1 *Selective sufficient reduction conditions*

The selective conditions herein proposed impose a sufficient reduction in one of the measures f or CV and aim to guarantee sufficient progress around the best point and around other points of attraction. They are implemented as follows. If both points in comparison are feasible, then x^i is considered to be preferred to x^j only if a sufficient decrease in f is verified

$$f(x^i) \leq (1-\gamma)\, f(x^j), \tag{4}$$

where $\gamma \in (0,1)$ is a fixed constant. On the other hand, if only one point is feasible or both points in comparison are infeasible then x^i is considered to be preferred to x^j if

$$\mathrm{CV}(x^i) \leq (1-\gamma)\,\mathrm{CV}(x^j) \tag{5}$$

holds. We remark that points are not compared in terms of both objective function value and constraints violation, in this adopted constraint-handling technique. In practice, we first measure constraints violation of all points in the population. If the point is feasible, the objective function value is evaluated. On the other hand, if the point is infeasible, its objective function value is not required.

3.2 *Moving the points*

The procedures that involve the initialization of the population, and the definition of the total force vectors (with charges computed as shown in (2)) are similar to those described in [5] and [26]. The herein adopted procedure to move the points considers a strategy commonly used in the interior point methods [31]. The total force vector, F^i, is used to move the point x^i in the direction of the force by a random step size as follows $x^i = x^i + \lambda\,\alpha^i_{\mathrm{max}}\,F^i$ (for $i = 1, \ldots, p_{\mathrm{size}}$ and $i \neq$ best) where λ is a uniformly distributed random variable in $(0,1)$ ($\lambda \sim U(0,1)$) and α^i_{max} is the longest step size that can be taken along the force vector before violating the bound constraints, i.e.,

$$\alpha^i_{\mathrm{max}} = \min_{1\leq k\leq n} \alpha^i_k \equiv \begin{cases} \dfrac{(u_k - x^i_k)}{F^i_k}, & \text{if } F^i_k > 0 \\ \dfrac{(l_k - x^i_k)}{F^i_k}, & \text{if } F^i_k < 0 \\ M, & \text{if } F^i_k = 0 \end{cases} \tag{6}$$

where M is a sufficiently large positive value. A pairwise comparison is then carried out to detect the best point of the new population imposing conditions (4) or (5).

3.3 *The local descent search*

This section gives a detailed description of a derivative-free heuristic method that produces an approximate descent direction and aims to generate a new trial point around the best point of the population. First, two exploring points are randomly generated in a neighborhood of the best point x^{best} using

$$x_k^{\text{rand},\,i} = x_k^{\text{best}} \pm \lambda \varepsilon_r, \text{ for } k = 1, 2, \dots, n \tag{7}$$

and $i = 1, 2$, where $\lambda \sim U(0, 1)$ and ε_r is a sufficiently small positive value. Then, an approximate descent direction d for the fitness function, see (3), at x^{best} is defined. Based on the two random points from (7), a descent direction is generated by

$$d = -\frac{1}{\sum_{j=1}^{2} |\Delta_j|} \sum_{i=1}^{2} \Delta_i \frac{x^{\text{best}} - x^{\text{rand},\,i}}{\|x^{\text{best}} - x^{\text{rand},\,i}\|}, \tag{8}$$

where $\Delta_j = \text{fitness}(x^{\text{best}}) - \text{fitness}(x^{\text{rand},\,j})$. Theoretical properties related to this direction vector are shown in [13], where the authors use this descent direction in a point-to-point search context, a simulated annealing method. It is shown in [13] that for a set of l exploring points close to x^{best}, (8) has a high ability of producing an approximate descent direction if: (i) they are randomly generated in a small neighborhood of x^{best} and $l = 2$, or (ii) they are in equal distance to x^{best}, define with x^{best} a set of orthogonal directions, and $l = n$.

A trial point is generated along the descent direction with a prescribed step size,

$$y = x^{\text{best}} + s\, \alpha_{\max}\, d, \tag{9}$$

where $s \in (0, 1]$ represents the step size, and $\alpha_{\max}$ is computed similarly to (6) with F^i replaced by d and x^i replaced by x^{best}. The selection of the step size uses a classical backtracking strategy. To decide if the trial point leads to a sufficient improvement when compared with the best point, one of the selective conditions (4) or (5) must hold.

Finally, we present a formal description of the local descent search. See Algorithm 3.1. First, we generate two exploring points and a descent direction. These two steps in the Algorithm 3.1 are executed whenever *flag* is set to 1. Then, a trial point y is calculated and, according to the selective sufficient reduction conditions, (5) or (4), either y or x^{best} is preferred. If x^{best} is the preferred point, then y is discarded, the step size is halved (i.e., $s \leftarrow s/2$) and a new point is evaluated along that descent direction (*flag* is set to 0 in the Algorithm 3.1). However, when y is preferred, another approximate descent direction is computed (*flag* is set to 1, and s is reset to 1) and the process is repeated.

3.4 *Dealing with integer variables*

Some engineering design problems have integer as well as continuous variables. The technique implemented in the new hybrid EM algorithm to deal with the integer variables can be summarized as follows. Whenever new points are evaluated, for example, after the initialization of the population, after the movement of the points

Algorithm 3.1 (Local descent search in the hybrid EM)

```
input: $LsIt_{\max}$, $x^{\text{best}}$, $\varepsilon_r$, $\gamma$
$flag \leftarrow 1$, $s \leftarrow 1$, iteration $\leftarrow 0$
while iteration $\leq LsIt_{\max}$ do
    if $flag = 1$ then
        Generate two random points using (7)
        Compute descent direction $d$ using (8)
    end if
Compute trial point $y$ using (9)
    if both $y$ and $x^{\text{best}}$ are feasible then
        if $f(y) \leq (1-\gamma)f(x^{\text{best}})$ then
            $x^{\text{best}} \leftarrow y$, $s \leftarrow 1$, $flag \leftarrow 1$
        else
            $s \leftarrow s/2$, $flag \leftarrow 0$
        end if
    else
        if $\text{CV}(y) \leq (1-\gamma)\text{CV}(x^{\text{best}})$ then
            $x^{\text{best}} \leftarrow y$, $s \leftarrow 1$, $flag \leftarrow 1$
        else
            $s \leftarrow s/2$, $flag \leftarrow 0$
        end if
    end if
iteration $\leftarrow$ iteration $+ 1$
end while
```

and during the local search, the coordinates of each point that should be integer are rounded to the nearest integer. Then, the measure of violation CV of the new point is computed, and if the point is feasible, the objective function value is also evaluated, since this will be required during points comparison. All the other procedures in the algorithm proceed as before. This approach was also implemented in the EM algorithm that is based on the FAD rules for constraint-handling.

4. Numerical experiments

Problems of practical interest are important for assessing the effectiveness of a given approach. Thus, to evaluate the performance of the herein proposed hybrid electromagnetism-like algorithm for constrained problems, a set of 12 benchmark engineering problems is used. The algorithm is coded in the C programming language and it contains an interface to connect to AMPL so that the problems coded in AMPL could be easily read and solved [12]. The set of AMPL coded problems may be obtained from the first author upon request. We tested the original EM algorithm incorporating the FAD rules for constraint-handling, denoted in the subsequent tables only by ”EM”, and compared with the herein proposed hybrid EM algorithm, ”hybridEM”. A comparison with other published results is also included.

Some parameters are set to their standard values reported in the literature. The maximum number of iterations imposed on the Algorithm 3.1 is $LsIt_{\max} = 10$, like in [5]. We follow the suggestion made in [13] for the ray of the neighborhood of x^{best}, $\varepsilon_r = 0.001$. The selected value for the constant γ is proposed in [31], $\gamma = 0.00001$. Both implemented algorithms terminate after 1000 iterations, since this is the most common value used in the literature, related to the stochastic population-based methods that we use for comparison. To obtain with stochastic algorithms reliable estimates of the average performance, with an approximate normal distribution, one has to perform a large enough number of runs with each problem (more than 30). We run each problem 50 times, each starting from a random population with a different seed. In our study we use the number of points in the population dependent on n, $p_{\text{size}} = 10n$.

Values listed in the Tables 1-12 correspond to: (i) the obtained optimal design variables; (ii) the best objective function value achieved after 50 independent runs, f_{best}; (iii) the number of objective function evaluations of the best run, nf_{eval}; (iv) the standard deviation of the function values over the 50 runs, SD; (v) the number of points in the population, p_{size}; and (vi) the number of runs, "runs". In the tables, "-" means unavailable information.

4.1 *Design of a welded beam*

The design of a welded beam [2, 10, 14, 15, 18–20, 22, 24] is the most used problem to assess the effectiveness of an algorithm. The objective is to minimize the cost of a welded beam, subject to constraints on the shear stress, bending stress in the beam, buckling load on the bar, end deflection of the beam, and side constraints. There are 4 design variables and 5 inequality constraints. We remark that this problem has also been formulated as a multiobjective optimization problem. See, for example, [23] and [25]. In this situation, the problem deals with the design of a welded beam with a minimum cost and minimum end deflection.

Table 1 contains the values of the design variables at the best solution found in the 50 runs. For comparative purposes, we include similar results published in [10], which implements a genetic-based algorithm, in [2] and [24], where a social-behavioural simulation algorithm is used, in [14], where a simulated annealing based method is used (a point-to-point search method), in [18], where a harmony search method is tested, and in [32], with a hybrid evolutionary algorithm. The EM algorithm found a solution having objective function value within 2.2% of the best-known solution, 2.38, after 29985 function evaluations, while the hybrid EM found a solution within 0.26% of the best-known solution, after 28650 function evaluations. When solving this problem, hybrid EM outperforms the EM algorithm.

Table 1. Comparative results for the beam problem

Values				Best solution found				
	EM	hybridEM	in [2]	in [10]	in [14]	in [18]	in [24]	in [32]
x_1	0.235393	0.243532	0.2407	0.2088	0.244353	0.2442	0.244438	0.244369
x_2	5.844572	6.167268	6.4851	3.4205	6.215792	6.2231	6.237967	6.217518
x_3	9.069322	8.377163	8.2399	8.9975	8.293904	8.2915	8.288576	8.291477
x_4	0.239513	0.243876	0.2497	0.2100	0.244353	0.2443	0.244566	0.244369
f_{best}	2.431621	2.386269	2.4426	2.38119	2.381065	2.38	2.385435	2.380957
nf_{eval}	29985	28650	19259	40080	56243	110000	33095	30000
SD	5.6e-2	3.1e-2	-	-		-	-	-
p_{size}	40	40	100	80		20	40	100
runs	50	50	10	50	50	-	50	30

4.2 *Design of a disc brake*

The second example is a typical multiobjective optimization problem. In the design of a multiple disc brake, the objective is to minimize both the mass of the brake and the stopping time. The reader is referred to [23] for a full description. If the objective of stopping time minimization is dropped, then a constraint on maximum stopping time ought to be added to the set of constraints[1]. The 4 design variables are the inner radius of the discs, outer radius of the discs, the engaging force and the number of friction surfaces. The problem has 6 inequality constraints. The constraints include minimum distance between the radii, maximum length of the brake, pressure, temperature and torque limitations. Table 2 lists the optimal designs obtained by both EM and hybrid EM algorithms. Values in the first row are from the Pareto front of [23] with a swarm size of 500 points. Although both solutions are similar, we may conclude that the hybrid EM algorithm has a higher consistency solution due to the lower value of SD.

Table 2. Comparative results for the brake problem

	x_1	x_2	x_3	x_4	f_{best}	nf_{eval}	SD	p_{size}	runs
in [23]	-	-	-	-	0.2-2.7†	6385	-	500	-
EM	55.00	75.00	3000.00	2.00	0.127400	33961	2.4e-4	40	50
hybridEM	55.00	75.00	1862.87	2.00	0.127400	43480	2.8e-7	40	50

† range of values in the Pareto front

4.3 *Design of a heat exchanger*

The design of a heat exchanger involves minimizing the sum of the heat transfer areas of the three exchangers [10, 18, 19]. The problem has 8 design variables and 6 inequality constraints. Our results are presented in Table 3. The best objective function value 7057.274, reported by Lee and Geem in [18] that use a new meta-heuristic based on the harmony search theory, is obtained after 150000 function evaluations. The solution reported in [10], which implements a genetic algorithm based on a penalty parameter approach, 7060.221, was obtained after 4000 iterations and 320080 function evaluations. When our algorithm is allowed to run for 4000 iterations, the best solution is 7358.973 after 139748 function evaluations. Although the solution obtained by the hybrid EM algorithm is within 5.9% of the best-known solution in [18], it has been reached with 18% of the function evaluations required in [18]. Using a population of 20 points and terminating the iterative process after 150000 function evaluations, the algorithm reached the solution 7318.825. The reported results in [19] correspond to a fuzzy-based method with a proportional-derivative controller.

4.4 *Design of a speed reducer*

The design of a speed reducer has been previously analyzed by other authors [2, 8, 19, 22, 24, 32]. The objective here is to minimize the total weight of a speed reducer, subject to constraints on bending stress of the gear teeth, surface stress, transverse deflections of the shafts and stresses in the shafts. There are 7 design variables and 11 inequality constraints. Variable x_3 is integer. Here, we implemented the strategy previously described at the end of Section 3. The solution found by the run that reached the best function value is registered in the Table 4. The solutions obtained by the EM and hybrid EM algorithms are comparable to the others - both within 0.2% of the best-known solution 2988.49 achieved in [8] after 300 iterations. (The number of function evaluations is not therein reported.) Chen, Ge and Wei

[1] We used a value of 32 for the maximum stopping time constraint, taken from the Pareto front in [23].

Table 3. Comparative results for the heat problem

Values	Best solution found				
	EM	hybridEM	in [10]	in [18]	in [19]
x_1	423.769	607.211	-	500.004	951.8
x_2	1090.221	1560.399	-	1359.31	1529.5
x_3	5760.479	5303.680	-	5197.96	4807.3
x_4	166.3717	173.3244	-	174.726	206.6
x_5	272.9555	287.9510	-	292.0817	307.9
x_6	230.5197	205.4482	-	224.7054	193.4
x_7	288.5768	284.1103	-	282.6446	298.7
x_8	371.4911	387.9249	-	392.0817	407.8
f_{best}	7274.468	7471.29	7060.221	7057.274	7288.8
nf_{eval}	2683	27050	320080	150000	-
SD	1.6e3	1.4e3	-	-	
p_{size}	80	80	80	20	
runs	50	50	50	-	

[8] propose an improved particle swarm optimization (PSO) algorithm that uses a random mutation modification of the movement particle equation.

Table 4. Comparative results for the speed problem

Values	Best solution found						
	EM	hybridEM	in [2]	in [8]	in [19]	in [24]	in [32]
x_1	3.500008	3.500062	3.506122	3.500	3.5197	3.500000	3.500023
x_2	0.700000	0.700000	0.700006	0.700	0.7039	0.700000	0.700000
x_3	17	17	17	17	17.3831	17	17.000013
x_4	7.300004	7.367704	7.549126	7.300	7.3000	7.327602	7.300428
x_5	7.715424	7.731763	7.859330	7.800	7.7152	7.715322	7.715377
x_6	3.350228	3.351341	3.365576	3.350	3.3498	3.350267	3.350231
x_7	5.286655	5.286937	5.289773	5.274	5.2866	5.286655	5.286664
f_{best}	2994.365	2995.804	3008.08	2988.49	3007.8	2994.744	2994.499
nf_{eval}	27668	51989	19154	-	-	54456	40000
SD	2.3e-2	1.3e0	-	-		-	-
p_{size}	70	70	100	-		70	100
runs	50	50	10	40		50	30

4.5 *Design of a tension/compression spring*

The problem that considers the design of a tension/compression spring minimizes the weight of the spring, subject to constraints on the minimum deflection, shear stress, surge frequency, limits on outside diameter and on the design variables [9, 14, 15, 19, 20, 24]. The problem has 3 design variables and 4 inequality constraints. Table 5 contains the results of the spring design problem. The best solutions found by both EM algorithms, 0.01266765 and 0.01266707, are between the best-known solutions in the literature, and were found after 5379, in one case, and 9605, in the other, objective function evaluations. We can see that the results are consistent in the sense that the standard deviations of the 50 solutions are small, in particular in the hybrid EM algorithm. Results taken from [9] correspond to a genetic-based algorithm and those from [15] and [30] correspond to PSO-type algorithms.

4.6 *Design of a tanker fleet*

This multiobjective optimization problem considers the minimization of cost, which includes the cost of fuel, cost of hull and cost of machinery, and the maximization of cargo transportation capacity [25]. It has 9 decision variables (8 continuous and

Table 5. Comparative results for the spring problem

Values				Best solution found				
	EM	hybridEM	in [9]	in [14]	in [15]	in [24]	in [30]	in [32]
x_1	0.361564	0.353534	0.351661	0.358005	0.351384	0.368159	0.310414	0.356729
x_2	0.051891	0.051557	0.051480	0.051743	0.051466	0.052160	0.05	0.051690
x_3	11.01150	11.47952	11.63220	11.21391	11.60866	10.64844	15.06	11.28829
f_{best}	0.012668	0.012667	0.012705	0.012665	0.012666	0.012669	0.013193	0.012665
nf_{eval}	5379	9605	-	49531	-	25167	757800	24000
SD	2.3e-4	8.0e-6	-		-	-	-	
p_{size}	30	30	60		20	30	-	100
runs	50	50	11	30	11	50	-	30

1 integer) and 13 inequality constraints. Here, we consider the uniobjective formulation of the tanker design optimization problem (where the objective of cargo capacity maximization is dropped and the constraint on minimum annual cargo transport capacity is maintained). The variable x_5 is integer and it was dealt as previously described. The results reported in [25] consider both the multiobjective and uniobjective optimization formulations. In the latter, the found optimal design is registered in Table 6 and has a cost of 135 500000 dollars. In both cases, Ray and Tai [25] use an evolutionary algorithm with a multilevel pairing strategy. Both herein tested algorithms reached the optimal designs listed in Table 6, with costs around 24 025120 and 21 216265 dollars. The 50 runs were carried out with a population of 90 points.

Table 6. Comparative results for the tanker problem

Method	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	f_{best}	nf_{eval}	SD
in [25]	27.63	12.090	15200.000	165.200	44	7.4060	0.928	10.910	22660	135500000	-	-
EM	39.83650	17.07769	51561.901	227.4075	17	10.78351	0.707421	11.02364	60083.19	24025120	1140	8.5e6
hybridEM	48.31752	19.95852	79071.998	279.4188	8	12.41858	0.734665	14.49254	118646.56	21216265	2452	6.1e6

4.7 *Design of a gear train*

In the design of a gear train, the cost of a gear ratio is minimized, subject to constraints on the design variables [20]. The problem has 4 integer design variables. We implemented the strategy previously described at the end of Section 3. The obtained solutions are reported in Table 7. EM found a solution in 43042 function evaluations, while the hybrid EM needed 51040 objective function evaluations. In this problem, the least standard deviation is obtained by the EM algorithm. For comparison, the best result reported in [20], a PSO-type algorithm, is also included. No information about the optimal design variables is therein included.

Table 7. Comparative results for the train problem

Method	x_1	x_2	x_3	x_4	f_{best}	nf_{eval}	SD	p_{size}	runs
in [20]	-	-	-	-	2.70085e-12	-	-	20	100
EM	43	19	16	49	2.700857e-12	43042	1.6e-27	40	50
hybridEM	49	19	16	43	2.700857e-12	51040	3.5e-10	40	50

4.8 *Design of three-bar truss*

This problem considers the minimization of the volume of a 3-bar truss structure, subject to stress constraints. The problem is fully described in [24], has 2 design variables, representing cross-sectional areas of two bars (two of the bars are equal) and 3 inequality constraints. The results reported in Table 8 correspond to the best designs found in [24, 32] and in our study. Our results are competitive. When comparing both EM algorithms, we observe a slight reduction in the standard deviation of the hybrid EM. We remark that the author in [19] uses a formulation that minimizes the weight of the truss structure.

Table 8. Comparative results for the 3-truss problem

Method	x_1	x_2	f_{best}	nf_{eval}	SD	p_{size}	runs
in [19]	0.7511	0.5262	2.6507	-			
in [24]	0.788621	0.408401	263.8958	17610	-	20	50
in [32]	0.788680	0.408234	263.8959	15000	-	100	30
EM	0.788666	0.408274	263.8959	9747	1.9e-2	20	50
hybridEM	0.788764	0.408000	263.8960	17479	6.5e-3	20	50

4.9 *Design of a four-bar truss*

This is a problem where the structural volume and the displacement at a particular joint, of a 4-bar truss structure, are to be minimized subject to the stress constraints on the members. This multiobjective optimization problem is shown by Ray and Liew in [23]. If the objective of displacement at the particular joint minimization is dropped while a constraint on the maximum displacement (at that particular joint) is added to the constraints[2], a uniobjective formulation is obtained. The cross sectional areas of the members are the 4 design variables. The problem has 1 inequality constraint. Results obtained by our study are listed in Table 9. With less function evaluations and a standard deviation of zero, the EM algorithm that is based on the FAD rules outperforms the hybrid EM algorithm.

Table 9. Comparative results for the 4-truss problem

Method	x_1	x_2	x_3	x_4	f_{best}	nf_{eval}	SD	p_{size}	runs
in [23]	-	-	-	-	1400-3000†	2525	-	100	-
EM	1	1.414214	1.414214	1	1400.000	17969	0	40	50
hybridEM	1.000003	1.414214	1.414214	1	1400.001	51106	2.4e-3	40	50

† range of values in the Pareto front

4.10 *Design of a tubular column*

The design of a tubular column is described in full detail in [19, 22]. The objective in this problem is to minimize the total cost of the material and construction of a tubular column. The problem has 2 design variables and 2 inequality constraints. The results reported in Table 10 for comparison are taken from [19], a fuzzy-based method, and [22], a deterministic-type algorithm. Both EM algorithms give similar optimal designs.

[2] We used a value of 0.04 for the maximum joint displacement constraint, taken from the Pareto front [23].

Table 10. Comparative results for the tubular problem

Method	x_1	x_2	f_{best}	nf_{eval}	SD	p_{size}	runs
in [19]	5.4507	0.2920	25.5316	-			
in [22]	5.44	0.293	26.53	-			
EM	5.452383	0.29190	26.53380	17308	7.7e-2	20	50
hybridEM	5.451083	0.29199	26.53227	25136	3.5e-3	20	50

4.11 *Design of a cylindrical vessel*

This example is the design of a cylindrical pressure vessel with both ends capped with a hemispherical head [2, 8, 9, 14, 15, 18, 20]. This problem consists of minimizing the total cost of the material, forming and welding of the cylindrical vessel, and has 4 design variables and 4 inequality constraints. Variables x_1 and x_2 are integer multiples of 0.0625. For this problem, we consider $x_i = 0.0625n_i, (i = 1, 2)$ and work with the integer variables n_1 and n_2. We implemented the strategy mentioned in Subsection 3.4. The results obtained when solving the pressure vessel design problem are reported in Table 11. The solution obtained by EM is 6071.167 (with 24182 function evaluations). The hybrid EM found the solution 6072.232 after 20993 function evaluations. Our solutions are competitive with those in the literature that use the same bound constraints (for example, [2],[9],[30]).

Table 11. Comparative results for the vessel problem

Values				Best solution found				
	EM	hybridEM	in [2]	in [8]	in [9]	in [14]	in [18]	in [30]
x_1	0.8125	0.8125	0.8125	0.75	0.8125	0.768326	1.125	0.778169
x_2	0.4375	0.4375	0.4375	0.375	0.4375	0.379784	0.625	0.384649
x_3	42.00476	42.07007	41.9768	38.860	40.3239	39.80962	58.2789	40.3196
x_4	177.8015	177.3762	182.2845	221.582	200.0	207.2256	43.7549	200.0
f_{best}	6071.167	6072.232	6171.000	5854.738	6288.745	5868.765	7198.433	5885.33
nf_{eval}	24182	20993	12630	-	-	108883	-	879000
SD	1.3e2	5.3e1	-	-	-		-	-
p_{size}	40	40	100	-	60		20	-
runs	50	50	10	40	11	30	-	-

4.12 *Design of a water distribution system*

The last problem is related with the design of a water distribution system in a building. The case herein solved is fully described in [16] and has 21 pipes. The decision variables x_i, $i = 1, \ldots, 21$, are the interior pipe diameters. Bounds for the variables and data concerning the length and the design flow for each pipe are reported in [16]. We solved the problem 50 times and recorded the optimal design variables that gave the least function value. The best solutions obtained by both EM algorithms are reported in columns ”optimal” of Table 12 ($f_{\text{best}} = 2673.580$ for EM and $f_{\text{best}} = 2660.577$ for hybrid EM).

Table 12. Comparative results for the water distribution problem

Values	EM			hybridEM			in [16]	
	optimal	real	optimal	real	optimal	real	optimal	real
f_{best}	2673.580	2726.08	2660.577	2726.08	2661.976	2726.08	2714.177	2726.08
nf_{eval}	174331		184577		44804		-	-
p_{size}	210		210		50			

We then need to adjust the diameters to the real commercially available diameters[3]. Our strategy considers rounding each coordinate of the solution to the nearest real diameter value that belongs to the set Ω. These values, for both EM and hybrid EM algorithms, are $x_1 = x_3 = x_4 = 0.0603, x_2 = 0.0721, x_5 = x_6 = x_8 = 0.0516, x_7 = x_9 = x_{10} = 0.0396, x_{11} = x_{12} = x_{13} = x_{14} = 0.0330, x_{15} = x_{16} = x_{17} = x_{18} = x_{19} = 0.0264, x_{20} = x_{21} = 0.0166$, and correspond to $f_{\text{best}} = 2726.080$, see Table 12 under the columns headed by ”real”. These results are equal to those reported in [16] where a branch-and-bound type algorithm is used. The solution obtained by the hybrid EM algorithm required 184577 function evaluations and the EM algorithm needed 174331 function evaluations (for a population of 210 points). When $p_{\text{size}} = 50$, the algorithm reached the solution 2661.976 after 44804 function evaluations. After adjusting the optimal diameters to their real available values, the solution 2726.080 was obtained.

5. Conclusions

This paper presents a new version of the electromagnetism-like optimization algorithm for solving global constrained optimization problems. The new version incorporates selective conditions that aim to detect the best point of the population, to attract points to promising regions and to guarantee progress around the best point. The selective conditions herein proposed impose a sufficient reduction either in the constraints violation or in the objective function value, instead of implementing a penalty technique [4], avoiding therefore the update of the penalty parameter that is associated with the penalization of the constraints in a penalty function. Further, we hybridize the electromagnetism-like mechanism with a random local search procedure that is able to produce an approximate descent direction and refine locally the best point in the population. A backtracking line search technique is also incorporated into the local search procedure to give faster progress towards optimality.

To assess the performance of the herein proposed hybrid EM algorithm, a set of twelve constrained engineering problems of practical interest is solved. A comparison with results from other stochastic-type methods is included. The results show the effectiveness of our hybrid EM method. Analyzing the standard deviation of the function values, one is able to conclude about the consistency of the solutions. We observe that the solutions reached by the new hybrid EM algorithm have in general slightly smaller standard deviations than those of the EM FAD-based algorithm. Together with the accuracy, this is a measure of solutions quality. The computational costs, measured by the number of function evaluations required to achieve a high accuracy solution, of the herein proposed hybrid EM are in some cases lower than those of other population-based algorithms. It is however noteworthy the reduced computational costs of the EM FAD-based algorithm when compared with the others. It seems that the efficiency of the hybrid EM is worse than that of the EM FAD-based algorithm in most of experiments in terms of the number of function evaluations. Since computational cost is an important factor considered in optimization algorithms, future developments will focus on this issue. Solving NP-problems with integer/binary variables, namely asymmetric traveling salesman problems, using this new hybrid algorithm is a matter for future research.

[3] $\{0.0138, 0.0166, 0.0206, 0.0264, 0.0330, 0.0396, 0.0516, 0.0603, 0.0721, 0.0849, 0.104\}$, as shown in [16].

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