



Hydraulic Fracture Mechanics

Peter Valkó and Michael J. Economides

*Texas A & M University,
College Station,
USA*

JOHN WILEY & SONS

Chichester • New York • Brisbane • Toronto • Singapore

CONTENTS

Preface

xi

List of Notation

xiii

1	Hydraulically Induced Fractures in the Petroleum and Related Industries	1
1.1	Fractures in Well Stimulation	1
1.2	Fluid Flow Through Porous Media	2
1.2.1	The Near-well Zone	4
1.3	Flow from a Fractured Well	5
1.4	Hydraulic Fracture Design	7
1.5	Treatment Execution	11
1.5.1	Fracturing Fluids	11
1.5.2	Proppants	13
1.6	Data Acquisition and Evaluation for Hydraulic Fracturing	14
1.6.1	Well Log Measurements	14
1.6.2	Core Measurements	15
1.6.3	Well Testing	15
1.7	Mechanics in Hydraulic Fracturing	15
	References	16
2	Linear Elasticity, Fracture Shapes and Induced Stresses	19
2.1	Force and Deformation	19
2.1.1	Stress	19
2.1.2	Strain	21
2.2	Material Properties	23
2.2.1	Linear Elastic Material	23
2.2.2	Material Behavior Beyond Perfect Elasticity	26
2.3	Plane Elasticity	27
2.3.1	Plane Stress	27
2.3.2	Stresses Relative to an Oblique Line (Force Balance I)	28
2.3.3	Equilibrium Relations (Force Balance II)	30
2.3.4	Plane Strain	30
2.3.5	Boundary Conditions	32

2.4	Pressurized Crack	32
2.4.1	Solution of the Line Crack Problem	32
2.4.2	Constant Pressure	34
2.4.3	Polynomial Pressure Distribution	35
2.4.4	"Zipper" Cracks	37
2.4.5	"Zipper" Crack with Polynomial Pressure Distribution	40
2.5	Stress Concentration and Stress Intensity Factor	41
2.5.1	Stress Intensity Factor, Symmetric Loading	42
2.5.2	Stress Intensity Factor, non-symmetric Loading	43
2.6	Fracture Shape in the Presence of Far-field Stress. The Concept of Net Pressure	43
2.7	Circular Crack	45
2.8	Volume and Strain Energy	47
2.9	Computational Methods	49
	References	50
3	Stresses in Formations	53
3.1	Basic Concepts	53
3.2	Stresses at Depth	55
3.3	Near-wellbore Stresses	59
3.4	Stress Concentrations for an Arbitrarily Oriented Well	63
3.5	Vertical Well Breakdown Pressure	65
3.6	Breakdown Pressure for an Arbitrarily Oriented Well	66
3.7	Limiting Case: Horizontal Well	69
3.7.1	Arbitrarily Oriented Horizontal Well	70
3.8	Permeability and Stress	71
3.8.1	Stress-sensitive Permeability	72
3.9	Measurement of Stresses	73
3.9.1	Small Interval Fracture Injection Tests	74
3.9.2	Acoustic Measurements	75
3.9.3	Determination of the Closure Pressure	76
3.9.4	Core Stress Measurements	77
3.9.5	Critique and Applicability of Techniques	79
	References	80
4	Fracture Geometry	83
4.1	The Perkins and Kern and Khristianovich and Zhel'tov Geometries	83
4.1.1	The Consequences of the Plane Strain Assumption	86
4.2	Fracture Initiation vs. Propagation Direction	88
4.2.1	Fractures in Horizontal Wells	90
4.3	Fracture Profiles in Multi-layered Formations	92
	References	95
5	Rheology and Laminar Flow	97
5.1	Basic Concepts	97
5.1.1	Material Behavior and Constitutive Equations	98
5.1.2	Force Balance	103

5.2	Slot Flow	105
5.2.1	Derivation of the Basic Relations	105
5.2.2	Equivalent Newtonian Viscosity	111
5.3	Flow in Circular Tube	112
5.3.1	Basic Relations	112
5.3.2	Flow Curve	115
5.3.3	Equivalent Newtonian Viscosity for Tube Flow	119
5.4	Flow in Other Cross Sections	122
5.4.1	Flow in Annulus	122
5.4.2	Flow in Elliptic Cross Section	123
5.4.3	Limiting Ellipsoid Cross Section	124
	References	128
6	Non-laminar Flow and Solids Transport	131
6.1	Non-laminar Flow	131
6.1.1	Newtonian Fluid	131
6.1.2	General Fluid	132
6.1.3	Drag Reduction	134
6.1.4	Turbulent Flow in Other Geometries	137
6.2	Solids Transport	138
6.2.1	Settling of an Individual Sphere	139
6.2.2	Effect of Shear Rate Induced by Flow	141
6.2.3	Effect of Slurry Concentration	142
6.2.4	Wall Effects	143
6.2.5	Agglomeration Effects	145
	References	145
7	Advanced Topics of Rheology and Fluid Mechanics	147
7.1	Foam Rheology	147
7.1.1	Quality Based Correlations	148
7.1.2	Volume Equalized Constitutive Equations	148
7.1.3	Volume Equalized Power Law	151
7.1.4	Turbulent Flow of Foam	152
7.2	Accounting for Mechanical Energy	153
7.2.1	Basic Concepts	153
7.2.2	Incompressible Flow	154
7.2.3	Foam Flow	154
7.3	Rheometry	156
7.3.1	Pipe Viscometry	156
7.3.2	Slip Correction	157
	References	162
8	Material Balance	165
8.1	The Conservation of Mass and Its Relation to Fracture Dimensions	165
8.2	Fluid Leakoff and Spurt Loss as Material Properties	169
8.2.1	Carter Equation I	169
8.2.2	Formal Material Balance. The Opening Time Distribution Factor	171

8.3	The Constant Width Approximation (Carter Equation II)	172
8.4	The Power Law Approximation to Surface Growth	174
	8.4.1 The Consequences of the Power Law Assumption	174
	8.4.2 The Combination of the Power Law Assumption with Interpolation	178
8.5	Numerical Material Balance	179
8.6	Differential Material Balance	181
8.7	Leakoff as Flow in the Porous Medium	183
	8.7.1 Filter-cake Pressure Drop	184
	8.7.2 Pressure Drop in the Reservoir	185
	8.7.3 Leakoff Rate from Combining the Resistances (Ehlig-Economides <i>et al.</i> [6])	187
	References	187
9	Coupling of Elasticity, Flow and Material Balance	189
9.1	Width Equations of the Early 2D Models	189
	9.1.1 Perkins-Kern Width Equation	189
	9.1.2 Geertsma-de Klerk Width Equation	192
	9.1.3 Radial Width Equation	195
9.2	Algebraic (2D) Models as Used in Design	196
	9.2.1 PKN-C	196
	9.2.2 KGD-C	199
	9.2.3 PKN-N and KGD-N	200
	9.2.4 PKN- α and KGD- α	201
	9.2.5 Radial Model	202
	9.2.6 Non-Newtonian Behavior	202
9.3	Numerical Material Balance (NMB) with Width Growth	204
9.4	Differential 2D Models	205
	9.4.1 Nordgren Equation	206
	9.4.2 Differential Horizontal Plane Strain Model	209
9.5	Models With Detailed Leakoff Description	210
9.6	Pressure Decline Analysis	211
	9.6.1 Nolte's Pressure Decline Analysis (Power Law Assumption)	212
	9.6.2 The No-spurt-loss Assumption (Shlyapobersky method)	217
	9.6.3 Material Balance and Propagation Pressure Estimates of the Spurt Loss	218
	9.6.4 Resolving Contradictions	227
	9.6.5 Pressure Decline Analysis With Detailed Leakoff Description (Mayerhofer <i>et al.</i> Technique)	230
	References	232
10	Fracture Propagation	235
10.1	Fracture Mechanics	237
	10.1.1 Griffith's Analysis of Crack Stability	238
	10.1.2 Mott's Theory for the Rate of Crack Growth	241
10.2	Classical Crack Propagation Criterion for Hydraulic Fracturing	242
	10.2.1 Fracture Toughness Criterion	242
	10.2.2 The Injection Rate Dependence Paradox	243

10.3	Retarded Fracture Propagation	245
10.3.1	Fluid Lag	245
10.3.2	Tip Dilatancy	245
10.3.3	Apparent Fracture Toughness	246
10.3.4	Process Zone Concept	246
10.3.5	The Reopening Paradox	247
10.4	Continuum Damage Mechanics in Hydraulic Fracturing	247
10.4.1	Tip Propagation Velocity from CDM	247
10.4.2	CDM-NK Model	249
10.4.3	CDM-PKN Design Model	252
10.5	Pressure Decline Analysis and Tip Retardation	256
10.5.1	Resolving Contradictions with Continuum Damage Mechanics	258
	References	263

11 Fracture Height Growth (3D and P-3D Geometries) 267

11.1	Equilibrium Fracture Height	269
11.1.1	Reverse Application of the Net-pressure Concept	269
11.1.2	Different Systems of Notation	270
11.1.3	Basic Equations	272
11.1.4	The Effect of Hydrostatic Pressure	276
11.2	Three-dimensional Models	278
11.2.1	Surface Integral Method	279
11.2.2	The Stress Intensity Factor Paradox	281
11.3	Pseudo-three-dimensional Models	283
	References	284

Appendix: Comparison Study of Hydraulic Fracturing Models: Input Data and Results 287

	References	294
--	------------	-----

Index 295