HYDRODYNAMIC DERIVATIVES ON SHIP MANOEUVRING

by

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Summary

Model experiments have been carried out to investigate the hydrodynamic forces acting on ship in even keel and trimmed conditions. The relations between this force and load condition of ship were proved, and the linear and non-linear derivatives in manoeuvring mathematical models for fitting the measured results to describe the forces were also examined semi-empirically by using the model tests of various kinds of ships and applying a theoretical approach. Finally, the approximate formulae for practical application in estimation the lateral force and moment on ship in manoeuvring motion.

1. Introduction

In the field of ship manoeuvrability, it is of elementary importance to know the hydrodynamic forces acting on ship, and these forces are indispensable factor for the evaluation of the manoeuvring simulation, course stability or analysis of ship motion etc..

The hydrodynamic force acting on ship may be generally divided into three parts, that is on hull, rudder and propeller and their interferences. On these forces, many works have already done theoretically and experimentally, however, many problems to be solved will still remain. We can find, for example, the problem of differing polynomial expression or perturbation models in manoeuvring mathematical one for fitting those forces obtained from captive model tests, a problem of comparison with the theoretical values and measured one, and model/full-scale correlations etc.. Under such a background, the description of the hydrodynamic forces acting on hull, rudder and propeller and their interference have discussed in a work group named MMG (was specially organized in the Manoeuvrability Sub-Committee of JTTC), and one of the results of discussions has been reported [1].

However, for the estimation of the force acting on bare hull, being the main part, it will be required to obtain the more useful description with as high accuracy as possible. The authors proposed a method estimating the linear derivatives of the force acting on bare hull in expression based on perturbation model, by using the non-linear lifting surface theory and the measured results. We can not, however, analyze the ship manoeuvring by only means of the linear terms of the forces, especially when ship is in manoeuvring motion with large rudder angle, because the non-linear terms will account for the main part in this case. Furthermore, the force acting on hull will also depend on load condition, that is full, half and ballast load conditions or trimmed condition. Some results for this problem have already been reported [2, 3, 4, 5] by the authors.

In the past works, the hydrodynamic force acting on bare hull has been almost discussed in even keel condition, but ship manoeuvring will be greatly affected by load condition. For example, it has been well known that the course stability has quite difference between even keel and trimmed conditions.

In this paper, the relations between ship dimension and load condition on the hydrodynamic force acting on bare hull were proved, and also the linear and non-linear derivatives in manoeuvring mathematical model for fitting the measured forces were obtained semi-empirically as simple form by using the model tests of various kinds of ships. And finally the approximate formulae for practical applications to estimate the force and moment on ship in manoeuvring motion are proposed.

2. Linear derivatives of hydrodynamic force in even keel condition

In general, the full-scale manoeuvring trials for oil tanker will be carried out in full load condition, but almost for general cargo ships in ballast condition by the several reasons. And consequently, we must estimate the manoeuvring performance of ship in full load condition from the results in ballast load condition, or vice versa. Because of this, it will be required to know the hydrodynamic forces acting on ship in even keel condition as a function of draft to evaluate the manoeuvring performance of a full-scale ship. Another importance is the problem of mathematical model for expression of these forces, it may be said that means a relation between linear or non-linear terms of these forces and load condition.

In this paper, lateral force and moment on ship are here measured by the model ships of several kinds as a function of draft. And the linear terms in the manoeuvring mathematical model for these forces are here examined.

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2.1. Model test

The model ships used here to measure the hydrodynamic force are three oil tankers, three dry cargo ships, a container ship, a LNG tanker, a RO/RO ship and a car carrier as shown in Table 1. And the lateral force (Y) and yaw moment (N) around midship acting on bare hull without rudder and propeller were measured in three cases of load condition, that are full, half and ballast load conditions. Its detail items are also shown in Table 1. The Froude number in the test condition was set to about 0.06 for the all model ships. To measure the lateral force and yaw moment in deep water, the depth of water was set to give water/draft (H/d) value 9.8 in full load condition, which was no effects of water depth. These experiments were carried out at the seakeeping and manoeuvring basin in Kyushu University, by means of Rotating Arm Test and Oblique Towing Test. The coordinate system used in measurement is shown in Figure 2.1.

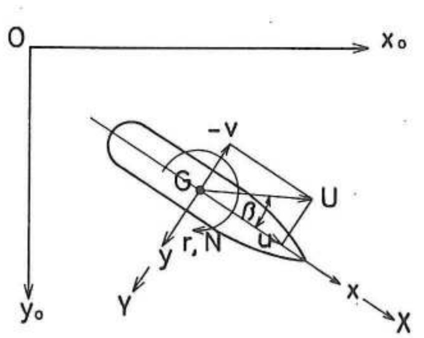

Figure 2.1. Coordinate systems

2.2. Measured results

The non-dimensional lateral force (Y') and yaw moment (N') around midship are here assumed as follows,

$$Y'(\beta,r') = Y'_\beta \beta + Y'_r r' + f_Y(\beta,r') \tag{2-1}$$

$$N'(\beta,r') = N'_\beta \beta - N'_r r' + f_N(\beta,r') \tag{2-2}$$

where $f_Y(\beta,r')$ and $f_N(\beta,r')$ represent the non-linear term respectively. The linear derivatives are shown in Figures 2.2 ~ 2.5 as function of aspect ratio by fitting the measured results to the above equations. All symbols be written in these figures represent the ships shown in Table 1. In these figures, the change of load condition in each ship is represented by a function of aspect ratio. According to these results, it should be noted that the linear derivatives except Y'_β may be represented by a fixed line with a function of aspect ratio, and furthermore on Y'_β the slope of the chain line which connected points depend on load condition are almost equivalent to that in each ship.

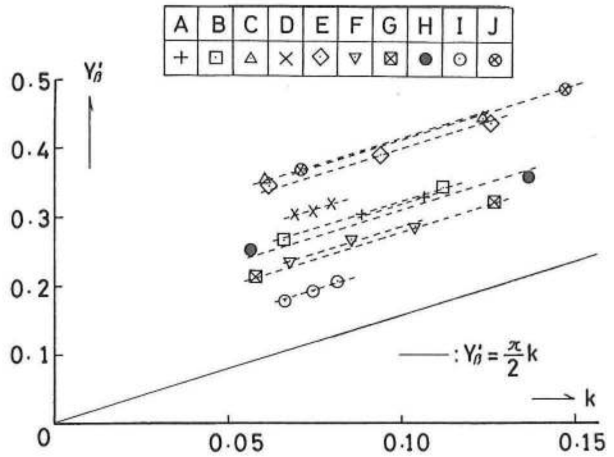

Figure 2.2. Derivative of lateral force in even keel condition for oblique motion.

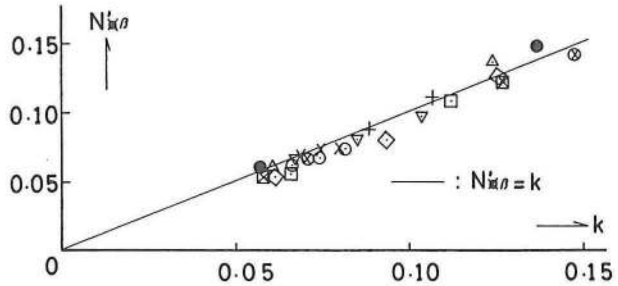

Figure 2.3. Derivative of yaw moment in even keel condition for oblique motion.

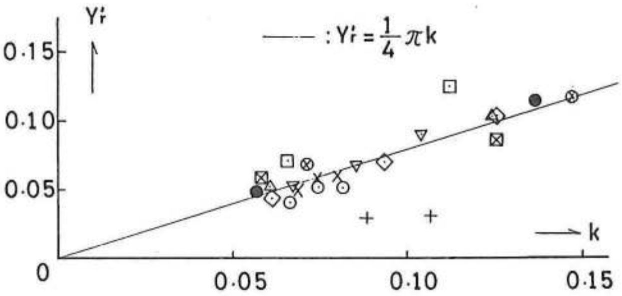

Figure 2.4. Derivative of lateral force in even keel condition for turning motion.

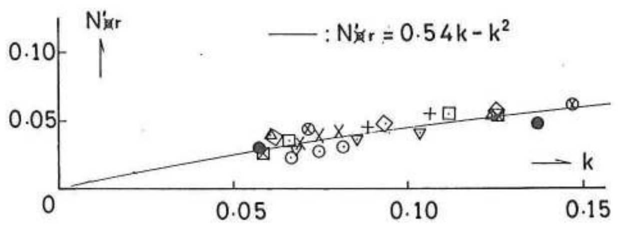

Figure 2.5. Derivative of yaw moment in even keel condition for turning motion.

One of the authors has proposed the approximate formulae for estimating the linear derivatives of the lateral force and yaw moment on bare hull by applying the lifting surface theory as follows,

Table 1

Main particulars of model ship

Name of ship		A	B	C	D	E	F	G	H	I	J
L(m)		2.500	2.500	2.500	2.500	2.500	2.500	2.500	2.500	2.500	2.500
B(m)		0.482	0.419	0.466	0.409	0.436	0.386	0.376	0.408	0.367	0.556
L/B		5.188	5.962	5.368	6.111	5.741	6.474	6.649	6.124	6.812	4.500
	dm(m)	0.134	0.140	0.156	0.100	0.157	0.130	0.158	0.171	0.102	0.183
	C_B	0.522	0.698	0.835	0.714	0.802	0.566	0.651	0.773	0.557	0.821
Full even	Δ(Kg)	84.25	102.98	151.26	73.04	137.52	71.20	96.62	135.32	52.50	209.04
	k	0.107	0.112	0.125	0.080	0.126	0.140	0.126	0.137	0.082	0.147
	$\otimes G/L$	+0.0198	−0.0060	−0.0268	−0.0056	−0.0299	+0.0229	+0.0147	−0.0116	+0.0357	−0.0400
	dm(m)				0.093	0.117	0.107			0.093	
	C_B				0.707	0.782	0.540			0.537	
Half even	Δ/Kg				67.25	99.76	55.77			45.85	
	k				0.074	0.094	0.086			0.074	
	$\otimes G/L$				−0.0060	−0.0379	+0.0187			+0.0324	
	dm(m)	0.111	0.082	0.076	0.086	0.077	0.085	0.072	0.071	0.083	0.089
	C_B	0.491	0.666	0.802	0.703	0.761	0.516	0.574	0.771	0.512	0.783
Ballast even	Δ(Kg)	65.90	57.51	71.00	62.12	63.60	42.32	39.09	51.80	39.13	96.62
	k	0.089	0.066	0.061	0.069	0.061	0.068	0.058	0.057	0.067	0.071
	$\otimes G/L$	+0.0126	−0.0122	−0.0380	−0.0062	−0.0435	+0.0167	+0.0070	−0.0250	+0.0300	−0.0540
Ballast trim	τ/dm	+0.225	+0.608	+0.329	+0.289	+0.326	+0.295	+0.691	+0.700	+0.300	+0.281
	$\otimes G/L$	+0.0273	+0.0200	−0.0170	+0.0112	−0.0241	+0.0310	+0.0421	+0.0156	+0.0506	−0.0362

$$Y'_\beta \doteqdot K_1 \pi k$$
$$N'_\beta \doteqdot k$$
$$Y'_r \doteqdot \pi k/4 \qquad (2\text{-}3)$$
$$N'_r \doteqdot 0.54\, k$$

The above formulae will be available for full load and even keel conditions comparing with the measured results, but not so available for another condition, especially on Y'_β. However, the results obtained by using the non-linear lifting surface theory of Bollay [6] are fairly available for fine ship in even keel condition. We have used K_1 as the form factor for Y'_β in the above formulae for applications to all kind of ships.

As shown in Figure 2.2, the slope of Y'_β as function of aspect ratio will be almost $\pi/2$. It is interesting that this value is the same with one obtained by the low aspect ratio wing theory of Weisinger [7]. In the derivatives except Y'_β, the measured values agree well with the theoretical results by this method.

Now, it may be considered that Y'_β will be represented by the term depend on lift and cross flow, and the latter will depend on 'Newtonian Law', namely

$$Y'_\beta = \pi\, k/2 + f \qquad (2\text{-}4)$$

The function f will depend on the difference between lateral forces acting on starboard and port sides at the near bow of ship, and also depend on the water plane and transverse plane at the near bow without depending on aspect ratio. Furthermore, if this is assumed to be a function of $C_B B/L$, f may be represented by the following approximate formulae as shown in Figure 2.6.

$$f = 1.4\, C_B B/L$$

According to the measured results, the estimation shown in equation (2-3) will be almost useful for practical application to the derivatives except Y'_β.

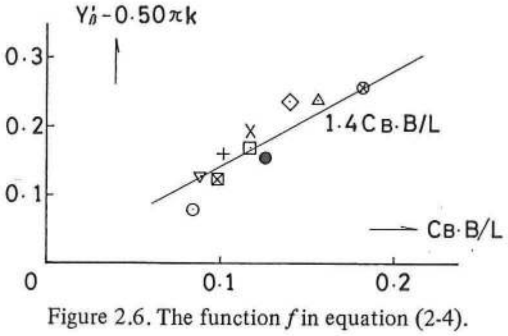

Figure 2.6. The function f in equation (2-4).

3. Non-linear derivatives of hydrodynamic force in even keel condition

As mentioned in the introduction, it will be not enough to analyze the ship manoeuvring by only means of the linear terms of the manoeuvring mathematical model, especially when the ship is in manoeuvring motion with a large rudder angle. In this case, the non-linear terms in mathematical model for the force and yaw moment will account for the main part. On the other hand, the problem on description of hydrodynamic force acting on bare hull has also been discussed. We can say, for example, whether the non-linear term in mathematical model to describe it should be represented on practical applications by mainly 2nd, 3rd or much more power terms in function of drift angle β and angular velocity r'. However, it will be not so easy to make clear this non-linear terms by the pure theoretical approach, especially on comparison the measured results by the model ship with the theoretical results.

Therefore, the non-linear terms of lateral force and yaw moment acting on bare hull are here investigated by using ten model ships shown in Table 1.

3.1. *Model test and measured results*

Model ships used here are shown in Table 1, and the experiments are carried out by the above method. If the equation (2-1) and (2-2) are used for the lateral force and yaw moment acting on hull, the linear terms are rewritten by the results of the chapter 2 as follows,

$$Y'_\beta \doteqdot \frac{1}{2}\pi k + f(C_B B/L)$$
$$Y'_r \doteqdot \frac{1}{4}\pi k$$
$$N'_\beta \doteqdot k \qquad (3\text{-}1)$$
$$N'_r \doteqdot 0.54\, k - k^2$$

where the approximate formula for N'_r, which will be affected considerably by plan form of force and aft parts of ship, is corrected to let agree with the measured results.

It will be supposed that f_Y and f_N, non-linear terms, depend on the terms of β^2, $\beta r'$ and r'^2 in the main by applying the cross flow theory or Newtonian Law. From the measured results, f_Y also may be described by the terms of β^2, $\beta r'$ and r'^2, however it is not enough to describe f_N by only those terms. As a result of some examinations for fitting the measured results to the perturbation model, f_Y and f_N may be described as follows,

$$f_Y = Y_{\beta\beta}\beta|\beta| + Y_{\beta r}\beta|r'| + Y_{rr}r'|r'|$$
$$f_N = N_{rr}r'|r'| + (N_{rr\beta}r' + N_{\beta\beta r}\beta)\,\beta r' \qquad (3\text{-}2)$$

The derivatives obtained by using the above description are shown in Figures 3.1 to 3.6 as function of $(1 - C_B)/(B/d)$. It goes without saying that we may be able to find out much better description with another function. A difference of load condition in each ship is expressed by a parameter d, and a solid line shows the average of measurements.

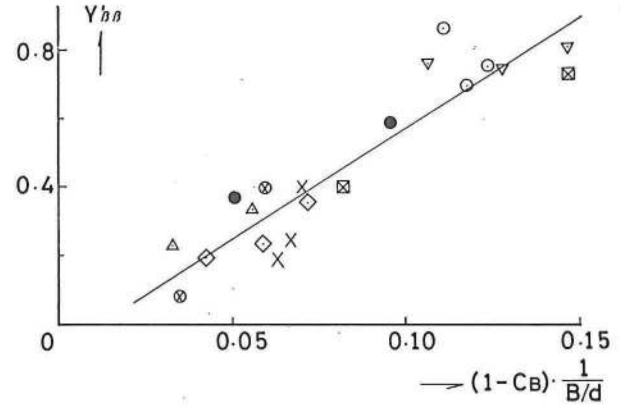

Figure 3.1. Non-linear derivative of lateral force as function of $d(1-C_B)/B$.

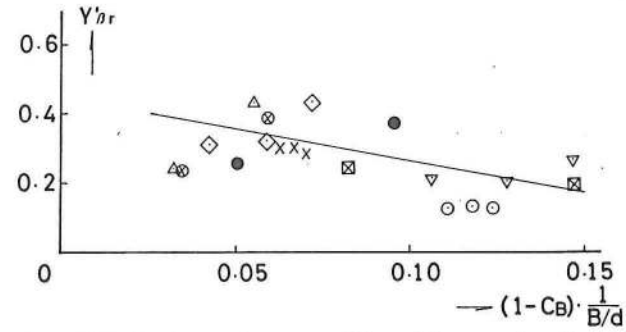

Figure 3.2. Non-linear derivative of lateral force as function of $d(1-C_B)/B$.

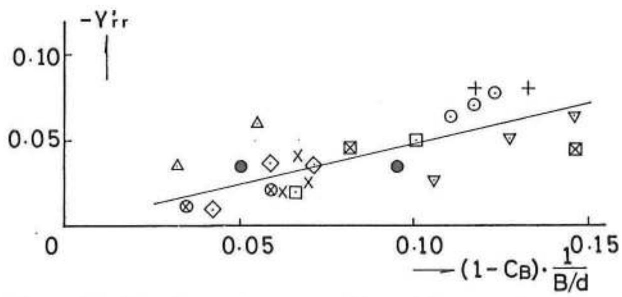

Figure 3.3. Non-linear derivative of lateral force as function of $d(1-C_B)/B$.

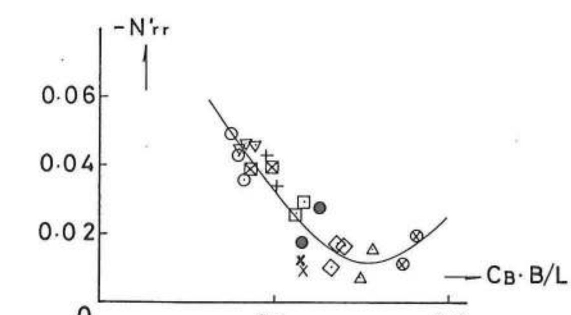

Figure 3.4. Non-linear derivative of yaw moment as function of $C_B B/L$.

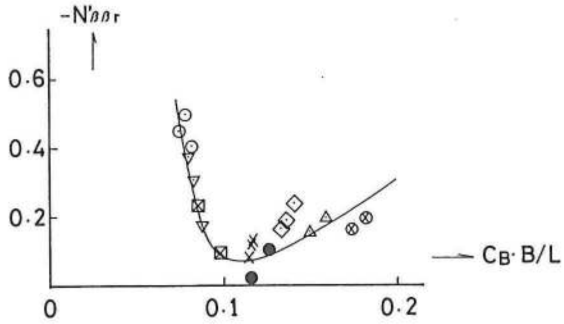

Figure 3.5. Non-linear derivative of yaw moment as function of $C_B B/L$.

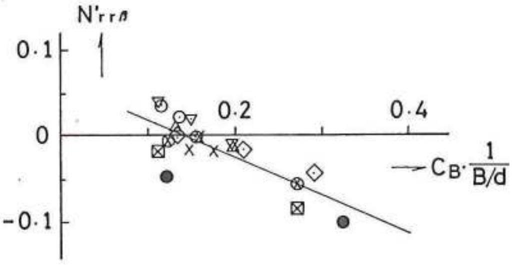

Figure 3.6. Non-linear derivative of yaw moment as function of $d \cdot C_B/B$.

3.2. *Measured and approximate results*

We can now estimate the hydrodynamic force acting on bare hull by using formulae (3-1) and (3-2) representing linear term and non-linear term respectively. The comparison the measured results of model ships with the results used by the above approximate formulae on a cargo ship and a oil tanker are shown in Figures 3.7 ~ 3.14 as a function of drift angle and angular velocity. According to these figures, the almost close agreement between measured and calculated values is obtained, but the agreement between those is not so well for N value of cargo ship in ballast load and even keel conditions in large angular velocity. However this agreement is not so serious problem because the force in large angular velocity will be not required for cargo ship.

Among the non-linear terms in formulae (3-2) when as a example $r' = 1.0$ and $\beta = 20°$, the second term in f_Y, namely $Y_{\beta r}\beta|r'|$, dominates and accounts for 50 percent of the total value in non-linear terms. And in this case, the value of linear term in Y' is the same order with it of the non-linear terms, but in N' non-linear term shows a value ten times as much as linear term.

4. Hydrodynamic derivatives in trimmed condition

We will be able to evaluate the manoeuvrability of ship by full-scale manoeuvring trials. As one of examples, the trial condition in general cargo ship will be almost ballast and trim by stern conditions. And the

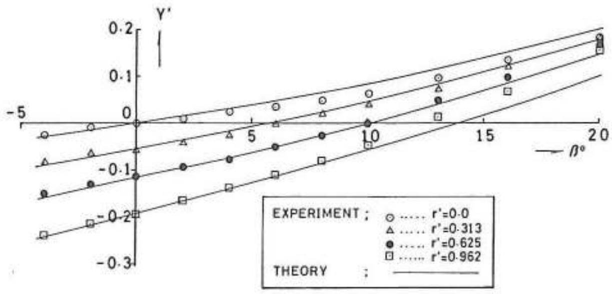

Figure 3.7. Comparison the measured results with the computed results by using the approximate formulae for the lateral force acting on a cargo ship 'H' in full load and even keel conditions.

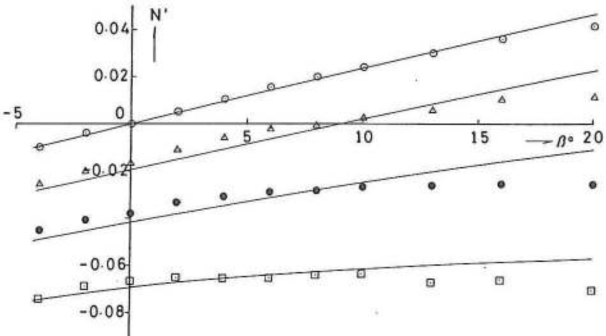

Figure 3.8. Comparison the measured results with the computed results by using the approximate formulae for the moment acting on a cargo ship 'H' in full load and even keel conditions.

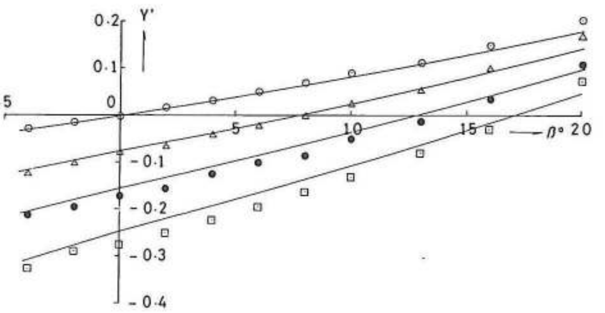

Figure 3.9. Comparison the measured results with the computed results by using the approximate formulae for the lateral force acting on a oil tanker 'C' in full load and even keel conditions.

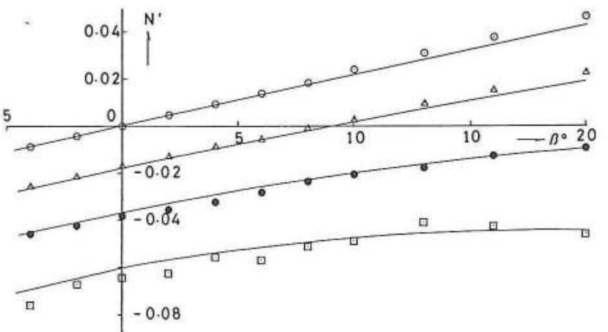

Figure 3.10. Comparison the measured results with the computed results by using the approximate formulae for the moment acting on a oil tanker 'C' in full load and even keel conditions.

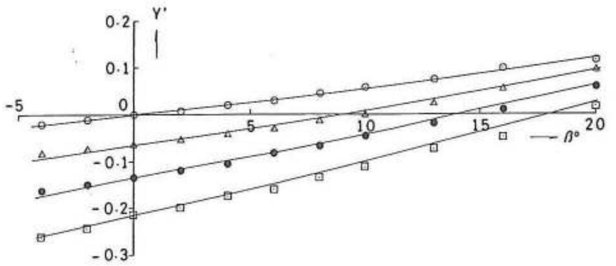

Figure 3.11. Comparison the measured results with the computed results by using the approximate formulae for the lateral force acting on a cargo ship 'H' in ballast load and even keel conditions.

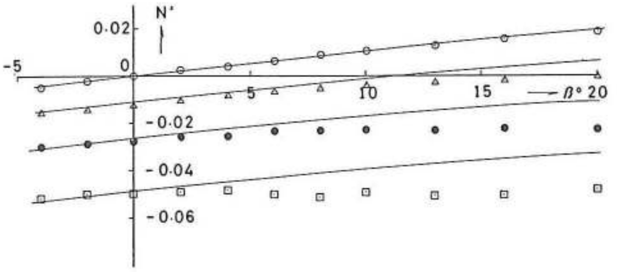

Figure 3.12. Comparison the measured results with the computed results by using the approximate formulae for the moment acting on a cargo ship 'H' in ballast load and even keel conditions.

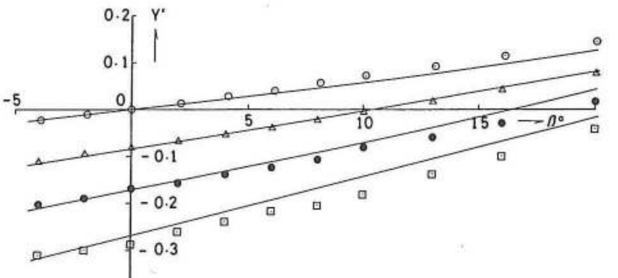

Figure 3.13. Comparison the measured results with the computed results by using the approximate formulae for the lateral force acting on a oil tanker 'C' in ballast load and even keel conditions.

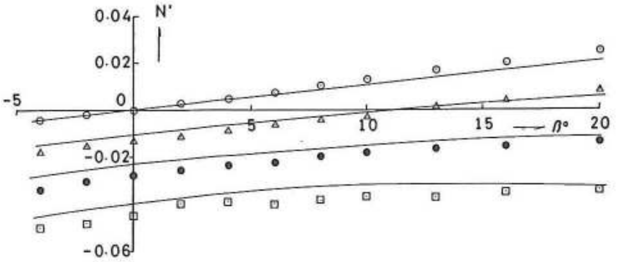

Figure 3.14. Comparison the measured results with the computed results by using the approximate formulae for the acting on a oil tanker 'C' in ballast load and even keel conditions.

trials for oil tanker will be carried out in full load condition. We can not evaluate the manoeuvrability of ship in full load condition from the data of trial in ballast load condition. However, we have to know the manoeuvrability in full load, also ballast load and any other load condition for evaluation of ship performance.

And this manoeuvrability can be estimated by using the hydrodynamic force acting on ship. We can understand in the preceding chapter that the hydrodynamic force on ship is greatly affected by a load condition. This will make no difference for a ship being in even keel and also trimmed conditions. In this chapter, it has made clear the relations between this force and trimmed condition experimentally, and simultaneously the theoretical approach about it has been executed.

4.1. *Model test*

Model tests were carried out by ships of Todd's Series 60 as shown in Table 2 and by several ships used in chapter 2. And the measurement is executed by means of Rotating Arm and Oblique Towing Tests as shown in preceding chapter. Test conditions are taken in trim by stern in the main to examine the relations between the hydrodynamic force acting on ship and the change of trim at the fixed displacement. The results are shown in Figures 4.1 ~ 4.4, 4.8 ~ 4.11 and 4.16 ~ 4.23.

4.2. *Theoretical approach*

The hydrodynamic forces acting on ship in trimmed and even keel conditions were dealt experimentally in the preceding section. Now, we try to estimate this force by the theoretical approach. We have well known some available theories, by the use of slender body theory or low aspect ratio lifting surface theory, to estimate hydrodynamic force on ship. The authors have also proposed the theoretical method based upon Bollay's low aspect ratio wing theory for estimation of hydrodynamic derivatives on ship manoeuvrability in deep water [8], shallow water [9] and narrow waterways [10, 11].

On the other hand, it is difficult to see some examples of work which dealt with hydrodynamic force on ship in trimmed condition. Accordingly, this section makes clear the relations between this force and trimmed condition by the theory based upon a slender body. This theoretical approach can be also applied for estimation of force on ship in even keel condition.

As fundamental conditions of the flow model, the following assumptions are here used for the flow around a hull.

Table 2

Main particulars of model ship

Todd Series 60 4210 W				
	L(m) = 2.500			
	B(m) = 0.333			
	L/B = 7.500			
Full even	dm(m)	0.133		
	C_B	0.600		
	Δ(Kg)	66.493		
	k	0.106		
		0.5d	0.6d	0.7d
even	dm(m)	0.067	0.080	0.093
	C_B	0.540	0.554	0.567
	Δ(Kg)	30.120	36.896	43.938
	k	0.053	0.064	0.075
	$\otimes G/L$	+0.006	+0.007	+0.008
trim	τ/dm	−0.233	−0.195	−0.168
	$\otimes G/L$	−0.004	−0.002	−0.001
	τ/dm	+0.233	+0.195	+0.168
	$\otimes G/L$	+0.017	+0.017	+0.017
	τ/dm	+0.467	+0.414	+0.337
	$\otimes G/L$	+0.028	+0.027	+0.026
	τ/dm	+0.933	+0.781	+0.672
	$\otimes G/L$	+0.054	+0.049	+0.044

Todd Series 60 4214 WB−4				
	L(m) = 2.500			
	B(m) = 0.385			
	L/B = 6.500			
Full even	dm(m)	0.154		
	C_B	0.800		
	Δ(Kg)	118.580		
	k	0.1232		
		0.5d	0.6d	0.7d
even	dm(m)	0.077	0.092	0.108
	C_B	0.762	0.771	0.779
	Δ(Kg)	56.474	68.272	80.977
	k	0.062	0.074	0.086
	$\otimes G/L$	−0.033	−0.032	−0.031
trim	τ/dm	−0.203	−0.170	−0.144
	$\otimes G/L$	−0.045	−0.042	−0.036
	τ/dm	+0.203	+0.170	+0.144
	$\otimes G/L$	−0.020	−0.020	−0.022
	τ/dm	+0.406	+0.340	+0.290
	$\otimes G/L$	−0.006	−0.009	−0.012
	τ/dm	+0.812	+0.679	+0.579
	$\otimes G/L$	+0.019	+0.016	+0.013

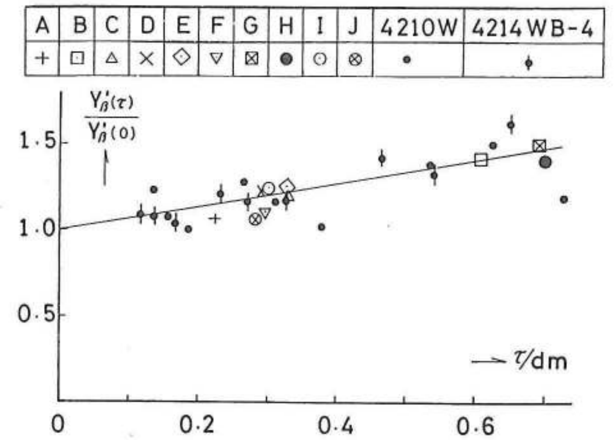

Figure 4.1. Derivative of lateral force in trimmed condition for oblique motion.

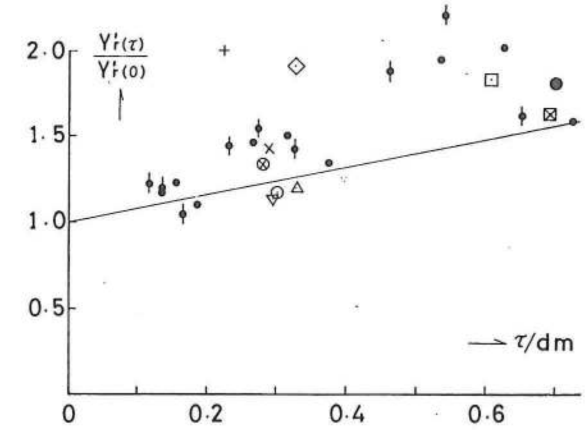

Figure 4.3. Rotary derivative of lateral forces in trimmed condition.

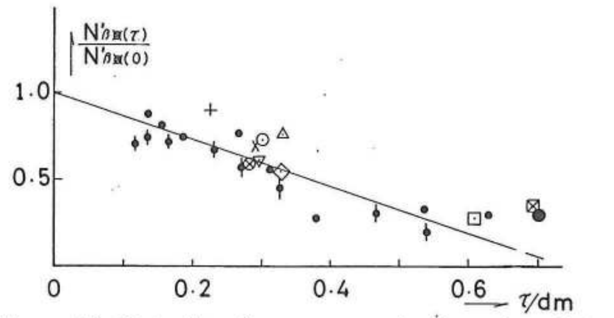

Figure 4.2. Derivative of yaw moment in trimmed condition for oblique motion.

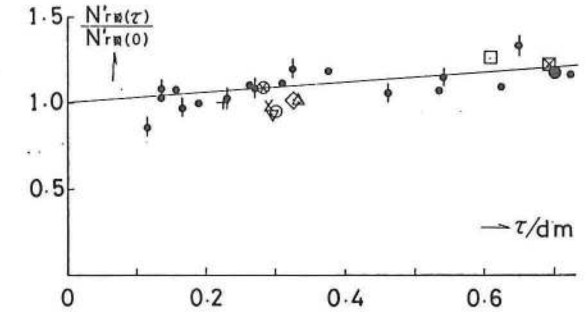

Figure 4.4. Rotary derivative of yaw moment in trimmed condition.

1. A bare hull has large length compared to her lateral dimensions, so called 'slender body'.
2. Froude number is sufficiently so small that free surface effects can be ignored.
3. Fluid is inviscid and incompressible.

In view of the rigid free surface assumption, a double body model can be analyzed, and the method used here such as reported in ITTC Proceeding [12] applied the method of Fuwa [13], and based on the slender body theory of Newman [14].

We consider now that the body motions are assumed to consist of a constant forward velocity U which parallel to the x_0 – axis of a space fixed coordinate system (x_0, y_0, z_0), and a small lateral motion in the y_0-direction. The lateral motion of the body is described by its displacement η from the x_0-axis. And the boundary conditions will take the following forms for the velocity potential due to the presence of the body and for the body surface S_B, as shown in Figures 4.5 and 4.6.

$$[L] \quad \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{4-1}$$

$$[S_B] \quad \frac{D}{Dt} F(x_0 - Ut, y_0 - \eta, z_0) = 0 \tag{4-2}$$

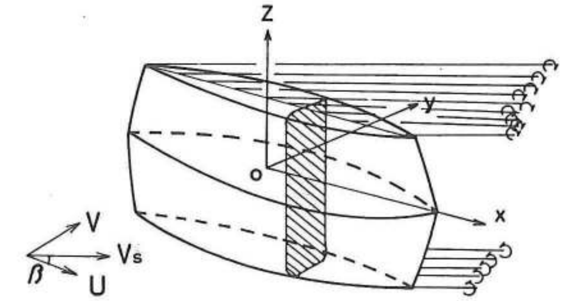

Figure 4.5. Coordinate system and flow model.

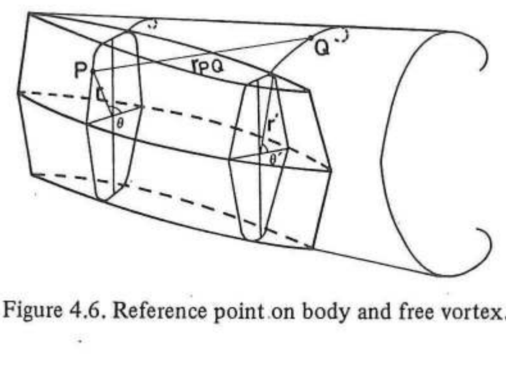

Figure 4.6. Reference point on body and free vortex.

$$[\infty] \quad \phi \to 0 \quad \text{at far field} \tag{4-3}$$

Here ϕ is the velocity potential due to the presence of the body, which defined initially with respect to the fixed coordinates. From the relation between the fixed and moving coordinates, we can obtain the following formula at the near body surface by using the slenderness approximation.

$$\frac{\partial^2\phi}{\partial y_0^2}+\frac{\partial^2\phi}{\partial z_0^2}=0 \tag{4-4}$$

Therefore, ϕ may be rewritten by the two dimensional velocity potential Φ.

If n is the unit normal vector into the body, the boundary condition (4.2) is rewritten as follows,

$$\frac{\partial\phi}{\partial n}=Un_x+\left(\frac{\partial\eta}{\partial t}-U\frac{\partial\eta}{\partial x}+\frac{\partial\phi}{\partial x_0}\frac{\partial\eta}{\partial x}\right)n_y \tag{4-5}$$

By the slenderness approximation, we get

$$\frac{\partial\Phi}{\partial N}=Un_x+\left(\frac{\partial\eta}{\partial t}-\frac{\partial\eta}{\partial x}\right)n_y \tag{4-6}$$

where N is the unit normal vector into the body in the cross section on body. And the following equations are here assumed.

$$\frac{\partial\Phi_1}{\partial n}=n_x \tag{4-7}$$

$$\frac{\partial\Phi_2}{\partial n}=n_y \tag{4-8}$$

$$V(x,t)=\frac{\partial\eta}{\partial t}-U\frac{\partial\eta}{\partial x} \tag{4-9}$$

If we can describe the following condition by the above assumptions,

$$\Phi = U\Phi_1+V\Phi_2 \tag{4-10}$$

the Φ will satisfy the boundary conditions (4.6) on the body surface. Therefore, the velocity potential Φ_1 due to the longitudinal motion and Φ_2 to the lateral motion will be that

$$\begin{aligned}&[L]\quad \frac{\partial^2\Phi_1}{\partial y^2}+\frac{\partial^2\Phi_1}{\partial z^2}=0\\ &[S_B]\quad \frac{\partial\Phi_1}{\partial N}=n_x\\ &[\infty]\quad \Phi_1\to 0\end{aligned} \tag{4-11}$$

and

$$\begin{aligned}&[L]\quad \frac{\partial^2\Phi_2}{\partial y^2}+\frac{\partial^2\Phi_2}{\partial z^2}=0\\ &[S_B]\quad \frac{\partial\Phi_2}{\partial N}=n_y\\ &[\infty]\quad \Phi_2\to 0\end{aligned} \tag{4-12}$$

By using the slenderness approximation, the problem on the near field about a body can be translated to two dimensional boundary value problem. The first approximate solution $\Phi^{(1)}$ will be a solution of the following equation.

$$\begin{aligned}&[L]\quad \frac{\partial^2\Phi_1^{(1)}}{\partial y^2}+\frac{\partial^2\Phi_1^{(1)}}{\partial z^2}=0\\ &[S_B]\quad \frac{\partial\Phi_1^{(1)}}{\partial n}=\frac{dn}{dx}=n_x\end{aligned} \tag{4-13}$$

This approximate solution by means of conformal mapping may be obtained. For also Φ_2, this conformal mapping will be taken directly.

Now, the wake vortex is assumed to leave as straight trailing vortex at some angle to the body surface from keel center line of the body on the flow field due to the wake as shown in Figure 4.1 by using Fuwa's model. Noting the point $P(x,r,\theta)$ on body surface and $Q(x',r',\theta')$ on the wake vortex sheet, the induced velocity $\vec{V}(P)$ due to the wake vortex at the point P may be written by the form

$$V(P)=\frac{1}{4\pi}\iint\frac{\vec{\omega}(Q)\times\vec{r}_{QP}}{r_{QP}^3}\,dS$$

$$=\frac{1}{4\pi}\int_{x_0}^{x_\infty}\int_{\theta_1}^{\theta_2}\frac{\vec{\omega}(Q)\times\vec{r}_{QP}}{r_{QP}^3}\,r'd\theta'dx' \tag{4-14}$$

where

$$\vec{r}_{QP}=(x-x')\vec{i}+(r\cos\theta-r'\cos\theta')\vec{j}+(r\sin\theta-r'\sin\theta')\vec{k}$$

$$r_{QP}=[(x-x')^2+r^2+r'^2-2rr'\cos(\theta-\theta')]^{1/2}$$

$\vec{i},\vec{j},\vec{k}$ represent unit vector.

And applying some assumptions of slenderness and the wake vortex, we can rewrite the equation (4-14) as follows by replacing

$$\vec{\omega}(\theta)=(\omega_1,\omega_2,\omega_3)$$

$$\vec{V}(P)\fallingdotseq$$

$$\frac{1}{2\pi}\int_{\theta_1}^{\theta_2}\frac{\omega_1^*r^*[-\vec{j}(r\sin\theta-r^*\sin\theta')+\vec{k}(r\cos\theta-r^*\cos\theta')]}{r^2+r^{*2}-2rr^*\cos(\theta-\theta')}d\theta' \tag{4-15}$$

where

$$r^*\equiv r(x,\theta')\qquad \omega_1^*\equiv\omega_1(x,\theta')$$

Therefore, the induced velocity due to the wake vortex at a point $P(x,r,\theta)$ in $x=x$ section can be obtained by x-axis component ω_1^* of the vorticity $\vec{\omega}(x,\theta')$ in $x'=x$ section.

We may consider approximately that the velocity induced on the body surface is induced only by the vortex on cross sectional plane of the body. Therefore, the integral equation (4-14) will be solved on each cross sectional plane to decide the vortex distribution. Then, the velocity potential represented the flow field due to the vortex sheet is also solved by means of conformal mapping. Simultaneously, after making some assumptions for the separation condition, the total velocity potential represented the flow field about body is decided.

4.3. *Calculation of hydrodynamic force*

Generally, the hydrodynamic force acting on the body in the body surface S_B to the control plane S_C is described by

$$\vec{F} = -\rho \frac{d}{dt_0} \iint_{S_B} \phi \vec{n}\, ds - \rho \iint_{S_C} \left(\nabla\phi \frac{\partial \phi}{\partial n} - \frac{1}{2} \nabla\phi \cdot \nabla\phi \cdot \vec{n} \right) dS \tag{4-16}$$

As shown in Figure 4.7, the hydrodynamic force acting on the unit length dx_0 of the body at $x = x$ is

$$F'_y dx_0 = -\rho \frac{d}{dt_0} \iint_{S_B} \phi\, n_y dS - \rho \iint_{S_0} \frac{\partial \phi}{\partial x_0} \cdot \frac{\partial \phi}{\partial y_0} dS - \rho \iint_{S_\infty} \left(\frac{\partial \phi}{\partial n} \frac{\partial \phi}{\partial y_0} - \frac{1}{2} n_y \nabla\phi\, \nabla\phi \right) dS \tag{4-17}$$

where

$dS = dy_0 dz_0$ on S_0 $\quad dS = dx_0 dl$ on S_∞

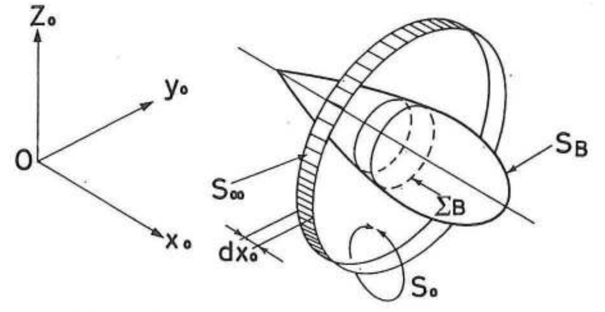

Figure 4.7. Coordinate system in space fixed.

By using the slenderness assumption, the relation between coordinates, and steady motion, the force may be

$$F'_y = \rho U \frac{\partial}{\partial x} \oint \phi\, n_y dl \tag{4-18}$$

Therefore, the lateral force (Y) acting on the body is described by

$$Y = \int_L F'_y\, dx$$

$$= \int_L \rho U \frac{d}{dx} (\oint \phi\, n_y\, dl)\, dx \tag{4-19}$$

Simultaneously, the yaw moment (N) around the mid-body is

$$N = \int_L (-x) F'_y\, dx \tag{4-20}$$

Thus the above mentioned procedure is restrained on the oblique motion of a body. In general case of manoeuvring motion, we must consider the parameters of drift angle and angular velocity. If the parameters β and r' are not so large, the drift angle at the point of x on keel center line of the body may be represented as follows,

$$\beta(x) = \beta + \frac{x}{R} = \beta + \frac{xr}{U} \tag{4-21}$$

By using this drift angle, we will be able to obtain the force acting on a body in turning motion. In this paper, it is assumed that the leaving angle of the wake vortex to the body surface is $\beta/2$ when the body is in oblique motion. And it is assumed in trimmed condition that the wake vortex flow along the bottom of ship.

4.4. *Measured and computed results*

The measured results are shown in Figures 4.1 ~ 4.4, on twelve model ships, which the ordinate represents the ratio of linear derivative for lateral force and moment in trimmed condition to that in even keel condition, the abscissa trim quantity. The abscissa, τ/d_m, expresses trim by stern for positive value.

The numerical calculations are carried out on the model of Todd's Series 60, and its results as shown in Figures 4.8 ~ 4.19 are almost in close agreement with

Model	Mean Draft	Aspect Ratio	Theory	Experi-ment
	0.5d	0.0543	—·—	○
4210W	0.6d	0.0640	----	△
	0.7d	0.0747	——	□

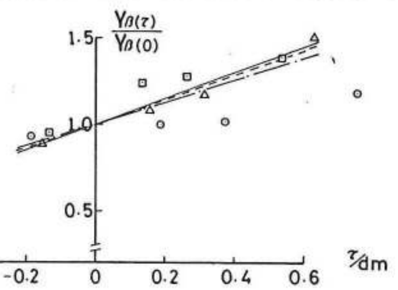

Figure 4.8. Derivative of lateral force in trimmed condition for oblique motion.

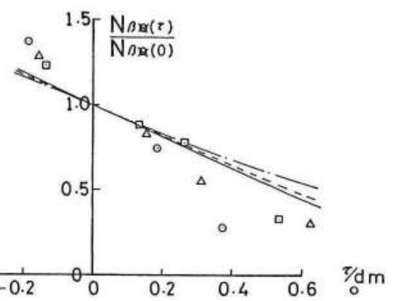

Figure 4.9. Derivative of yaw moment in trimmed condition for oblique motion.

Model	Mean Draft	Aspect Ratio	Theory	Experi-ment
4210W	0.5 d	0.0543	—·—	○
	0.6 d	0.0640	----	△
	0.7 d	0.0747	——	□

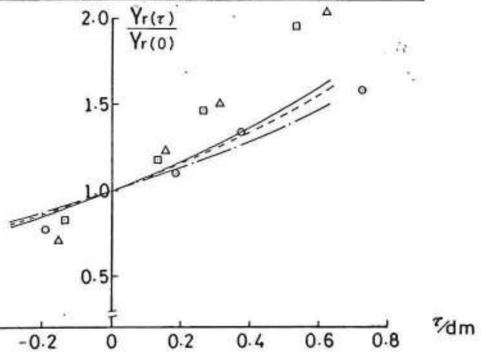

Figure 4.10. Rotary derivative of lateral force in trimmed condition.

Model	Mean Draft	Aspect Ratio.	Theory
4212W	0.5 d	0.0572	—·—
	0.6 d	0.0686	----
	0.7 d	0.0800	——

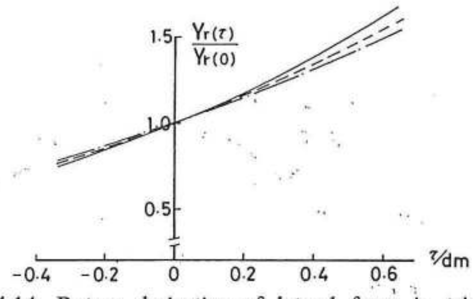

Figure 4.14. Rotary derivative of lateral force in trimmed condition.

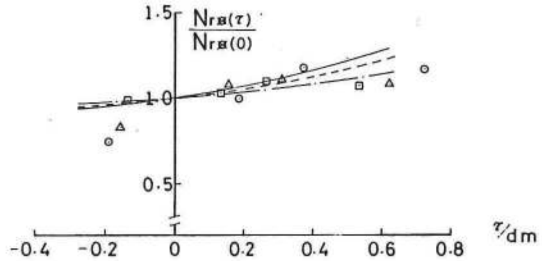

Figure 4.11. Rotary derivative of yaw moment in trimmed condition.

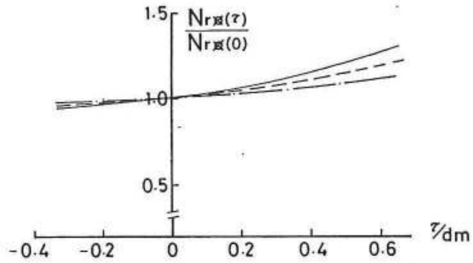

Figure 4.15. Rotary derivative of yaw moment in trimmed conditon.

Model	Mean Draft	Aspect Ratio	Theory
4212W	0.5 d	0.0572	—·—
	0.6 d	0.0686	----
	0.7 d	0.0800	——

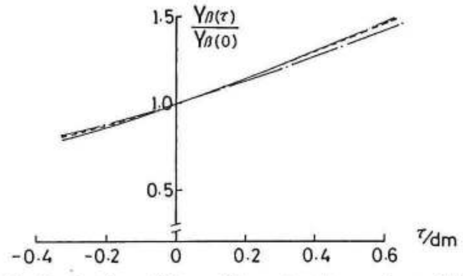

Figure 4.12. Derivative of lateral force in trimmed condition for oblique motion.

Model	Mean Draft	Aspect Ratio	Theory	Experi-ment
4214WB-4	0.5 d	0.0616	—·—	●
	0.6 d	0.0739	-----	▲
	0.7 d	0.0862	——	■
SR-154	1.0 d	0.1465		×
TANKER		0.0800		⌖

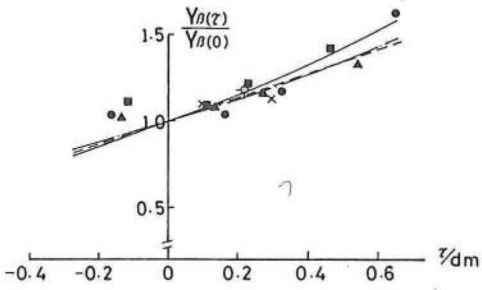

Figure 4.16. Derivative of lateral force in trimmed condition for oblique motion.

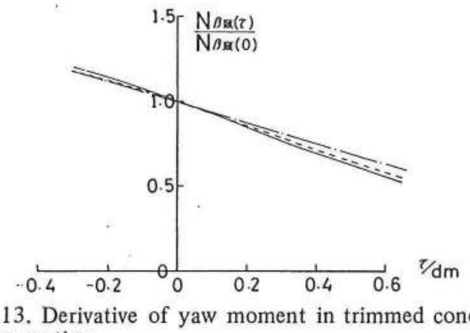

Figure 4.13. Derivative of yaw moment in trimmed condition for oblique motion.

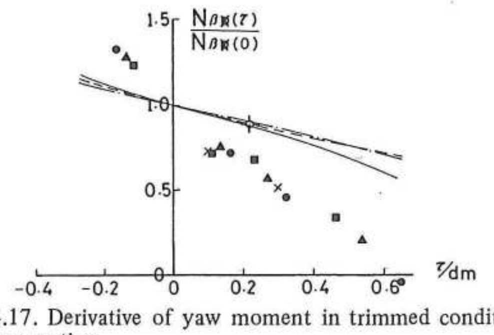

Figure 4.17. Derivative of yaw moment in trimmed condition for oblique motion.

Model	Mean Draft	Aspect Ratio	Theory	Experi -ment
4214WB -4	0.5 d	0.0616	— · —	●
	0.6 d	0.0739	— — — —	▲
	0.7 d	0.0862	———	■
SR-154	1.0 d	0.1465		×
TANKER		0.0800		✧

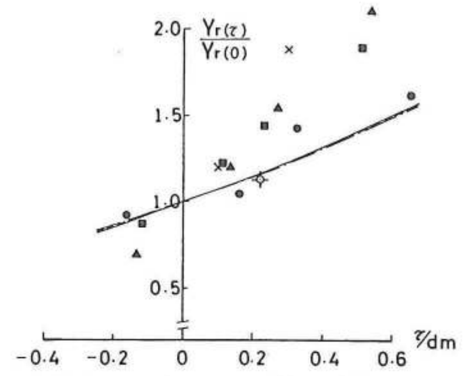

Figure 4.18. Rotary derivative of lateral force in trimmed condition for turning motion.

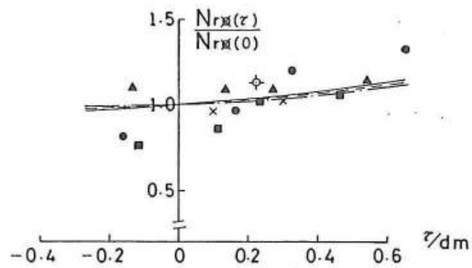

Figure 4.19. Rotary derivative of yaw moment in trimmed condition for turning motion.

that of the measured one except $N'_\beta(\tau)$. We understand that the hydrodynamic derivatives on ship manoeuvring are also greatly affected by trim quantity, and it can be considered that there is no effect of change of draft at a fixed τ/d_m to the linear derivatives in theoretical values.

Some investigations are here given to this problem because we can see noticeable errors between measured and computed results in the value of $N_\beta(\tau)/N_\beta(0)$. The moment due to the horizontal motion generally may be represented by two terms, one is the term due to Munk's moment and the other to lift. Now if Munk's moment is assumed to be not affected by trim quantity, we may consider the correction in the center of pressure of hydrodynamic force generated by vorticities.

With these results and some assumptions, we may represent the following approximate formulae based upon this theory for estimation a hydrodynamic derivative on ship manoeuvring in trimmed condition, as shown in Figures 4.20 ~ 4.23.

Model	Mean Draft	Aspect Ratio	Experi -ment
4210W	0.5 d	0.0543	○
	0.6 d	0.0640	△
	0.7 d	0.0747	□
4214WB -4	0.5 d	0.0616	●
	0.6 d	0.0739	▲
	0.7 d	0.0862	■
SR-154	1.0 d	0.1465	×
TANKER		0.0800	✧

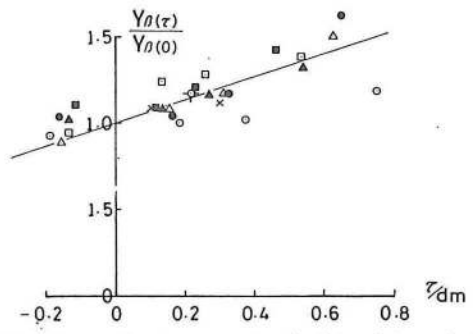

Figure 4.20. Approximate expression for lateral force derivative in trimmed condition.

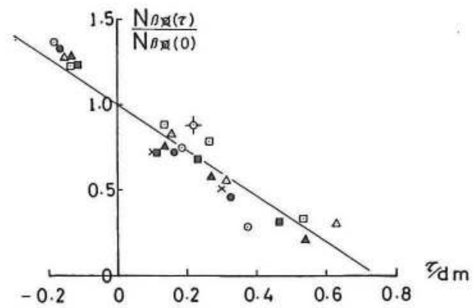

Figure 4.21. Approximate expression for yaw moment derivative in trimmed condition.

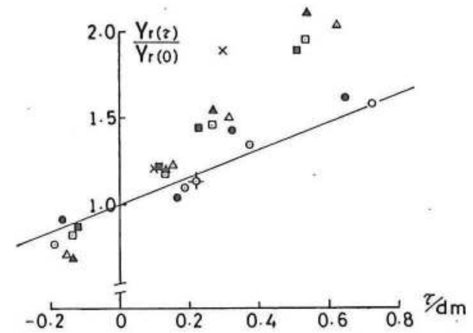

Figure 4.22. Approximate expression for lateral force derivative in trimmed condition.

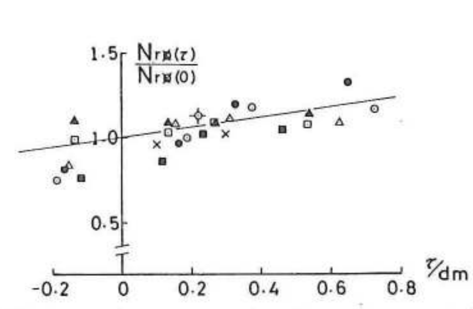

Figure 4.23. Approximate expression for yaw moment derivative in trimmed condition.

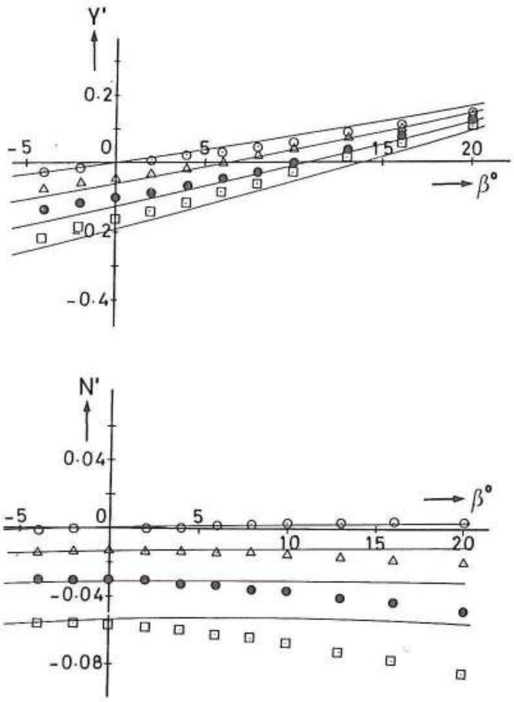

Figure 4.25. Comparison the measured with the computed results by using the approximate formulae for the lateral force and moment acting on a oil tanker 'C' in ballast and trimmed (1%L) conditions.

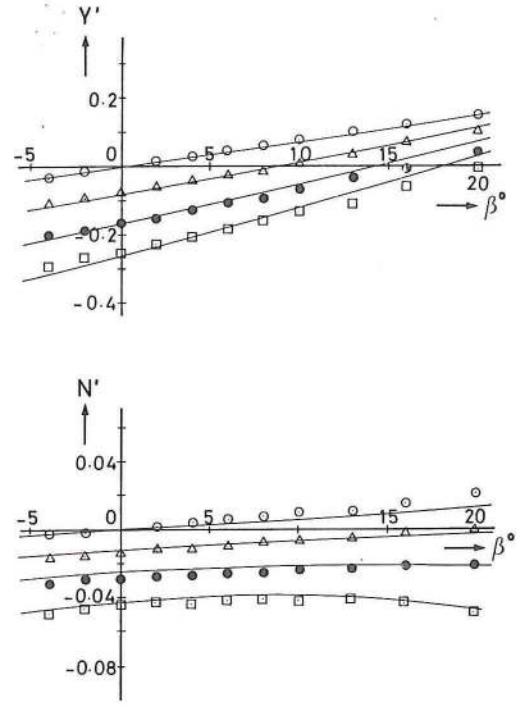

Figure 4.24. Comparison the measured with the computed results by using the approximate formulae for the lateral force and moment acting on a cargo ship 'H' in ballast and trimmed (2%L) conditions.

$$
\begin{aligned}
Y'_\beta(\tau) &= Y'_\beta(0)\left[1.0 + \frac{2\tau}{3d_m}\right] \\
N'_\beta(\tau) &= N'_\beta(0)\left[1.0 - \frac{0.27\tau}{l_\beta d_m}\right] \\
Y'_r(\tau) &= Y'_r(0)\left[1.0 + 0.80\frac{\tau}{d_m}\right] \\
N'_r(\tau) &= N'_r(0)\left[1.0 + 0.30\frac{\tau}{d_m}\right]
\end{aligned}
\tag{4-22}
$$

where $l_\beta = N'_\beta(0)/Y'_\beta(0)$

These approximate formulae show in almost close agreement with the measured results. The examples of comparison the measured results of model ships with the results used the above approximate formulae for this force on a cargo ship and a oil tanker are shown in Figures 4.24 and 4.25.

5. Concluding remarks

Thus the relations between derivatives of hydrodynamic force acting on ship in even keel and in trimmed conditions, or linear term and non-linear term for description of the force, are here made clear. We understand that this force is greatly affected by load condition as parameters of draft and trim. And this force may be sufficiently described by a linear term and almost 2nd and 3rd power terms for the perturbation model as the non-linear term of the manoeuvring mathematical model for the practical applications to analyze the ship manoeuvrability.

Making the approximate formulae to estimate the hydrodynamic force acting on ship form the whole measured and computed results, therefore, it will be obtained the following form.

$$
\begin{aligned}
Y' = {} & \left[\frac{1}{2}\pi k + f(C_B B/L)\right]\left(1 + \frac{2\tau}{3d_m}\right)\beta \\
& + \frac{\pi k}{4}\left(1 + 0.8\frac{\tau}{d_m}\right)r' \\
& + Y'_{\beta\beta}\beta|\beta| + Y'_{\beta r}\beta|r'| + Y'_{rr}r'|r'| \\
N' = {} & k\left(1 - \frac{0.27}{l_\beta}\frac{\tau}{d_m}\right)\beta \\
& - (0.54k - k^2)\left(1 + 0.30\frac{\tau}{d_m}\right)r' \\
& + N'_{rr}r'|r'| + (N'_{rr\beta}r' + N'_{\beta\beta r}\beta)\beta r'
\end{aligned}
$$

where $l_\beta = k/\left[\frac{1}{2}\pi k + f(C_B B/L)\right]$

These formulae include the function of draft, trim and aspect ratio of ship. The non-linear term in the above formulae can be estimated by the values shown in Figures 3.1 ~ 3.6.

Thus, we can analyze the ship's manoeuvrability, in also initial stage of ship design, by using the preceding results.

The next report will be presented about the prediction of manoeuvrability in full scale ship, simulations for turning test, spiral and zigzag tests etc., and will also present the available data for initial design of a ship.

Nomenclature

- L ship length
- B ship breadth
- d ship draft
- d_m mean draft of ship
- τ trim quantity (positive for trim by stern)
- C_B block coefficient
- U ship speed
- k aspect ratio
- β drift angle
- r angular velocity
- Y lateral force acting on ship
- N yaw moment about midship
- ϕ velocity potential
- n normal vector
- v velocity vector
- ω vorticity vector
- v_t^+ tangential velocoty vector on upper surface in vortex sheet
- v_t^- tangential velocity vector on lower surface in vortex sheet
- v_n^+ normal velocity vector on upper surface in vortex sheet
- v_n^- normal velocity vector on lower surface in vortex sheet
- τ_t tengential vector in separation line

Nondimension

$r' = r/R$ (R : turning circle of ship)

$$Y' = Y/\left(\frac{1}{2}\rho L d U^2\right)$$

$$N' = N/\left(\frac{1}{2}\rho L^2 d U^2\right)$$

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