LA-UR -84-2278

CONF-840519З--32

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36.

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LA-UR--84-2278

DE84 015513

Invited talk at the Conference on the Intersections Between Particle and Nuclear Physics, Steamboat Springs, Colorado, May 23-30, 1984.

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FORM NO 836 R4
ST NO 2629 5/81

HYDRODYNAMIC FLOW STRUCTURES IN QUANTUM FIELD THEORY

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ABSTRACT

The transport of matter is examined in the context of relativistic quantum transport theory for the case of neutral scalar fields. The goal is to formulate a theory valid off mass shell and out of equilibrium. We construct a conserved moment tensor which coincides with the ensemble average of the Noether tensor, or the improved energy-momentum tensor within an additive constant of the "improvement" term. Conditions for closure of the conservation equation are given for the ϕ^4 coupling.

Relativistic Quantum Transport Theory reformulates field theory in a kinetic form convenient for the exhibition of transport behavior and collective modes as well as classical behavior.[1,2] It also provides a natural bridge between field theory and hydrodynamics. Intuitive but mathematically complex, this approach to the relativistic many-body problem can, we believe, be used to place in context various models, limits, etc., which presently are used to obtain quantitative predictions. For a general density matrix ρ the single particle distribution function for a scalar field ϕ is defined by

$$F(p,R) = \int d^4r e^{ip\cdot r} \, \mathrm{Tr}\rho\phi(R - \tfrac{1}{2} r) \, \phi(R + \tfrac{1}{2} r) \tag{1}$$

where p is <u>not</u> necessarily on the mass shell (and is <u>not</u> an operator) and R is the effective space-time coordinate. The equation of motion for F is of kinetic form, leading to a hierarchy of coupled equations for distribution functions describing more and more "particles." As in nonrelativistic many-body theory, approximate methods can be used to identify (almost) normal modes (quasiparticles) in terms of which the dynamics of the system (sometimes) can be described. When ρ describes scattering ($\rho = |\psi_{in}\rangle\langle\psi_{in}|$ with normalized states), the knowledge of F(p,R) gives a complete knowledge of the single particle inclusive cross section. Hence, the solution to a kinetic problem for F(p,R) (which is an inclusive construct) allows the prediction of the inclusive cross section.

For simplicity we shall describe the dynamics of neutral scalar fields. However, the density matrix is typically nonequilibrium and lacks translation invariance. We are especially interested in the possibility of hydrodynamic behavior in the absence of LTE (local thermodynamic equilibrium) and for off-shell quanta. (If LTE exists in some region of space time, this can be ascertained by kinetic theory, although this problem is not addressed here.)

Throughout, our aim is to assess and correct the limiting assumptions in the usual hydrodynamic and kinetic theories.

Hydrodynamics uses local thermodynamic equilibrium, thermodynamics, equation of state (EOS), classical equations of motion and often omits viscosity, heat conduction and dynamical fluctuations. Conventional kinetic theory[3] uses mass shell particles, omits interaction energy in the energy-momentum tensor, and formulates dynamics in terms of (positive) single particle distribution, and often uses number currents (not always conserved) instead of energy momentum fluxes.

As stressed recently,[2] nonrelativistic quantum theory can be recast in fluid mechanics form regardless of equilibrium assumptions. For a one-particle problem the first three moments of the Wigner function $f(\underline{p},\underline{R},t)$ suffice. $\int d^3p f$ is the particle density, $\int d^3p p f$ is the current and $\int d^3p p_i p_j f$ gives the pressure tensor. In the N-particle problem, the problem is similar except that the 2-particle distribution occurs in the pressure tensor. Of course the hard work lies in calculating these distribution functions especially for the N-body problem.

In order to illuminate these ideas for a relativistic field theory, we consider a neutral scalar field ϕ with polynomial interaction L_I.

$$L = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + L_I \quad , \qquad L_I = \sum \frac{1}{d} g_d \phi^d \tag{2}$$

the sum proceeding over integral $d > 2$. We shall use the notation $j = \Sigma g_d \phi^{d-1}$ (Klein-Gordon current). The conserved (Noether's) energy momentum tensor for (1) is $T_{\mu\nu} \equiv T_{\mu\nu}^{K} + T_{\mu\nu}^{V}$

$$T_{\mu\nu}^{K} = \partial_\mu\phi\partial_\nu\phi + \frac{1}{2}m^2 g_{\mu\nu}\phi^2 - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2 \tag{3}$$

$$T_{\mu\nu}^{V} = -g_{\mu\nu}L_I$$

The breakup into kinetic (K) and potential (V) pieces is partly notational owing to the equations of motion. The Noether construction does not give a unique conserved tensor. Various improvements or corrections are needed (symmetrization terms, anomalies) and the present case (spin zero) is no exception. As defined by (3) $T_\mu{}^\mu$ is highly singular. This can be cured[4,5] by adding a conserved gradient term $\Delta T_{\mu\nu}$

$$T_{\mu\nu} \to \theta_{\mu\nu} = T_{\mu\nu} + \Delta T_{\mu\nu} \tag{4}$$

$$\Delta T_{\mu\nu} = \frac{1}{6}(g_{\mu\nu}\Box - \partial_\mu\partial_\nu)\phi^2 \tag{5}$$

The improved tensor $\theta_{\mu\nu}$ obeys the <u>canonical</u> trace theorem

$$\theta^\mu{}_\mu = \sum_d (4 - d)L^d \tag{6}$$

where L^d now includes the mass term. Canonically, $\theta^\mu{}_\mu$ differs from zero when scale breaking ($d \neq 4$) scalars appear in the Lagrangian. (However corrections due to anomalous dimension and perturbative anomalies must be made in general. These are ignored here.)

The generators of the conformal group have a simple form in terms of $\Theta_{\mu\nu}$. Here we assume that in flat space it is $\Theta_{\mu\nu}$ whose components give directly such physical quantities as pressure, energy density, etc.

Next consider momentum moments of the distribution function (1).

$$\int dp\ F(p,R) = \langle\phi^2(R)\rangle \tag{7}$$

$$\int dp p_\mu\ F(p,R) \equiv j_\mu(R) = -\frac{1}{2}\, i\langle\phi(R)\, \overleftrightarrow{\partial}_\mu \phi(R)\rangle = 0 \tag{8}$$

$$\int dp p_\mu p_\nu\ F(p,R) \equiv t_{\mu\nu}^{K} = \frac{1}{2}\langle\partial_\mu\phi\partial_\nu\phi - \phi\partial_\mu\partial_\nu\phi\rangle \tag{9}$$

Here dp is shorthand for $d^4p/(2\pi)^4$. Eq. (9) is the analoque of the classical energy momentum tensor. Calculating $\partial^\mu t_{\mu\nu}^{K}$ we can define a "potential" tensor

$$t_{\mu\nu}^{V} \equiv g_{\mu\nu} \sum_d \frac{d-2}{2}\langle L_I^d\rangle \quad , \qquad L_I^d = \frac{1}{d}\, g_d \phi^d \tag{10}$$

such that $t_{\mu\nu} = t_{\mu\nu}^{K} + t_{\mu\nu}^{V}$ is conserved

$$t_{\mu\nu} = \frac{1}{2}\langle\partial_\mu\phi\partial_\nu\phi - \phi\partial_\mu\partial_\nu\phi\rangle + g_{\mu\nu} \sum \frac{d-2}{2}\ \langle L_I^d\rangle \tag{11}$$

This expression looks quite different from $\langle T_{\mu\nu}\rangle \equiv \mathrm{Tr}\rho T_{\mu\nu}$ computed from (3):

$$\langle T_{\mu\nu}\rangle = \langle\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\, g_{\mu\nu}(\partial\phi)^2\rangle + \frac{1}{2}\, m^2 g_{\mu\nu}\langle\phi^2\rangle - g_{\mu\nu}\langle L_I\rangle \tag{12}$$

Nevertheless one easily establishes the connections

$$\begin{aligned} t_{\mu\nu} &= \langle T_{\mu\nu}\rangle + \frac{3}{2}\langle\Delta T_{\mu\nu}\rangle \\ &= \langle\theta_{\mu\nu}\rangle + \frac{1}{2}\langle\Delta T_{\mu\nu}\rangle \end{aligned} \tag{13}$$

where the difference depends on

$$\langle\Delta T_{\mu\nu}\rangle = (g_{\mu\nu}\Box - \partial_\mu\partial_\nu) \int dp\ F(p,R) \tag{14}$$

(the latter vanishing for a uniform system).

Note that to this point all equations are completely general for the chosen system. No assumption has been made that ρ corresponds to a "fluid"-type situation.

To condense our discussion we specialize to the ϕ^4 theory. Now we write ($g_4 \to \lambda$ here)

$$t_{\mu\nu} = \int dp\; p_\mu p_\nu\; F(p,R) + \lambda g_{\mu\nu} \int dp dp'\; F_2(pR,p'R) \tag{15}$$

$$F_2(pR,p'R') \equiv \int d^4r d^4r' e^{ip\cdot r + ip'\cdot r'} \langle\phi(R - \tfrac{1}{2}r)\, \phi(R + \tfrac{1}{2}r)\, \phi(R' + \tfrac{1}{2}r')\, \phi(R' - \tfrac{1}{2}r')\rangle \tag{16}$$

Thus the zeroth moment of the 1-particle distribution F_2 along with the second moment of the 2-particle distribution $F(p,R)$ are needed for the construction of $t_{\mu\nu}$. These objects are connected by transport-like equations of motion which shall not be discussed here. Note that the trace theorem gives[4,6,7]

$$\langle\theta_{00}\rangle - \sum \langle\theta_{ii}\rangle = m^2\langle\phi^2\rangle = m^2\int dp\; F(p,R) \tag{17}$$

with energy density $\varepsilon = \langle\theta_{00}\rangle$ and pressure density $\langle\theta_{xx}\rangle$. Referred to the local rest frame of the fluid, (17) is essentially an equation of state (without equilibrium) becoming "ideal" ($\varepsilon = 3p$) as $m \to 0$, despite the presence of interactions. Hence <u>an ideal equation of state does not imply the absence of interactions</u>, a point recently stressed for the case of the QCD plasma.[8]

Although expressions such as (15) are simple, they tacitly involve an infinite number of degrees of freedom. Not only the initial density matrix but the dynamical evolution of the system must be considered to determine the validity of truncation (to refer generically to the problem of the reduction of the number of degrees of freedom). For orientation refer to the perfect relativistic fluid with $T_{\mu\nu} = (\varepsilon + p)\, u_\mu u_\nu - g_{\mu\nu}p$ with ε and p Lorentz scalars (the local rest frame energy and momentum densities) and u^μ the fluid four-velocity $u^2 = 1$. These five independent variables subject to the four EOM $\partial^\mu T_{\mu\nu} = 0$ and the equation of state $p = p(\varepsilon)$ are soluble once boundary conditions are specified.

In the general case $t_{\mu\nu}$ has ten independent components subject to only four equations of motion. To obtain a closed system, what should be done? Without analyzing the most general case, consider the traditional method of ignoring two body correlations, i.e., $F_2(pR,p'R') = F(pR)\ F(p'R')$. In this case one can imagine solving the kinetic (Vlasov) equation for F(p,R) or better, its Fourier transform

$$2p\cdot q\ F(p,q) = \lambda \int dp'dq'\ F(p'q')[F(p - \frac{1}{2}q', q - q') - F(p + \frac{1}{2}q', q - q')] \tag{18}$$

for F(p,q) subject to a given boundary condition.

Since F is known, Eq. (17) provides one constraint on $t_{\mu\nu}$ (this is the generalized analogue of the EOS). Now $t_{\mu\nu}$ (Eq. 15) is expressed in terms of F alone. To investigate further constraints we define a unit four velocity via

$$u_\mu = \Theta_{\mu\nu}u^\nu/(u^u u^\nu \Theta_{\mu\nu}) \tag{19}$$

tacitly understanding that the ensemble average of $\Theta_{\mu\nu}$ is meant. Since $u^2 = 1$ we can transform to the comoving frame to determine constraints which are enforced in all frames by covariance. The assumption of isotropy implies the existence of only four independent components of $t^K_{\mu\nu}$, say t^K_{00}, t^K_{0x}, t^K_{xx}, t^K_{xy}. Hence the equations of motion involve the five functions $\int dp\ F$ and four independent components of $\int dp\ p_\mu p_\nu\ F$. Thus factorization plus comoving isotropy allow closure and hydrodynamic structure without imposing thermal equilibrium or mass shell moments. Isotropy is, of course, a rather strong assumption: among other things it precludes viscous shear stresses for a general flow. The aforementioned closure required the knowledge of the single particle distribution function. This knowledge allowed us to obtain hydrodynamic equations off-shell and out of equilibrium. But we have not yet succeeded in substituting a traditional hydrodynamic calculation for the kinetic theory. Analysis of this and other questions will appear elsewhere.

This research was supported by the U.S. Department of Energy.

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