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## HYDRODYNAMICS OF CIRCULATING FLUIDIZED BEDS: KINETIC THEORY APPROACH

by

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### ABSTRACT

Rigorous methods of kinetic theory were used to derive particular phase viscosities and granular conductivities. This new kinetic theory predicted flow behavior and oscillations in a complete loop of a CFB. The results were compared to computations with imposed gas phase turbulence in the riser. The computations were repeated for production of synthesis gas from char.

The hydrodynamics of solids processing plants, such as circulating fluidized bed combustors that are being built to burn high sulfur coals is not well understood. As an aid for better design for such systems, a particulate multiphase Navier-Stokes' equation solver was developed. Particulate viscosities and solids pressures can be computed by a subroutine that solves a fluctuating kinetic energy equation for the particles.

### Kinetic Theory Model

Savage (1) and Jenkins and Savage (2) have shown how to obtain constitutive equations and conservation laws for granular flow. Ding and Gidaspow (3) extended their approach to gas-particle flow. Since Ding and Gidaspow used a Maxwellian distribution their approach was restricted to dense flow. Recently Gidaspow extended this approach to cover dilute and dense particle-fluid flow. The mathematics is very similar to that described in detail in the classical text on kinetic theory by Chapman and Cowling (4). A summary of the techniques and the results is presented below.

Starting with the Boltzmann integral-differential equation for the frequency of particle velocity distributions,  $f$ , the dense phase transport theorem shown below was obtained by the authors cited above.

$$\frac{\partial}{\partial t} (n \langle \psi \rangle) + \nabla \cdot (n \langle \vec{c} \psi \rangle + P_c) = \langle n \vec{F}_s \cdot \frac{\partial \psi}{\partial \vec{c}} \rangle + N_c \quad (1)$$

where  $\vec{F}_s$  is the body and the drag force acting on the system,  $n = \int f d\vec{c}$  and  $\langle \psi \rangle = \frac{1}{n} \int \psi f d\vec{c}$

$$P_c = -\frac{1}{2} g_o(\epsilon_s) d_p^3 \int \int \int (\psi_1 - \psi'_1) f_1 f_2 \vec{k} (\vec{c}_{12} \cdot \vec{k}) d\vec{k} d\vec{c}_1 d\vec{c}_2 \quad (2)$$

$$-\frac{1}{4} g_o(\epsilon_s) d_p^4 \int \int \int (\psi_1 - \psi'_1) f_1 f_2 \nabla \ln \left( \frac{f_2}{f_1} \right) \vec{k} (\vec{c}_{12} \cdot \vec{k}) d\vec{k} d\vec{c}_1 d\vec{c}_2$$

and

$$N_c = \frac{d_p^2}{2} \int \int \int (\psi'_2 + \psi'_1 - \psi_2 - \psi_1)(\vec{c}_{12} \cdot \vec{k}) f^{(2)} d\vec{k} d\vec{c}_1 d\vec{c}_2 \quad (3)$$

The particle conservation of mass and momentum equations are obtained by setting  $\psi = m$  and  $\psi = m\vec{c}$ , respectively as done by Ding and Gidaspow and previous authors. The new part of the theory was to note that for the first perturbation upon the Maxwellian distribution the collision integral given by Equation (2) consists of two integrals which became as shown below.

$$P_c = P_{c_1} + P_{c_2} \quad (4)$$

$$P_{c_1} = \frac{2(1+e)}{5} g_o \rho_p \epsilon_s^2 (2 \langle \vec{C}\vec{C} \rangle + C^2 I) \quad (5)$$

$$P_{c_2} = 2\epsilon_s \mu_{s,c} \overset{o}{\nabla} \vec{v} + \epsilon_s \zeta_s \nabla \cdot \vec{v} I \quad (6)$$

where  $\overset{o}{\nabla}$  is the rate of shear tensor, [S] and  $\mu_{s,c}$  is the coefficient of viscosity, simply called viscosity by Ding and Gidaspow (3). For a Maxwellian distribution the first integral does not contribute to the viscosity since the non-diagonal terms of  $\langle CC \rangle$  vanish. The expressions for the viscosity and the granular conductivity given here have been generalized to the first perturbation for the frequency distribution. They are thus valid for dilute and dense flow. Equation (T1.5C) gives this shear viscosity. The standard dilute phase viscosity is given by Equation (T1.5d). Note that the kinematic viscosity is simply a product of the mean free path times a random velocity.

A conservation equation for this random velocity can be obtained by substituting  $\psi = \frac{1}{2} m \vec{c}^2$  into Equation (1). The result is the fluctuating energy equation for the granular temperature  $\theta$  given by Equation (T1.6). The conductivity given by Equation (T1.6c) was obtained from Equation (2) and the collisional dissipation  $\gamma$  due to the non-elasticity of the particles was obtained from Equation (3). The restitution coefficient "e" is the normal restitution coefficient. There also exists a tangential restitution coefficient associated with particle rotation that was not considered in the theory developed so far.

### CFB LOOP

The equations shown in Table 1 were solved for the loop shown in Fig 1. Fig 2 shows that 8 sec. after start-up, there is the expected dense flow in the downcomer, strong downflow at one wall of the riser, a reasonable value of the solids granular temperature and viscosity. Such reasonable viscosities and particle oscillations were achieved by adjusting the value of the restitution coefficient until the viscosity matched Miller's (5) measurements, as shown in Figure 2. Miller has also observed the fluctuations of the particle velocities, as computed in Fig. 4. The discharge velocity from the standpipe to the riser is shown in Fig. 5. The 1 m/sec velocity is of the order of discharge velocity from hoppers. The new kinetic theory model computations do not differ greatly from Ding's (6) earlier computations using the turbulent viscosity shown in Table 1 and dense kinetic theory (e.g. Fig. 5). Time averaged

values of the mass fluxes, solids concentrations and velocities are given in Figs. 6 to 9. At 2 m/sec there is a strong asymmetry caused by the inlet geometry. As expected, the bed is denser near the inlet. A color video shows the turbulent structure of the particles as a function of time.

### COAL REACTIONS

Ding's (6) turbulence model and dense kinetic theory were used to simulate the reactions of char particles for the conditions given in Fig. 1. Char was assumed to consist of 88% carbon. The CFB was initially filled with the  $N_2$  at  $1300^{\circ}K$ . At the riser bottom a mixture of 40%  $O_2$  and 60%  $H_2O$  entered at  $700^{\circ}K$ , at a pressure of 3.71 atm. Table 2 summarizes the energy and the species equations while Table 3 gives the principal reactions.

Figure 10 shows typical mole fractions profiles in the reactor 15 seconds after start-up. The oxygen was all consumed near the inlet. Fig. 11 shows some hot spots. Fig. 12 shows that the exit compositions reach a near steady state 10 seconds after start-up.

### ACKNOWLEDGEMENT

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### NOTATION

$c$ fluctuating velocity of particle	$P$ pressure tensor
$C$ fluctuating minus hydrodynamic velocity	$q$ flux vector of fluctuating energy
$C_d$ drag coefficient	$Re$ Reynolds number
$d_p$ particle diameter	$r_i$ reaction rate
$D$ diffusivity	$\underline{S}$ deformation rate tensor
$e$ coefficient of restitution	$T_g, T_s$ temperature of gas and solid
$g$ gravity	$u_g, u_s$ gas and solid velocity vector
$g_r$ radial distribution function	$Y_n$ weight fraction of gas of species $n$
$h_s$ volumetric heat transfer coefficient	$t$ time
$h$ enthalpy	
$\Delta H^0$ standard heat of reaction	<u>Greek Letters</u>
$K_{e,q}$ equilibrium constant of reaction	$\beta$ two phase drag coefficient
$K_{r,i}$ rate constant of reaction $i$	$\gamma$ collisional energy dissipation
$k_g, k_s$ thermal conductivities of the gas and solid	$\gamma_{nm}$ stoichiometric coefficient of species $n$
$M$ molecular weight	$\epsilon, \epsilon_s$ gas and solid volume fractions
$Nu$ gas particle Nusselt number	$\eta$ effectiveness factor for reaction $i$
$p$ pressure	$\kappa$ conductivity of granular temperature
	$\mu$ shear viscosity
	$\zeta$ bulk viscosity
	$\rho$ density
	$\theta$ granular temperature = $\frac{1}{3} \langle C^2 \rangle$

### LITERATURE CITED

1. Savage, S.B. in "Theory of Dispersed Multiphase Flow," ed. R.E. Meyer, Academic Press, (1983).
2. Jenkins, J.T. and Savage, S.B. *J. Fluid Mech.*, **130**, 187 (1983).
3. Ding, J. and D. Gidaspow, *AIChE Journal*, **36**, 523 (1990).
4. Chapman, S. and T.G. Cowling, "The Mathematical Theory of Non-Uniform Gases," 2nd Ed., Cambridge Univ. Press (1961).
5. Miller, Aubrey, Ph.D. Thesis, Illinois Institute of Technology, Chicago 1991.
6. Gidaspow, D., J. Ding and U.K. Jayaswal, in "Numerical Methods for Multiphase Flows," FED-Vol. 91, American Society of Mechanical Engineering, p. 47 (1990).

TABLE 1: KINETIC THEORY MODEL FOR MULTIPHASE FLOW

1. CONTINUITY EQUATION FOR PHASE k (= g, s)

$$\frac{\partial}{\partial t}(\epsilon_k \rho_k) + \nabla \cdot (\epsilon_k \rho_k v_k) = \dot{m}_k, \quad \sum_k \epsilon_k = 1 \quad (T1.1)$$

3. CONSTITUTIVE EQUATION FOR STRESS

$$\bar{\tau}_k = [-P_k + \xi_k \nabla \cdot v_k] \bar{I} + 2\mu_s \bar{S}_k \quad (T1.3)$$

where

$$\bar{S}_k = \frac{1}{2} [\nabla v_k + (\nabla v_k)^T] - \frac{1}{3} \nabla \cdot v_k \bar{I} \quad (T1.3a)$$

5. Solid Phase Stress

Kinetic Theory Model (Gidaspow's 1991 extension of Savage, 1983 and Ding and Gidaspow's, 1990 expressions to dilute and dense flow)

Solids Phase Pressure

$$P_s = \rho_p \epsilon_s \Theta [1 + 2(1 + e)g_n \epsilon_s] \quad (T1.5a)$$

Solids Phase Bulk Viscosity

$$\xi_s = \frac{4}{3} \epsilon_s^2 \rho_p d_p g_n (1 + e) \left(\frac{\Theta}{\pi}\right)^{\frac{1}{2}} \quad (T1.5b)$$

Solids Phase Shear Viscosity

$$\mu_s = \frac{2\mu_{s,dil}}{(1 + e)g_n} \left[1 + \frac{4}{5}(1 + e)g_n \epsilon_s\right]^2 + \frac{4}{5} \epsilon_s^2 \rho_p d_p g_n (1 + e) \left(\frac{\Theta}{\pi}\right)^{\frac{1}{2}} \quad (T1.5c)$$

Solids Phase Dilute Viscosity

$$\mu_{s,dil} = \frac{5\sqrt{\pi}}{96} \rho_p d_p \Theta^{\frac{1}{2}} \quad (T1.5d)$$

Radial Distribution Function

$$g_n = \frac{3}{5} \left[1 - \left(\frac{\epsilon_s}{\epsilon_{s,max}}\right)^4\right]^{-1} \quad (T1.5e)$$

7. GAS - SOLID DRAG COEFFICIENTS

For  $\epsilon_s < 0.8$  (based on Ergun equation)

$$\beta = 150 \frac{\epsilon_s^2 \mu_g}{\epsilon d_p^2} + 1.75 \frac{\rho_g \epsilon_s |v_g - v_s|}{d_p} \quad (T1.7a)$$

For  $\epsilon_s > 0.8$  (based on empirical correlation)

$$\beta = \frac{3}{4} C_d \frac{\epsilon_s \rho_g |v_g - v_s|}{d_p} \epsilon^{-2.65} \quad (T1.7b)$$

2. MOMENTUM EQUATION FOR PHASE k (= g, s; l = g, s)

$$\frac{\partial}{\partial t}(\epsilon_k \rho_k v_k) + \nabla \cdot (\epsilon_k \rho_k v_k v_k) = \epsilon_k \rho_k g + \nabla \cdot \bar{\tau}_k + \beta(v_l - v_k) + \dot{m}_k v_k \quad (T1.2)$$

4. Gas Phase Stress

$$\bar{\tau}_g = 2\epsilon_g \mu_{g,e} \bar{S}_g \quad (T1.1)$$

Turbulent Viscosity (Used for comparison only)

$$\mu_{g,e} = \mu_t + \mu_g = \rho_g (\epsilon_t \Delta)^2 (2\bar{S}_g \cdot \bar{S}_g)^{\frac{1}{2}} + \mu_g \quad (T1.4a)$$

with  $\epsilon_t = 0.1$  and  $\Delta = \sqrt{(\Delta x \Delta y)}$

6. FLUCTUATING ENERGY ( $\Theta = \frac{1}{3} \langle C^2 \rangle$ ) EQUATION

$$\frac{3}{2} \left[ \frac{\partial}{\partial t}(\epsilon_s \rho_s \Theta) + \nabla \cdot (\epsilon_s \rho_s v_s \Theta) \right] = \bar{\tau}_s : \nabla v_s - \nabla \cdot q - \gamma \quad (T1.6)$$

Collisional Energy Dissipation  $\gamma$

$$\gamma = 3(1 - e^2) \epsilon_s^2 \rho_s g_n \Theta \left( \frac{4}{d_p} \left(\frac{\Theta}{\pi}\right)^{\frac{1}{2}} - \nabla \cdot v_s \right) \quad (T1.6a)$$

Flux of Fluctuating Energy  $q$

$$q = -\kappa \nabla \Theta \quad (T1.6b)$$

Conductivity of Fluctuating Energy

$$\kappa = \frac{2}{(1 + e)g_n} \left[1 + \frac{6}{5}(1 + e)g_n \epsilon_s\right]^2 \kappa_{dil} + 2\epsilon_s^2 \rho_p d_p g_n (1 + e) \left(\frac{\Theta}{\pi}\right)^{\frac{1}{2}} \quad (T1.6c)$$

Dilute Phase ("Eddy Type") Granular Conductivity

$$\kappa_{dil} = \frac{75}{384} \sqrt{\pi} \rho_p d_p \Theta^{\frac{1}{2}} \quad (T1.6d)$$

where

$$C_d = \frac{24}{Re_p} \left[1 + 0.15 Re_p^{0.687}\right], \text{ for } Re_p < 1000 \quad (T1.7c)$$

$$C_d = 0.44, \text{ for } Re_p > 1000 \quad (T1.7d)$$

$$Re_p = \frac{\epsilon \rho_g |v_g - v_s| d_p}{\mu_g} \quad (T1.7e)$$

TABLE 2: CONSERVATION OF SPECIES AND ENERGY

1. CONSERVATION EQUATION OF GAS COMPONENT "n"

$$\frac{\partial}{\partial t}(\epsilon \rho_g Y_n) + \nabla \cdot (\epsilon \rho_g Y_n v_g) = M_n \sum \gamma_{nm} r_n \quad (2.1)$$

2. ENERGY EQUATIONS

Gas Phase

$$\frac{\partial}{\partial t}(\epsilon \rho_g h_g) + \nabla \cdot (\epsilon \rho_g h_g v_g) = \epsilon \left( \frac{\partial p}{\partial t} + v_g \cdot \nabla p \right) + \epsilon \sum r_n \Delta H_n^0 + h_v(T_g - T_s) + \nabla \cdot (k_g \epsilon \nabla T_g) \quad (2.2)$$

Solid Phase

$$\frac{\partial}{\partial t}(\epsilon_s \rho_s h_s) + \nabla \cdot (\epsilon_s \rho_s h_s v_s) = +h_v(T_g - T_s) + \nabla \cdot (k_s \epsilon_s \nabla T_s) \quad (2.3)$$

Gas - Particle Heat Transfer  $h_v$   
For  $\epsilon \leq 0.8$ ,

$$Nu_p = (2 + 1.1 Re^{0.6} Pr^{1/3}) S_p, \quad Re \geq 200$$

$$Nu_p = 0.123 \left( \frac{4Re}{d_p} \right)^{0.63} S_p^{0.17}, \quad 200 < Re \leq 2000$$

$$Nu_p = 0.61 Re^{0.67} S_p, \quad Re > 2000$$

$$\epsilon > 0.8$$

$$Nu_p = (2 + 0.16 Re^{0.67}) S_p, \quad Re \leq 200$$

$$Nu_p = 8.2 Re^{0.6} S_p, \quad 200 < Re \leq 1000$$

$$Nu_p = 1.06 Re^{0.4557} S_p, \quad Re > 1000$$

where

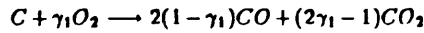
$$Re = \frac{d_p \rho_g |v_g - v_s|}{\mu_g}$$

$$S_p = \epsilon_s \frac{6}{d_p} \quad \text{and} \quad Nu_p = h_v d_p / k_g$$

TABLE 3. KEY REACTIONS

$$r_i = \frac{\epsilon_s (p_i - p_i^*)}{\delta K_{r,i} + \frac{d_p^2 (1-p) RT}{12 \rho D_{M,i}} + \frac{1}{np^2 K_{r,i} C_i^2}}$$

1. COMBUSTION REACTION



where  $\gamma_1$  is determined by

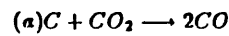
$$\frac{CO}{CO_2} = \frac{2(1 - \gamma_1)}{2\gamma_1 - 1} = 10^{3.4} \exp\left(\frac{-12400}{RT_s}\right)$$

$$K_{r,1} = 1.79 \times 10^6 \exp\left(\frac{-27000}{RT_s}\right)$$

$$\Delta H_{r,1}|_{298^0k} = (-97.7)(2\gamma_1 - 1) + 2(-28.714)$$

$$(1 - \gamma_1) + 2.4 \text{ and } p_{O_2}^* = 0.0$$

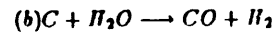
2. GASIFICATION REACTIONS



$$K_{r,2} = 930 \exp\left(\frac{-45000}{RT_s}\right)$$

$$K_{eq} = \frac{P_{CO}^2}{P_{CO_2}} = 1.22 \times 10^9 \exp\left(\frac{-40300}{RT_s}\right)$$

$$\Delta H_{r,2}|_{298^0k} = 40.3$$

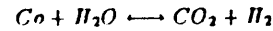


$$K_{r,3} = 930 \exp\left(\frac{-45000}{RT_s}\right)$$

$$K_{eq} = \frac{P_{H_2} P_{CO}}{(P_{H_2O})^2} = \exp\left(17.3 - \frac{16326.1}{T_s}\right)$$

$$\Delta H_{r,3}|_{298^0k} = 32.4$$

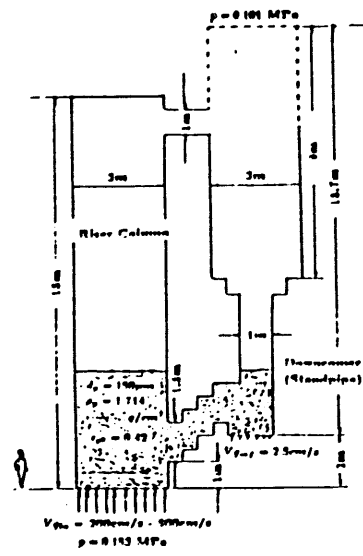
3. WATER SHIFT REACTION



$$r_8 = 0.775 \exp\left(\frac{-8421.3}{T_g}\right) p^{0.8 - \frac{x}{350}} (x_{CO} x_{H_2O} - \frac{x_{CO_2} x_{H_2}}{K_{wg}}) (1 - W_c) \rho_{coal} \epsilon_s$$

$$K_{wg} = 0.027 \exp\left(\frac{7860}{T_g}\right)$$

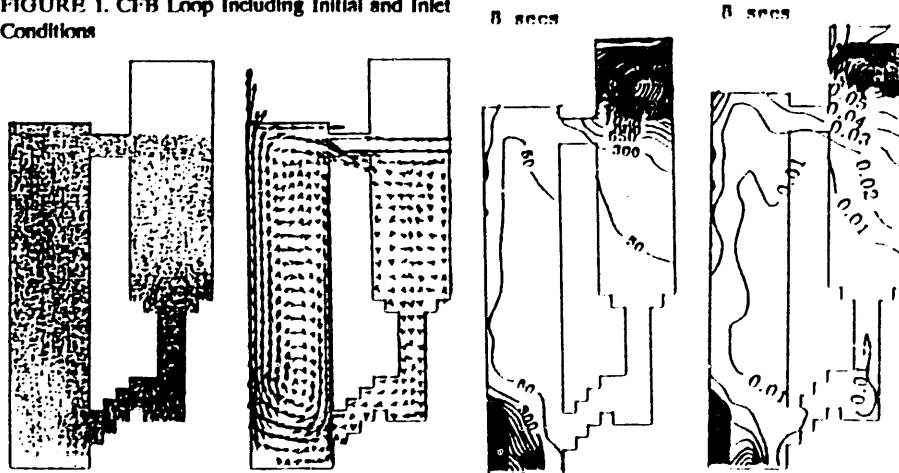
$$\Delta H_{r,4}|_{298^0k} = -9.84$$



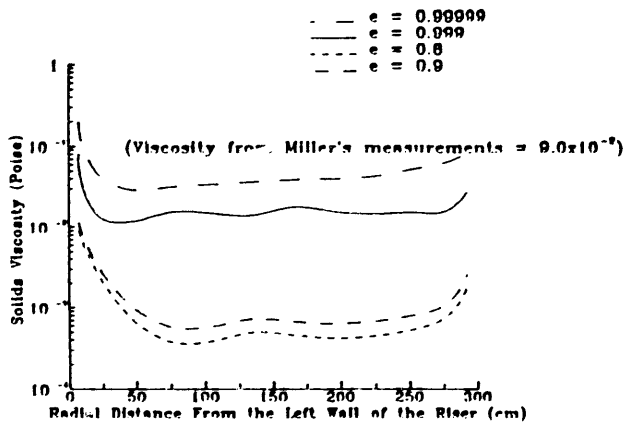
**Inlet Conditions**

Diameter of particles	= 150 $\mu\text{m}$
Density of particles	= 1714 $\text{Kg/m}^3$
Terminal velocity of particles	= 1.1 $\text{m/s}$
Gas flow rate at bottom of standpipe (Minimum fluidization velocity)	= 2.5 $\text{cm/s}$
Air inlet (riser) velocity	= 2 $\text{m/s}$
Initial bed height	= 4.5 $\text{m}$
Initial porosity	= 0.42

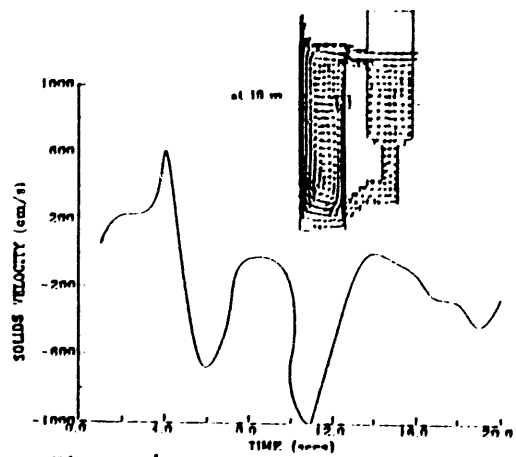
**FIGURE 1. CFB Loop Including Initial and Inlet Conditions**



**FIGURE 2. Particle Concentration, Velocities, Granular Temperatures ( $\text{cm/s}^2$ ) and Solid Viscosities (Poise)**



**Fig. 3. Effect of Restitution Coefficient on Solids Viscosity**



**Figure 4**

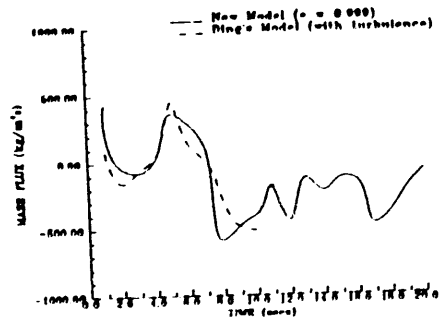
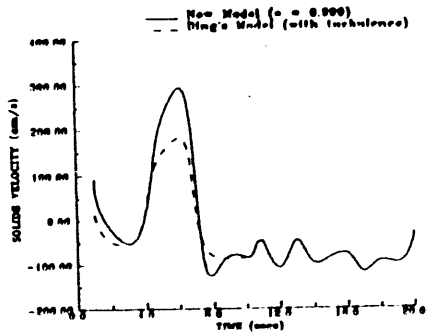


FIGURE 5. Discharge from Standpipe to River

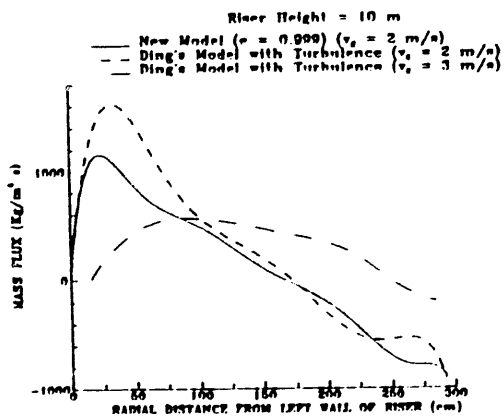


FIGURE 6. Time Averaged Radial Mass Flux Profile

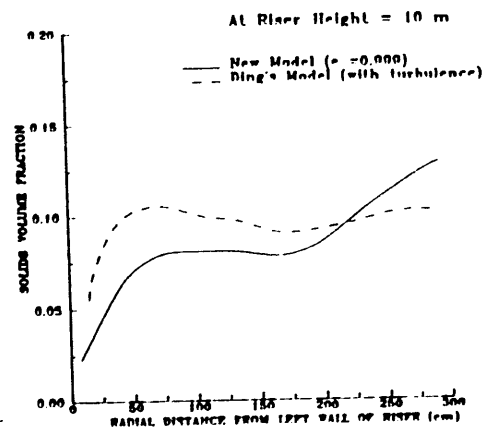


FIGURE 7. Time Averaged Radial Solids Volume Fraction Profile

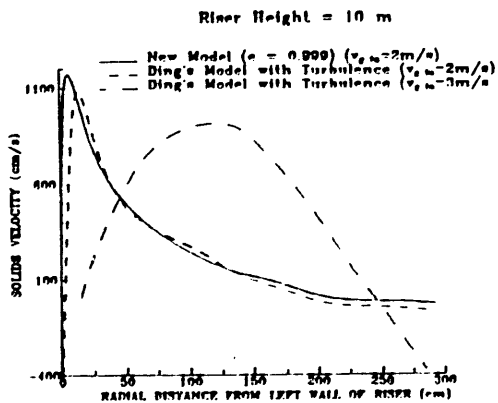


FIGURE 8. Time Averaged Radial Solids Velocity Profile

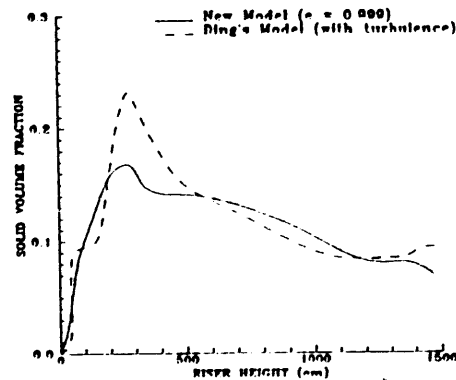


FIGURE 9. Area Averaged Axial Solids Volume Fraction Profile

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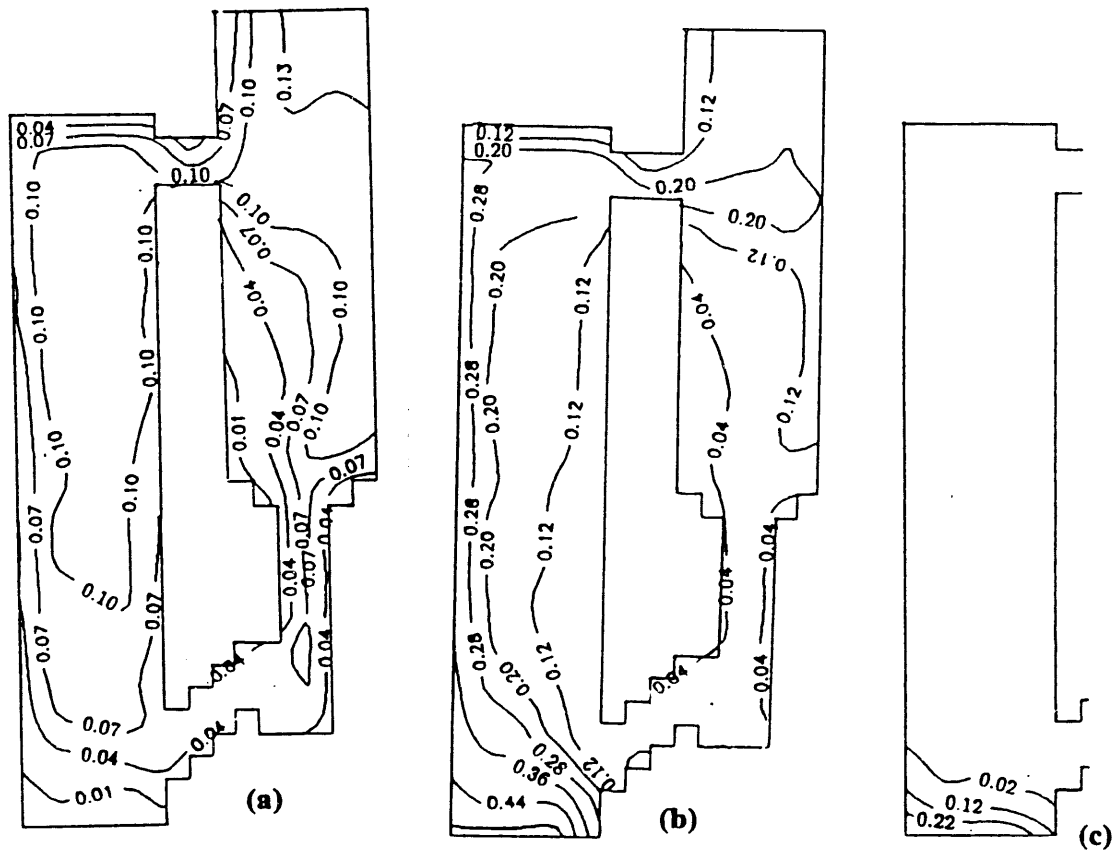


FIGURE 10. Mole Fractions CO<sub>2</sub>, Steam (b), and O<sub>2</sub> in the CFB gasifier at 15 seconds.

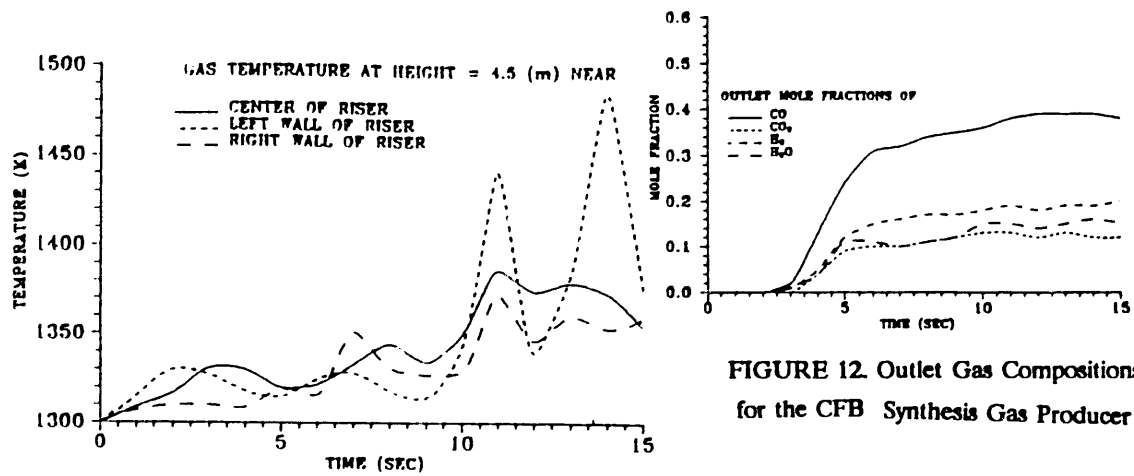


FIGURE 11. Gas Temperature

FIGURE 12. Outlet Gas Compositions for the CFB Synthesis Gas Producer



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