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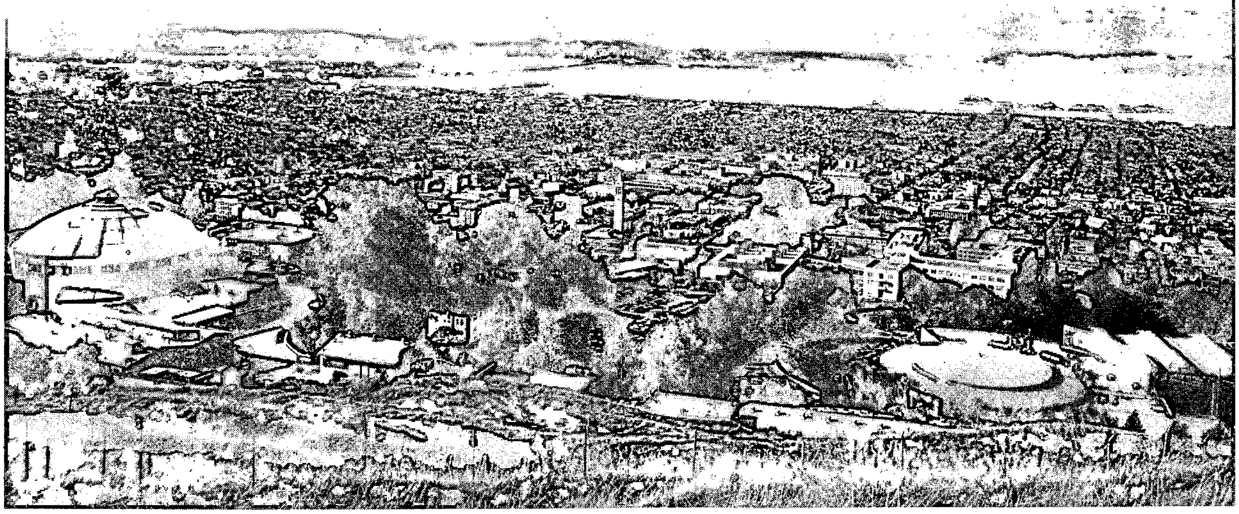

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HYDROLOGIC DETECTION OF ABANDONED WELLS NEAR PROPOSED INJECTION WELLS FOR HAZARDOUS WASTE DISPOSAL

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ABSTRACT

Deep saline aquifers are being used for disposal of hazardous liquid wastes. A thorough knowledge of the competency of such aquifers and their confining geologic beds in permanently isolating the hazardous substances is the key to successful disposal operations. Characterization of such systems, and in particular the detection of any conduit that may permit hydraulic communication between the host aquifer and nearby freshwater aquifers, must be carried out prior to the initiation of disposal projects. In deep, multi-aquifer systems, leaky faults, abandoned wells, highly conductive fractures or shear zones may all provide leakage paths. If not initially detected, such conduits may show no apparent effect until contaminants in the freshwater aquifer reach detectable levels at the discharge point. By then, of course, detection is generally too late.

This paper is an attempt to address the problem of initial detection of improperly plugged or open abandoned wells. A new analytic solution has been derived to calculate the amount of leakage from an abandoned well and the corresponding drawdown at monitoring wells. A method is proposed that can be used to detect such deep abandoned wells in the area of influence of a proposed deep injection well in a multiple-aquifer system.

INTRODUCTION

Injection into deep saline aquifers is one of several methods used for disposal of hazardous liquid wastes. Although the use of wells for subsurface disposal of industrial wastes has been known since early 1930's, its use was initially limited to brine disposal (Donaldson, 1964; and Warner and Orcutt, 1973). Injection of hazardous wastes in deep underground aquifers are documented from 1950 (Donaldson, 1964). Between 1950 and 1965 an average of only two wells per year were built for disposal of hazardous wastes. During the period of 1965-1980, however, stricter regulation of industrial waste disposal into surface water bodies resulted in renewed interest in deep-well disposal techniques leading to construction of more than 130 deep hazardous waste injection wells.

Hazardous waste injection wells are a subset of Class I wells. Class I refers to those wells used for disposal of municipal or industrial waste liquids that discharge below the deepest underground source of drinking water (USDW), (Federal Register, 1982). Figure 1 illustrates the increase in number of total Class I wells as well as the hazardous waste subset of them with time. It is estimated that 423 million gallons of non-aqueous hazardous waste with about 10 billion gallons of water was injected through 181 wells in 1983 (U.S. Environmental protection Agency, 1985). Deep-well disposal is used by chemical, petrochemical and pharmaceutical industries, refineries, steel mills, and photo-processing plants, among others. The depth of the wells is generally between 1000 and 10,000 ft; most of them are between 2000 and 6000 ft deep. Most of the injection wells are located in states with long history of oil and gas exploration (Office of Technology Assessment, 1983). Currently 15 states have active wells that inject hazardous liquid wastes (U.S. Environmental Protection Agency, 1985).

In an effort to protect underground sources of drinking water against the danger of contamination, the "area of review" concept, which is the major Underground Injection Control (UIC) requirement, has been devised (Anzzolin and Graham, 1984; Thornhill et al., 1982). The area of review process requires

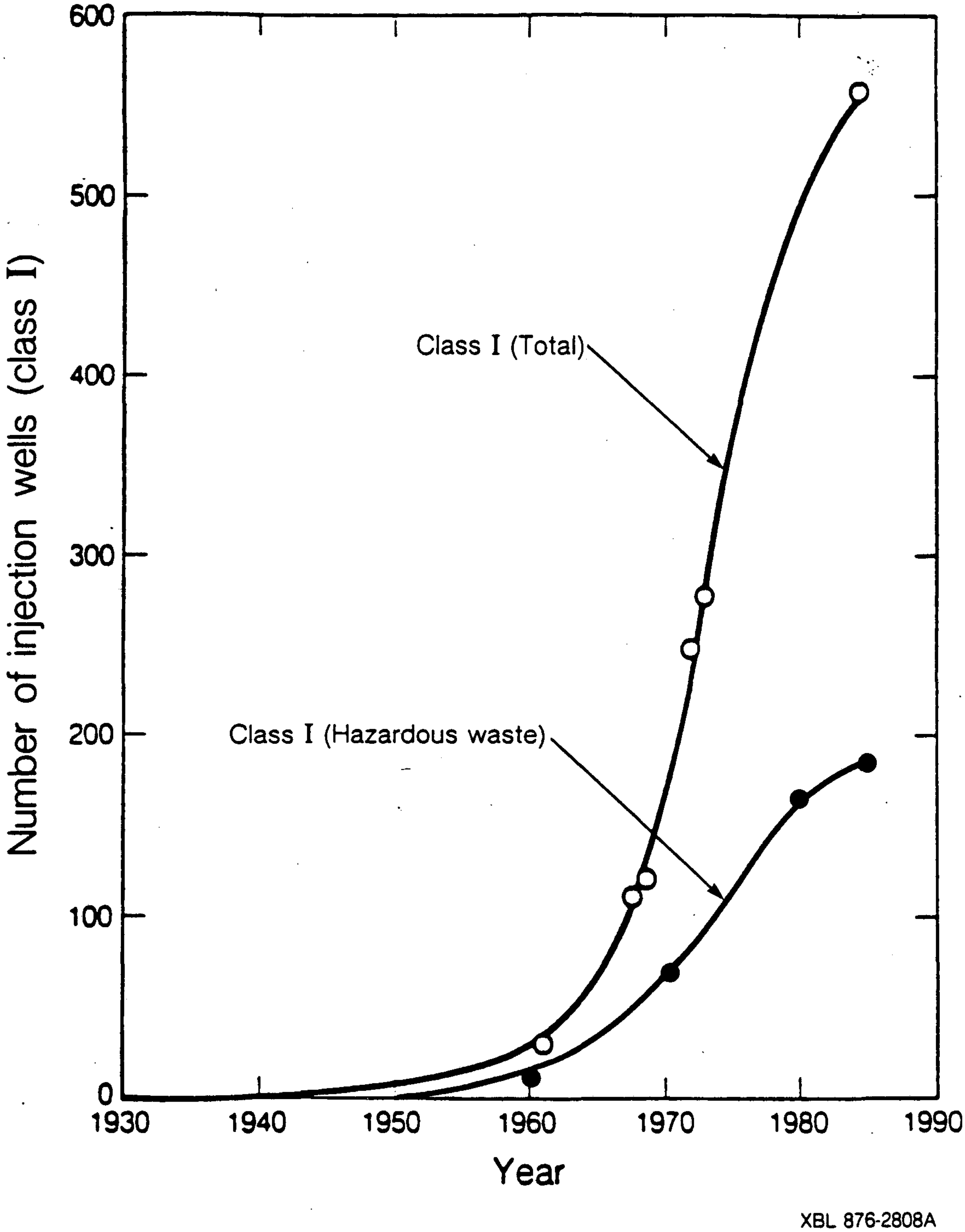

Figure 1. The trend of deep-well injection.

that the records of existing wells penetrating the injection zone be examined to ensure that wells are properly constructed or abandoned. However, the lack of existing records for thousands of the abandoned wells severely hampers the process.

The number of wells abandoned in the United States between 1859 and 1974 is estimated to be 1,647,661 (Anzzolin and Graham, 1984). Other estimates puts this number at 1,930,000 wells (Fryberger and Tinlin, 1984). Records on the locations and characteristics of many of these wells are non-existent (Canter, 1984). Such information is available for only about 1,200,000 abandoned wells. Approximately 450,000 or more of these wells were abandoned between 1859 and 1930. The majority of abandoned wells are also located in regions with long history of oil and gas exploration. It is estimated that most of wells abandoned before 1930 were probably improperly plugged by today's standards (Anzzolin and Graham, 1984). Thousands of these wells penetrate formations of both fresh and saline waters. The leakage of contaminated or highly mineralized water through abandoned wells and unplugged exploration holes has led to insidious groundwater pollution problems (Gass et al., 1977). Numerous examples of groundwater contamination related to abandoned wells have been reported (U.S. Environmental Protection Agency, 1977; Fairchild et al., 1981; Wait and McCollum, 1963).

It has been reported that leakage through undetected abandoned wells has led to the failure of a number of deep-well disposal projects (Gass et al., 1977). One case of hazardous waste leakage through abandoned wells is reported (U.S. Environmental Protection Agency, 1986). However, one must be concerned with the potential of leakage for future deep-well injection projects and also keep in mind that leakage through improperly plugged abandoned wells into USDW could continue for a long time before it can manifest itself at a discharge point. Therefore, prior knowledge of the location of the deep abandoned wells and their ability to conduct the liquid waste to shallow freshwater aquifers is a necessary step toward the design of a successful deep-well injection project.

In this paper present methods of locating abandoned wells will be briefly reviewed and their shortcomings discussed. A new method will then be introduced which should permit detection of leaky abandoned wells avoiding the problems encountered in the present methods.

Present Methods of Locating Abandoned Wells

Methods presently used for determining the location of abandoned wells are discussed in detail by Aller (1984). These methods may be divided into 4 groups:

(1) Search for existing records and contact local residents. Most states follow this method for locating wells in the area of review; however, records are missing for many wells drilled before 1930.

(2) Interpretation of historical aerial photographs. This could be a useful tool except that the use of photographic coverage was not widely employed in the United States before the 1930s (Avery, 1968).

(3) Geophysical methods. Magnetic, electrical resistivity and electromagnetic surveys can be used to detect cased wells. These methods do not work with uncased wells or those equipped with nonmetallic casings. Ground-penetrating radar may be used to find both cased and uncased wells. The major problem with all of these methods is the difficulty in determining whether a well, when located, is properly plugged, or subject to leakage.

(4) Hydrologic methods. Two hydrologic techniques have been discussed by Aller (1984). In the first method water levels are monitored in wells penetrating the freshwater aquifer overlying the injection formation in the vicinity of the abandoned well. Any major leakage from the abandoned wells should produce a water level anomaly in the fresh water aquifer. However, this method is only feasible if several water wells are already present in the vicinity of an unknown abandoned well. The second method involves injection of fluid into the injection zone. The presence of a leaky abandoned well is indicated if pressure resulting from fluid injection causes fluid to flow up through the abandoned well to the ground surface. However, if the conduit is not open to the surface or the induced pressure increase is insufficient, there may be no observable leakage to the surface.

PROPOSED NEW METHOD

Theory

Let us assume that Aquifer A is a shallow freshwater aquifer and Aquifer B is a deep aquifer underlying Aquifer A. Aquifer B is assumed to be homogeneous, isotropic, uniform in thickness and of infinite areal extent. An aquiclude with very low hydraulic conductivity separates these two aquifers (see Figure 2). Let us further assume that liquid is injected into Aquifer B through Well 1 at constant rate Q. Assume also that Well 2 is an abandoned well located at distance R from the injection well. Originally open in both aquifers, Well 2 has never been properly plugged. If sufficient hydraulic head builds up in Aquifer B, some fluid may migrate up through the abandoned well from Aquifer B into Aquifer A, perhaps even reaching the ground surface. We want to determine the effect of such migration at the injection well itself and at an appropriately located monitoring well (Well 3). This leads to a procedure for locating the abandoned well. First we will look at the effect at a monitoring well.

The change of hydraulic head at Well 2 due to injection at Well 1 is given by (Theis, 1935)

$$\Delta h_2 = -\frac{Q}{4\pi T}\,\mathrm{Ei}\left(-\frac{R^2}{4\alpha t}\right) \tag{1}$$

where T and α are the transmissivity and hydraulic diffusivity of Aquifer B and -Ei(-x) is the exponential integral. Assuming that at any moment the rate of leakage ,Q_2, from Aquifer B to A through the abandoned well is proportional to the difference between the hydraulic head at the two aquifers and inversely proportional to the resistance to the flow, Ω, one can write

$$Q_2(t) = \frac{\Delta h_2' - H_1}{\Omega} \tag{2}$$

where H_1 is the initial difference in the hydraulic head between the Aquifers A and B, ($h_A - h_B$), and $\Delta h_2'$ is the net increase in hydraulic head at the

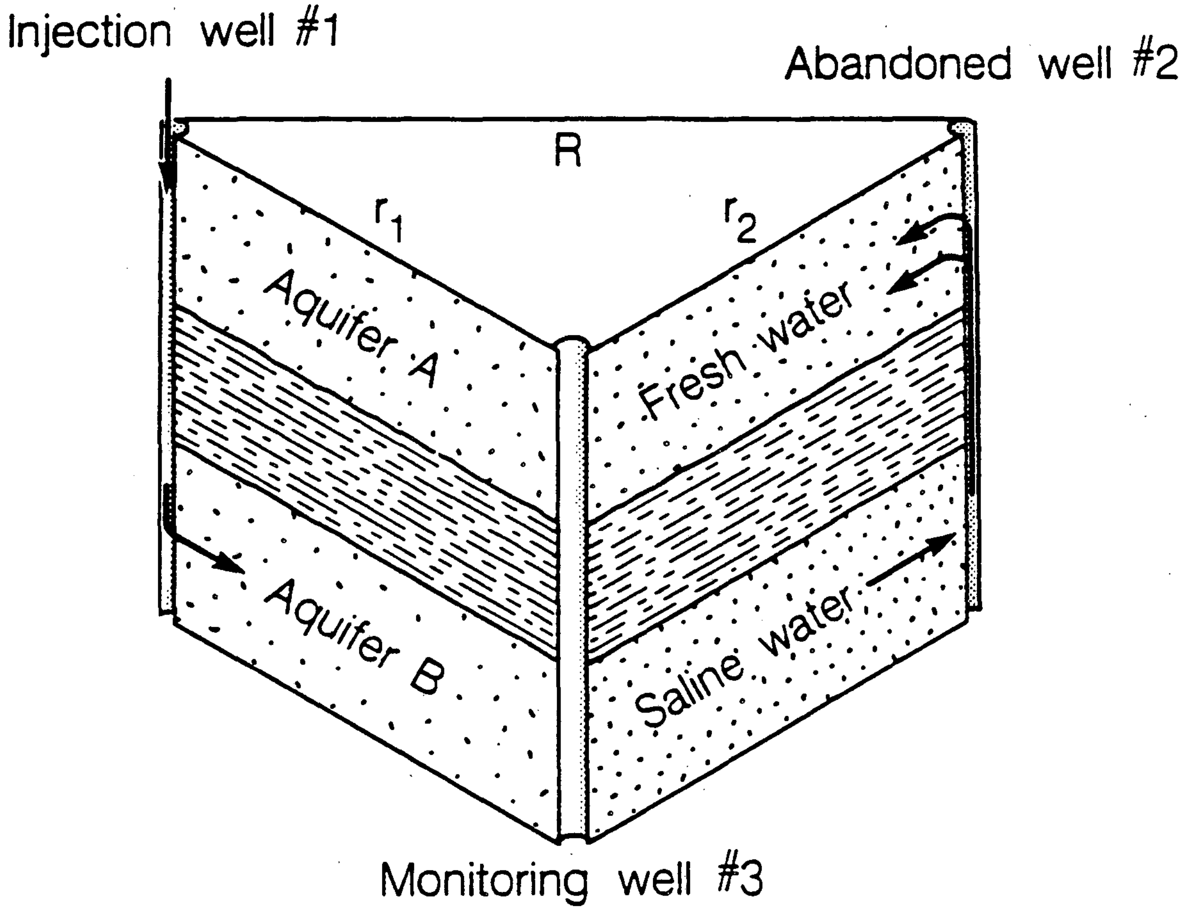

Figure 2. A schematic section showing the position of the wells in the hypothetical aquifer system.

abandoned well in Aquifer B. The expression for $\Delta h_2'$ may be written as

$$\Delta h_2'(t) = \Delta h_2(t) - s_w(t) \tag{3}$$

where $s_w(t)$ is the drawdown at the abandoned well due to leakage through that well. The expression for $s_w(t)$ may be written as

$$s_w(t) = \frac{1}{4\pi T}\int_0^t Q_2(t')\,\frac{\exp\left\{-\dfrac{r_w^2}{4\alpha(t-t')}\right\}}{(t-t')}\,dt' \tag{4}$$

Here $Q_2(t)$ is the rate of leakage from the abandoned well and r_w the effective radius of that well. Substituting for $\Delta h_2'$ in (2) yields

$$Q_2(t) = \frac{1}{4\pi T\Omega}\left\{-Q\,\mathrm{Ei}\left(-\frac{R^2}{4\alpha t}\right) - \int_0^t Q_2(t')\,\frac{\exp\left\{-\dfrac{r_w^2}{4\alpha(t-t')}\right\}}{(t-t')}\,dt' - 4\pi T H_1\right\} \tag{5}$$

If, for the sake of shortening the equations, we assume that the initial heads in both aquifers are the same, and thus, letting H_1 vanish, (5) may be written as

$$\Omega\,Q_2(t) + \frac{1}{4\pi T}\int_0^t Q_2(t')\,\frac{\exp\left\{-\dfrac{r_w^2}{4\alpha(t-t')}\right\}}{(t-t')}\,dt' = -\frac{Q}{4\pi T}\,\mathrm{Ei}\left\{\frac{-R^2}{4\alpha t}\right\} \tag{6}$$

One must first find $Q_2(t)$ in order to calculate the effect of such leakage at other points in the aquifer.

The Laplace transformation of (6) with respect to t is

$$\Omega\bar{Q}_2 + \frac{1}{4\pi T}\left\{2\,\bar{Q}_2\,K_0\left(r_w\sqrt{\frac{p}{\alpha}}\right)\right\} = \frac{2\,Q}{4\pi T p}\,K_0\left(R\sqrt{\frac{p}{\alpha}}\right) \tag{7}$$

where K_0 is the modified Bessel function of second kind and zero order and p is the Laplace transform parameter. Solving for $\bar{Q}_2$ we obtain

$$\bar{Q}_2 = \frac{\frac{Q}{2\pi T} \frac{K_0(R\sqrt{\frac{p}{\alpha}})}{p}}{\Omega + \frac{1}{2\pi T} K_0(r_w\sqrt{\frac{p}{\alpha}})} \tag{8}$$

Equation (8) gives the solution for the leakage rate from the abandoned well in the Laplace transform domain. To obtain the Laplace inversion of (8) we use the complex inversion integral shown below:

$$Q_2(t) = \frac{1}{2\pi i} \lim_{\beta\to\infty} \int_{\gamma-i\beta}^{\gamma+i\beta} \exp(\lambda t) \frac{\frac{Q}{2\pi T} \frac{K_0(R\sqrt{\frac{\lambda}{\alpha}})}{\lambda}}{\Omega + \frac{1}{2\pi T} K_0(r_w\sqrt{\frac{\lambda}{\alpha}})} d\lambda \tag{9}$$

where γ is so large that all singularities of the integrand in (9) lie to the left of the line (γ-i∞ , γ+i∞) on the complex plane. Appendix A shows the procedure for solving (9) and the result is given below.

$$Q_2(t) = Q - \frac{2Q}{\pi} \int_0^\infty e^{-\alpha u^2 t} \frac{J_0(uR)\left\{4T\Omega - Y_0(ur_w)\right\} + J_0(ur_w)\, Y_0(uR)}{\left\{4T\Omega - Y_0(ur_w)\right\}^2 + J_0^2(ur_w)} \frac{du}{u} \tag{10}$$

where J_0 and Y_0 are Bessel functions of first and second kind, respectively. Equation (10) gives the leakage rate from the abandoned well located at distance R from the injection well. In dimensionless form (10) may be written as

$$Q_D(t) = 1 - \frac{2}{\pi}\int_0^\infty e^{-v^2 t_D} \frac{J_0(v)\left\{\frac{2\Omega_D}{\pi} - Y_0(vr_{w_D})\right\} + J_0(vr_{w_D})Y_0(v)}{\left\{\frac{2\Omega_D}{\pi} - Y_0(vr_{w_D})\right\}^2 + J_0^2(vr_{w_D})} \frac{dv}{v} \quad (11)$$

where

$$Q_D(t) = \frac{Q_2(t)}{Q} \quad (12)$$

$$t_D = \frac{\alpha t}{R^2} \quad (13)$$

$$r_{w_D} = \frac{r_w}{R} \quad (14)$$

$$\Omega_D = 2\pi T\Omega \quad (15)$$

Drawdown due to Leakage

Drawdown due to leakage from an abandoned well, observed at a monitoring well located at distance r_2 from the abandoned well, may be obtained from the following equation:

$$h_2(t\ ,\ r_2) = \frac{1}{4\pi T}\int_0^t Q_2(t')\ \frac{e^{-\frac{r_2^2}{4\alpha(t-t')}}}{(t-t')}dt' \quad (16)$$

Substituting for Q_2 from (10) yields

$$h_2(t\ ,\ r_2) = \frac{Q}{4\pi T}\int_0^t \frac{e^{-\frac{r_2^2}{4\alpha(t-t')}}}{(t-t')}dt' - \frac{Q}{2\pi^2 T}\int_0^t \frac{e^{-\frac{r_2^2}{4\alpha(t-t')}}}{(t-t')}$$

$$\left\{\int_0^\infty e^{-\alpha u^2 t'} \frac{J_0(uR)\left\{4T\Omega - Y_0(ur_w)\right\} + J_0(ur_w)Y_0(uR)}{\left\{4T\Omega - Y_0(ur_w)\right\}^2 + J_0^2(ur_w)} \frac{du}{u}\right\} dt' \quad (17)$$

To obtain the corresponding drawdown at the injection well itself, we substitute R for r_2 in (17). As a result we obtain

$$h_2(t\,,R) = \frac{Q}{4\pi T}\int_0^t \frac{e^{-\frac{R^2}{4\alpha(t-t')}}}{(t-t')}dt' - \frac{Q}{2\pi^2 T}\int_0^t \frac{e^{-\frac{R^2}{4\alpha(t-t')}}}{(t-t')}$$

$$\left\{\int_0^\infty e^{-\alpha u^2 t'} \frac{J_0(uR)\left\{4T\Omega - Y_0(ur_w)\right\} + J_0(ur_w)Y_0(uR)}{\left\{4T\Omega - Y_0(ur_w)\right\}^2 + J_0^2(ur_w)} \frac{du}{u}\right\} dt' \qquad (18)$$

In dimensionless form (17) and (18) may be written as

$$h_{2_D}(t_D\,,R_D) = \frac{1}{2}\int_{\frac{R_D^2}{4t_D}}^\infty \frac{e^{-y}}{y}dy - \frac{1}{\pi}\int_{\frac{R_D^2}{4t_D}}^\infty \frac{e^{-y}}{y}$$

$$\left\{\int_0^\infty e^{-v^2\left(t_D - \frac{R_D^2}{4y}\right)} \frac{J_0(v)\left\{\frac{2\Omega_D}{\pi} - Y_0(vr_{w_D})\right\} + J_0(vr_{w_D})\,Y_0(v)}{\left\{\frac{2\Omega_D}{\pi} - Y_0(vr_{w_D})\right\}^2 + J_0^2(vr_{w_D})} \frac{dv}{v}\right\} dy \qquad (19)$$

and

$$h_{2_D}(t_D,1) = \frac{1}{2}\int_{\frac{1}{4t_D}}^\infty \frac{e^{-y}}{y}dy - \frac{1}{\pi}\int_{\frac{1}{4t_D}}^\infty \frac{e^{-y}}{y}$$

$$\left\{\int_0^\infty e^{-v^2\left(t_D - \frac{1}{4y}\right)} \frac{J_0(v)\left\{\frac{2\Omega_D}{\pi} - Y_0(vr_{w_D})\right\} + J_0(vr_{w_D})\,Y_0(v)}{\left\{\frac{2\Omega_D}{\pi} - Y_0(vr_{w_D})\right\}^2 + J_0^2(vr_{w_D})} \frac{dv}{v}\right\} dy \qquad (20)$$

where

$$h_{2_D} = \frac{2\pi T h_2}{Q} \qquad (21)$$

$$R_D = \frac{r_2}{R} \qquad (22)$$

DISCUSSION AND RESULTS

Equation (11) gives the amount of leakage that may occur through an improperly plugged abandoned well existing in the vicinity of an injection well. The abandoned well is assumed to be open in the injection formation and in an upper aquifer, thus providing a conduit between the formation and the aquifer that might be an underground source of drinking water (USDW). The amount of leakage is a function of time, the resistance of the abandoned well to flow, rate of injection, and the distance between the two wells. Hydrologic properties of the injected formation are also important parameters in controlling the leakage rate. All of these controlling factors have been combined into four dimensionless parameters: Q_D, t_D, Ω_D, and r_{w_D}.

Equation (11) has been evaluated for several values of r_{w_D} and a wide range of t_D and Ω_D. Table 1 shows values of $Q_D(t)$ for $r_{w_D} = 0.001$ and useful ranges of t_D and Ω_D. More extensive tables are presented elsewhere (Javandel et al., 1986). Figure 3 shows the variation of Q_D as a function of dimensionless time for values of Ω_D ranging between 0.01 and 100, and $r_{w_D} = .002$. Examination of results indicates that for a given value of time little change in Q_D is achieved by decreasing the resistance of the abandoned well beyond $\Omega_D = 0.01$. Figure 3 shows that for some unfavorable conditions where resistance to flow in the abandoned well is negligible, up to about 50% of the injection rate may leak through the this well. One may note that this is the upper limit of leakage and is unlikely to happen. When the leakage rate is relatively high, pressure buildup in the upper aquifer becomes appreciable and that will reduce the driving force causing the leakage. It must be emphasized that, at least at early times, the leaking fluid consists of the host-formation water, which although is free from injection fluid, is greatly inferior to drinking water quality.

If the distance between the abandoned well and the injection well is known, and an estimate of the hydraulic resistance of the abandoned well has been made, then the results from (11) can be used to obtain leakage rates and their variation

Table 1

Values of $Q_D(t)$ for $r_{w_D} = 0.001$

t_D/Ω_D	0.01	0.05	0.10	0.50	1.00	5.00
0.1	0.014	0.014	0.014	0.013	0.012	0.008
0.2	0.029	0.029	0.029	0.027	0.025	0.017
0.5	0.060	0.060	0.059	0.056	0.052	0.035
0.7	0.073	0.073	0.072	0.069	0.064	0.043
1.0	0.088	0.088	0.087	0.083	0.078	0.053
2.0	0.120	0.119	0.118	0.113	0.106	0.073
5.0	0.162	0.161	0.160	0.153	0.144	0.101
7.0	0.177	0.176	0.175	0.167	0.158	0.111
10.0	0.193	0.192	0.191	0.183	0.173	0.122
20.0	0.223	0.222	0.221	0.211	0.201	0.143
50.0	0.260	0.259	0.257	0.247	0.235	0.169
70.0	0.273	0.272	0.270	0.259	0.247	0.179
100.0	0.286	0.285	0.283	0.272	0.260	0.189
200.0	0.311	0.309	0.308	0.296	0.283	0.207
500.0	0.340	0.339	0.338	0.325	0.311	0.231
700.0	0.351	0.350	0.348	0.335	0.321	0.239
1000.0	0.362	0.360	0.359	0.346	0.331	0.248
2000.0	0.381	0.380	0.378	0.365	0.350	0.264
5000.0	0.406	0.404	0.403	0.389	0.374	0.284
7000.0	0.414	0.413	0.411	0.398	0.382	0.291
10000.0	0.423	0.421	0.420	0.406	0.391	0.299
20000.0	0.439	0.438	0.436	0.422	0.406	0.313
50000.0	0.459	0.458	0.456	0.442	0.426	0.330
70000.0	0.466	0.465	0.463	0.449	0.433	0.337

Values of $Q_D(t)$ for $r_{w_D} = 0.001$

t_D/Ω_D	10.00	20.00	50.00	100.00	200.00	500.00
0.1	0.005	0.003	0.002	0.001	0.000	0.000
0.2	0.012	0.007	0.003	0.002	0.001	0.000
0.5	0.025	0.016	0.007	0.004	0.002	0.001
0.7	0.031	0.019	0.009	0.005	0.003	0.001
1.0	0.038	0.024	0.011	0.006	0.003	0.001
2.0	0.052	0.033	0.016	0.009	0.004	0.002
5.0	0.073	0.047	0.023	0.012	0.006	0.003
7.0	0.081	0.052	0.025	0.014	0.007	0.003
10.0	0.089	0.058	0.028	0.015	0.008	0.003
20.0	0.105	0.069	0.034	0.018	0.009	0.004
50.0	0.126	0.083	0.041	0.022	0.012	0.005
70.0	0.133	0.088	0.044	0.024	0.012	0.005
100.0	0.141	0.093	0.046	0.025	0.013	0.005
200.0	0.156	0.104	0.052	0.028	0.015	0.006
500.0	0.174	0.117	0.059	0.032	0.017	0.007
700.0	0.181	0.122	0.062	0.034	0.018	0.007
1000.0	0.188	0.127	0.064	0.035	0.019	0.008
2000.0	0.201	0.137	0.070	0.038	0.020	0.008
5000.0	0.218	0.149	0.077	0.042	0.022	0.009
7000.0	0.224	0.154	0.079	0.044	0.023	0.010
10000.0	0.231	0.158	0.082	0.045	0.024	0.010
20000.0	0.243	0.168	0.087	0.048	0.026	0.011
50000.0	0.258	0.179	0.094	0.052	0.028	0.011
70000.0	0.263	0.183	0.096	0.053	0.028	0.012

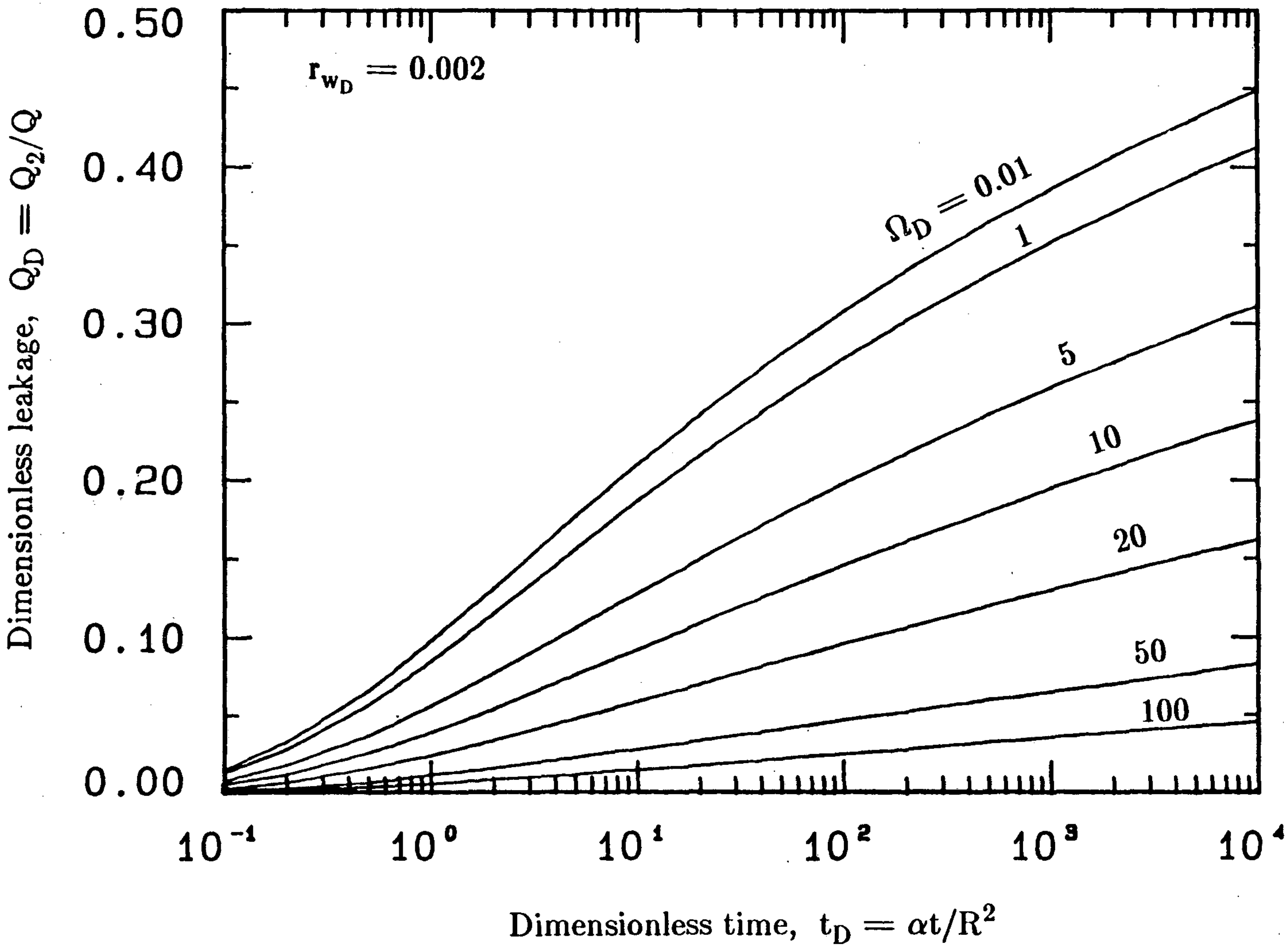

Figure 3. Variation of leakage-rate ratio with time as a function of Ω_D, and for $r_{w_D} = 0.002$.

with time.

An examination of (19) reveals that h_{2_D} is a function of dimensionless time t_D, dimensionless hydraulic resistance Ω_D, dimensionless radii R_D, and r_{w_D}. Therefore, values of h_{2_D} have been calculated and presented for a large range of t_D, and some reasonable values of R_D, Ω_D and for $r_{w_D} = 0.002$. Table 2 presents values of net drawdown caused by the leakage of fluid through the abandoned well at a monitoring well tapping the injection zone for $R_D = 0.5$. Tables of h_{2_D} for other values of R_D are available elsewhere (Javandel et al., 1986).

Figure 4 presents variations of head with time observed in a monitoring well located 100 m from the injection well. An abandoned well is located 300 m from both the injection and monitoring wells. The injection rate is about 3.79 x 10^{-2} m^3/s (600 gpm). The no-leakage curve in Figure 4 represents the increase of head with time at the monitoring well if the abandoned well is properly plugged. The other curves show the expected values of increased head in the monitoring well for different values of dimensionless hydraulic resistance of the abandoned well. An interesting result which is apparent in this figure is that the departure time of all the curves is practically the same, regardless of the magnitude of hydraulic resistance. Note that Ω_D changes by 2 orders of magnitude.

Figure 5 presents the expected values of increased head at the same monitoring well. But here, the abandoned well assumes different positions, such that the dimensionless distance R_D varies between 0.5 and 0.9. When R is fixed, the larger values of R_D correspond to larger distances between the abandoned well and the monitoring wells. This requires more time for the response of the abandoned well to be detected at the monitoring well. For this reason the curves for larger values of R_D tend to depart from the no-leakage curve at later times. Therefore, the time of departure from the no-leakage curve is an indication of the spread in the distances between these three wells.

One may note that for problems such as this where there are several running parameters on the dimensionless plot, the type curve superposition technique can

Table 2

Values of h_{2_D} for

$R_D = 0.5$ and $r_{w_D} = 0.002$

t_D/Ω_D	0.01	0.05	0.10	0.50	1.00	5.00
0.1	0.0056	0.0056	0.0054	0.0051	0.0047	0.0029
0.2	0.0188	0.0186	0.0185	0.0173	0.0161	0.0102
0.5	0.0614	0.0610	0.0605	0.0570	0.0531	0.0343
0.8	0.0981	0.0975	0.0968	0.0913	0.0853	0.0558
1.0	0.1195	0.1188	0.1179	0.1114	0.1041	0.0684
2.5	0.2346	0.2333	0.2317	0.2196	0.2061	0.1382
5.0	0.3498	0.3480	0.3457	0.3283	0.3090	0.2100
7.5	0.4275	0.4252	0.4225	0.4019	0.3787	0.2593
10.0	0.4868	0.4843	0.4813	0.4581	0.4322	0.2974
25.0	0.6969	0.6935	0.6894	0.6580	0.6226	0.4351
50.0	0.8745	0.8705	0.8655	0.8276	0.7846	0.5544
75.0	0.9849	0.9805	0.9750	0.9332	0.8858	0.6297
100.0	1.0659	1.0612	1.0554	1.0108	0.9602	0.6855
250.0	1.3372	1.3316	1.3246	1.2713	1.2104	0.8750
500.0	1.5544	1.5480	1.5402	1.4803	1.4117	1.0297
750.0	1.6856	1.6789	1.6706	1.6069	1.5337	1.1244
1000.0	1.7806	1.7735	1.7649	1.6984	1.6222	1.1933
2500.0	2.0917	2.0839	2.0741	1.9993	1.9130	1.4222
5000.0	2.3354	2.3269	2.3163	2.2353	2.1417	1.6041
7500.0	2.4809	2.4720	2.4610	2.3764	2.2785	1.7138
10000.0	2.5854	2.5762	2.5649	2.4778	2.3770	1.7930
25000.0	2.9246	2.9147	2.9024	2.8075	2.6973	2.0528
50000.0	3.1872	3.1767	3.1637	3.0631	2.9461	2.2564
75000.0	3.3430	3.3322	3.3187	3.2149	3.0940	2.3782

Values of h_{2_D} for

$R_D = 0.5$ and $r_{w_D} = 0.002$

t_D/Ω_D	10.00	20.00	50.00	100.00	200.00	500.00
0.1	0.0020	0.0012	0.0005	0.0003	0.0001	0.0001
0.2	0.0070	0.0043	0.0020	0.0010	0.0005	0.0002
0.5	0.0238	0.0148	0.0069	0.0037	0.0019	0.0008
0.8	0.0389	0.0243	0.0114	0.0061	0.0031	0.0013
1.0	0.0479	0.0300	0.0141	0.0075	0.0039	0.0016
2.5	0.0978	0.0618	0.0293	0.0156	0.0081	0.0033
5.0	0.1499	0.0954	0.0456	0.0244	0.0126	0.0052
7.5	0.1860	0.1188	0.0570	0.0305	0.0158	0.0065
10.0	0.2140	0.1371	0.0660	0.0354	0.0184	0.0075
25.0	0.3162	0.2044	0.0992	0.0534	0.0278	0.0114
50.0	0.4056	0.2640	0.1289	0.0696	0.0362	0.0149
75.0	0.4626	0.3021	0.1481	0.0801	0.0417	0.0171
100.0	0.5049	0.3307	0.1625	0.0879	0.0459	0.0188
250.0	0.6500	0.4292	0.2126	0.1154	0.0603	0.0248
500.0	0.7695	0.5111	0.2546	0.1387	0.0726	0.0299
750.0	0.8431	0.5620	0.2809	0.1532	0.0802	0.0330
1000.0	0.8969	0.5993	0.3003	0.1640	0.0859	0.0354
2500.0	1.0768	0.7248	0.3659	0.2005	0.1053	0.0434
5000.0	1.2210	0.8263	0.4195	0.2304	0.1212	0.0500
7500.0	1.3084	0.8882	0.4524	0.2489	0.1310	0.0541
10000.0	1.3718	0.9333	0.4764	0.2623	0.1382	0.0571
25000.0	1.5807	1.0827	0.5566	0.3075	0.1623	0.0672
50000.0	1.7456	1.2016	0.6210	0.3440	0.1818	0.0753
75000.0	1.8447	1.2734	0.6601	0.3662	0.1937	0.0803

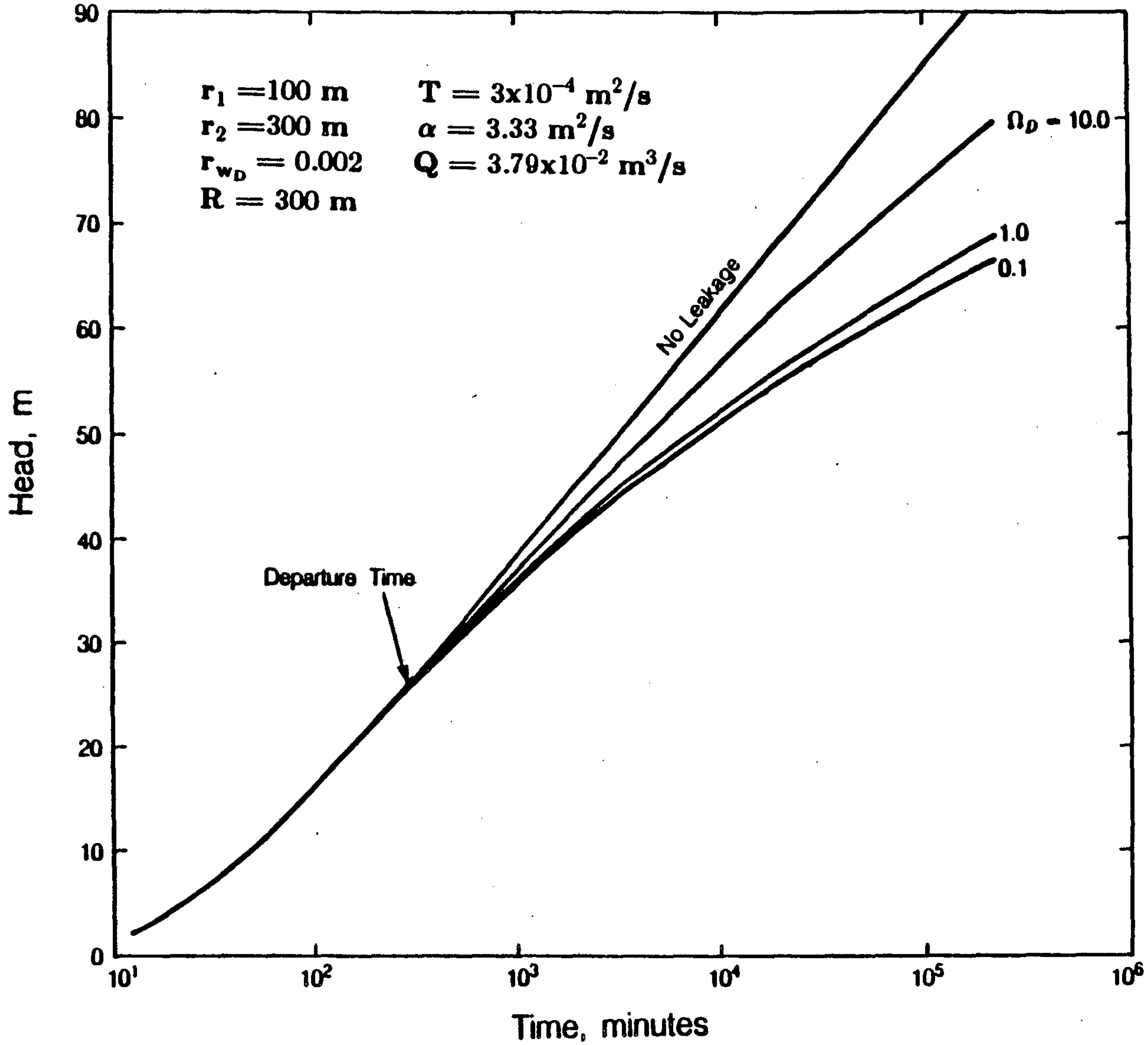

Figure 4. The effect of hydraulic resistance of the abandoned well on the slope of the buildup curves.

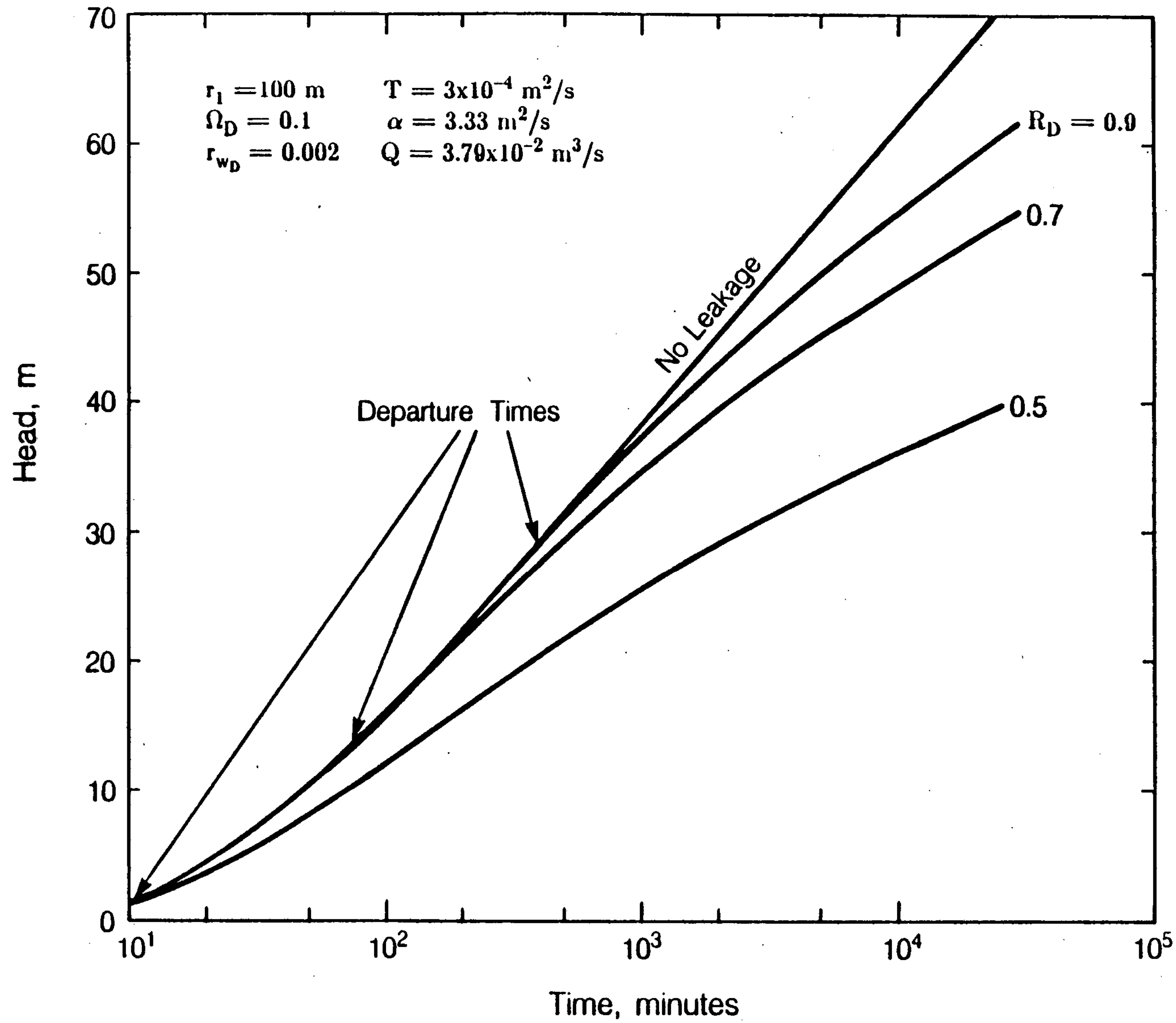

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Figure 5. The effect of dimensionless distance on the departure times from the no-leakage curve.

not be correctly applied. Therefore, one must modify the type-curve-matching analysis.

Since the cost of drilling and construction of a separate monitoring well is relatively high, one may be able to use the injection well for measuring the pressure changes with time. This will fix the value of R_D at unity and as a result eliminate one of the dimensionless parameters.

Figure 6 illustrates the effect of r_{w_D} on the plot of dimensionless drawdown versus dimensionless time for $R_D = 1$ and $\Omega_D = 10$. This figure shows that the two curves covering a reasonable range of r_{w_D} run parallel and are close to each other. Similar studies for other values of Ω_D show the same results. Therefore, for the sake of simplicity, for each Ω_D, one may replace two curves with an averaged one, and thus eliminate the parameter r_{w_D}. Figure 7 shows a set of type curves representing the variation of h_{2_D} versus t_D for $R_D = 1$ and r_{w_D} ranging between 0.001 and 0.0001. This figure, which is based on one running parameter, Ω_D, can be used for analysis of data obtained from the injection well.

Procedure for Finding Abandoned Well and its Leakage Rate

The following is the step-by-step procedure for determining the location of an unplugged or improperly plugged abandoned well that may establish a conduit between a deep injection zone and a more shallow freshwater aquifer.

1. Inject water at constant rate Q in the proposed deep disposal well. The rate Q should be equal to or greater than the maximum injection rate planned for that well.

2. Record the change of hydraulic head in the injection well with time.

3. Having obtained the hydraulic properties (T, S) of the injection zone from conventional pump tests or other sources, calculate the expected hydraulic head buildup at the injection well in the absence of leakage.

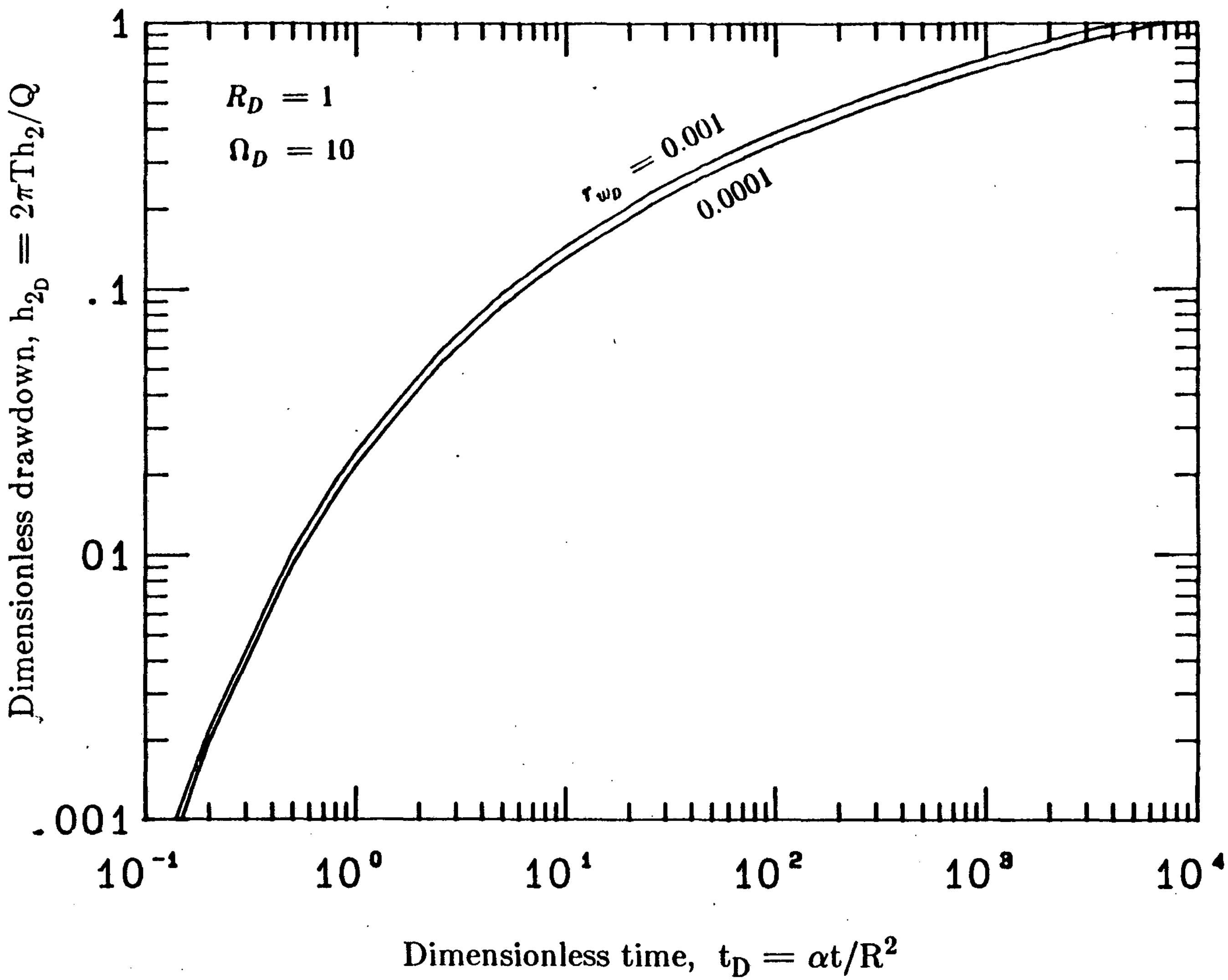

Figure 6. Effect of r_{w_D} on the drawdown calculated for the injection well and $\Omega_D = 10$.

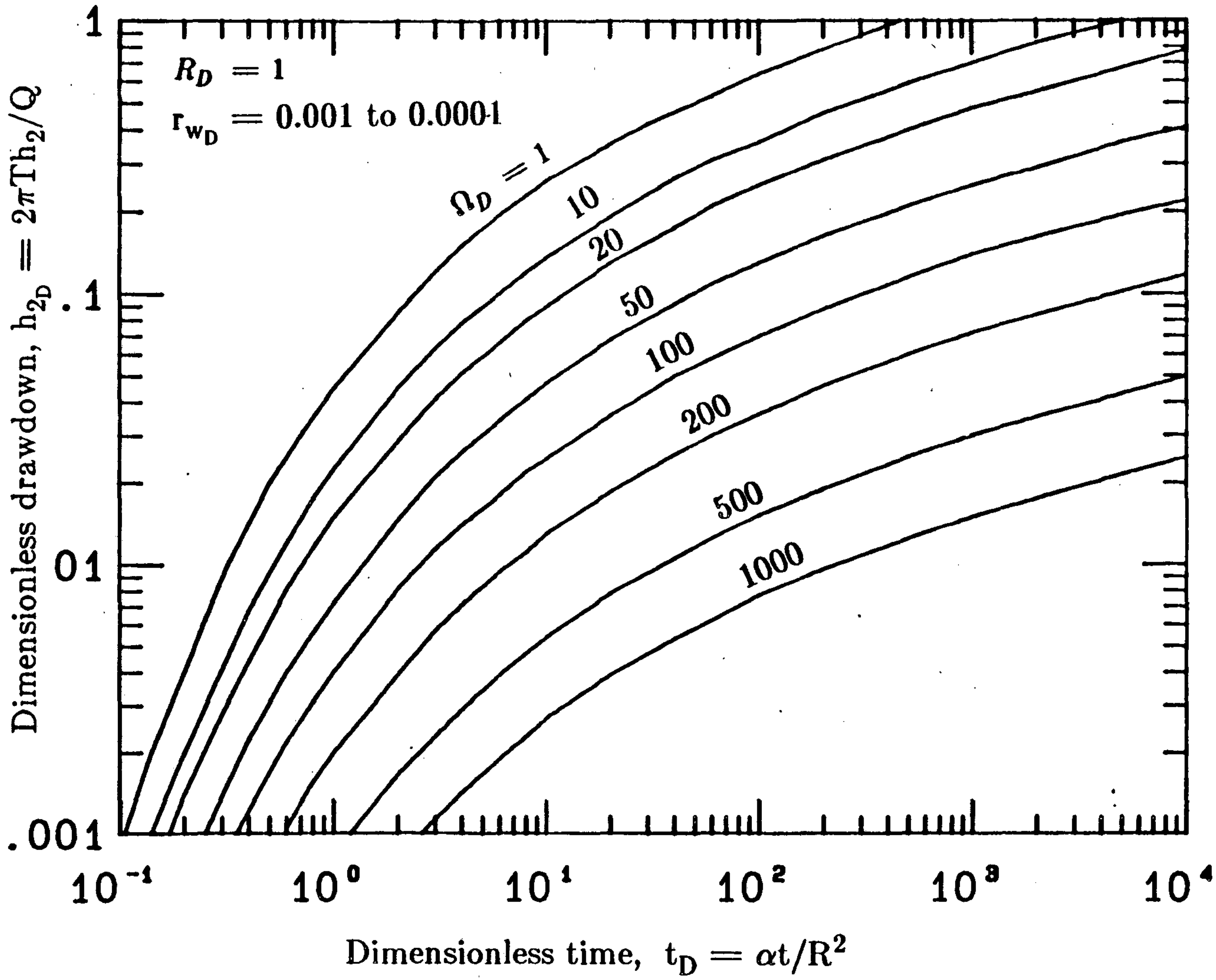

Figure 7. Type curves of dimensionless drawdown versus dimensionless time for the injection well and several values of Ω_D ranging between 1 and 1000 and r_{w_D} between 10^{-3} and 10^{-4}.

4. Find h_2, the difference between the calculated values of buildup obtained in Step 3 and the measured values from Step 2.

5. Convert the drawdowns obtained in Step 4 into dimensionless form using (21).

$$h_{2_D} = \frac{2\pi T h_2}{Q} \tag{21}$$

6. Plot values for h_{2_D} vs time on log-log paper having the scale used in Figure 7.

7. Superimpose the plot obtained in Step 6 over the log-log type curves in Figure 7. Keeping the horizontal axis of both plots coincident (make sure that horizontal axes refer to the same value of h_{2_D}) shift the top plot horizontally until you obtain the best match with one of the curves on Figure 7. If the best match happens to fall between two of the curves, additional curves can be plotted from available data. Because the plots can not be shifted vertically over each other, the amount of error in this process is usually not appreciable. Once the best match is obtained, read the value of Ω_D from the curve that best matches the measured data, and read values of t and t_D from any matching point.

8. Calculate the value of R from

$$R = \left(\frac{\alpha t}{t_D}\right)^{1/2} \tag{23}$$

R is the estimated distance between the injection well and the leaky abandoned well.

9. Estimate the magnitude of r_w and calculate r_{w_D} from

$$r_{w_D} = \frac{r_w}{R} \tag{14}$$

10. Choose the appropriate table of variation of Q_D versus t_D for the known value of Ω_D. Table 1 is a sample for $r_{w_D} = 0.001$. For other values of r_{w_D}

refer to Javandel et al. (1986).

11. Tabulate variation of Q_2 versus time by using the definition of dimensionless parameters.

Further information about the location of the abandoned well can be obtained if an observation well is available. Let r_1 represent the distance between the injection and observation well. To find the distance between the observation well and the abandoned well, r_2, the following steps may be carried out.

1. Plot the variation of the h_{2_D}, from Table 2 or Javandel et al. (1986), vs t_D on a log-log paper for the known value of Ω_D (calculated above) and several values of R_D.
2. Calculate the net drawdown due to leakage from the abandoned well at the observation well (follow Steps 2-4 above).
3. Convert h_2 and t from Step 2 into dimensionless parameters h_{2_D} and t_D.
4. Plot h_{2_D} vs t_D obtained in Step 3 on log-log paper with the scale used in Step 1.
5. Superimpose the plot prepared in Step 4 onto the one obtained in Step 1. Keeping both axes of both plots coincident read the appropriate R_D from the curve matching the observed curve.
6. Calculate r_2 from

$$r_2 = R_D \cdot R \tag{24}$$

7. Using the location of the injection well as a center, draw a circle of radius R; using the location of the observation well as a center draw another circle with a radius of r_2. As shown in Figure 8 the two circles will intersect at two points, one of which will be the location of the abandoned well.

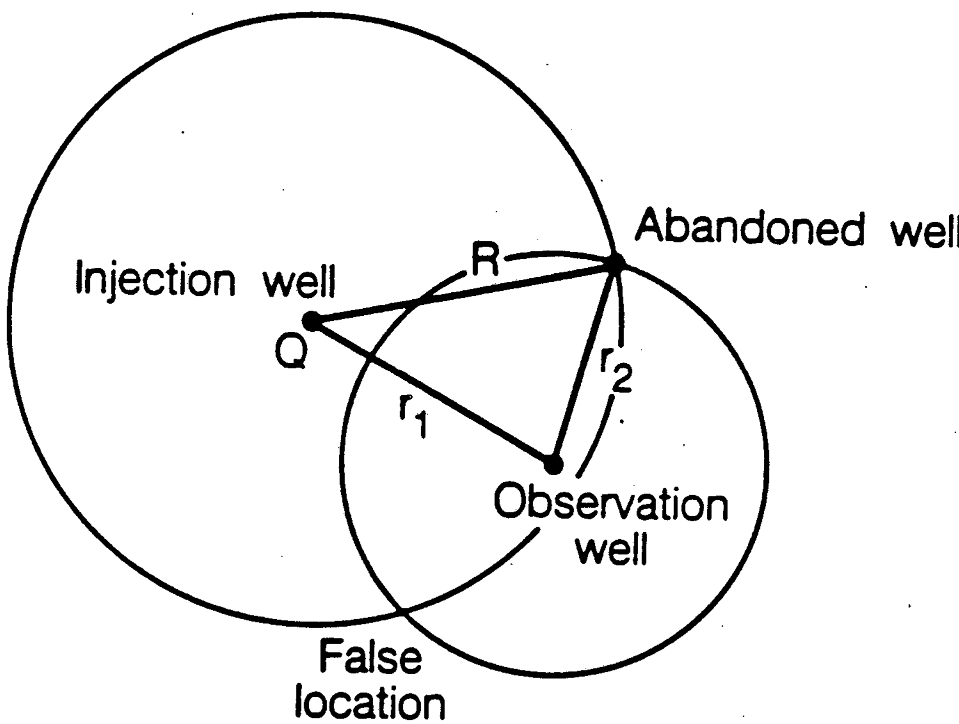

Figure 8. Locating the abandoned well.

8. If surface investigation fails to identify the true position of the abandoned well from these two points, then another observation well is needed to estimate the true position of the abandoned well.

Example 1.

This example is designed to show how the results of this study may be used to estimate the time variation of drawdown at the injection well due to leakage from the abandoned well and thus examine the limitation of this method.

Consider a deep sandstone formation intercalated between two impermeable layers. The sandstone has a thickness of 30 m (98.4 ft). The hydraulic conductivity and storage coefficient of the sandstone are assumed to be 10^{-6}m/s (0.283 ft/d) and 9 x 10^{-5} respectively. Let us suppose that we want to test an injection well at a rate of 3.79 x 10^{-2}m^3/s (600 gpm). We also assume that an unplugged or improperly plugged abandoned well is located at 200 m (656 ft) from the injection well. Examine a wide range of Ω between 1 and 10^7 s/m^2. The lower limit of Ω represents the open casing and the upper limit represents the case where the well is filled with materials having a permeability in the order of darcies. Estimate the variation of drawdown in the injection well with time due to leakage from the abandoned well.

Dimensionless parameters corresponding to the given case are calculated. Because we are monitoring the injection well itself, $R = r_2$, thus

$$R_D = \frac{r_2}{R} = 1 \qquad (25)$$

Estimating a radius of 0.2 m for the abandoned well leads to

$$R_{w_D} = \frac{0.2}{200} = 0.001 \qquad (26)$$

Values of Ω_D vary between 1.9 x 10^{-4} and 1880. Table 3 gives the values of dimensionless drawdown versus dimensionless time for the above parameters. Conversion factors for changing values of dimensionless time and dimensionless

Table 3

Values of h_{2_D} for $R_D = 1.0$ and $r_{w_D} = 0.001$

t_D/Ω_D	0.10E-03	0.10E-02	0.10E-01	0.50E-01	0.10E+00	0.50E+00	0.10E+01
0.1	0.0012	0.0012	0.0012	0.0012	0.0012	0.0011	0.0010
0.2	0.0056	0.0056	0.0056	0.0056	0.0056	0.0052	0.0049
0.5	0.0252	0.0252	0.0252	0.0251	0.0249	0.0236	0.0221
0.8	0.0453	0.0453	0.0453	0.0450	0.0447	0.0424	0.0398
1.0	0.0579	0.0579	0.0578	0.0575	0.0571	0.0542	0.0510
2.5	0.1324	0.1324	0.1323	0.1316	0.1308	0.1245	0.1174
5.0	0.2145	0.2145	0.2143	0.2132	0.2119	0.2022	0.1912
7.5	0.2726	0.2726	0.2723	0.2710	0.2694	0.2573	0.2436
10.0	0.3182	0.3181	0.3178	0.3163	0.3145	0.3005	0.2848
25.0	0.4852	0.4851	0.4847	0.4825	0.4799	0.4596	0.4366
50.0	0.6316	0.6316	0.6309	0.6283	0.6249	0.5995	0.5705
75.0	0.7244	0.7243	0.7236	0.7206	0.7169	0.6883	0.6556
100.0	0.7931	0.7930	0.7923	0.7890	0.7850	0.7541	0.7188
250.0	1.0268	1.0267	1.0258	1.0218	1.0168	0.9785	0.9346
500.0	1.2170	1.2169	1.2159	1.2113	1.2055	1.1616	1.1110
750.0	1.3331	1.3330	1.3319	1.3269	1.3207	1.2735	1.2190
1000.0	1.4175	1.4173	1.4162	1.4110	1.4045	1.3549	1.2976
2500.0	1.6963	1.6962	1.6949	1.6889	1.6815	1.6244	1.5582
5000.0	1.9168	1.9166	1.9151	1.9086	1.9005	1.8378	1.7650
7500.0	2.0492	2.0490	2.0475	2.0406	2.0320	1.9661	1.8895
10000.0	2.1445	2.1444	2.1428	2.1356	2.1268	2.0586	1.9793
25000.0	2.4558	2.4557	2.4539	2.4460	2.4363	2.3609	2.2730
50000.0	2.6983	2.6981	2.6962	2.6878	2.6774	2.5967	2.5025
75000.0	2.8427	2.8425	2.8406	2.8319	2.8210	2.7373	2.6395

Values of h_{2_D} for $R_D = 1.0$ and $r_{w_D} = 0.001$

t_D/Ω_D	0.50E+01	0.10E+02	0.50E+02	0.10E+03	0.50E+03	0.10E+04	0.20E+04
0.1	0.0007	0.0005	0.0001	0.0001	0.0000	0.0000	0.0000
0.2	0.0032	0.0022	0.0007	0.0003	0.0001	0.0000	0.0000
0.5	0.0147	0.0104	0.0031	0.0017	0.0003	0.0002	0.0001
0.8	0.0268	0.0191	0.0057	0.0031	0.0006	0.0003	0.0002
1.0	0.0345	0.0246	0.0074	0.0040	0.0008	0.0004	0.0002
2.5	0.0808	0.0582	0.0179	0.0096	0.0020	0.0010	0.0005
5.0	0.1331	0.0965	0.0301	0.0162	0.0035	0.0017	0.0009
7.5	0.1707	0.1243	0.0391	0.0211	0.0045	0.0023	0.0011
10.0	0.2005	0.1464	0.0463	0.0250	0.0053	0.0027	0.0013
25.0	0.3117	0.2296	0.0739	0.0400	0.0086	0.0043	0.0022
50.0	0.4112	0.3048	0.0993	0.0539	0.0116	0.0058	0.0029
75.0	0.4751	0.3535	0.1160	0.0630	0.0135	0.0068	0.0034
100.0	0.5229	0.3900	0.1286	0.0700	0.0151	0.0076	0.0038
250.0	0.6875	0.5168	0.1730	0.0945	0.0204	0.0103	0.0052
500.0	0.8239	0.6227	0.2109	0.1154	0.0250	0.0126	0.0063
750.0	0.9080	0.6885	0.2346	0.1286	0.0279	0.0141	0.0071
1000.0	0.9696	0.7368	0.2522	0.1384	0.0300	0.0152	0.0076
2500.0	1.1754	0.8993	0.3123	0.1720	0.0374	0.0189	0.0095
5000.0	1.3404	1.0306	0.3617	0.1997	0.0436	0.0220	0.0111
7500.0	1.4405	1.1106	0.3921	0.2168	0.0474	0.0240	0.0121
10000.0	1.5129	1.1687	0.4144	0.2294	0.0502	0.0254	0.0128
25000.0	1.7515	1.3611	0.4891	0.2716	0.0596	0.0302	0.0152
50000.0	1.9395	1.5138	0.5493	0.3058	0.0673	0.0341	0.0171
75000.0	2.0524	1.6059	0.5860	0.3267	0.0719	0.0364	0.0183

drawdown can be calculated from

$$t = \frac{R^2}{\alpha}\, t_D = \frac{(200\ \text{m})^2}{(0.333\ \text{m}^2/\text{s})\,(60\ \text{s/min})}\, t_D = 2000\ t_D \quad \text{min} \tag{27}$$

$$h_2 = \frac{Q}{2\pi T}\, h_{2_D} = \frac{(3.79 \text{ x } 10^{-2}\ \text{m}^3/\text{s})}{2\pi(3 \text{ x } 10^{-5}\ \text{m}^2/\text{s})}\, h_{2_D} = 201.06\ h_{2_D} \quad \text{m} \tag{28}$$

Applying the above two equations, Table 3 gives the net time variation of drawdown at the injection well as a function of Ω_D. The results are shown in Table 4.

Obviously in this example when permeability of the fill material in the abandoned well is less than tenth of one darcy the magnitude of the leakage is very small. As a result, the corresponding drawdown observed at the injection well is too small to give reliable results, since errors in field measurement of hydraulic head can overshadow these small drawdowns.

Example 2

This hypothetical example is intended to illustrate the detection of a leaky abandoned well in the vicinity of a proposed deep injection well, and the estimation of the distance between the two wells. The leakage rate from the abandoned well is also calculated.

Table 5 shows values of the net drawdown (difference between the observed and calculated nonleaky values of buildup) in an injection well, located in a sandstone aquifer having the properties described in Example 1. If the injection rate is again 3.79 x 10^{-2} m^3/s (600 gpm), estimate the distance of the unplugged, or improperly plugged, abandoned well from the injection well, and the time variation of leakage rate in the abandoned well.

The procedure is as follows:

1. Calculate the corresponding dimensionless drawdown h_{2_D} for each value of drawdown given in Table 5 as given by

$$h_{2_D} = \frac{2\pi\ T\ h_2}{Q} = \frac{2\pi(3 \times 10^{-5}\ \text{m}^2/\text{s})}{3.79 \times 10^{-2}\ \text{m}^3/\text{s}}\, h_2 = 4.97 \times 10^{-3}\, h_2\ (\text{m}) \tag{29}$$

Table 4
Time variation of drawdown expected at the injection well for Example 1

Time, (min)	Drawdown (m)								
	$\Omega_D =$ 0.0001	$\Omega_D =$ 0.001	$\Omega_D =$ 0.01	$\Omega_D =$ 0.1	$\Omega_D =$ 1.	$\Omega_D =$ 10.	$\Omega_D =$ 100.	$\Omega_D =$ 1000.	$\Omega_D =$ 2000.
200	0.24	0.24	0.24	0.24	0.2	0.1	0.02	0	0
400	1.12	1.12	1.12	1.12	0.98	0.44	0.06	0	0
1,000	5.06	5.06	5.06	5.00	4.44	2.09	0.34	0.04	0.02
2,000	11.64	11.64	11.64	11.48	10.25	4.94	0.80	0.08	0.04
5,000	26.62	26.62	26.6	26.29	23.60	11.70	1.93	0.20	0.10
10,000	43.12	43.12	43.12	42.60	38.44	19.40	3.25	0.34	0.18
20,000	63.95	63.95	63.89	63.23	57.26	29.43	5.02	0.54	0.26
50,000	97.55	97.55	97.45	96.48	87.78	46.16	8.00	0.86	0.44
100,000	126.98	126.98	126.84	125.6	114.6	61.28	10.83	1.16	0.58

Table 5.

Values of the net drawdown calculated from the measured data at the injection well for Example 2

Time (min)	Drawdown (m)
1,000	0.34
2,000	0.80
5,000	1.93
10,000	3.25
20,000	5.02
50,000	8.00
100,000	10.83

Table 6

Calculated values of h_{2_D} as a function of time for Example 2

Time (min)	h_{2_D}
1,000	0.0017
2,000	0.0040
5,000	0.0096
10,000	0.0162
20,000	0.0250
50,000	0.0400
100,000	0.0539

Table 6 shows the calculated values of h_{2_D} versus time.

2. Plot h_{2_D} vs time on log-log paper with the scale used in Figure 7.

3. Superimpose this plot on Figure 7 and keep the horizontal axes ($h_{2_D} = 10^{-3}$) of both figures coincident. Shift the top figure horizontally until the plot of the measured data matches one of the type curves in Figure 7. Read the values of t and t_D from any arbitrary match point on both figures. The value of t corresponding to $t_D = 1$ would be 2000 minutes.

4. Calculate the distance between the abandoned and injection well from

$$R = \left(\frac{\alpha t}{t_D}\right)^{1/2} = \left[\frac{(0.333\ m^2/s)\ (2 \times 10^3\ min)(60\ s/min)}{1}\right]^{1/2} = 200\ m \quad (30)$$

5. The value of Ω_D from the matching type curve is

$$\Omega_D = 100 \quad (31)$$

6. If we assume a radius of 0.2 m for the abandoned well the corresponding value of r_{w_D} would become $0.2/200 = 0.001$.

7. Find values of Q_D as a function of t_D for $r_{w_D} = 0.001$ and $\Omega_D = 100$ using Table 1. Convert t_D and Q_D to t and Q_2 using (13) and (12). The results are tabulated in Table 7.

Table 7

Time variation of leakage from the abandoned well for Example 2

t_D	t,min	Q_D	Q_2, gpm
0.1	200	0.001	0.6
0.2	400	0.002	1.2
0.5	1,000	0.004	2.4
0.7	1,400	0.005	3.0
1.	2,000	0.006	3.6
2.	4,000	0.009	5.4
5.	10,000	0.012	7.2
7.	14,000	0.014	8.4
10.	20,000	0.015	9.0
20	40,000	0.018	10.8
50	1×10^5	0.022	13.2
70	1.4×10^5	0.024	14.4
100	2×10^5	0.025	15.0
500	1×10^6	0.032	19.2
1,000	2×10^6	0.035	21.0
5,000	1×10^7	0.042	25.2
10,000	2×10^7	0.045	27.0

SUMMARY AND CONCLUSIONS

For thousands of abandoned wells drilled prior to 1930 there are no records on their location and characteristics. It is believed that the majority of these wells are improperly plugged. Many of the wells are located in regions in which underground deep-disposal facilities have or are being established. Consequently, the deep disposal facilities in these regions are subject to the risk of leakage through the nearby abandoned wells. It is therefore very important to detect any of these wells that may be within the area of influence of a proposed deep injection well.

In this report a new method is proposed for detecting the presence of an improperly plugged abandoned well near a proposed injection well. The method is based in pump testing the injection well. Measurement of pressure variations within the injection and/or observation well(s), coupled with the use of the new set of type curves given in this paper, should reveal the distance of a leaky abandoned well to an injection well. Extensive tables provided by this work enable one to determine the magnitude of leakage from the abandoned well and its variation with time. Furthermore, approximate location of the leaky well can be determined by measuring pressure variations in an observation well located in the vicinity of the injection well.

There are two conditions under which this method may not be applicable:

(1) If the confining layers leak. In this case, since the effect of leakage from the confining layers overshadows the effect of leakage from the improperly plugged well, one may not be able to detect the abandoned well. However, this is beyond our scope of interest, because if the layers above and/or below the injection zones leak, the zone is not suitable for disposal of hazardous liquid wastes anyway.

(2) If the abandoned well is filled with very low permeability materials (on the order of one tenth of a Darcy or less). In this case, the amount of leakage from the well is so small that the corresponding drawdown measured in the

observation or injection well is too small to be detected. However, the solution derived in this work and the resulting tables should enable one to estimate the magnitude and variation of leakage from a hypothetical well located at a given distance and the materials filling the borehole have a given permeability.

APPENDIX A

THE LAPLACE INVERSION OF EQUATION (8)

As was noted the Laplace inversion of (8) may be obtained through the application of the complex inversion integral shown below:

$$Q_2(t) = \frac{1}{2\pi i} \lim_{\beta \to \infty} \int_{\gamma - i\beta}^{\gamma + i\beta} \exp(\lambda t) \frac{\frac{Q}{2\pi T} \frac{K_0\left(R\sqrt{\frac{\lambda}{\alpha}}\right)}{\lambda}}{\Omega + \frac{1}{2\pi T} K_0\left(r_w \sqrt{\frac{\lambda}{\alpha}}\right)} d\lambda \qquad \text{(A1)}$$

where γ is so large that all singularities of the integrand in (A1) lie to the left of the line $(\gamma - i\infty\ ,\ \gamma + i\infty)$. To perform the above integration it is customary to choose a new path shown by the contour given in Figure A1. The value of the integral over the line AB when $\bar{R}$ tends to infinity gives the solution for $Q_2(t)$. Since the integral over the large circle with the radius $\bar{R}$ vanishes when $\bar{R}$ tends to infinity, the value of integral in (A1) is equal to the integral over CD, EF, and the small circle DE.

On CD we let $\lambda = \alpha u^2 e^{i\pi}$. Then

$$\frac{d\lambda}{\lambda} = \frac{2\alpha u e^{i\pi} du}{\alpha u^2 e^{i\pi}} = \frac{2du}{u}$$

and if the integral on CD, when $\bar{R}$ tends to infinity, is represented by I_{CD}, then

$$I_{CD} = \frac{Q}{2\pi^2 T i} \int_0^\infty e^{-\alpha u^2 t} \frac{K_0\left(Rue^{i\frac{\pi}{2}}\right)}{\Omega + \frac{1}{2\pi T} K_0\left(r_w u e^{i\frac{\pi}{2}}\right)} \frac{du}{u} \qquad \text{(A2)}$$

Substituting for K_0 in terms of J_0 and Y_0 we obtain

$$I_{CD} = \frac{Q}{2\pi^2 T i} \int_0^\infty e^{-\alpha u^2 t} \frac{-\pi i \left\{ J_0(Ru) - iY_0(Ru) \right\}}{\Omega - \frac{i}{4T}\left\{ J_0(r_w u) - iY_0(r_w u) \right\}} \frac{du}{2u} \qquad \text{(A3)}$$

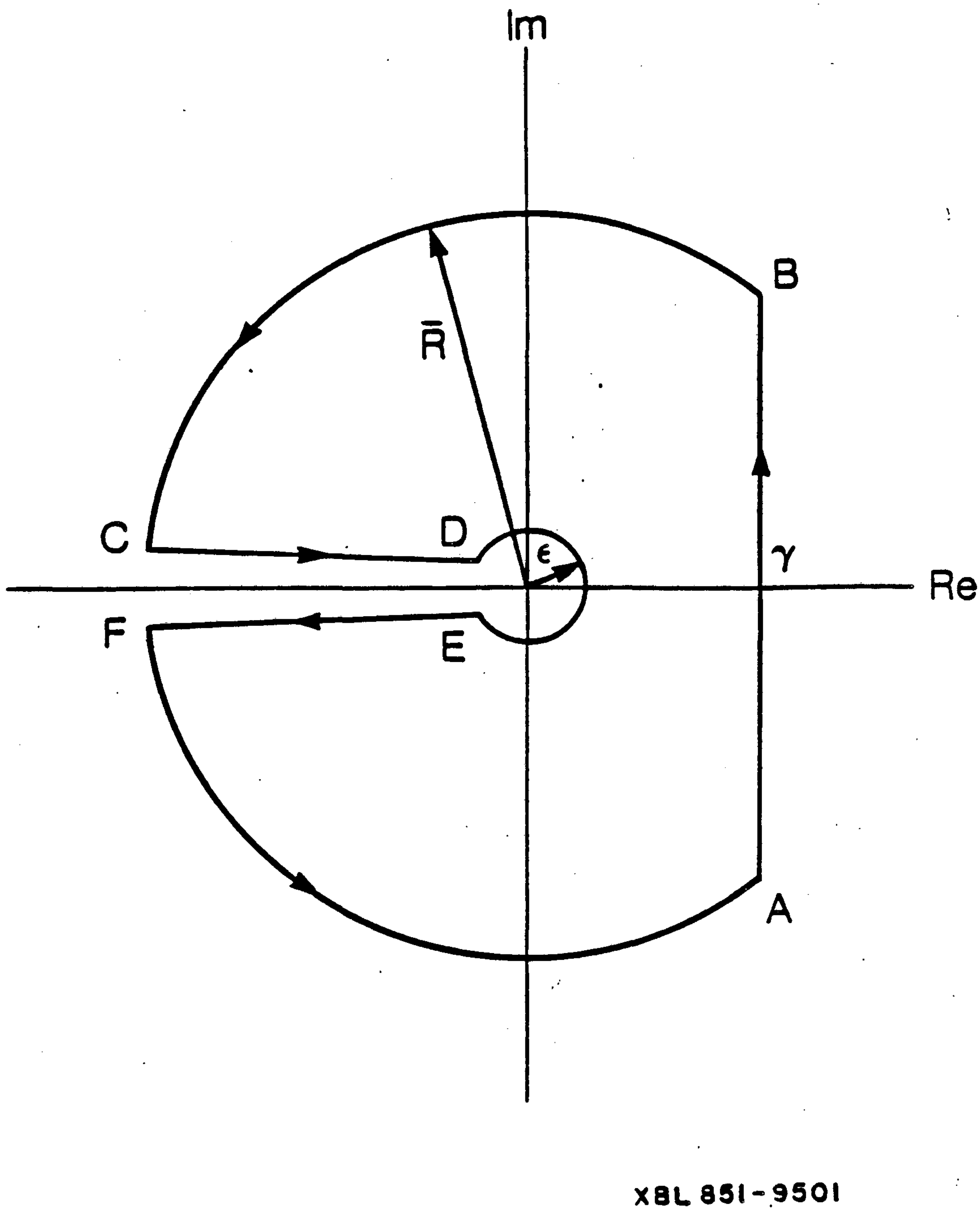

Figure A1. Contour path for Laplace inversion

The corresponding integral over EF may be written as

$$I_{EF} = \frac{Q}{2\pi^2 T\, i} \int_{\infty}^{0} e^{-\alpha u^2 t} \frac{K_0(Rue^{-i\frac{\pi}{2}})}{\Omega + \frac{1}{2\pi T} K_0(r_w ue^{-i\frac{\pi}{2}})} \frac{du}{u} \tag{A4}$$

or

$$I_{EF} = -\frac{Q}{2\pi^2 Ti} \int_{0}^{\infty} e^{-\alpha u^2 t} \frac{i\pi\left\{ J_0(Ru) + iY_0(Ru)\right\}}{\Omega + \frac{i}{4\pi}\left\{ J_0(r_w u) + iY_0(r_w u)\right\}} \frac{du}{2u} \tag{A5}$$

On the small circle, $\lambda = \alpha u^2 e^{i\theta}$. Therefore

$$I_{DE} = \lim_{u \to 0} \frac{Q}{4\pi^2 Ti} \int_{-\pi}^{\pi} e^{\alpha u^2 e^{i\theta} t} \frac{K_0(Rue^{i\frac{\theta}{2}})}{\Omega - \frac{1}{2\pi T} K_0(r_w ue^{i\frac{\theta}{2}})} i\, d\theta \tag{A6}$$

Noting that when $z \to 0$, $K_0(z) \approx -\ln z$ one can write

$$I_{DE} = \lim_{u \to 0} \frac{Q}{4\pi^2 T} \int_{-\pi}^{\pi} \frac{-\ln(Rue^{i\frac{\theta}{2}})}{\Omega - \frac{1}{2\pi T} \ln(r_w ue^{i\frac{\theta}{2}})} d\theta \tag{A7}$$

Using L'Hospital's rule it can be shown that $I_{DE} = Q$. Therefore,

$$Q_2(t) = Q - \frac{Q}{4\pi T} \int_{0}^{\infty} e^{-\alpha u^2 t} \frac{J_0(Ru) - iY_0(Ru)}{\Omega - \frac{i}{4T}\left\{ J_0(r_w u) - iY_0(r_w u)\right\}} \frac{du}{u}$$

$$- \frac{Q}{4\pi T} \int_{0}^{\infty} e^{-\alpha u^2 t} \frac{J_0(Ru) + iY_0(Ru)}{\Omega + \frac{i}{4T}\left\{ J_0(r_w u) + iY_0(r_w u)\right\}} \frac{du}{u} \tag{A8}$$

Simplifying (A8) we obtain

$$Q_2(t) = Q - \frac{2Q}{\pi} \int_{0}^{\infty} e^{-\alpha u^2 t} \frac{J_0(uR)\left\{ 4T\Omega - Y_0(ur_w)\right\} + J_0(ur_w)\, Y_0(uR)}{\left\{ 4T\Omega - Y_0(ur_w)\right\}^2 + J_0^2(ur_w)} \frac{du}{u} \tag{A9}$$

Equation (A9) gives the leakage rate from the abandoned well located at a distance R from the injection well.

NOTATION

$-Ei(-x)$	$= \int_x^\infty \frac{e^{-y}}{y} dy$, exponential integral
H_1	initial head difference between Aquifers A and B in the vicinity of the abandoned well ($h_A - h_B$), m
h_2	drawdown due to leakage through the abandonded well, observed at a monitoring well, m
h_A, h_B	initial hydraulic heads in Aquifers A and B, m
Δh_2	change of hydraulic head at well 2 due to injection at Well 1, m
$\Delta h_2'$	net increase in hydraulic head at the abandoned well(Well 2) in Aquifer B, m
$J_0(x)$	Bessel function of first kind and zero order
$K_0(x)$	Modified Bessel function of second kind and zero order
p	Laplace transform parameter
Q	injection rate through Well 1, m^3/s
Q_2	leakage rate through the abandoned well, m^3/s
Q_D	Q_2/Q, dimensionless leakage rate
R	distance between injection and abandoned wells, m
R_D	r_2/R, dimensionless radius
r_w	effective radius of the abandoned well, m
r_1	distance between abandonded and injection well, m
r_2	distance between abandoned and monitoring well, m
r_{w_D}	r_w/R , dimensionless radius of abandoned well

s_w drawdown at the abandoned well due to leakage through it, m

T transmissivity of the Aquifer B (injection zone), m^2/s

t time since the start of injection, s

t_D $\alpha t/R^2$, dimensionless time

$Y_0(x)$ Bessel function of second kind and zero order

α diffusivity of Aquifer B (injection zone), m^2/s

Ω hydraulic resistance of the abandoned well between Aquifer A and Aquifer B, s/m^2

Ω_D $2\pi T\Omega$, dimensionless hydraulic resistance of the abandoned well

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