HYDROMAGNETIC BOUNDARY LAYER MICROPOLAR FLUID FLOW OVER A STRETCHING SURFACE EMBEDDED IN A NON-DARCIAN POROUS MEDIUM WITH RADIATION

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Received 31 January 2006; Accepted 16 July 2006

We have studied the effects of radiation on the boundary layer flow and heat transfer of an electrically conducting micropolar fluid over a continuously moving stretching surface embedded in a non-Darcian porous medium with a uniform magnetic field. The transformed coupled nonlinear ordinary differential equations are solved numerically. The velocity, the angular velocity, and the temperature are shown graphically. The numerical values of the skin friction coefficient, the wall couple stress, and the wall heat transfer rate are computed and discussed for various values of parameters.

1. Introduction

Eringen [7] introduced the concept of micropolar fluid in an attempt to explain the behavior of a certain fluid containing polymeric additives and naturally occurring fluids such as the phenomenon of the flow of colloidal fluids, real fluid with suspensions, liquid crystals, and animal blood. The theory of thermomicropolar fluids has been developed by Eringen [8], taking into account the effect of microelements of fluids on both the kinematics and conduction of heat. Micropolar fluid theory has been used to describe in detail the effect of dirt in journal bearing, see [2, 11, 14, 22]. The review articles by Ariman et al. [4, 5] describe some of the various applications which have been explored.

Boundary layer on continuous surface is an important type of flow occurring in a number of technical problems. Examples may be found in continuous casting, glass fiber production, metal extrusion, hot rolling, textiles, and wire drawing (see [3, 20]). Sakiadis [17] initiated the theoretical study of boundary layer on a continuous semi-infinite sheet moving steadily through an otherwise quiescent fluid environment, whereas its heat transfer aspect was studied by Tsou et al. [23]. Karwe and Jaluria [9] carried out a numerical study of the transport arising due to the movement of a continuous heated body.

Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2006, Article ID 39392, Pages 1–10
DOI 10.1155/MPE/2006/39392

The boundary layer flow of a micropolar fluid past a semi-infinite plate has been studied by Peddieson and Mcnitt [13] whereas a similarity solution for boundary layer flow near stagnation point was presented by Ebert [6]. The boundary layer flow of micropolar fluids past a semi-infinite plate was studied by Ahmadi [1], taking into account the gyration vector normal to the xy-plane and the microinertia effects. Flow and heat transfer of a micropolar fluid past a continuously moving plate are studied by Takhar and Soundalgekar [18, 21]. By drawing the continuous strips through a quiescent electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and final product of desired characteristics can be achieved. Kelson and Farrell [10] studied micropolar flow over a porous stretching sheet with strong suction or injection.

Flow and heat transfer through porous media have several practical engineering applications such as transpiration cooling, packed bed chemical reactors, geothermal systems, crude oil extraction, ground water hydrology, and building thermal insulation. We know that the radiation effect is important under many nonisothermal situations. If the entire system involving the polymer extrusion process is placed in a thermally controlled environment, then radiation could become important. The knowledge of radiation heat transfer in the system can perhaps lead to a desired product with sought characteristic. Recently, the effects of radiation on the flow and heat transfer of a micropolar fluid past a continuously moving plate have been studied by many authors, see [12, 15]. Raptis [16] studied the boundary layer flow of a micropolar fluid through non-Darcian porous medium. The problem of hydromagnetic boundary layer micropolar fluid flow over a continuously moving stretching surface through a fluid saturated porous medium with radiation is therefore an important one. It is now proposed to study the flow and heat transfer of an electrically conducting micropolar fluid on a continuously moving plate embedded in a non-Darcian porous medium in the presence of a uniform magnetic field and radiation.

2. Mathematical formulation

Consider a steady, two-dimensional laminar flow of an incompressible, electrically conducting micropolar fluid over a continuously moving stretching surface embedded in a non-Darcian porous medium which issues from a thin slit. The x-axis is taken along the stretching surface in the direction of the motion and y-axis is perpendicular to it. We assume that the velocity is proportional to its distance from the slit. A uniform magnetic field B_0 is imposed along y-axis. Then under the usual boundary layer approximations, the flow and heat transfer of a micropolar fluid in porous medium with the non-Darcian effects included are governed by the following equations [16].

(i) The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{2.1}$$

(ii) The equation of momentum is

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+k_1\frac{\partial N}{\partial y}-\frac{\nu\varphi}{k}u-c\varphi u^2-\frac{\sigma B_0^2}{\rho}u. \tag{2.2}$$

(iii) The equation of angular momentum is

$$G_1 \frac{\partial^2 N}{\partial y^2} - 2N - \frac{\partial u}{\partial y} = 0. \tag{2.3}$$

(vi) The equation of energy is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}. \tag{2.4}$$

(v) The boundary conditions are

$$\begin{gathered} y = 0 : u = ax, \qquad v = 0, \qquad T = T_w, \qquad N = 0, \\ y \longrightarrow \infty : u \longrightarrow 0, \qquad T \longrightarrow T_\infty, \qquad N \longrightarrow 0, \end{gathered} \tag{2.5}$$

where $\nu = (\mu + S)/\rho$ is the apparent kinematic viscosity, μ is the coefficient of dynamic viscosity, S is a constant characteristic of the fluid, N is the microrotation component, $k_1 = S/\rho(> 0)$ is the coupling constant, $G_1(> 0)$ is the microrotation constant, ρ is the fluid density, u and v are the components of velocity along x and y directions, respectively, φ is the porosity, k is the permeability of the porous medium, c is Forchheimer's inertia coefficient, T is the temperature of the fluid in the boundary layer, T_∞ is the temperature of the fluid far away from the plate, T_w is the temperature of the plate, K is the thermal conductivity, C_p is the specific heat at constant pressure, σ is the electrical conductivity, B_0 is an external magnetic field, and q_r is the radiative heat flux.

We now introduce the following transformations:

$$\begin{gathered} \eta = \left(\frac{a}{\nu}\right)^{1/2} y, \qquad \psi = (a\nu)^{1/2} x f(\eta), \\ N = \left(\frac{a^3}{\nu}\right)^{1/2} x g(\eta), \qquad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \\ u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}. \end{gathered} \tag{2.6}$$

Using the Rosselant approximation [19], we have

$$q_r = \left(-\frac{4\sigma_0}{3k_0}\right)\frac{\partial T^4}{\partial y}, \tag{2.7}$$

where σ_0 is the Stefan-Boltzmann constant and k_0 is the mean absorption coefficient.

Substituting expressions in (2.6)-(2.7) into (2.1)–(2.5), we have

$$f''' + ff'' + Lg' - (D_a^{-1} + R)f' - (1+\alpha)f'2 = 0, \tag{2.8}$$

$$Gg'' - (2g + f'') = 0, \tag{2.9}$$

$$3F\theta'' + 3FP_r f\theta' + 4P_r\left[(1 + r\theta)^3\theta'' + 3r(1 + r\theta)^2\theta'2\right] = 0, \tag{2.10}$$

where $L = k_1/\nu$ denotes the coupling constant parameter; $D_a^{-1} = \varphi\nu/ka$ denotes the inverse Darcy number; $R = (\sigma_0 B_0^2)/\rho a$ denotes the magnetic parameter; $\alpha = c\varphi x$ denotes the inertia coefficient parameter; $G = G_1 a/\nu$ denotes the microrotation parameter; $P_r = (\nu\rho C_p)/k$ denotes the Prandtl number; $F = (\rho C_p k_0 \nu)/(4\sigma_0 T_\infty^3)$ denotes the radiation parameter; and $r = (T_w - T_\infty)/T_\infty$ is the relative difference between the temperature of the surface and the temperature far away from the surface.

The corresponding boundary conditions are

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad g(0) = 0,$$
$$f'(\infty) = 0, \quad \theta(\infty) = 0, \quad g(\infty) = 0. \tag{2.11}$$

In the above equations, a prime denotes differentiation with respect to η. In the case of $L = 0$ and $\alpha = 0$, (2.8) together with the boundary conditions $f(0) = 0$, $f'(0) = 1$, and $f'(\infty) = 0$ has an exact solution in the form

$$f(\eta) = \frac{1}{\sqrt{1 + D_a^{-1} + R}}\left(1 - e^{(-\sqrt{1+D_a^{-1}+R})\eta}\right). \tag{2.12}$$

The shear stress at the surface of the plate is given by [9],

$$\tau_w = -\left|(\mu + S)\frac{du}{dy} + SN\right|_{y=0}. \tag{2.13}$$

The skin-friction coefficient is given by

$$c_f = \left(\frac{2\tau}{\rho U^2}\right)_{y=0} = -2\sqrt{R_{ex}}f''(0), \tag{2.14}$$

where $R_{ex} = Ux/\nu$ is the local Reynolds number.

The couple stress at the wall is given by the following:

$$m_w = G_1\left(\frac{\partial N}{\partial y}\right)_{y=0} = R_{ex}\left(\frac{G_1 U}{x^2}\right)g'(0). \tag{2.15}$$

From the temperature field, we can now study the rate of heat transfer. It is given by

$$q_w = -K\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{2.16}$$

From (2.6), (2.15) is reduced to

$$q_w = -K(T_w - T_\infty)\sqrt{\frac{u_0}{2\nu x}}\theta'(0). \tag{2.17}$$

The numerical values of $f''(0)$, $\theta'(0)$, and $g'(0)$ are displayed in Table 3.1.

Table 3.1. Values of $-f''(0)$, $g'(0)$, and $-\theta'(0)$ with $L = 0.1$, $G = 2$, $\alpha = 0.2$, and $r = 0.3$.

D_a^{-1}	R	P_r	F	$-f''(0)$	$g'(0)$	$-\theta'(0)$
2	1	7	1	2.02742	0.334229	0.272683
2	1	7	0.1	2.02742	0.334229	0.203909
2	1	7	1	2.02742	0.334229	0.272683
2	1	0.72	1	2.02742	0.334229	0.245378
2	1	7	1	2.02742	0.334229	0.272683
2	2	7	1	2.26029	0.346146	0.265989
2	1	7	1	2.02742	0.334229	0.272683
0.5	1	7	1	1.61666	0.307951	0.288118

Table 3.2. Comparison between analytical and numerical values of $f''(0)$ for various values of D_a^{-1} and R with $L = 0$ and $\alpha = 0$.

D_a^{-1}	R	Analytical	Numerical
1	0	−1.41421	−1.41442
1	2	−2.00000	−2.00000
1	3	−2.23607	−2.23607
2	3	−2.44949	−2.44949
1	3	−2.23607	−2.23607
0.5	3	−2.12132	−2.12132

3. Solutions and discussion

The system of coupled nonlinear ordinary differential equation (2.8)–(2.10) together with the boundary conditions (2.11) is solved numerically by using the fourth-order Runge-kutta method along with the shooting technique. In order to assess the accuracy of the present numerical method, we have compared our numerical results obtained for the skin-friction coefficient taking $L = 0$ and $\alpha = 0$ in (2.8) with those obtained analytically. The analytical and numerical values of $-f''(0)$ for various values of D_a^{-1} and R are tabulated in Table 3.2. The numerical values of $-f''(0)$ are in good agreement with the obtained analytical values.

We have considered in some detail the influence of the physical parameters D_a^{-1}, R, P_r, and F on the velocity, microrotation, and temperature distributions which are shown in Figures 3.1–3.6. Figures 3.1 and 3.2 show the velocity, angular velocity, and temperature profiles for various values of the magnetic parameter R, respectively. Application of a transverse magnetic field normal to the flow direction gives rise to a resistive drag-like force acting in a direction opposite to that of flow. This has a tendency to reduce both the fluid velocity and angular velocity and increase the fluid temperature. This is indicative from the decreases in the fluid velocity f, the angular velocity g and increases in the temperature θ as shown in Figures 3.1 and 3.2, respectively.

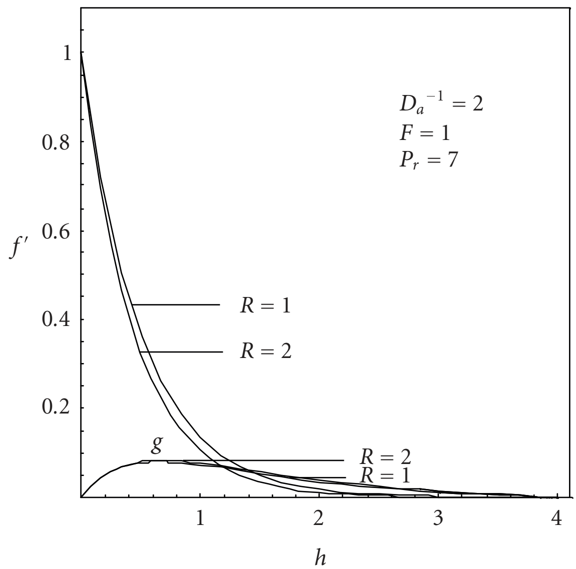

Figure 3.1. Velocity and microrotation distributions for various values of R.

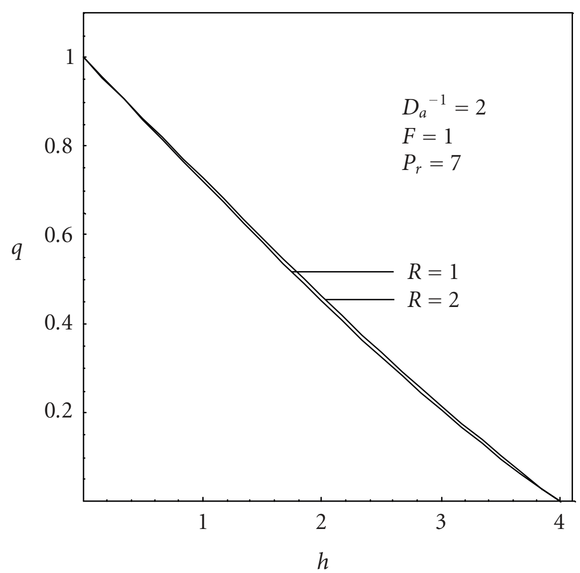

Figure 3.2. Temperature distributions for various values of R.

Figures 3.3 and 3.4 display the influence of the inverse Darcy number D_a^{-1} on the velocity and the temperature profiles, respectively. It is obvious that the presence of porous medium causes higher restriction to the fluid, which reduces both the velocity and the

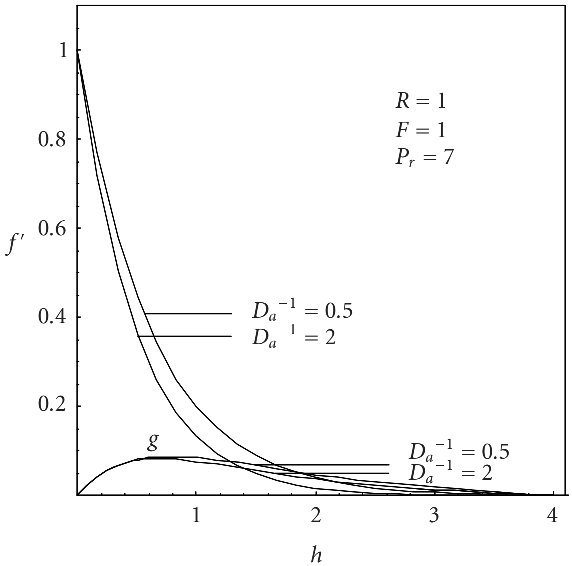

Figure 3.3. Velocity and microrotation distributions for various values of D_a^-1.

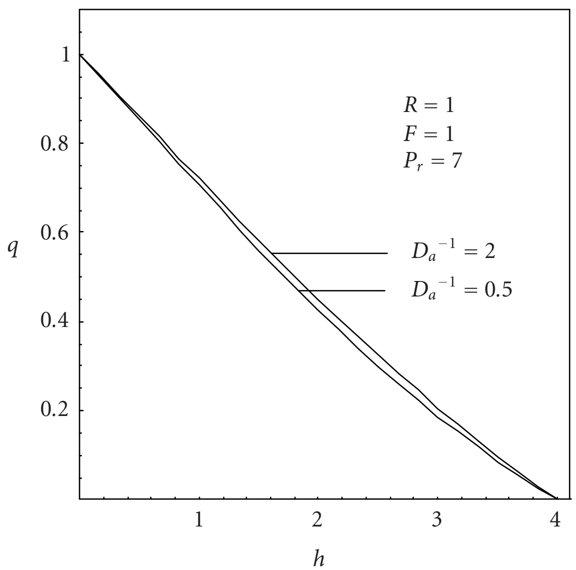

Figure 3.4. Temperature distributions for various values of D_a^-1.

angular velocity and enhanced the temperature. Figures 3.5 and 3.6 depect the influence of the Prandtl number P_r and the radiation parameter F on the temperature distributions, respectively. It is observed that the temperature at fixed values of η decreases

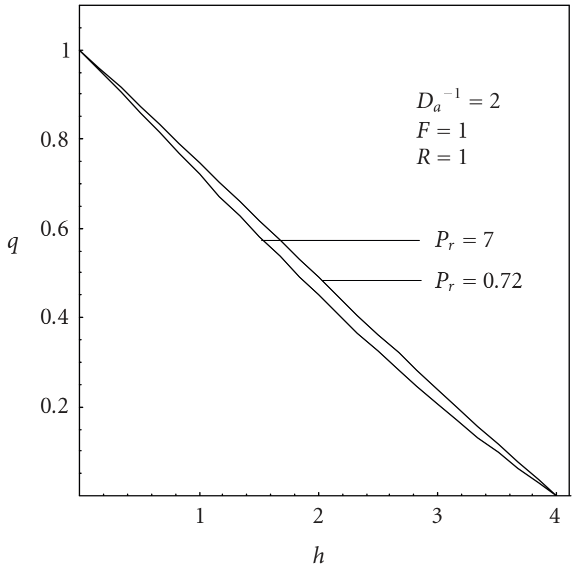

Figure 3.5. Temperature distributions for various values of P_r.

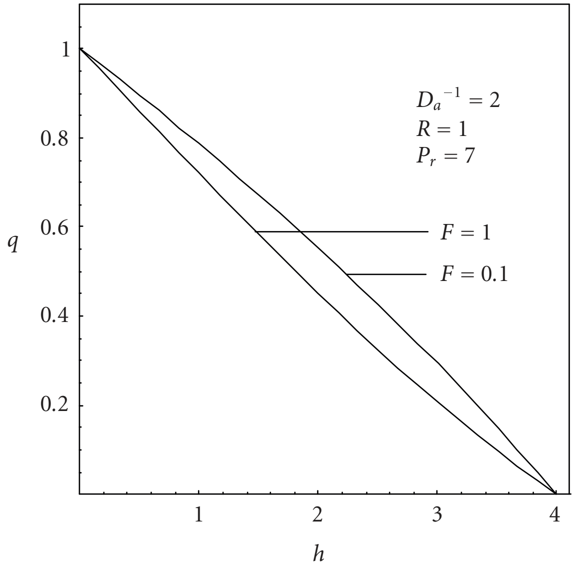

Figure 3.6. Temperature distributions for various values of F.

with an increase in the Prandtl number P_r as shown in Figure 3.5. This is in agreement with the physical fact that the thermal boundary thickness decreases with the increase of P_r. Figure 3.6 displays the variation of temperature for different values of the radiation parameter F. We observe that the temperature decreases as the radiation parameter F increase.

Table 3.1 displays numerical results for the skin friction coefficient, wall couple stress, and the wall heat transfer rate for various values of D_a^{-1}, R, P_r, and F. We observe from this table that the skin friction coefficient and wall couple stress increase with the increase of R or D_a^{-1} and the wall heat transfer rate decreases with the increase of R or D_a^{-1}. This is because, as mentioned before, increases in R or D_a^{-1} cause respective decreases in the velocity, the angular velocity, and the temperature, respectively. This results in increasing and decreasing the slopes of velocity and temperature, respectively. This has the direct effect of increasing the skin-friction coefficient, the wall couple stress and decreasing the rate of wall heat transfer as shown in Table 3.1. Also the wall heat transfer rate increases with the increase of P_r. Furthermore, the negative values of the wall temperature gradient, for all values of the parameters, are indicative of the physical fact that the heat flows from the surface to the ambient fluid.

4. Conclusions

The problem of hydromagnetic boundary layer flow and heat transfer of an electrically conducting micropolar fluid on a continuously moving stretching surface embedded in a non-Darcian porous medium in the presence of radiation was investigated. The resulting partial differential equations, which describe the problem, are transformed into ordinary differential equations by using similarity transformations. Numerical evaluations were performed and graphical results were obtained.

It was found that both the velocity and the angular velocity are decreased as either the inverse Darcy number or the magnetic parameter was increased. Also it is observed that the temperature decreased as the Prandtl number or the radiation parameter increased. Analysis of the tables shows that the skin-fricition coefficient is found to increase with the increase of magnetic parameter or of the inverse Darcy number. Also, we found that the wall heat transfer rate increased as the Prandtl number or the radiation parameter increased, while it decreased as the magnetic parameter or the inverse Darcy number increased.

References

[1] G. Ahmadi, *Self-similar solution of imcompressible micropolar boundary layer flow over a semi-infinite plate*, International Journal of Engineering Science **14** (1976), no. 7, 639–646.

[2] S. J. Allen and K. A. Kline, *Lubrication theory for micropolar fluids*, Journal of Applied Mechanics: Transactions of the ASME **38** (1971), no. 3, 646–650.

[3] T. Altan, S. Oh, and H. Gegel, *Metal Forming Fundamentals and Applications*, American Society of Metals, Ohio, 1979.

[4] T. Ariman, M. A. Turk, and N. D. Sylvester, *Microcontinuum fluid mechanics—a review*, International Journal of Engineering Science **11** (1973), no. 8, 905–930.

[5] ———, *Applications of microcontinuum fluid mechanics*, International Journal of Engineering Science **12** (1974), no. 4, 273–293.

[6] F. Ebert, *A similarity solution for the boundary layer flow of a polar fluid*, The Chemical Engineering Journal **5** (1973), no. 1, 85–92.

[7] A. C. Eringen, *Theory of micropolar fluids*, Journal of Mathematics and Mechanics **16** (1966), 1–18.

[8] ______, *Theory of thermomicro fluids*, Journal of Mathematical Analysis and Applications **38** (1972), no. 9, 480–492.

[9] M. V. Karwe and Y. Jaluria, *Thermal transport from a heated moving surface*, Journal of Heat Transfer: Transactions of the ASME **108** (1986), no. 4, 728–733.

[10] N. A. Kelson and T. W. Farrell, *Micropolar flow over a porous stretching sheet with strong suction or injection*, International Communications in Heat and Mass Transfer **28** (2001), no. 4, 479–488.

[11] M. M. Khonsari, *On the self-excited whirl orbits of a journal in a sleeve bearing lubricated with micropolar fluids*, Acta Mechanica **81** (1990), no. 3-4, 235–244.

[12] Y. J. Kim and A. G. Fedorov, *Transient mixed radiative convection flow of a micropolar fluid past a moving, semi-infinite vertical porous plate*, International Journal of Heat and Mass Transfer **46** (2003), no. 10, 1751–1758.

[13] J. Peddieson and R. P. Mcnitt, *Boundary layer theory for a micropolar fluid*, Recent Advances in Engineering Science **5** (1970), 405–426.

[14] J. Prakash and P. Sinha, *Lubrication theory for micropolar fluids and its application to a journal bearing*, International Journal of Engineering Science **13** (1975), no. 3, 217–232.

[15] A. Raptis, *Flow of a micropolar fluid past a continuously moving plate by the presence of radiation*, International Journal of Heat and Mass Transfer **41** (1998), no. 18, 2865–2866.

[16] ______, *Boundary layer flow of a micropolar fluid through a porous medium*, Journal of Porous Media **3** (2000), no. 1, 95–97.

[17] B. C. Sakiadis, *Boundary-layer behavior on continuous solid surfaces: I. Boundary-layer equations for two-dimensional and axisymmetric flow*, AIChE Journal **7** (1962), no. 1, 26–28.

[18] V. M. Soundalgekar and H. S. Takhar, *Flow of micropolar fluid past a continuously moving plate*, International Journal of Engineering Science **21** (1983), no. 8, 961–965.

[19] E. M. Sparrow and R. D. Cess, *Radiation Heat Transfer*, Hemisphere, Washington, DC, 1978.

[20] Z. Tadmor and I. Klein, *Engineering Principles of Plasticating Extrusion*, Polymer Science and Van nostrand Reinhold, New York, 1970.

[21] H. S. Takhar and V. M. Soundalgekar, *Flow and heat transfer of a micropolar fluid past a continuously moving porous plate*, International Journal of Engineering Science **23** (1985), no. 2, 201–205.

[22] N. Tipei, *Lubrication with micropolar liquids and its application to short bearings*, Journal of Lubrication Technology: Transactions of the ASME **101** (1979), no. 3, 356–363.

[23] F. K. Tsou, E. M. Sparrow, and R. J. Goldstein, *Flow and heat transfer in the boundary layer on a continuous moving surface*, International Journal of Heat and Mass Transfer **10** (1967), no. 2, 219–235.

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