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# Hydrostatic Equilibrium of Hypothetical Quark Stars

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Ambartsumyan and Saakyan[1] initiated the study of the degenerate superdense gas of elementary particles taking into account various hyperons. Afterwards many authors[2] have investigated hyperon stars further either by adding newly discovered elementary particles or by assuming some interactions between these elementary particles. A primitive and straightforward question then arises: What state occurs at the density higher than hyperon stars? No one can answer this question now, as our knowledge of strong interaction physics is very incomplete.

If a baryon consists of some fundamental particles, it may be possible that unbound fundamental particles will exist in the interior of superdense stars. Ivanenko and Kurdgelaidze[3] suggested the existence of a quarkian core in the interior of hypothetical super-stars of very large mass. In this note we shall investigate the hydrostatic equilibrium of quark stars by the method of Oppenheimer and Volkoff[4] assuming the conventional quark model.[5] The interaction between quarks is not well known at present, therefore, we shall neglect the interaction between quarks. This assumption is not permissible near the normal nuclear density, for quarks are considered to interact each other strongly at this density.

For the statistical property of quarks, we shall consider two alternative possibilities, namely para-fermi statistics of order $q=3$[6] and fermi statistics. The equation of state for completely degenerate fermions[7] can be readily extended to a para-fermion case with the mass $m$. One quantum state can be occupied by less than or equal to $q$ para-fermions. Putting $q=1$, we have the usual fermion case. The ratio of the limiting momentum $p_0$ to $mc$, $x\equiv p_0/mc$, is related to the number density of para-fermions, $n$, by

$$n=q\frac{8\pi m^3c^3}{3h^3}x^3. \quad (1)$$

The pressure is given by

$$P=q\frac{\pi m^4c^5}{3h^3}f(x) \quad (2)$$

with the function

$$f(x)=x(2x^2-3)(x^2+1)^{1/2}+3\sinh^{-1}x. \quad (3)$$

The internal energy of the gas $U_{\rm kin}$ is given by

$$U_{\rm kin}=q\frac{\pi m^4c^5}{3h^3}Vg(x), \quad (4)$$

where

$$g(x)=8x^3\{(x^2+1)^{1/2}-1\}-f(x). \quad (5)$$

We shall consider a quark star consisting of an equal amount of $u$, $d$ and $s$-quarks for simplicity. The condition of the charge neutrality is fulfilled in this case. We shall further assume that the masses of three kinds of quarks are equal and treat them as noninteracting particles. The macroscopic energy density of a quark star is given by

$$\rho_q = \frac{3N_b}{V} m_q c^2 + 3q \frac{\pi m_q{}^4 c^5}{3h^3} g(x_q), \qquad (6)$$

where $N_b/V$ is the baryon number density. The value $x_q$ is related with $N_b/V$ by

$$\frac{N_b}{V} = q \frac{8\pi m_q{}^3 c^3}{3h^3} x_q{}^3. \qquad (7)$$

On the other hand, we have an expression for the pressure:

$$P_q = 3q \frac{\pi m_q{}^4 c^5}{3h^3} f(x_q). \qquad (8)$$

The situation is quite similar to that of Oppenheimer and Volkoff. The unit of mass in our case is

$$b = \frac{1}{(3q)^{1/2}\pi} \left(\frac{h}{m_q c}\right)^{3/2} \frac{c^3}{(m_q G^3)^{1/2}}, \qquad (9)$$

where $G$ is the gravitational constant. In Fig. 1, the mass and the central baryon number density relation for quark stars is shown in the case of para-fermi statistics of order $q=3$ and fermi statistics. For the numerical calculation the mass of a quark was assumed to be $10\,\mathrm{GeV}/c^2$.

It is noticeable that the order of magnitude of the mass of quark stars is $10^{-3}M_\odot$. The fact that the maximum mass of quark stars is much less than the maximum mass of neutron stars, which is the order of magnitude of $1M_\odot$, shows that masses larger than the maximum mass of neutron stars are not permissible in the hydrostatic equilibrium even at higher densities and will do a gravitational collapse.

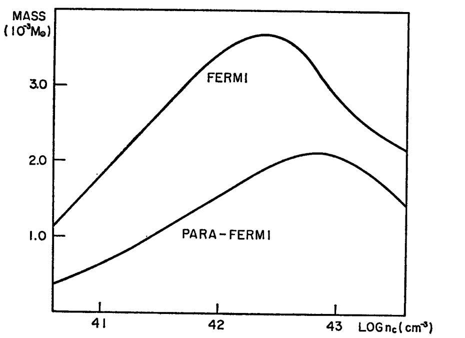


Fig. 1. The relation between the mass of quark stars and the central baryon number density. Two curves correspond to the case of para-fermi statistics of order $q=3$ and the case of fermi statistics.

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